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The application of fuzzy analytic hierarchy process (FAHP) approach to selection of optimum underground mining method for Jajarm Bauxite Mine, Iran

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A B S T R A C T

Underground mining method selection is one of the most crucial decisions that should be made by mining engineers. Choosing a suitable underground mining method to carry out extraction from a mineral deposit is very important in terms of the economics, safety and the productivity of mining operations. The aim of this study is developing a fuzzy model to selection the optimum mining method by using effective and major criteria and at the same time, taking subjective judgments of decision makers into consideration. Proposed approach is based on the combination of fuzzy analytic hierarchy process (FAHP) method with an advanced type of conventional AHP. FAHP is used in determining of the weights of the criteria by decision makers and then rankings of the methods are determined by AHP. The proposed method is applied for Jajarm Bauxite Mine in Iran and finally the most appropriate mining methods for this mine are ranked.

1. Introduction

Once an ore body has been probed and outlined and sufficient information has been collected to warrant further analysis, the important process of selecting the most appropriate method or methods of mining can begin. Mineral exploitation in which all extractions are carried out beneath the earth's surface is termed as underground mining. Underground methods are employed when the depth of the deposit, the stripping ratio of overburden to ore (or coal or stone), or both become excessive for surface exploitation. Once economic analysis points to underground methods, the choice of a proper mining procedure hinges mainly on determining the appropriate form of ground support, if necessary, or its absence; and designing the openings and their sequence of extraction to conform to the spatial characteristics of the mineral deposit (Hartman & Mutmansky, 2002).

To make a right decision on underground mining method selection, all known criteria related to the problem should be analyzed. Although an increasing in the number of related criteria makes the problem more complicated and more difficult to reach a solution, this may also increase the correctness of the decision made because of those criteria. Due to the arising complexity in the decision process, many conventional methods are able to consider limited criteria and may be generally deficient. Therefore, it is clearly seen that assessing all of the known criteria connected to the mining method selection by combining the decision making process is extremely significant (Hartman & Mutmansky, 2002).

The aim of this paper is to compare the many different geological, geotechnical, economical and technical aspects in the selection of the most appropriate underground mining method for Jajarm Bauxite Mine in Iran, with reference to some different extraction methods. The comparison has been performed with the combination of the analytic hierarchy process (AHP) and fuzzy logic. The analysis is one of the multi-criteria techniques that provides useful support in the choice among several alternatives with different objectives and criteria. The overall process is nominated fuzzy AHP (FAHP). FAHP method has been used in determining the weights of the criteria by decision makers and then ranking of the methods has been determined by conventional AHP method. The study was supported by results that were obtained from a questionnaire carried out to know the opinions of the experts in this subject.

The remainder of this paper is organized as follows. In Section 2, a brief review is done on concept of the fuzzy sets and fuzzy numbers. In Section 3, FAHP method is illustrated. This section is included the literature review and also the methodology of FAHP method. In Section 4, after explanation of mine information and the hierarchy structure of problem, the FAHP method is applied for determination of the weights of the criteria given by experts. Then, subsequent calculations and analyses are done and finally the optimum mining method is selected. Eventually, in Section 5, results of the application are presented. This section concludes the paper.

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According to the authors' knowledge, underground mining method selection using the FAHP is a unique research.

2. Fuzzy sets and fuzzy numbers

To deal with vagueness of human thought, Zadeh (1965) first introduced the fuzzy set theory, which was oriented to the rationality of uncertainty due to imprecision or vagueness. A major contribution of fuzzy set theory is its capability of representing vague data. The theory also allows mathematical operators and programming to apply to the fuzzy domain. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function, which assigns to each object a grade of membership ranging between zero and one.

With different daily decision making problems of diverse intensity, the results can be misleading if the fuzziness of human decision making is not taken into account (Tsaur, Chang, & Yen, 2002). Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world (Ertugrul & Tus, 2007). Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for common-sense reasoning in decision making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution (Bojadziev & Bojadziev, 1998). Fuzzy set theory is a better means for modeling imprecision arising from mental phenomena which are neither random nor stochastic. Human beings are heavily involved in the process of decision analysis. A rational approach toward decision making should take into account human subjectivity, rather than employing only objective probability measures. This attitude, towards imprecision of human behavior led to study of a new decision analysis filed fuzzy decision making (Lai & Hwang, 1996).

A tilde '$\sim$' will be placed above a symbol if the symbol represents a fuzzy set. A triangular fuzzy number (TFN), $\widetilde{M}$ is shown in Fig. 1. A TFN is denoted simply as $(l|m, m|u)$ or (l, m, u). The parameters l, m and u, respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event.

Each TFN has linear representations on its left and right side such that its membership function can be defined as

$$\mu(x|\widetilde{M}) = \begin{cases} 0, & x < l, \\ (x-l)/(m-l), & l \le x \le m, \\ (u-x)/(u-m), & m \le x \le u, \\ 0, & x > u. \end{cases} \tag{1}$$

A fuzzy number can always be given by its corresponding left and right representation of each degree of membership:

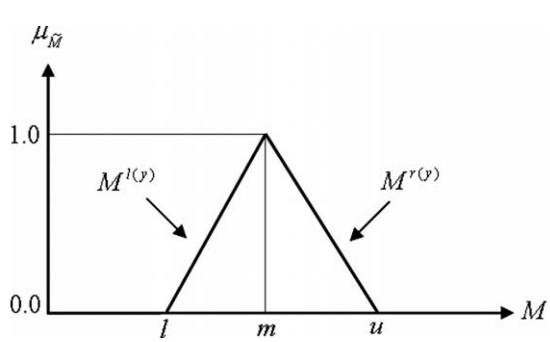

Fig. 1. A triangular fuzzy number, $\widetilde{M}$.

$$\widetilde{M} = (M^{l(y)}, M^{r(y)}) = (l + (m-l)y, u + (m-u)y), \quad y \in [0,1], \tag{2}$$

where $l(y)$ and $r(y)$ denote the left side representation and the right side representation of a fuzzy number, respectively. Many ranking methods for fuzzy numbers have been developed in the literature. These methods may give different ranking results and most methods are tedious in graphic manipulation requiring complex mathematical calculation. The algebraic operations with fuzzy numbers can be found in Kahraman (2001) and Kahraman, Ruan, and Tolga (2002).

3. Fuzzy analytic hierarchy process

The analytic hierarchy process (AHP) is an approach that is suitable for dealing with complex systems related to making a choice from among several alternatives and which provides a comparison of the considered options, firstly proposed by Saaty (1980). The AHP is based on the subdivision of the problem in a hierarchical form. In fact, the AHP helps organize the rational analysis of the problem by dividing it into its single parts; the analysis then supplies an aid to the decision makers who, making several pair-wise comparisons, can appreciate the influence of the considered elements in the hierarchical structure; the AHP can also give a preference list of the considered alternative solutions (Bentivegna, Mondini, Nati Poltri, & Pii, 1994; Roscelli, 1990; Saaty, 1980; Saaty & Vargas, 1990).

The AHP is a tool that can be used for analyzing different kinds of social, political, economic and technological problems, and it uses both qualitative and quantitative variables. The fundamental principle of the analysis is the possibility of connecting information, based on knowledge, to make decisions or previsions; the knowledge can be taken from experience or derived from the application of other tools. Among the different contexts in which the AHP can be applied, mention can be made of the creation of a list of priorities, the choice of the best policy, the optimal allocation of resources, the prevision of results and temporal dependencies, the assessment of risks and planning (Saaty & Vargas, 1990).

Although the AHP is to capture the expert's knowledge, the traditional AHP still cannot really reflect the human thinking style (Kahraman, Cebeci, & Ulukan, 2003). The traditional AHP method is problematic in that it uses an exact value to express the decision maker's opinion in a comparison of alternatives (Wang & Chen, 2007). And AHP method is often criticized due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process (Deng, 1999). To overcome all these shortcomings, FAHP was developed for solving the hierarchical problems. Decision makers usually find that it is more confident to give interval judgments than fixed value judgments. This is because usually he/she is unable to explicit his/her preference to explicit about the fuzzy nature of the comparison process (Kahraman, Ruan, & Dogan, 2003). This paper proposes the use of FAHP for determining the weights of the main criteria.

3.1. A brief review on the past researches

There are many fuzzy AHP methods and applications in the literature proposed by various authors. Van Laarhoven and Pedrcyz (1983) proposed the first studies that applied fuzzy logic principle to AHP. Buckley (1985) initiated trapezoidal fuzzy numbers to express the decision maker's evaluation on alternatives with respect to each criterion while Van Laarhoven and Pedrcyz (1983) were using triangular fuzzy numbers. Chang (1996) introduced a new approach for handling FAHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of FAHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons. Deng (1999) presented a fuzzy approach for tackling qualitative multi-criteria analysis problems in a simple and straightforward manner. Zhu, Jing, and Chang (1999) proved the basic theory of the triangular fuzzy number and improved the formulation of comparing the triangular fuzzy number's size. On this basis, they introduced a practical example on petroleum prospecting. Leung and Cao (2000) proposed a fuzzy consistency definition with consideration of a tolerance deviation. Essentially, the fuzzy ratios of relative importance, allowing certain tolerance deviation, were formulated as constraints on the membership values of the local priorities. Chou and Liang (2001) proposed a fuzzy multi-criteria decision making model by combining fuzzy set theory, AHP and concept of entropy, for shipping company performance evaluation. Bozdag, Kahraman, and Ruan (2003) proposed four different fuzzy multi-attribute group decision making methods to select the best computer integrated manufacturing system. One of these methods is FAHP and the others are Yager's weighted goals method, Blin's approach and fuzzy synthetic evaluation. Chang, Cheng, and Wang (2003) developed a methodology for performance evaluation of airports. They used the gray statistics method in selecting the criteria, and FAHP method in determining the weights of criteria. And finally they adopted fuzzy synthetic and TOPSIS approach for the ranking of airport performance. Kahraman, Cebeci, et al. (2003) used FAHP to select the best supplier firm providing the most satisfaction for the criteria determined. Hsieh, Lu, and Tzeng (2004) presented a fuzzy multi-criteria analysis approach for selecting of planning and design alternatives in public office buildings. The FAHP method is used to determine the weightings for evaluation criteria among decision makers. Mikhailov and Tsvetinov (2004) applied a new fuzzy modification of the AHP for evaluating services. The proposed fuzzy prioritization method uses fuzzy pair-wise comparison judgments rather than exact numerical values of the comparison ratios and transforms initial fuzzy prioritization problem into non-linear program. Enea and Piazza (2004) focused on the constraints that have to be considered within FAHP. They used constrained FAHP in project selection. Kahraman, Cebeci, and Ruan (2004) used the FAHP for comparing catering firms in Turkey. The means of the triangular fuzzy numbers produced by the customers and experts for each comparison were successfully used in the pair-wise comparison matrices. Tang and Beynon(2005) used FAHP method for the application and development of a capital investment study. They tried to select the type of fleet car to be adopted by a car rental company. Tolga, Demircan, and Kahraman (2005) used fuzzy replacement analysis and AHP in the selection of operating system. The economic part of the decision process had been developed by fuzzy replacement analysis. Non-economic factors and financial figures had been combined by using a FAHP approach. Basligil (2005) provided an analytical tool to select the best software providing the most customer satisfaction. Tang, Kuo, and Lee (2005) proposed a multi-objective model for Taiwan notebook computer distribution problem. Their model involves a mixed integer programming and fuzzy analytic hierarchy process approach. Gu and Zhu (2006) constructed fuzzy symmetry matrix as attribute evaluation space based on fuzzy decision matrix and improved the FAHP method using the approximate fuzzy eigenvector of such fuzzy symmetry matrix. Tuysuz and Kahraman (2006) provided an analytical tool to evaluate the project risks under incomplete and vague information. They used FAHP to evaluate the riskiness of an information technology project of a Turkish company. Ayag and Ozdemir (2006) proposed an intelligent approach based on FAHP for evaluating machine tool alternatives. They firstly used FAHP to weight the alternatives under multiple attributes and then carried out benefit/cost ratio analysis by using both the FAHP score and procurement cost of each alternative. Ertugrul and Karakasoglu (2006) proposed to use FAHP to select the best supplier for a textile firm in Turkey. Haq and Kannan(2006) proposed a structured model for evaluating vendor selection using AHP and FAHP. Chan and Kumar (2007) proposed a model for providing a framework for an organization to select the global supplier by considering risk factors. They used fuzzy extended analytic hierarchy process in the selection of global supplier. Finally, Ertugrul and Karakasoglu (2009) utilized both FAHP and TOPSIS methods for performance evaluation of Turkish cement firms.

3.2. Methodology of FAHP

In this study the extent FAHP is utilized, which was originally introduced by Chang (1996). Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ an object set, and $G = \{g_1, g_2, g_3, \ldots, g_n\}$ be a goal set. According to the method of Chang's extent analysis, each object is taken and extent analysis for each goal performed respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$M_{gi}^1, M_{gi}^2, \ldots, M_{gi}^m, \quad i = 1, 2, \ldots, n,$$

where M_{gi}^j $(j = 1, 2, \ldots, m)$ all are TFNs. The steps of Chang's extent analysis (Chang, 1996) can be given as in the following:

Step 1. The value of fuzzy synthetic extent with respect to the i th object is defined as

$$S_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^j\right]^{-1}. \tag{3}$$

To obtain $\sum_{j=1}^{m} M_{gi}^j$, the fuzzy addition operation of m extent analysis values for a particular matrix is performed such as

$$\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right), \tag{4}$$

and to obtain $[\sum_{j=1}^{n}\sum_{j=1}^{m} M_{gi}^j]^{-1}$, the fuzzy addition operation of M_{gi}^j $(j = 1, 2, \ldots, m)$ values is performed such as

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right), \tag{5}$$

and then the inverse of the vector above is computed, such as

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right). \tag{6}$$

Step 2. As $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as

$$V(M_2 \geq M_1) = \sup_{y \geq x} \left\lfloor \min(\mu_{M_1}(x), \mu_{M_2}(y)) \right\rfloor, \tag{7}$$

and can be expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) \tag{8}$$

$$= \begin{cases} 1 & \text{if } M_2 \geq M_1 \\ 0 & \text{if } l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & \text{otherwise} \end{cases} \tag{9}$$

Fig. 2 illustrates Eq. (9) where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2} to compare M_1 and M_2, we need both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step 3. The degree possibility for a convex fuzzy number to be greater than k convex fuzzy $M_i (i = 1, 2, \ldots, k)$ numbers can be defined by

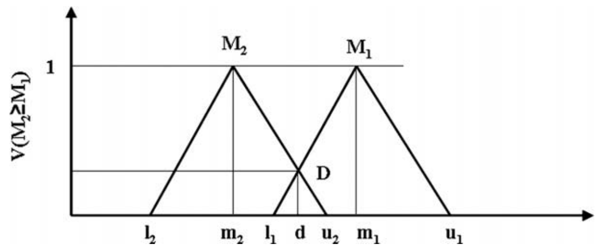

Fig. 2. The intersection between M_1 and M_2 (Chang, 1996).

$$V(M \geq M_1, M_2, \ldots, M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots \text{ and } (M \geq M_k)] = \min V(M \geq M_i), \quad i = 1, 2, 3, \ldots, k. \tag{10}$$

Assume that $d(A_i) = \min V(S_i \geq S_k)$ for $k = 1,2,\ldots,n$; $k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^{\mathrm{T}}, \tag{11}$$

where A_i ($i = 1,2,\ldots,n$) are n elements.

Step 4. Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \ldots, d(A_n))^{\mathrm{T}}, \tag{12}$$

where W is a non-fuzzy number.

4. Application

The purpose of this paper was to selection of the optimum underground mining method for a bauxite mine in Iran, with the help of mine information and appropriate factors. Firstly, a comprehensive questionnaire including main criteria of mining method selection is designed to understand and quantify the affecting factors in the process. Then, fifteen decision makers from different areas evaluate the importance of these factors with the help of mentioned questionnaire. FAHP is utilized for determining the weights of main criteria. Finally, AHP approach is employed for the rest of selection process. By this way, the ranking of considered underground mining methods according to their overall efficiency for Jajarm Bauxite Mine is obtained.

There are too many factors that have relation with mining method selection such as, geological and geotechnical properties, economic parameters and geographical factors. It is very difficult to formulate definite criteria for the method selection by changing in condition from one to another part of mine which can satisfy all conditions of the mining simultaneously. Therefore it seems clear that only an experienced engineer, who has improved his experience by working in several mines and gaining skills in different methods, can make a logical decision about mining method selection.

Characteristics which have major impacts on mining method selection include:

(1) Physical and mechanical characteristics of the deposit such as ground conditions of the ore zone, hanging wall, footwall, ore thickness, general shape, dip, plunge, depth, grade distribution, quality of resource, etc. The basic components that define the ground conditions are rock material shear strength, natural fractures and discontinuities shear strength, orientation, length, spacing and location of major geologic structures, in situ stress, hydrologic conditions, etc.
(2) Economic factors such as capital cost, operating cost, mineable ore tonnage, ore body grades and mineral value.
(3) Technical factors such as mine recovery, flexibility of methods, machinery and mining rate.

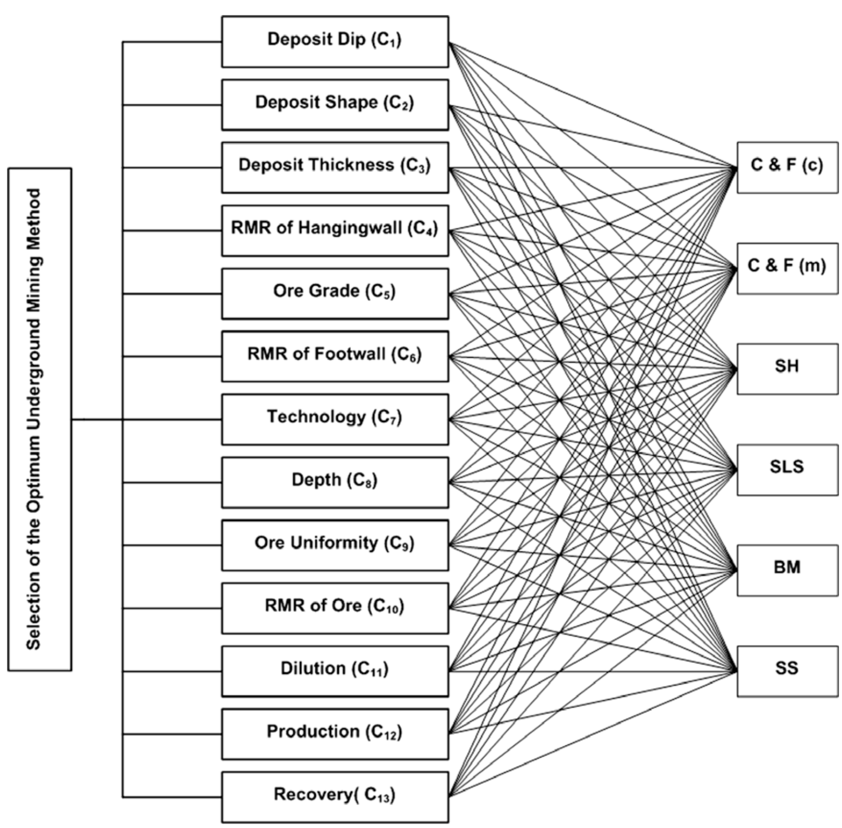

Fig. 3. Hierarchical structure of problem in application.

(4) Productivity factors such as annual productivity, equipment, efficiency and environmental considerations.

Regarding the mentioned factors, thirteen criteria were selected. These criteria can be found in hierarchical structure of the problem shown in Fig. 3.

4.1. Determination of criteria's weights

Because different groups inside and outside the mining projects have varying objectives and expectations, they approach mining method selection from different perspectives. So, affecting criteria have different level of significance for different users. For this reason, fifteen decision makers are selected from different areas and these decision makers evaluate the criteria. FAHP is proposed to take the decision makers subjective judgments into consideration and to reduce the uncertainty and vagueness in the decision process.

Decision makers from different backgrounds may define different weight vectors. They usually cause not only the imprecise evaluation but also serious persecution during decision process. For this reason, we proposed a group decision based on FAHP to improve pair-wise comparison. Firstly each decision maker (D_p), individually carry out pair-wise comparison by using Saaty's (Saaty, 1980) 1–9 scale (Table 1).

One of these pair-wise comparisons is shown here as example:

$$
\begin{array}{c}
\\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ D_1 = c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \\ c_{13}
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} & c_{13} \end{array} \\
\begin{bmatrix}
1 & 3 & 1 & 1 & 7 & 3 & 3 & 5 & 3 & 1 & 7 & 7 & 7 \\
1/3 & 1 & 1/3 & 1/3 & 5 & 1 & 1 & 3 & 1 & 1/3 & 5 & 5 & 5 \\
1 & 3 & 1 & 1 & 7 & 3 & 3 & 5 & 3 & 1 & 7 & 7 & 7 \\
1 & 3 & 1 & 1 & 7 & 3 & 3 & 5 & 3 & 1 & 7 & 7 & 7 \\
1/7 & 1/5 & 1/7 & 1/7 & 1 & 1/5 & 1/5 & 1/3 & 1/5 & 1/7 & 1 & 1 & 1 \\
1/3 & 1 & 1/3 & 1/3 & 5 & 1 & 1 & 3 & 1 & 1/3 & 5 & 5 & 5 \\
1/3 & 1 & 1/3 & 1/3 & 5 & 1 & 1 & 3 & 1 & 1/3 & 5 & 5 & 5 \\
1/5 & 1/3 & 1/5 & 1/5 & 3 & 1/3 & 1/3 & 1 & 1/3 & 1/5 & 3 & 3 & 3 \\
1/3 & 1 & 1/3 & 1/3 & 5 & 1 & 1 & 3 & 1 & 1/3 & 5 & 5 & 5 \\
1 & 3 & 1 & 1 & 7 & 3 & 3 & 5 & 3 & 1 & 7 & 7 & 7 \\
1/7 & 1/5 & 1/7 & 1/7 & 1 & 1/5 & 1/5 & 1/3 & 1/5 & 1/7 & 1 & 1 & 1 \\
1/7 & 1/5 & 1/7 & 1/7 & 1 & 1/5 & 1/5 & 1/3 & 1/5 & 1/7 & 1 & 1 & 1 \\
1/7 & 1/5 & 1/7 & 1/7 & 1 & 1/5 & 1/5 & 1/3 & 1/5 & 1/7 & 1 & 1 & 1
\end{bmatrix}
\end{array}
$$

Then, a comprehensive pair-wise comparison matrix is built as in Table 2 by integrating fifteen decision makers' grades through Eq. (12) (Chen, Lin, & Huang, 2006). By this way, decision makers' pair-wise comparison values are transformed into triangular fuzzy numbers as in Table 1,

Table 1
Pair-wise comparison scale (Saaty, 1980).

Comparison index	Score
Extremely preferred	9
Very strongly preferred	7
Strongly preferred	5
Moderately preferred	3
Equal	1
Intermediate values between the two adjacent judgments	2,4,6,8

Table 2
Fuzzy pair-wise comparison matrix.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}
c_1	(1,1,1)	(0.6,1.07,1.67)	(0.6,0.98,1.25)	(0.6,1.047,1.67)	(0.6,1.363,4)	(0.8, 1.35, 2.5)	(0.8,1.34,2)	(0.8,1.38,2.5)	(0.6,1.3,2.5)	(0.75,1.3,2.5)	(0.6,1.6,4)	(0.6,1.5,4)	(0.8,1.7,5)
c_2	(0.6,0.99,1.67)	(1,1,1)	(0.6,0.95,1.25)	(0.6,1.019,1.67)	(0.75,1.253,3)	(0.6, 1.34,2.5)	(0.75,1.3,2)	(0.75,1.33,2.5)	(1,1.2,2)	(0.75,1.24,2.5)	(0.8,1.5,3)	(0.8,1.4,3)	(0.8,1.6,5)
c_3	(0.8,1.05,1.67)	(0.8,1.1,1.67)	(1,1,1)	(0.8,1.064,1.33)	(0.8,1.381,4)	(0.8,1.39,2.5)	(0.8,1.42,2.5)	(0.8,1.41,2.5)	(0.8,1.4,2.5)	(0.8,1.31,2)	(0.8,1.6,4)	(0.8,1.5,4)	(0.8,1.6,4)
c_4	(0.6,1.01,1.67)	(0.6,1.1,1.67)	(0.75,0.957,1.25)	(1,1,1)	(0.6,1.334,4)	(0.6,1.33,2.5)	(0.6,1.36,2.5)	(0.6,1.36,2.5)	(0.6,1.3,2.5)	(0.75,1.23,2)	(0.6,1.6,4)	(0.6,1.4,4)	(0.6,1.5,4)
c_5	(0.25,0.88,1.67)	(0.33,0.9,1.33)	(0.25,0.83,1.25)	(0.25,0.893,1.67)	(1,1,1)	(0.33,1.14,2.5)	(0.33,1.16,2.5)	(0.5,1.12,1.67)	(0.3,1.1,2.5)	(0.25,1.09,2)	(1,1.2,2.5)	(0.6,1.1,2)	(0.8,1.3,4)
c_6	(0.4,0.84,1.25)	(0.4,0.9,1.67)	(0.4,0.803,1.25)	(0.4,0.859,1.67)	(0.4,1.083,3)	(1,1,1)	(0.4,1.16,2.5)	(0.67,1.06,1.5)	(0.4,1.1,2.5)	(0.5,1.02,1.7)	(0.4,1.3,3)	(0.4,1.2,3)	(0.5, 1.2,3)
c_7	(0.5,0.8,1.25)	(0.5,0.8,1.33)	(0.4,0.79,1.25)	(0.4,0.849,1.67)	(0.4,1.061,3)	(0.4,1.1,2.5)	(1,1,1)	(0.5,1.09,1.67)	(0.5,1.2)	(0.5,1.04,2.5)	(0.5,1.2,3)	(0.4,1.1,3)	(0.5,1.3,5)
c_8	(0.4,0.8,1.25)	(0.4,0.8,1.33)	(0.4,0.763,1.25)	(0.4,0.821,1.67)	(0.6,0.983,2)	(0.67,0.99,1.5)	(0.6,1.06,2)	(1,1,1)	(0.4,1,2)	(0.5,0.98,1.5)	(0.6,1.2,2)	(0.4,1.1,2)	(0.5,1.2,3)
c_9	(0.4,0.85,1.67)	(0.5,0.9,1)	(0.4,0.807,1.25)	(0.4,0.873,1.67)	(0.4,1.086,3)	(0.4,1.16,2.5)	(0.5,1.1,2)	(0.5,1.15,2.5)	(1,1,1)	(0.4,1.07,2)	(0.7,1.3,3)	(0.4,1.2,3)	(0.5,1.3,4)
c_{10}	(0.4,0.86,1.33)	(0.4,0.9,1.33)	(0.5,0.817,1.25)	(0.5,0.857,1.33)	(0.5,1.158,4)	(0.6,1.11,2)	(0.4,1.14,2)	(0.67,1.14,2)	(0.5,1.1,2.5)	(1,1,1)	(0.7,1.3,4)	(0.5,1.2,4)	(0.7,1.3,4)
c_{11}	(0.25,0.76,1.67)	(0.33,0.8,1.33)	(0.25,0.717,1.25)	(0.25,0.778,1.67)	(0.4,0.88,1)	(0.33,1,2.5)	(0.33,0.99,2)	(0.5,0.96,1.67)	(0.3,0.9,1.5)	(0.25,0.93,1.5)	(1,1,1)	(0.4,1,2)	(0.5,1.1,3)
c_{12}	(0.25,0.83,1.67)	(0.33,0.8,1.33)	(0.25,0.79,1.25)	(0.25,0.838,1.67)	(0.5,0.976,1.67)	(0.33,1.12,2.5)	(0.33,1.09,2.5)	(0.5,1.09,2.5)	(0.3,1.1,2.5)	(0.25,1.02,2)	(0.5,1.2,2.5)	(1,1,1)	(0.7,1.2,3)
c_{13}	(0.2,0.78,1.33)	(0.2,0.8,1.33)	(0.25,0.737,1.25)	(0.25,0.783,1.67)	(0.25,0.918,1.33)	(0.33,1,2)	(0.2,1.04,2)	(0.33,1.01,2)	(0.3,1,2)	(0.25,0.93,1.5)	(0.3,1.1,2)	(0.3,1.1,1.5)	(1,1,1)

$$(\widetilde{x}_{ij}) = (a_{ij}, b_{ij}, c_{ij})$$

$$l_{ij} = \min_k\{a_{ijk}\}, \quad m_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk}, \quad u_{ij} = \max_k\{d_{ijk}\} \tag{13}$$

After forming fuzzy pair-wise comparison matrix, weights of all criteria and sub-criteria are determined by the help of FAHP. According to the FAHP method, firstly synthesis values must be calculated. From (Table 2), synthesis values respect to main goal are calculated like in Eq. (4):

$$S_{c_1} = (9.10, 16.88, 34.58) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0248, 0.0915, 0.3749),$$
$$S_{c_2} = (9.65, 16.07, 31.08) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0263, 0.0871, 0.3369),$$
$$S_{c_3} = (10.60, 17.25, 33.67) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0289, 0.0935, 0.3650),$$
$$S_{c_4} = (8.50, 16.50, 33.58) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0232, 0.0894, 0.3640),$$
$$S_{c_5} = (6.18, 13.72, 26.58) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0169, 0.0744, 0.2882),$$
$$S_{c_6} = (6.27, 13.58, 27.00) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0171, 0.0736, 0.2927),$$
$$S_{c_7} = (6.50, 13.29, 29.17) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0177, 0.0720, 0.3162),$$
$$S_{c_8} = (6.87, 12.69, 22.50) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0187, 0.0688, 0.2439),$$
$$S_{c_9} = (6.47, 13.73, 28.58) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0176, 0.0744, 0.3098),$$
$$S_{c_{10}} = (7.30, 13.96, 30.75) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0199, 0.0756, 0.3333),$$
$$S_{c_{11}} = (5.13, 11.76, 22.08) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0140, 0.0637, 0.2394),$$
$$S_{c_{12}} = (5.50, 13.01, 26.08) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0150, 0.0705, 0.2827),$$
$$S_{c_{13}} = (4.18, 12.07, 20.92) \otimes (1/366.58, 1/184.52, 1/92.25) = (0.0114, 0.0654, 0.2267).$$

These fuzzy values are compared by using Eq. (9) and these values are obtained:

$$V(S_{c_1} \geq S_{c_2}) = 1, \quad V(S_{c_1} \geq S_{c_3}) = 0.90, \quad V(S_{c_1} \geq S_{c_4}) = 1, \quad V(S_{c_1} \geq S_{c_5}) = 1,$$
$$V(S_{c_1} \geq S_{c_6}) = 1, \quad V(S_{c_1} \geq S_{c_7}) = 1, \quad V(S_{c_1} \geq S_{c_8}) = 1, \quad V(S_{c_1} \geq S_{c_9}) = 1,$$
$$V(S_{c_1} \geq S_{c_{10}}) = 1, \quad V(S_{c_1} \geq S_{c_{11}}) = 1, \quad V(S_{c_1} \geq S_{c_{12}}) = 1, \quad V(S_{c_1} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_2} \geq S_{c_1}) = 0.85, \quad V(S_{c_2} \geq S_{c_3}) = 0.83, \quad V(S_{c_2} \geq S_{c_4}) = 0.86, \quad V(S_{c_2} \geq S_{c_5}) = 1,$$
$$V(S_{c_2} \geq S_{c_6}) = 1, \quad V(S_{c_2} \geq S_{c_7}) = 1, \quad V(S_{c_2} \geq S_{c_8}) = 1, \quad V(S_{c_2} \geq S_{c_9}) = 1,$$
$$V(S_{c_2} \geq S_{c_{10}}) = 1, \quad V(S_{c_2} \geq S_{c_{11}}) = 1, \quad V(S_{c_2} \geq S_{c_{12}}) = 1, \quad V(S_{c_2} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_3} \geq S_{c_1}) = 1, \quad V(S_{c_3} \geq S_{c_2}) = 1, \quad V(S_{c_3} \geq S_{c_4}) = 1, \quad V(S_{c_3} \geq S_{c_5}) = 1,$$
$$V(S_{c_3} \geq S_{c_6}) = 1, \quad V(S_{c_3} \geq S_{c_7}) = 1, \quad V(S_{c_3} \geq S_{c_8}) = 1, \quad V(S_{c_3} \geq S_{c_9}) = 1,$$
$$V(S_{c_3} \geq S_{c_{10}}) = 1, \quad V(S_{c_3} \geq S_{c_{11}}) = 1, \quad V(S_{c_3} \geq S_{c_{12}}) = 1, \quad V(S_{c_3} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_4} \geq S_{c_1}) = 0.89, \quad V(S_{c_4} \geq S_{c_2}) = 1, \quad V(S_{c_4} \geq S_{c_3}) = 0.84, \quad V(S_{c_4} \geq S_{c_5}) = 1,$$
$$V(S_{c_4} \geq S_{c_6}) = 1, \quad V(S_{c_4} \geq S_{c_7}) = 1, \quad V(S_{c_4} \geq S_{c_8}) = 1, \quad V(S_{c_4} \geq S_{c_9}) = 1, \quad V(S_{c_4} \geq S_{c_{10}}) = 1,$$
$$V(S_{c_4} \geq S_{c_{11}}) = 1, \quad V(S_{c_4} \geq S_{c_{12}}) = 1, \quad V(S_{c_4} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_5} \geq S_{c_1}) = 0.79, \quad V(S_{c_5} \geq S_{c_2}) = 0.80 \quad V(S_{c_5} \geq S_{c_3}) = 0.77, \quad V(S_{c_5} \geq S_{c_4}) = 0.81,$$
$$V(S_{c_5} \geq S_{c_6}) = 1, \quad V(S_{c_5} \geq S_{c_7}) = 1, \quad V(S_{c_5} \geq S_{c_8}) = 1, \quad V(S_{c_5} \geq S_{c_9}) = 1,$$
$$V(S_{c_5} \geq S_{c_{10}}) = 0.87, \quad V(S_{c_5} \geq S_{c_{11}}) = 1, \quad V(S_{c_5} \geq S_{c_{12}}) = 1, \quad V(S_{c_5} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_6} \geq S_{c_1}) = 0.80, \quad V(S_{c_6} \geq S_{c_2}) = 0.80, \quad V(S_{c_6} \geq S_{c_3}) = 0.77, \quad V(S_{c_6} \geq S_{c_4}) = 0.81,$$
$$V(S_{c_6} \geq S_{c_5}) = 0.89, \quad V(S_{c_6} \geq S_{c_7}) = 1, \quad V(S_{c_6} \geq S_{c_8}) = 1, \quad V(S_{c_6} \geq S_{c_9}) = 0.88,$$
$$V(S_{c_6} \geq S_{c_{10}}) = 0.94, \quad V(S_{c_6} \geq S_{c_{11}}) = 1, \quad V(S_{c_6} \geq S_{c_{12}}) = 1, \quad V(S_{c_6} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_7} \geq S_{c_1}) = 0.81, \quad V(S_{c_7} \geq S_{c_2}) = 0.81, \quad V(S_{c_7} \geq S_{c_3}) = 0.78, \quad V(S_{c_7} \geq S_{c_4}) = 0.82,$$
$$V(S_{c_7} \geq S_{c_5}) = 0.89, \quad V(S_{c_7} \geq S_{c_6}) = 0.89, \quad V(S_{c_7} \geq S_{c_8}) = 1, \quad V(S_{c_7} \geq S_{c_9}) = 0.89,$$
$$V(S_{c_7} \geq S_{c_{10}}) = 1, \quad V(S_{c_7} \geq S_{c_{11}}) = 1, \quad V(S_{c_7} \geq S_{c_{12}}) = 1, \quad V(S_{c_7} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_8} \geq S_{c_1}) = 0.75, \quad V(S_{c_8} \geq S_{c_2}) = 0.75, \quad V(S_{c_8} \geq S_{c_3}) = 0.72, \quad V(S_{c_8} \geq S_{c_4}) = 0.77,$$
$$V(S_{c_8} \geq S_{c_5}) = 0.85, \quad V(S_{c_8} \geq S_{c_6}) = 0.85, \quad V(S_{c_8} \geq S_{c_7}) = 1, \quad V(S_{c_8} \geq S_{c_9}) = 0.85,$$
$$V(S_{c_8} \geq S_{c_{10}}) = 0.83, \quad V(S_{c_8} \geq S_{c_{11}}) = 1, \quad V(S_{c_8} \geq S_{c_{12}}) = 1, \quad V(S_{c_8} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_9} \geq S_{c_1}) = 0.81, \quad V(S_{c_9} \geq S_{c_2}) = 0.81, \quad V(S_{c_9} \geq S_{c_3}) = 0.78, \quad V(S_{c_9} \geq S_{c_4}) = 0.82,$$
$$V(S_{c_9} \geq S_{c_5}) = 1, \quad V(S_{c_9} \geq S_{c_6}) = 1, \quad V(S_{c_9} \geq S_{c_7}) = 1, \quad V(S_{c_9} \geq S_{c_8}) = 1,$$
$$V(S_{c_9} \geq S_{c_{10}}) = 0.88, \quad V(S_{c_9} \geq S_{c_{11}}) = 1, \quad V(S_{c_9} \geq S_{c_{12}}) = 1, \quad V(S_{c_9} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_{10}} \geq S_{c_1}) = 0.82, \quad V(S_{c_{10}} \geq S_{c_2}) = 0.83, \quad V(S_{c_{10}} \geq S_{c_3}) = 0.80, \quad V(S_{c_{10}} \geq S_{c_4}) = 0.84,$$
$$V(S_{c_{10}} \geq S_{c_5}) = 1, \quad V(S_{c_{10}} \geq S_{c_6}) = 1, \quad V(S_{c_{10}} \geq S_{c_7}) = 1, \quad V(S_{c_{10}} \geq S_{c_8}) = 1,$$
$$V(S_{c_{10}} \geq S_{c_9}) = 1, \quad V(S_{c_{10}} \geq S_{c_{11}}) = 1, \quad V(S_{c_{10}} \geq S_{c_{12}}) = 1, \quad V(S_{c_{10}} \geq S_{c_{13}}) = 1;$$
$$V(S_{c_{11}} \geq S_{c_1}) = 0.74, \quad V(S_{c_{11}} \geq S_{c_2}) = 0.74, \quad V(S_{c_{11}} \geq S_{c_3}) = 0.71, \quad V(S_{c_{11}} \geq S_{c_4}) = 0.75,$$

$V(S_{c_{11}} \geq S_{c_5}) = 0.83, \quad V(S_{c_{11}} \geq S_{c_6}) = 0.83, V(S_{c_{11}} \geq S_{c_7}) = 0.84,$
$V(S_{c_{11}} \geq S_{c_8}) = 0.83,$
$V(S_{c_{11}} \geq S_{c_9}) = 0.81, \quad V(S_{c_{11}} \geq S_{c_{10}}) = 0.81, \quad V(S_{c_{11}} \geq S_{c_{12}}) = 0.86,$
$V(S_{c_{11}} \geq S_{c_{13}}) = 0.90;$
$V(S_{c_{12}} \geq S_{c_1}) = 0.78, \quad V(S_{c_{12}} \geq S_{c_2}) = 0.79, \quad V(S_{c_{12}} \geq S_{c_3}) = 0.76,$
$V(S_{c_{12}} \geq S_{c_4}) = 0.80,$
$V(S_{c_{12}} \geq S_{c_5}) = 0.88, \quad V(S_{c_{12}} \geq S_{c_6}) = 0.88, \quad V(S_{c_{12}} \geq S_{c_7}) = 0.88,$
$V(S_{c_{12}} \geq S_{c_8}) = 1,$
$V(S_{c_{12}} \geq S_{c_9}) = 0.85, \quad V(S_{c_{12}} \geq S_{c_{10}}) = 0.85, \quad V(S_{c_{12}} \geq S_{c_{11}}) = 1,$
$V(S_{c_{12}} \geq S_{c_{13}}) = 1;$
$V(S_{c_{13}} \geq S_{c_1}) = 0.73, \quad V(S_{c_{13}} \geq S_{c_2}) = 0.73, \quad V(S_{c_{13}} \geq S_{c_3}) = 0.69,$
$V(S_{c_{13}} \geq S_{c_4}) = 0.74,$
$V(S_{c_{13}} \geq S_{c_5}) = 0.83, \quad V(S_{c_{13}} \geq S_{c_6}) = 0.83, \quad V(S_{c_{13}} \geq S_{c_7}) = 0.83,$
$V(S_{c_{13}} \geq S_{c_8}) = 0.83,$
$V(S_{c_{13}} \geq S_{c_9}) = 0.81, \quad V(S_{c_{13}} \geq S_{c_{10}}) = 0.80, \quad V(S_{c_{13}} \geq S_{c_{11}}) = 1,$
$V(S_{c_{13}} \geq S_{c_{12}}) = 0.86$

Then priority weights are calculated by using Eq. (10):

$$d'(c_1) = \min(1, 0.90, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1) = 0.90,$$
$$d'(c_2) = \min(0.85, 0.83, 0.86, 1, 1, 1, 1, 1, 1, 1, 1, 1) = 0.83,$$
$$d'(c_3) = \min(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1) = 1,$$
$$d'(c_4) = \min(0.89, 1, 0.84, 1, 1, 1, 1, 1, 1, 1, 1, 1) = 0.84,$$
$$d'(c_5) = \min(0.79, 0.80, 0.78, 0.81, 1, 1, 1, 1, 0.87, 1, 1, 1) = 0.78,$$
$$d'(c_6) = \min(0.80, 0.80, 0.77, 0.81, 0.89, 1, 1, 0.88, 0.94, 1, 1, 1) = 0.77,$$
$$d'(c_7) = \min(0.81, 0.81, 0.78, 0.82, 0.89, 0.89, 1, 0.89, 1, 1, 1, 1) = 0.78,$$
$$d'(c_8) = \min(0.75, 0.75, 0.72, 0.77, 0.85, 0.85, 1, 0.85, 0.83, 1, 1, 1) = 0.72,$$
$$d'(c_9) = \min(0.81, 0.81, 0.78, 0.82, 1, 1, 1, 1, 0.88, 1, 1, 1) = 0.78,$$
$$d'(c_{10}) = \min(0.82, 0.83, 0.80, 0.84, 1, 1, 1, 1, 1, 1, 1, 1) = 0.80,$$
$$d'(c_{11}) = \min(0.74, 0.74, 0.71, 0.75, 0.83, 0.83, 0.84, 0.83, 0.81, 0.81, 0.86, 0.90) = 0.71,$$
$$d'(c_{12}) = \min(0.78, 0.79, 0.76, 0.80, 0.88, 0.88, 0.88, 1, 0.85, 0.85, 1, 1) = 0.76,$$
$$d'(c_{13}) = \min(0.73, 0.73, 0.69, 0.74, 0.83, 0.83, 0.83, 0.83, 0.81, 0.80, 1, 0.86) = 0.69.$$

Priority weights form $W' = (0.90, 0.83, 1.00, 0.84, 0.78, 0.77, 0.83, 0.72, 0.78, 0.80, 0.71, 0.76, 0.69)$ vector. After the normalization of these values priority weights respect to main goal are calculated as (0.086, 0.080, 0.096, 0.081, 0.075, 0.074, 0.080, 0.069, 0.075, 0.077, 0.068, 0.073, 0.066). Mentioned priority weights have indicated for each criterion in Table 3.

4.2. Selection of mining method

According to the mine and ore body conditions, six mining methods that were possible and appropriate to this mine, considered. These methods are

(A) Conventional cut and fill ($C\&F_C$),
(B) mechanized cut and fill ($C\&F_M$),
(C) shrinkage stoping (SH),
(D) sublevel stoping (SLS),
(E) bench mining (BM),
(F) stull stoping (SS).

In this step, considered methods should be comprised with reference to each criterion. For this purpose, Liberatore's (Liberatore, Nydick, & Sanchez, 1992) five-point rating scale of outstanding (O), good (G), average (A), fair (F), and poor (P) was used to determine the pair-wise comparison judgment matrix (PCJM) as shown in Table 4. It is assumed that the difference in relative importance between two adjacent scales with respect to a particular scale is constant at two times, and obtains the corresponding PCJM for the rating scales. The matrix is then translated into the largest eigenvalue problem and the resulting priority weights of outstanding, good, average, fair, and poor are found as 0.513, 0.261, 0.129, 0.063, and 0.034, respectively.

Table 4
Pair-wise comparison judgment matrix for a five-point rating scale (Liberatore et al., 1992).

	O	G	A	F	P
O	1	3	5	7	9
G	1/3	1	3	5	7
A	1/5	1/3	1	3	5
F	1/7	1/5	1/3	1	3
P	1/9	1/7	1/5	1/3	1

Table 3
Priority weights for criteria.

Criteria	Local weights	Global wights
Deposit thickness	0.90	0.086
RMR of hanging wall	0.83	0.080
Deposit dip	1.00	0.096
Deposit shape	0.84	0.081
RMR of ore	0.78	0.075
Ore grade	0.77	0.074
Ore uniformity	0.83	0.080
Recovery	0.72	0.069
Production	0.78	0.075
RMR of footwall	0.80	0.077
Technology	0.71	0.068
Depth	0.76	0.073
Dilution	0.69	0.066

Table 5
Ranking of main criteria.

Rank	Criteria	Global weights
1	Deposit dip	0.096
2	Deposit thickness	0.086
3	Deposit shape	0.081
4	RMR of hanging wall	0.080
5	Ore uniformity	0.080
6	RMR of footwall	0.077
7	RMR of ore	0.075
8	Production	0.075
9	Ore grade	0.074
10	Depth	0.073
11	Recovery	0.069
12	Technology	0.068
13	Dilution	0.066

Table 6
Application of the AHP model to underground mining method selection of the Jajarm Bauxite Mine.

Main criteria	Global weights	Method A ($C\&F_C$)			Method B ($C\&F_M$)			Method C (SH)			Method D (SLS)			Method E (BM)			Method F (SS)		
		Rating	Score	xGW	Rating	Score	xGW	Rating	Score	xGW	Rating	Score	xGW	Rating	Score	xGW	Rating	Score	xGW
c_1	0.086	O	0.513	0.0441	A	0.129	0.0111	A	0.129	0.0111	A	0.129	0.0111	P	0.034	0.0029	P	0.034	0.0029
c_2	0.080	G	0.261	0.0209	G	0.261	0.0209	A	0.129	0.0103	A	0.129	0.0103	A	0.129	0.0103	A	0.129	0.0103
c_3	0.096	A	0.129	0.0124	A	0.129	0.0124	A	0.129	0.0124	A	0.129	0.0124	F	0.063	0.0060	A	0.129	0.0124
c_4	0.081	G	0.261	0.0211	G	0.261	0.0211	G	0.261	0.0211	F	0.063	0.0051	F	0.063	0.0051	A	0.129	0.0104
c_5	0.075	P	0.034	0.0026	A	0.129	0.0097	A	0.129	0.0097	O	0.513	0.0385	P	0.034	0.0026	F	0.063	0.0047
c_6	0.074	G	0.261	0.0193	G	0.261	0.0193	A	0.129	0.0095	F	0.063	0.0047	P	0.034	0.0025	A	0.129	0.0095
c_7	0.080	A	0.129	0.0103	A	0.129	0.0103	A	0.129	0.0103	A	0.129	0.0103	A	0.129	0.0103	A	0.129	0.0103
c_8	0.069	O	0.513	0.0354	G	0.261	0.0180	A	0.129	0.0089	A	0.129	0.0089	P	0.034	0.0023	P	0.034	0.0023
c_9	0.075	G	0.261	0.0196	G	0.261	0.0196	P	0.034	0.0026	F	0.063	0.0047	A	0.129	0.0097	F	0.063	0.0047
c_{10}	0.077	A	0.129	0.0099	A	0.129	0.0099	A	0.129	0.0099	A	0.129	0.0099	A	0.129	0.0099	A	0.129	0.0099
c_{11}	0.068	G	0.261	0.0177	G	0.261	0.0177	A	0.129	0.0088	A	0.129	0.0088	P	0.034	0.0023	P	0.034	0.0023
c_{12}	0.073	G	0.261	0.0191	G	0.261	0.0191	P	0.034	0.0025	A	0.129	0.0094	P	0.034	0.0025	F	0.063	0.0046
c_{13}	0.066	G	0.261	0.0172	G	0.261	0.0172	A	0.129	0.0085	A	0.129	0.0085	F	0.063	0.0042	F	0.063	0.0042
Total scores				0.2496			0.2064			0.1256			0.1426			0.0707			0.0887
Renormalized scores				0.2825			0.2335			0.1422			0.1614			0.0800			0.1004

Table 7
Final ranking of the mining methods.

Rank	Methods	Final ratings
1	Cut and fill (conventional)	0.2825
2	Cut and fill (mechanized)	0.2335
3	Sublevel stoping	0.1614
4	Shrinkage stoping	0.1422
5	Stull stoping	0.1004
6	Bench mining	0.0800

After computing the normalized priority weights for each PCJM of the AHP hierarchy, the next phase is to synthesize the solution for the mining method selection problem. The normalized local priority weights of criteria obtained from the fuzzy calculations are normalized to obtain the global priority weights. After calculating the global weights, they are rearranged in the descending order of priority, as shown in Table 5. It can be seen that the deposit dip occupies the top-most rankings in the list, followed by deposit thickness and deposit shape. Once the global priority weights of all criteria and ratings of mining methods are transferred to a spreadsheet, the global priority weight of each method can be found by multiplying the global priority weight of each criterion with the global priority weight of the mining method rating, and adding the resulting values. Since the priority weights of each rating is already determined, they can be used against each criterion on a spreadsheet format to determine the global priority weights of the six mining methods as shown in Table 6. Notice that these global priority weights need to be normalized as shown in Table 6. Based on the global priority weights of the six mining methods shown in Table 6, method of conventional cut and fill had the highest weight. Therefore, it was selected as the optimum method to satisfy the goals and objectives of the mining project.

The ranking of the mining methods are also shown in Table 7 in the descending order of priority. After the possibility evaluation of considered underground mining methods for Jajarm Bauxite by taking interfered effective criteria into account, the order of the methods are found as in Table 7.

In this approach of selection, the decision makers' priorities have had a great effect on the ranking of mining methods. If there will be a difference in the priority of the decision makers, the ranking may change. For this reason decision maker should know his priority properly and then determine the weights of the criteria.

5. Conclusion

There is no single appropriate mining method for a deposit. Usually, two or more feasible methods are possible. Each method entails some inherent problems. Consequently, the optimal method is one that offers the least problems. Selection of an appropriate mining method is a complex task that requires consideration of many technical, economical, political, social, and historical factors. The appropriate mining method is the method which is technically feasible for the ore geometry and ground conditions, while also being a low-cost operation. In this paper, a decision support system for underground mining method selection designed and developed to eliminate the difficulties in taking into consideration many decision criteria simultaneously in the underground mining method selection process and to guide the decision makers to select the optimal underground mining method. In this study, fuzzy AHP (FAHP) method is used to determine the degree of importance of the effective factors in the model. Humans are often uncertain in assigning the evaluation scores in conventional AHP. Fuzzy AHP can capture this difficulty. This method has the ability to capture the vagueness of human thinking style and effectively solve multi-criteria decision making problems. By using FAHP and appropriate calculations, the conventional cut and fill method was selected as optimum underground mining method for Jajarm Bauxite Mine.

In future studies, other multi-criteria methods can be used to selection of optimum underground mining method. And also the proposed method can be applied for selection of the methods in other sectors.

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