

## **Report**

### **Information About the Program:**

Each rendition of the puzzle is represented as an array. A global stack has these arrays added, and removed when necessary to be looked at. The initial configuration is first added to the stack, popped off, and used as input in the mrv function. At a high level view, this function creates lists for each unknown location of possible values that they can hold. Depending on if forward checking, or constraint propagation are implemented, these lists will be altered. If forward checking is implemented, the program will look to see if there are any empty spaces in which there are no possible values for that location. If this is found an empty array will be returned, and due to such nothing will be added to the stack, therefore, the stack will have simply been decreased by a length of one. On the other hand, if constraint propagation is implemented, the program will then proceed to do arc consistency. On a high level view, the function will look to see if there are any locations which have only one possible value, and proceed to remove this value from possible values of the row, column, and box. This will continuously be done until there are no changes made in the row, column and box values. This will again be repeated for all single possible value locations.

## Findings:

### Test 1-Easy Example

#### Input:

6,2,3  
\_,\_,5,3,\_,1  
\_,3,4,\_,\_,\_  
\_,6,\_,\_,5,3  
3,5,\_,\_,6,\_  
\_,\_,\_,5,3,\_  
5,\_,3,2,\_,\_

#### Output:

6, 2, 5, 3, 4, 1  
1, 3, 4, 6, 2, 5  
4, 6, 2, 1, 5, 3  
3, 5, 1, 4, 6, 2  
2, 1, 6, 5, 3, 4  
5, 4, 3, 2, 1, 6

#### 1. Backtracking + MRV heuristic:

Number of Consistency Checks: 20

#### 2. Backtracking + MRV + Forward Checking:

Number of Consistency Checks: 20

#### 3. Backtracking + MRV + Constraint Propagation:

Number of Consistency Checks: 20

### Test 2-Medium to Easy Example:

#### Input:

9,3,3  
\_,\_,8,\_,\_,\_,6,\_,\_  
\_,6,\_,\_,1,3,\_,5,2  
\_,\_,\_,4,\_,2,\_,\_,\_  
\_,\_,5,\_,\_,\_,7,6,8  
4,\_,3,\_,\_,\_,\_,\_  
2,\_,\_,\_,\_,7,\_,\_,\_  
\_,\_,\_,4,\_,\_,1,\_,\_  
\_,\_,\_,5,\_,8,\_,6  
\_,5,\_,1,\_,8,2,\_,\_

#### Output:

1, 2, 8, 9, 7, 5, 6, 4, 3  
7, 6, 4, 8, 1, 3, 9, 5, 2

5, 3, 9, 4, 6, 2, 1, 8, 7  
 9, 1, 5, 3, 2, 4, 7, 6, 8  
 4, 7, 3, 6, 8, 1, 5, 2, 9  
 2, 8, 6, 5, 9, 7, 4, 3, 1  
 8, 9, 2, 7, 4, 6, 3, 1, 5  
 3, 4, 1, 2, 5, 9, 8, 7, 6  
 6, 5, 7, 1, 3, 8, 2, 9, 4

1. Backtracking + MRV heuristic:

Number of Consistency Checks: 98

2. Backtracking + MRV + Forward Checking:

Number of Consistency Checks: 66

3. Backtracking + MRV + Constraint Propagation:

Number of Consistency Checks: 56

Test 3-Medium Example:

Input:

9,3,3  
 \_ \_ 9,7,5, \_ \_ \_ \_  
 \_ \_ \_ \_ \_ \_ \_ \_  
 \_ \_ 5,3,8,2, \_ \_ \_  
 \_ 1, \_ \_ \_ \_ \_ 3  
 \_ \_ 2, \_ \_ \_ 9, \_ 8  
 4, \_ 6, \_ \_ \_ \_ \_  
 9, \_ \_ \_ 4, \_ 1, 3, \_  
 7, \_ \_ \_ \_ 6,5,4,9  
 \_ \_ \_ 2, \_ \_ \_ \_ \_

Output:

3, 8, 9, 7, 5, 1, 6, 2, 4  
 2, 7, 1, 4, 6, 9, 3, 8, 5  
 6, 4, 5, 3, 8, 2, 7, 9, 1  
 8, 1, 7, 9, 2, 5, 4, 6, 3  
 5, 3, 2, 6, 7, 4, 9, 1, 8  
 4, 9, 6, 1, 3, 8, 2, 5, 7  
 9, 6, 8, 5, 4, 7, 1, 3, 2  
 7, 2, 3, 8, 1, 6, 5, 4, 9  
 1, 5, 4, 2, 9, 3, 8, 7, 6

1. Backtracking + MRV heuristic:

Number of Consistency Checks: 7855

2. Backtracking + MRV + Forward Checking:

Number of Consistency Checks: 939

### 3. Backtracking + MRV + Constraint Propagation:

Number of Consistency Checks: 527

### Test 4-Hard Example:

#### Input:

12, 3, 4  
\_, 11, \_, \_, \_, \_, \_, 4, \_  
7, \_, 2, 6, \_, 3, 5, 11, \_  
\_, 6, 9, 1, 12, \_, 7, 10, \_  
\_, 4, 1, \_, \_, 10, 8, 6, \_  
\_, 8, 9, \_, 12, 10, \_, \_  
2, \_, 11, 1, 9, \_, \_  
\_, 8, 2, 4, \_, 7  
\_, 3, 5, 12, 4, \_  
\_, 7, 4, 12, 6, 8, \_  
\_, 5, 8, 10, 7, 11, 1, \_  
\_, 1, 11, 3, 5, 2, 6  
\_, 3, \_, \_, 12, \_

#### Output:

8, 11, 12, 1, 7, 10, 5, 2, 6, 4, 3, 9  
7, 10, 4, 2, 6, 8, 9, 3, 5, 12, 11, 1  
3, 6, 9, 5, 1, 12, 4, 11, 7, 2, 10, 8  
11, 4, 1, 12, 9, 5, 2, 10, 8, 7, 6, 3  
6, 8, 5, 9, 4, 3, 7, 12, 10, 1, 2, 11  
2, 3, 7, 10, 11, 6, 1, 8, 4, 9, 5, 12  
5, 12, 8, 6, 10, 2, 11, 4, 1, 3, 9, 7  
1, 9, 11, 3, 5, 7, 8, 6, 12, 10, 4, 2  
10, 7, 2, 4, 12, 1, 3, 9, 11, 6, 8, 5  
12, 5, 6, 8, 2, 9, 10, 7, 3, 11, 1, 4  
9, 1, 10, 11, 3, 4, 12, 5, 2, 8, 7, 6  
4, 2, 3, 7, 8, 11, 6, 1, 9, 5, 12, 10

### 1. Backtracking + MRV heuristic:

Number of Consistency Checks: 54633

### 2. Backtracking + MRV + Forward Checking:

Number of Consistency Checks: 526

### 3. Backtracking + MRV + Constraint Propagation:

Number of Consistency Checks: 325

As you can see from the above examples, the best techniques are Backtracking + MRV + Constraint Propagation, followed by Backtracking + MRV + Forward Checking, followed by Backtracking + MRV Heuristic. For the easy examples, all three have the same number of consistency checks because upon each iteration there is a location which can only take one possible value, therefore no backtracking is done.