Mini-projet: Méthodes numériques avancées

Sujet: Pricing des options via différents modèles discrets et continus

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- Classe: **5DS2**/**Groupe 1**

Outils

In [31]:

```
install.packages("fOptions")
library(fOptions)
```

```
Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)
```

Contenu:

Modélisation numérique du modèle Cox Ross Rubinstein.

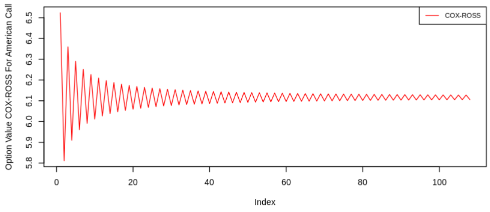
Implémentation à l'aide du package "fOptions"

Call Américain

In [32]:

```
par(mfrow = c(2, 1), cex = 0.7)
   steps = 110
   CRROptionValue  = rep(NA, times = steps)
   for (n in 3:steps) {
     CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "ca", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
   plot(CRROptionValue[3:steps], type = "l", col = "red", ylab = "Option Value COX-ROSS For American Call" )
legend("topright" ,legend=c("COX-ROSS "),col=c("red"), lty=1:2, cex=0.8)
```

Out[32]:



In [33]:

```
CRRBinomialTreeOption(TypeFlag = "ca", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)
```

Out[33]:
```
Title:
 CRR Binomial Tree Option

Call:
 CRRBinomialTreeOption(TypeFlag = "ca", S = 50, X = 50, Time = 0.4167,
     r = 0.1, b = 0.1, sigma = 0.4, n = n)

Parameters:
          Value:
 TypeFlag ca
 S        50
 X        50
 Time     0.4167
 r        0.1
 b        0.1
 sigma    0.4
 n        110

Option Price:
 6.105224

Description:
 Sun Dec  6 09:30:30 2020
```

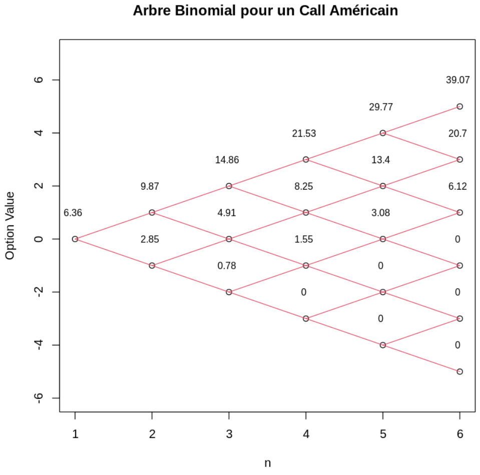
- Arbre binomial d'un **Call Américain** (N=5)

In [34]:
```
CRRBinomialTreeOption(TypeFlag = "ca", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
```

Out[34]:
```
Title:
 CRR Binomial Tree Option

Call:
 CRRBinomialTreeOption(TypeFlag = "ca", S = 50, X = 50, Time = 0.4167,
     r = 0.1, b = 0.1, sigma = 0.4, n = 5)

Parameters:
          Value:
 TypeFlag ca
 S        50
 X        50
 Time     0.4167
 r        0.1
 b        0.1
 sigma    0.4
 n        5

Option Price:
 6.359834

Description:
 Sun Dec  6 09:30:30 2020
```

In [35]:
```
CRRTree = BinomialTreeOption(TypeFlag = "ca", S = 50, X = 50,
     Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
   BinomialTreePlot(CRRTree, dy = 1, cex = 0.8, ylim = c(-6, 7),
     xlab = "n", ylab = "Option Value")
   title(main = "Arbre Binomial pour un Call Américain ")
```

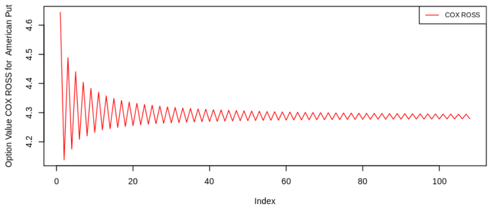
Out[35]:



Put Américain

In [36]:
```
par(mfrow = c(2, 1), cex = 0.7)
   steps = 110
   CRROptionValue_p  = rep(NA, times = steps)
   for (n in 3:steps) {
     CRROptionValue_p[n] = CRRBinomialTreeOption(TypeFlag = "pa", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
   plot(CRROptionValue_p[3:steps], type = "l", col = "red", ylab = "Option Value COX ROSS for  American Put")
legend("topright", legend=c("COX ROSS "),col=c("red"), lty=1:2, cex=0.8)
```

Out[36]:



In [37]:

```
CRRBinomialTreeOption(TypeFlag = "pa", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)
```

Out[37]:

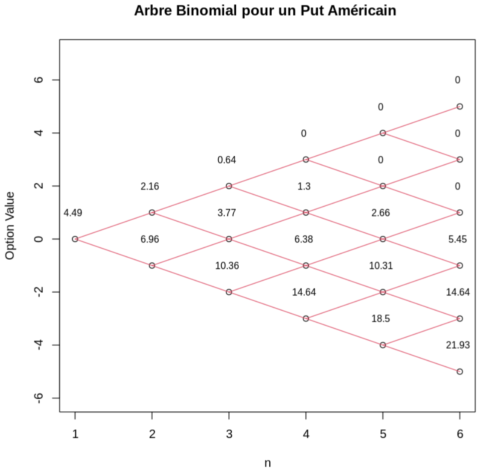
```
Title:
 CRR Binomial Tree Option

Call:
 CRRBinomialTreeOption(TypeFlag = "pa", S = 50, X = 50, Time = 0.4167,
     r = 0.1, b = 0.1, sigma = 0.4, n = n)

Parameters:
          Value:
 TypeFlag pa
 S        50
 X        50
 Time     0.4167
 r        0.1
 b        0.1
 sigma    0.4
 n        110

Option Price:
 4.278776

Description:
 Sun Dec  6 09:30:30 2020
```

- Arbre binomial d'un **Put Américain** (N=5)

In [38]:

```
CRRTree = BinomialTreeOption(TypeFlag = "pa", S = 50, X = 50,
     Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
   BinomialTreePlot(CRRTree, dy = 1, cex = 0.8, ylim = c(-6, 7),
     xlab = "n", ylab = "Option Value")
   title(main = "Arbre Binomial pour un Put Américain ")
```

Out[38]:

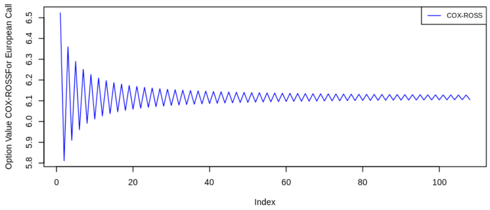


Call Européen

In [39]:

```
par(mfrow = c(2, 1), cex = 0.7)
   steps = 110
   CRROptionValue  = rep(NA, times = steps)
   for (n in 3:steps) {
     CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "ce", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
   plot(CRROptionValue[3:steps], type = "l", col = "Blue", ylab = "Option Value COX-ROSSFor European Call")
legend("topright", legend=c("COX-ROSS"),col=c("Blue"), lty=1:2, cex=0.8)
```

Out[39]:



In [40]:
```
CRRBinomialTreeOption(TypeFlag = "ce", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)
```

Out[40]:
```
Title:
 CRR Binomial Tree Option

Call:
 CRRBinomialTreeOption(TypeFlag = "ce", S = 50, X = 50, Time = 0.4167,
     r = 0.1, b = 0.1, sigma = 0.4, n = n)

Parameters:
          Value:
 TypeFlag ce
 S        50
 X        50
 Time     0.4167
 r        0.1
 b        0.1
 sigma    0.4
 n        110

Option Price:
 6.105224

Description:
 Sun Dec  6 09:30:31 2020
```

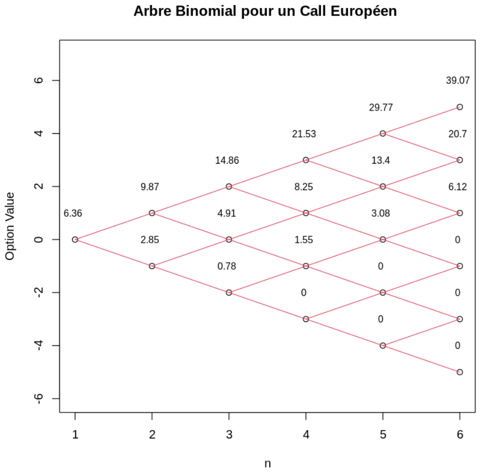
- Arbre binomial d'un **Call Européen** (N=5)

In [41]:
```
CRRBinomialTreeOption(TypeFlag = "ce", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
```

Out[41]:
```
Title:
 CRR Binomial Tree Option

Call:
 CRRBinomialTreeOption(TypeFlag = "ce", S = 50, X = 50, Time = 0.4167,
     r = 0.1, b = 0.1, sigma = 0.4, n = 5)

Parameters:
          Value:
 TypeFlag ce
 S        50
 X        50
 Time     0.4167
 r        0.1
 b        0.1
 sigma    0.4
 n        5

Option Price:
 6.359834

Description:
 Sun Dec  6 09:30:31 2020
```

In [42]:
```
CRRTree = BinomialTreeOption(TypeFlag = "ce", S = 50, X = 50,
     Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
   BinomialTreePlot(CRRTree, dy = 1, cex = 0.8, ylim = c(-6, 7),
     xlab = "n", ylab = "Option Value")
   title(main = "Arbre Binomial pour un Call Européen ")
```

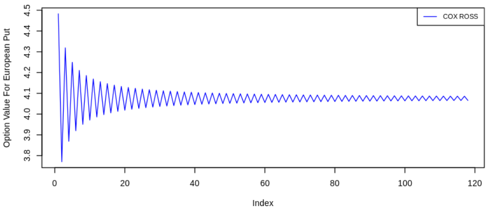
Out[42]:



Put Européen

In [43]:
```
par(mfrow = c(2, 1), cex = 0.7)
   steps = 120
   CRROptionValue =  JROptionValue = TIANOptionValue =
     rep(NA, times = steps)
   for (n in 3:steps) {
     CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "pe", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
    plot(CRROptionValue[3:steps], type = "l", col = "blue", ylab = "Option Value For European Put")
    legend("topright", legend=c("COX ROSS "),col=c("blue"), lty=1:2, cex=0.8)
```

Out[43]:



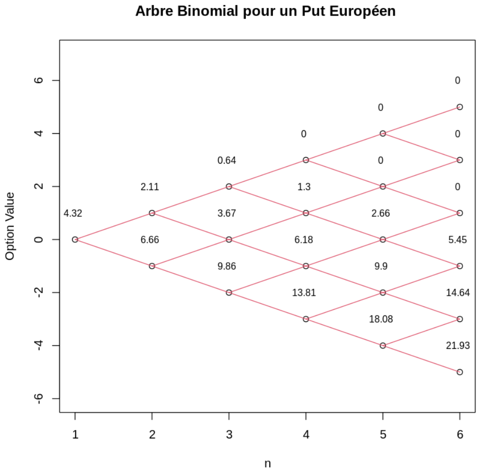
In [44]:
```
CRRBinomialTreeOption(TypeFlag = "pe", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)
```

Out[44]:
```
Title:
 CRR Binomial Tree Option

Call:
 CRRBinomialTreeOption(TypeFlag = "pe", S = 50, X = 50, Time = 0.4167,
     r = 0.1, b = 0.1, sigma = 0.4, n = n)

Parameters:
          Value:
 TypeFlag pe
 S        50
 X        50
 Time     0.4167
 r        0.1
 b        0.1
 sigma    0.4
 n        120

Option Price:
 4.0655

Description:
 Sun Dec  6 09:30:31 2020
```

- Arbre binomial d'un **Put Européen** (N=5)

In [45]:
```
CRRTree = BinomialTreeOption(TypeFlag = "pe", S = 50, X = 50,
     Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
   BinomialTreePlot(CRRTree, dy = 1, cex = 0.8, ylim = c(-6, 7),
     xlab = "n", ylab = "Option Value")
   title(main = "Arbre Binomial pour un Put Européen ")
```

Out[45]:



Implémentation du modèle de Cox-Ross-Rubinstein from scratch

In [88]:

```
CRR_Model = function(n,S,K,r,sigma,T,typeOption, PutOrCall){
    At = T/n
    u = exp(sigma*sqrt(At)) #up
    d = 1./u #down
    R = exp(r*At)
    p = (R-d) / (u-d) #probabilité risque neutre
    q = 1-p

    prix_sj = matrix(0, nrow = n+1, ncol = n+1)
    prix_sj[1,1] = S
    for (i in 2:(n+1)){
        prix_sj[i,1] = prix_sj[i-1,1]*u
        for (j in 2:i){
            prix_sj[i,j] = prix_sj[i-1,j-1]*d
        }
    }
    prix_option = matrix(0, nrow = n+1, ncol = n+1)
    for (j in 1:(n+1)){
        if(PutOrCall=="C"){
            prix_option[n+1,j] = max(0, prix_sj[n+1,j]-K)
        }else if(PutOrCall=="P"){
            prix_option[n+1,j] = max(0, K-prix_sj[n+1,j])
        }
    }

    if (typeOption == "E"){
        for (i in seq(n,1,-1)){
            for (j in 1:i){
                prix_option[i,j] = max(0,1/R*(p*prix_option[i+1,j] + q*prix_option[i+1,j+1]))
            }
        }
    }else if (typeOption == "A"){
        for(i in seq(n,1,-1)){
            for (j in 1:i){
                if (PutOrCall=="P"){
                     prix_option[i,j] = max(0, K-prix_sj[i,j], 1/R*(p*prix_option[i+1,j]+q*prix_option[i+1,j+1]))
                }else if (PutOrCall=="C"){
                    prix_option[i,j] = max(0, prix_sj[i,j]-K, 1/R*(p*prix_option[i+1,j]+q*prix_option[i+1,j+1]))
                }
            }
        }
    }
    return (prix_option[1,1])


 }
```

In [47]:

```
print("Cas d'un call Européen d'horizon N=5:")
print(CRR_Model(n=5,S=50,K=50,r=0.1,sigma=0.4,T=0.4167,typeOption="E", PutOrCall="C"))
print("-----------------------------------------")
print("Cas d'un put Européen d'horizon N=5:")
print(CRR_Model(n=5,S=50,K=50,r=0.1,sigma=0.4,T=0.4167,typeOption="E", PutOrCall="P"))
print("-----------------------------------------")
print("Cas d'un call Américain d'horizon N=5:")
print(CRR_Model(n=5,S=50,K=50,r=0.1,sigma=0.4,T=0.4167,typeOption="A", PutOrCall="C"))
print("-----------------------------------------")
print("Cas d'un put Américain d'horizon N=5:")
print(CRR_Model(n=5,S=50,K=50,r=0.1,sigma=0.4,T=0.4167,typeOption="A", PutOrCall="P"))
print("-----------------------------------------")
```

```
[1] "Cas d'un call Européen d'horizon N=5:"
[1] 6.359834
[1] "-----------------------------------------"
[1] "Cas d'un put Européen d'horizon N=5:"
[1] 4.319147
[1] "-----------------------------------------"
[1] "Cas d'un call Américain d'horizon N=5:"
[1] 6.359834
[1] "-----------------------------------------"
[1] "Cas d'un put Américain d'horizon N=5:"
[1] 4.488599
[1] "-----------------------------------------"
```

Modélisation numérique du modèle de Black & Scholes

Option Européenne sans dividende

In [48]:

```
BlackScholes <- function(S, K, r, T, sig, type){

  if(type=="C"){
  d1 <- (log(S/K) + (r + sig^2/2)*T) / (sig*sqrt(T))
  d2 <- d1 - sig*sqrt(T)

  value <- S*pnorm(d1) - K*exp(-r*T)*pnorm(d2)
  return(value)}

  if(type=="P"){
  d1 <- (log(S/K) + (r + sig^2/2)*T) / (sig*sqrt(T))
  d2 <- d1 - sig*sqrt(T)

  value <-  (K*exp(-r*T)*pnorm(-d2) - S*pnorm(-d1))
  return(value)}
}
```

- Cas d'un Call

In [49]:

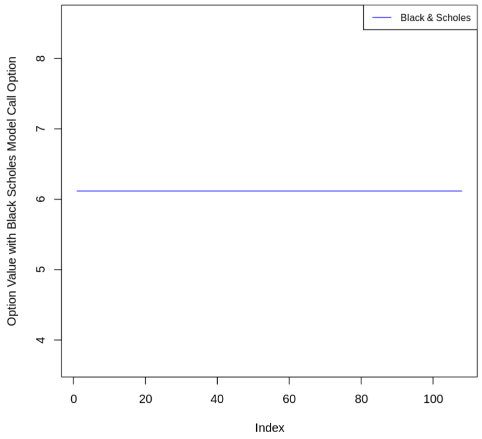
```
BS_call=BlackScholes(50,50,0.1,0.4167,0.4,"C")
BS_call
```

Out[49]: 6.11678761795579

In [50]:

```
steps = 110
   BSOptionValue  = rep(NA, times = steps)
   for (n in 3:steps) {
     BSOptionValue[n] = BlackScholes(50,50,0.1,0.4167,0.4,"C")

   }
plot(BSOptionValue[3:steps],type = "l",ylab = "Option Value with Black Scholes Model Call Option",
     col="Blue")
legend("topright", legend=c("Black & Scholes"),col=c("blue"), lty=1:2, cex=0.8)
```

Out[50]:



- Cas d'un Put

In [51]:

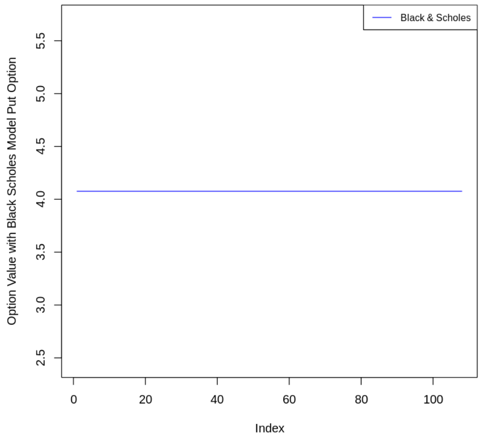
```
BS_put=BlackScholes(50,50,0.1,0.4167,0.4,"P")
BS_put
```

Out[51]: 4.07610060876962

In [52]:

```
steps = 110
   BSOptionValue  = rep(NA, times = steps)
   for (n in 3:steps) {
     BSOptionValue[n] = BlackScholes(50,50,0.1,0.4167,0.4,"P")

   }
plot(BSOptionValue[3:steps],type = "l",ylab = "Option Value with Black Scholes Model Put Option", col="Blue")
legend("topright", legend=c("Black & Scholes"),col=c("blue"), lty=1:2, cex=0.8)
```

Out[52]:



Option Européenne avec dividende

In [53]:

```
BlackScholesDiv <- function(S, K, r, T, sig, type,q){

  d1 <- (log(S/K) + (r - q + sig^2/2)*T) / (sig*sqrt(T))
  d2 <- d1 - sig*sqrt(T)

  if(type=="C"){

  value <- (S*exp(-q * T)*pnorm(d1) - K*exp(-r * T) * pnorm(d2))
  return(value)}

  if(type=="P"){
  value <-  (K*exp(-r * T)*pnorm(-d2) - S*exp(-q * T)*pnorm(-d1))
  return(value)}
}
```

In [54]:

```
q = 0.0205 #dividende
```

- Cas d'un Call

In [55]:

```
call_div = BlackScholesDiv(S=50, K=50, r=0.1, T=0.4167, sig=0.4, type="C",q=0.0205)
print(call_div)
```

```
[1] 5.858232
```

- Cas d'un Put

In [56]:

```
put_div=BlackScholesDiv(S=50, K=50, r=0.1, T=0.4167, sig=0.4, type="P",q=0.0205)
print(put_div)
```

```
[1] 4.242843
```

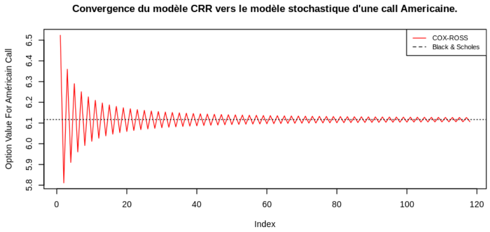
Convergance du modèle de CRR vers le modèle de B&S

Convergence d'un Call Américain

In [57]:

```
par(mfrow = c(2, 1), cex = 0.7)
steps = 120
CRROptionValue =  JROptionValue = TIANOptionValue = rep(NA, times = steps)
for(n in 3:steps) {
     CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "ca", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
plot(CRROptionValue[3:steps], type = "l", col = "red", ylab = "Option Value For Américain Call")

   # Add Result from BAW Approximation:
BAWValue =  BAWAmericanApproxOption(TypeFlag = "c", S = 50, X = 50,
Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4)@price
abline(h = BAWValue, lty = 3)
title(main = "Convergence du modèle CRR vers le modèle stochastique d'une call Americaine.")


legend("topright", legend=c("COX-ROSS ","Black & Scholes"),col=c("red","black"), lty=1:2, cex=0.8)
```

Out[57]:

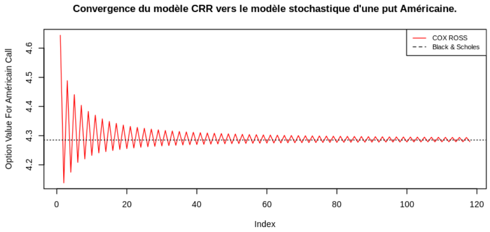


Convergence d'un Put Américain

In [58]:

```
par(mfrow = c(2, 1), cex = 0.7)
   steps = 120
   CRROptionValue =  JROptionValue = TIANOptionValue =
     rep(NA, times = steps)
   for (n in 3:steps) {
     CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "pa", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
    plot(CRROptionValue[3:steps], type = "l", col = "red", ylab = "Option Value For Américain Call")
    legend("topright", legend=c("COX ROSS ","Black & Scholes"),col=c("red","black"), lty=1:2, cex=0.8)

   # Add Result from BAW Approximation:
   BAWValue =  BAWAmericanApproxOption(TypeFlag = "p", S = 50, X = 50,
     Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4)@price
   abline(h = BAWValue, lty = 3)
   title(main = "Convergence du modèle CRR vers le modèle stochastique d'une put Américaine.")
```

Out[58]:

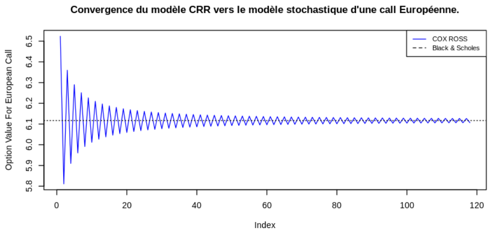


Convergence d'un call Européen

In [59]:

```
par(mfrow = c(2, 1), cex = 0.7)
   steps = 120
   CRROptionValue =  JROptionValue = TIANOptionValue =
     rep(NA, times = steps)
   for (n in 3:steps) {
     CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "ce", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
    plot(CRROptionValue[3:steps], type = "l", col = "Blue", ylab = "Option Value For European Call")
legend("topright", legend=c("COX ROSS ","Black & Scholes"),col=c("Blue","black"), lty=1:2, cex=0.8)

   # Add Result from BAW Approximation:
   BAWValue =  BAWAmericanApproxOption(TypeFlag = "c", S = 50, X = 50,
     Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4)@price
   abline(h = BAWValue, lty = 3)
   title(main = "Convergence du modèle CRR vers le modèle stochastique d'une call Européenne.")
```

Out[59]:

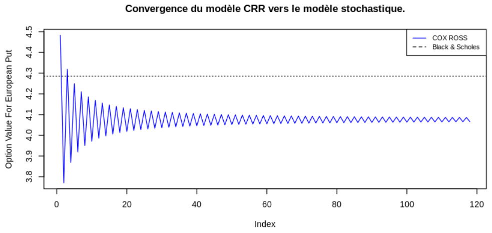


Convergence Put Européen

In [60]:

```
par(mfrow = c(2, 1), cex = 0.7)
   steps = 120
   CRROptionValue =  JROptionValue = TIANOptionValue =
     rep(NA, times = steps)
   for (n in 3:steps) {
     CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "pe", S = 50,
       X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price

   }
    plot(CRROptionValue[3:steps], type = "l", col = "blue", ylab = "Option Value For European Put")
legend("topright", legend=c("COX ROSS ","Black & Scholes"),col=c("blue","black"), lty=1:2, cex=0.8)

   # Add Result from BAW Approximation:
   BAWValue =  BAWAmericanApproxOption(TypeFlag = "p", S = 50, X = 50,
     Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4)@price
   abline(h = BAWValue, lty = 3)
   title(main = "Convergence du modèle CRR vers le modèle stochastique.")
```

Out[60]:



Correction

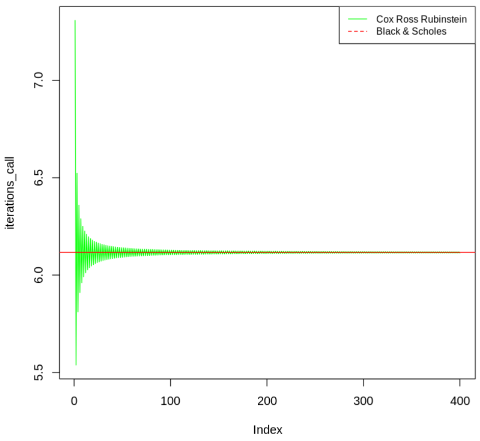
In [91]:

```
CRR_Model_corr = function(n,S,K,r,sigma,T,typeOption, PutOrCall){
    At = T/n
    u = exp(sigma*sqrt(At)+1/n*log(K/S)) #up
    d = 1./u #down
    R = exp(r*At)
    p = (R-d) / (u-d) #probabilité risque neutre
    q = 1-p

    prix_sj = matrix(0, nrow = n+1, ncol = n+1)
    prix_sj[1,1] = S
    for (i in 2:(n+1)){
        prix_sj[i,1] = prix_sj[i-1,1]*u
        for (j in 2:i){
            prix_sj[i,j] = prix_sj[i-1,j-1]*d
        }
    }
    prix_option = matrix(0, nrow = n+1, ncol = n+1)
    for (j in 1:(n+1)){
        if(PutOrCall=="C"){
            prix_option[n+1,j] = max(0, prix_sj[n+1,j]-K)
        }else if(PutOrCall=="P"){
            prix_option[n+1,j] = max(0, K-prix_sj[n+1,j])
        }
    }

    if (typeOption == "E"){
        for (i in seq(n,1,-1)){
            for (j in 1:i){
                prix_option[i,j] = max(0,1/R*(p*prix_option[i+1,j] + q*prix_option[i+1,j+1]))
            }
        }
    }else if (typeOption == "A"){
        for(i in seq(n,1,-1)){
            for (j in 1:i){
                if (PutOrCall=="P"){
                     prix_option[i,j] = max(0, K-prix_sj[i,j], 1/R*(p*prix_option[i+1,j]+q*prix_option[i+1,j+1]))
                }else if (PutOrCall=="C"){
                    prix_option[i,j] = max(0, prix_sj[i,j]-K, 1/R*(p*prix_option[i+1,j]+q*prix_option[i+1,j+1]))
                }
            }
        }
    }
    return (prix_option[1,1])


 }
```

In [93]:

```
iterations_call = rep(NA,400)
for (n in 1:400){
iterations_call[n] = CRR_Model_corr(n=n,S=50,K=50,r=0.1,sigma=0.4,T=0.4167,typeOption="E", PutOrCall="C")
}
plot(iterations_call, type="l", col="green")
abline(h=BlackScholes(50,50,0.1,0.4167,0.4,"C"), col="red")
legend("topright", legend=c("Cox Ross Rubinstein", "Black & Scholes"),
       col=c("green", "red"), lty=1:2, cex=0.8)
```

Out[93]:



In [0]: