

Animation

CS 4620 Lecture 18

The artistic process of animation

- What are animators trying to do?
 - Important to understand in thinking about what tools they need
- Basic principles are universal across media
 - 2D hand-drawn animation
 - 2D computer animation
 - 3D computer animation
- Widely cited set of principles laid out by Frank Thomas and Ollie Johnston in *The Illusion of Life* (1981)

Timing

sad



happy



Timing

sad



happy

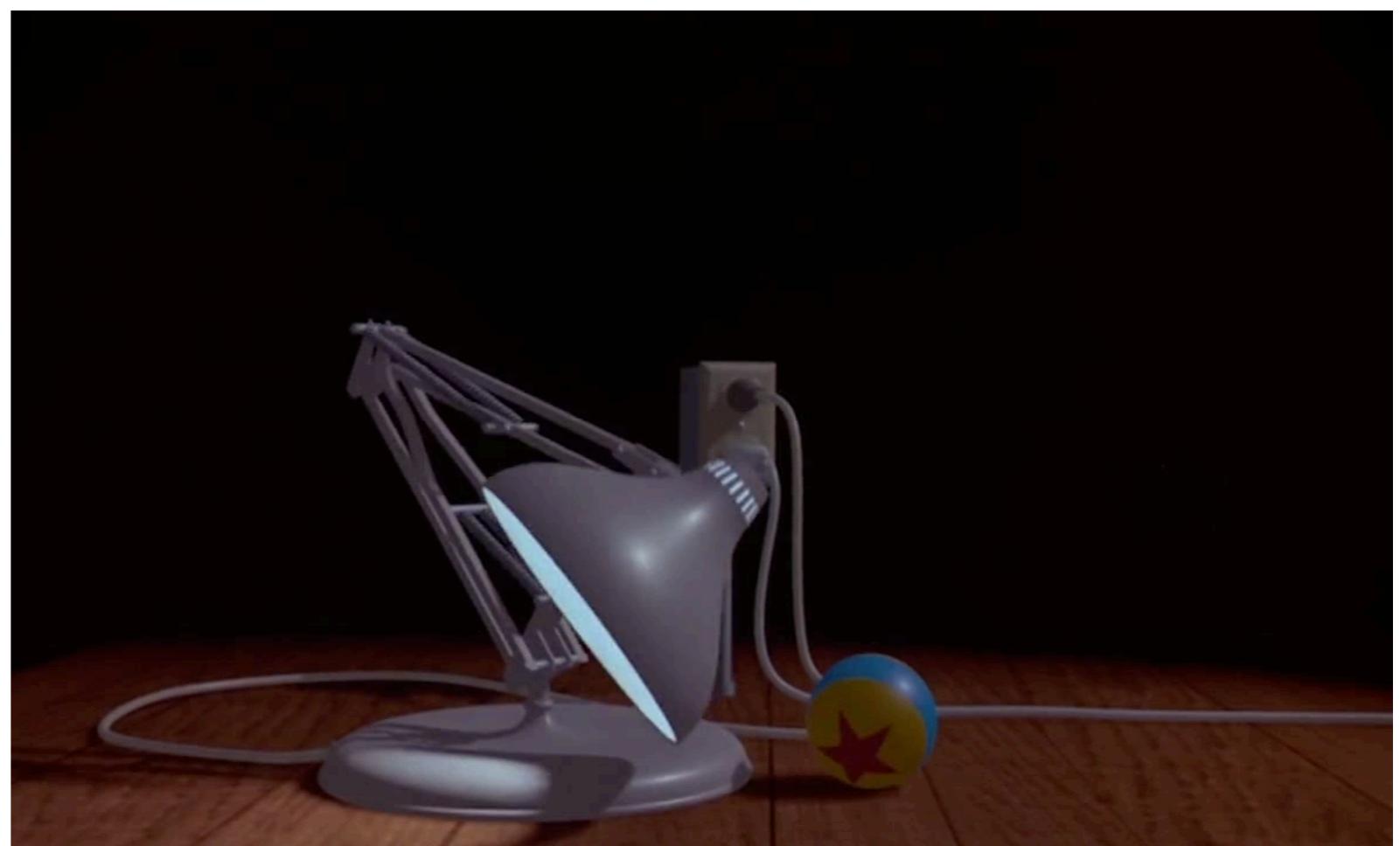


Anticipation & Follow-through

cautious



enthusiastic

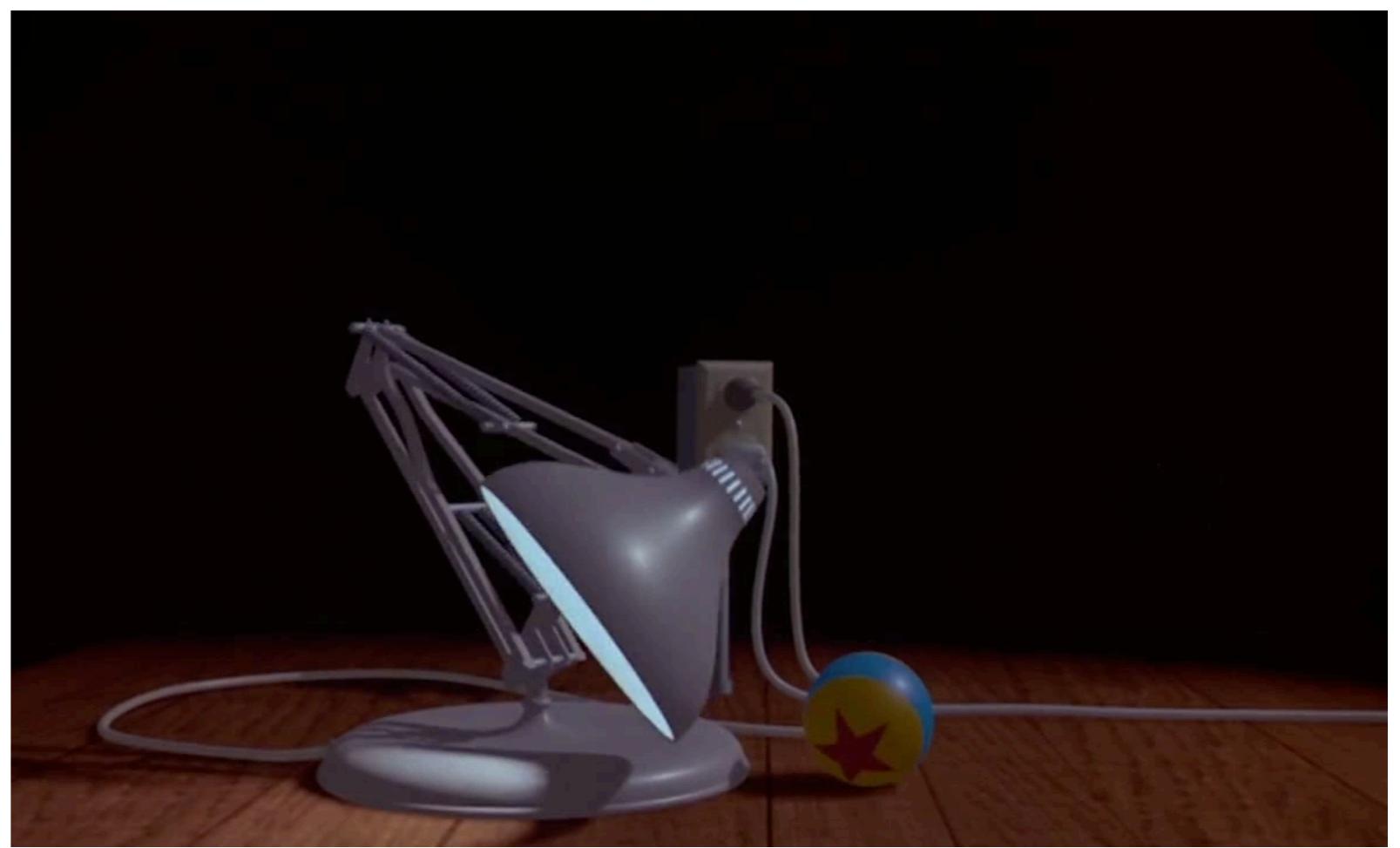


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Follow-through



Follow-through



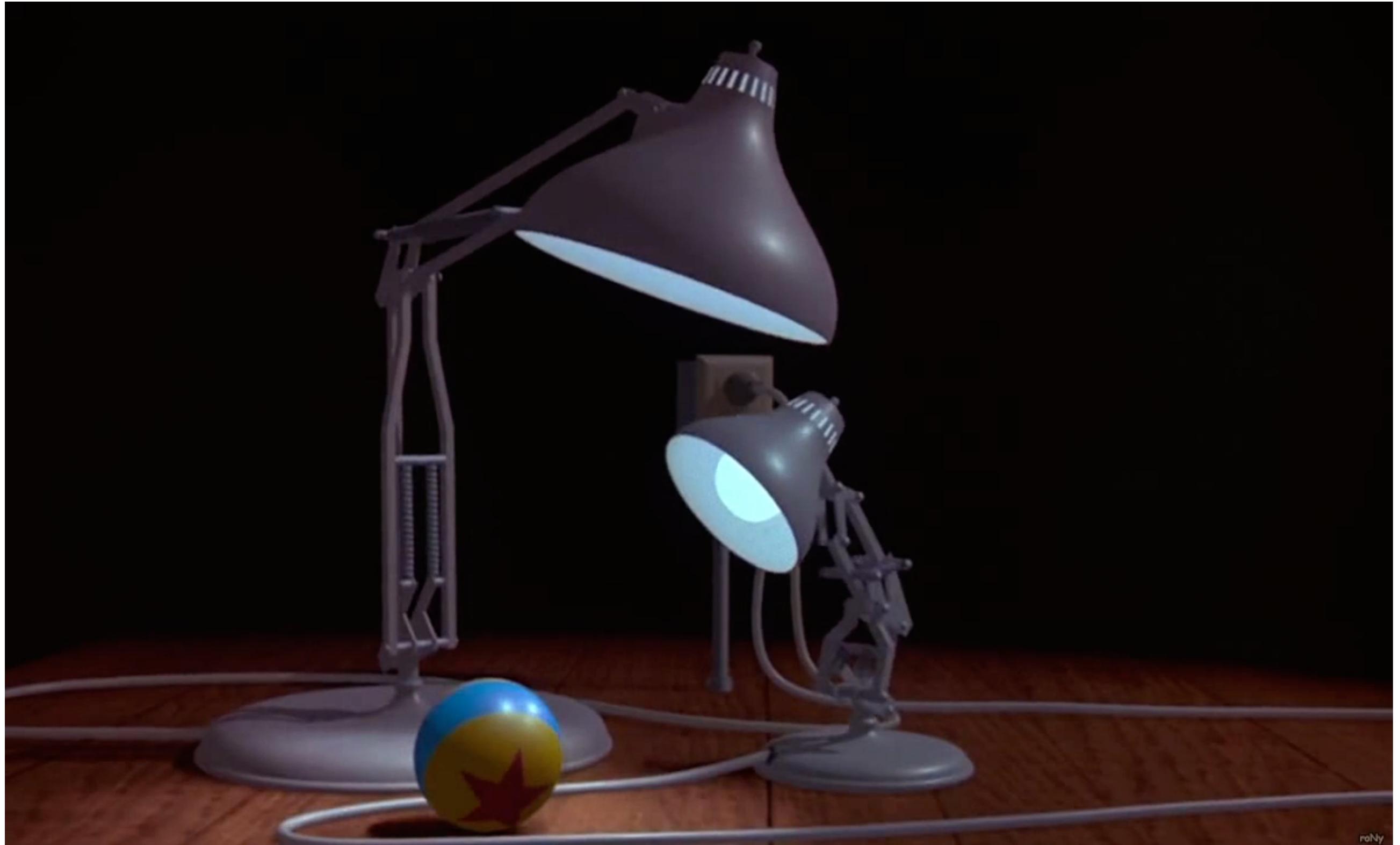
Overlapping action



Overlapping action



Staging and composition



What is animation?

- Modeling = specifying shape
 - using all the tools we've seen: hierarchies, meshes, curved surfaces...
- Animation = specifying shape as a function of time
 - just modeling done once per frame?
 - yes, but need smooth, concerted movement

Keyframes in hand-drawn animation

- End goal: a drawing per frame, with nice smooth motion
- “Straight ahead” is drawing frames in order (using a lightbox to see the previous frame or frames)
 - but it is hard to get a character to land at a particular pose at a particular time
- Instead use *key frames* to plan out the action
 - draw important poses first, then fill in the *in-betweens*



animation by Ollie Johnston, © Disney

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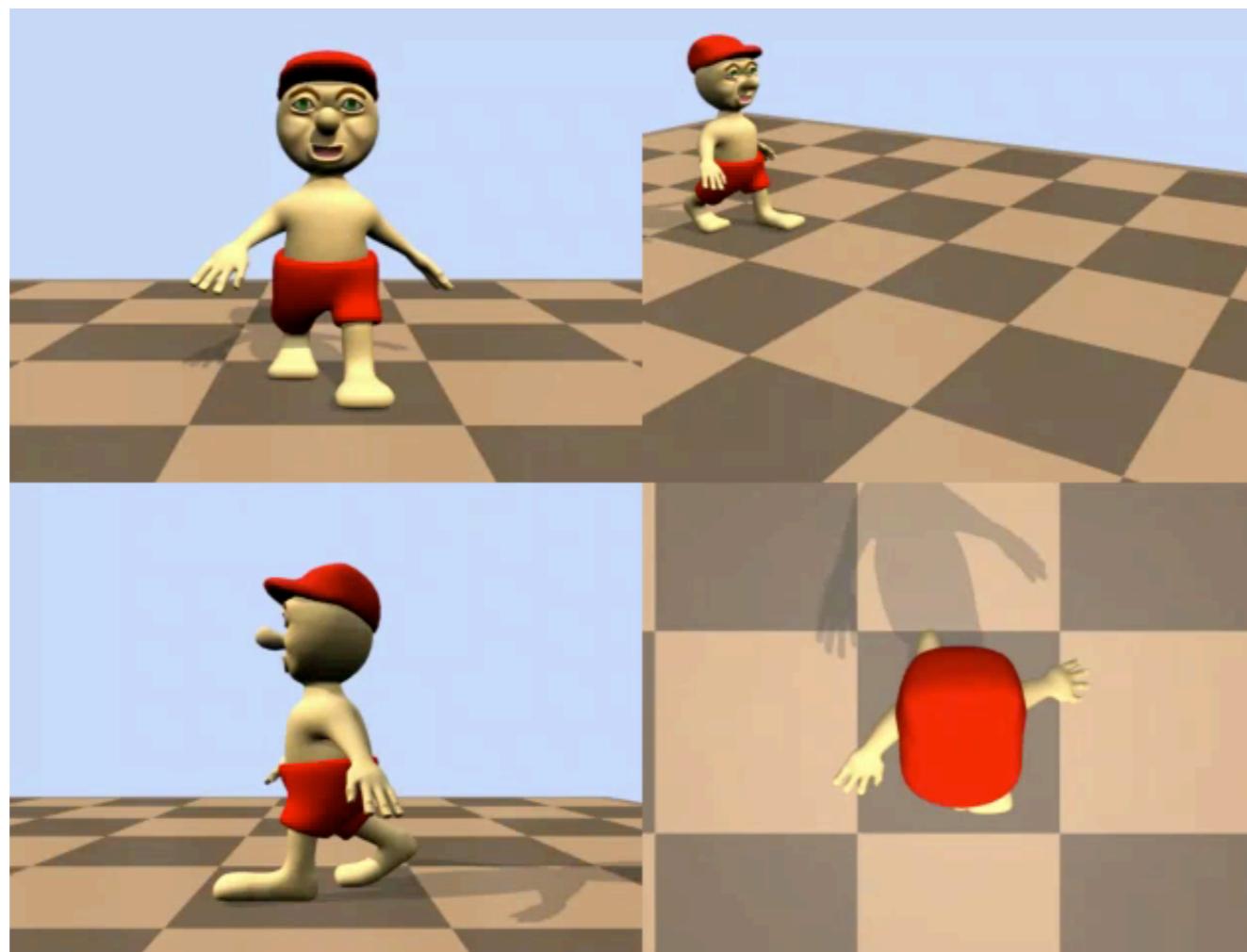
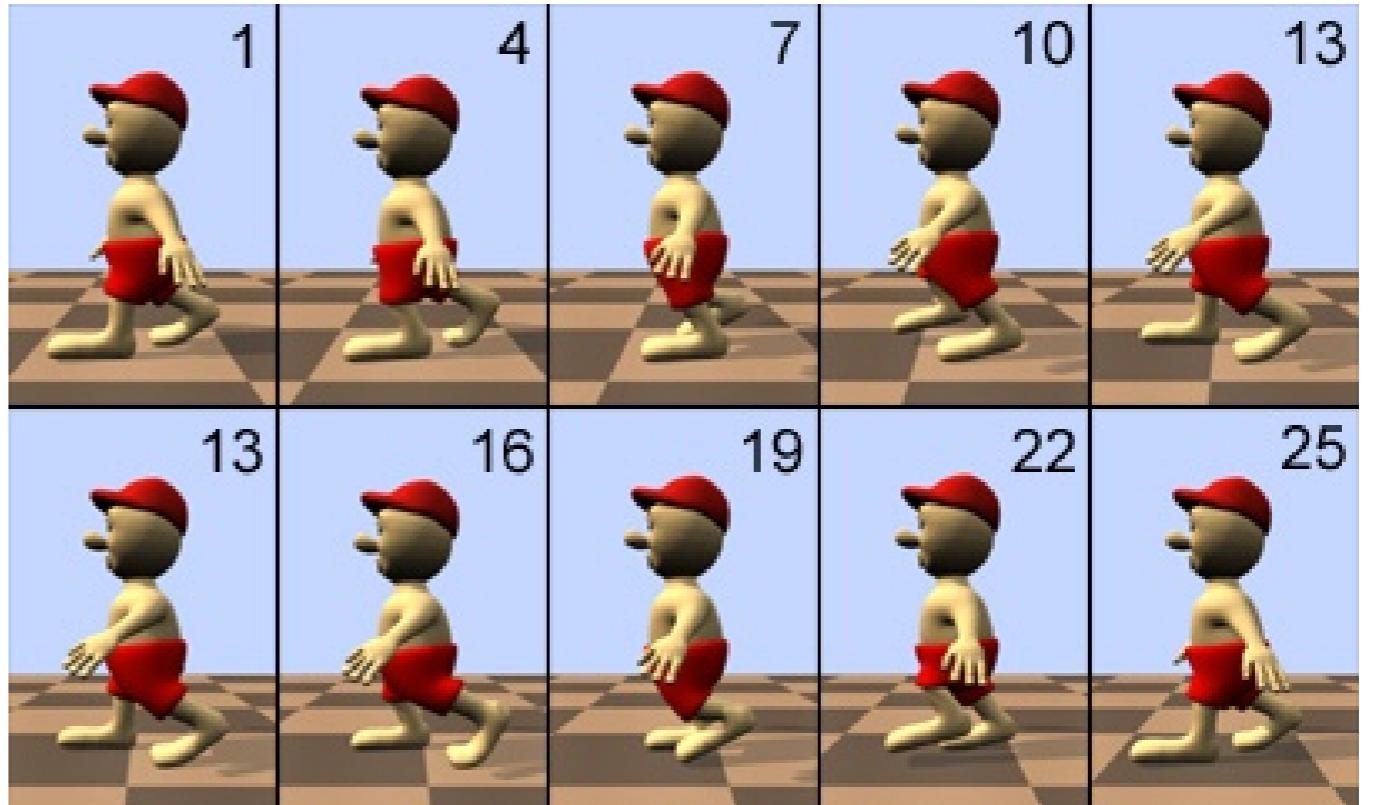


animation by Ollie Johnston, © Disney

Keyframe animation

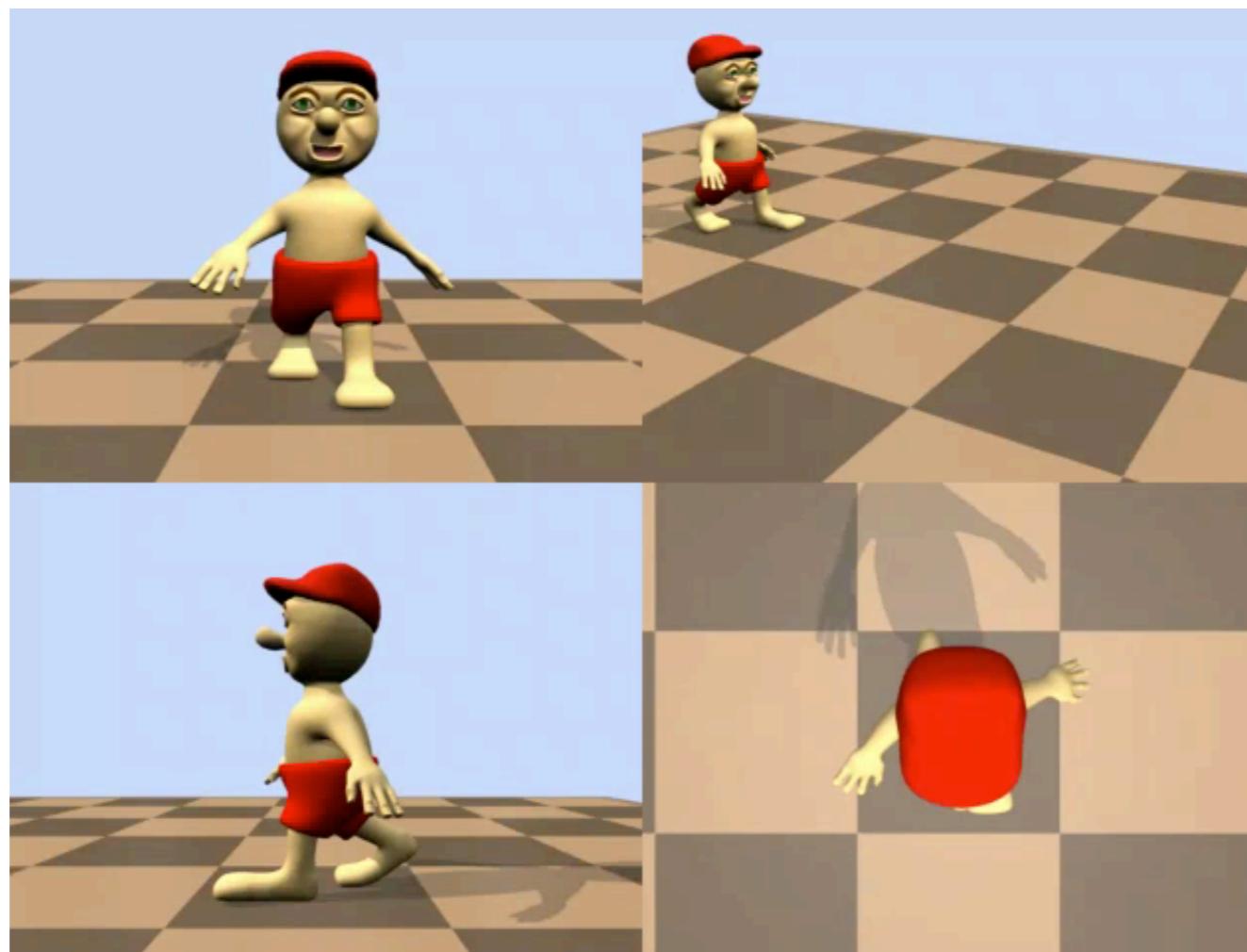
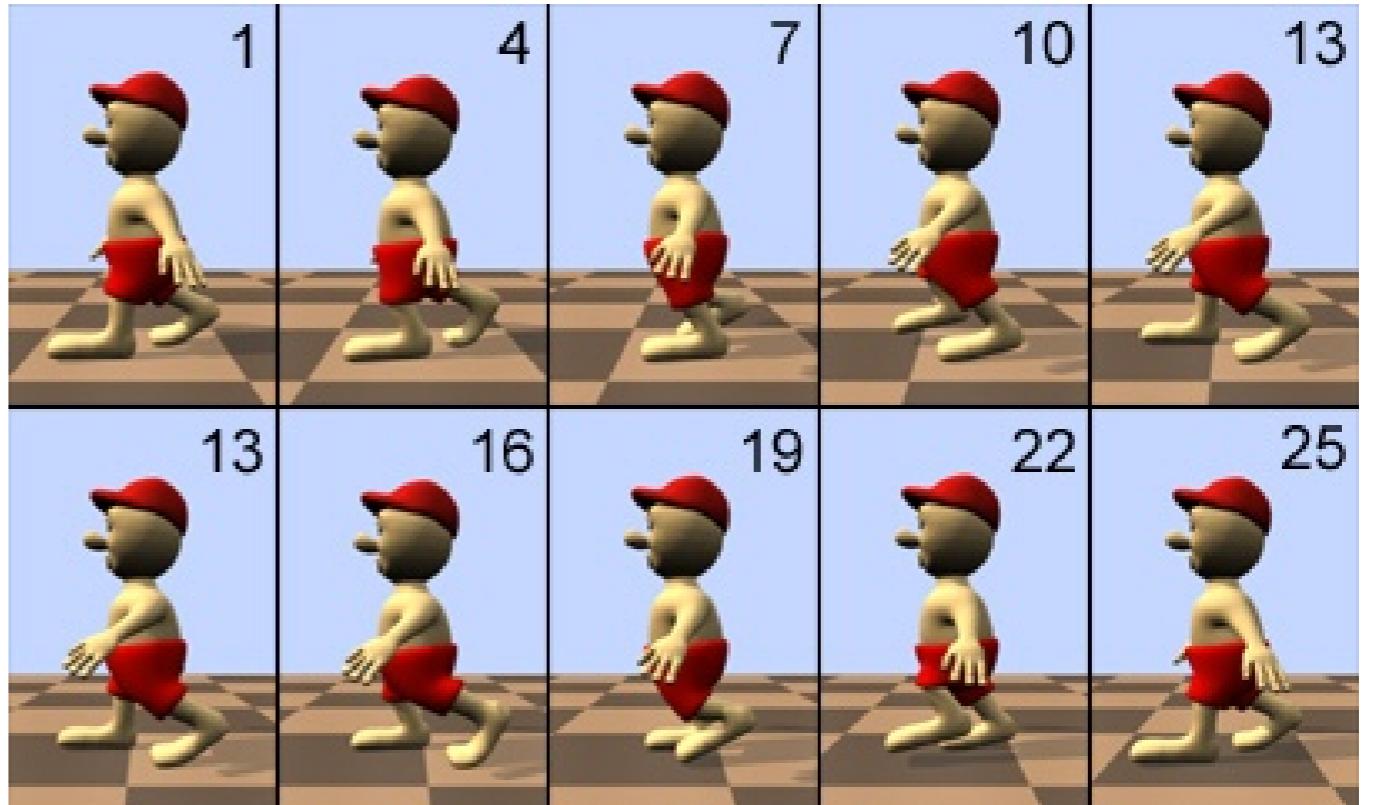
- Hand-drawn animation
 - head animator draws key poses—just enough to indicate what the motion is supposed to be
 - assistants do “in-betweening” and draw the rest of the frames
- Computer animation
 - in computer animation substitute “user” and “animation software”
 - *interpolation* is the principal operation
- Key ideas of computer animation
 - create *high-level* controls for adjusting geometry
 - *interpolate* these controls over time between keyframes

Walk cycle



[Christopher Lutz <http://www.animationsnippets.com>]

Walk cycle



[Christopher Lutz <http://www.animationsnippets.com>]

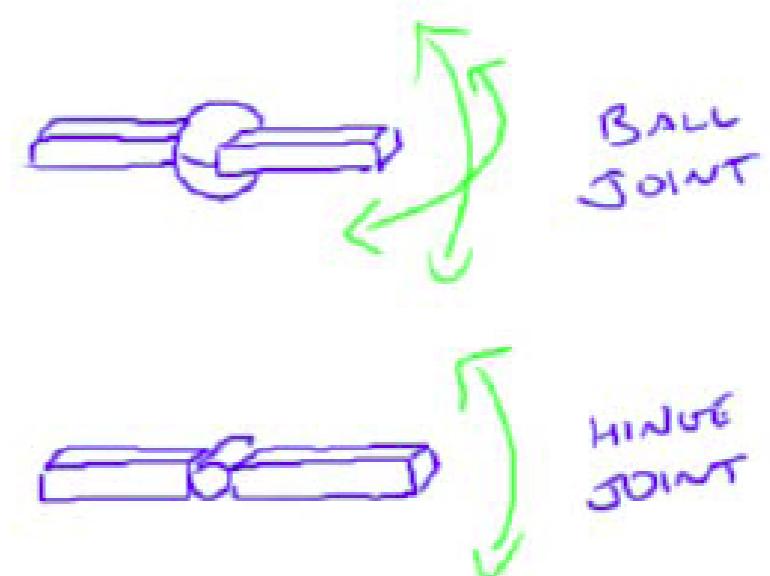
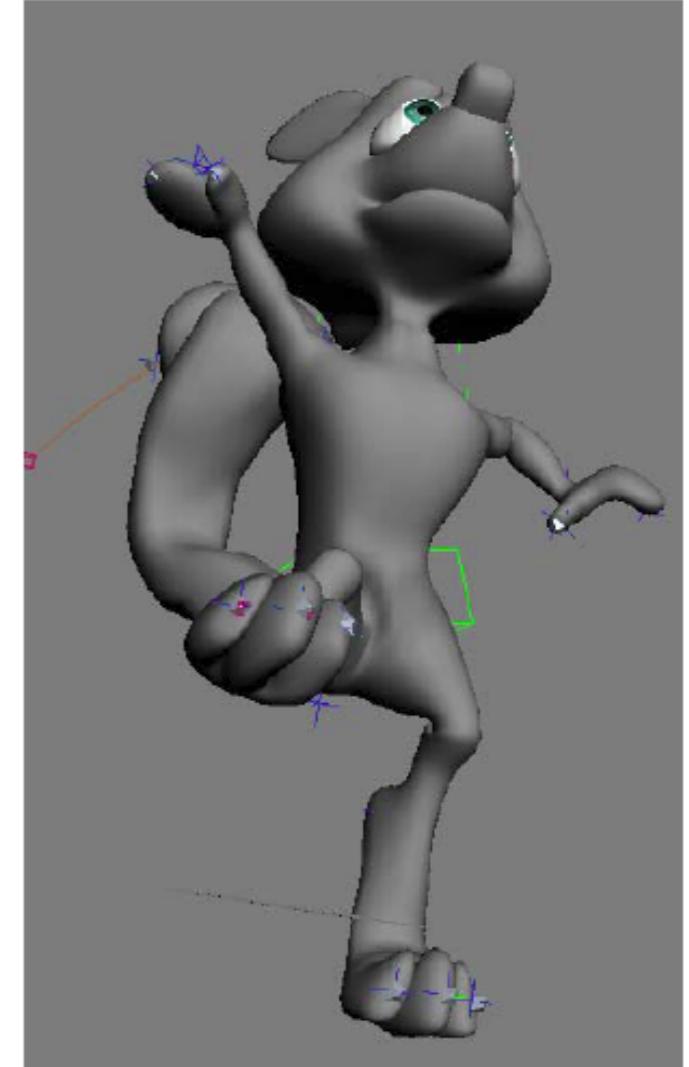
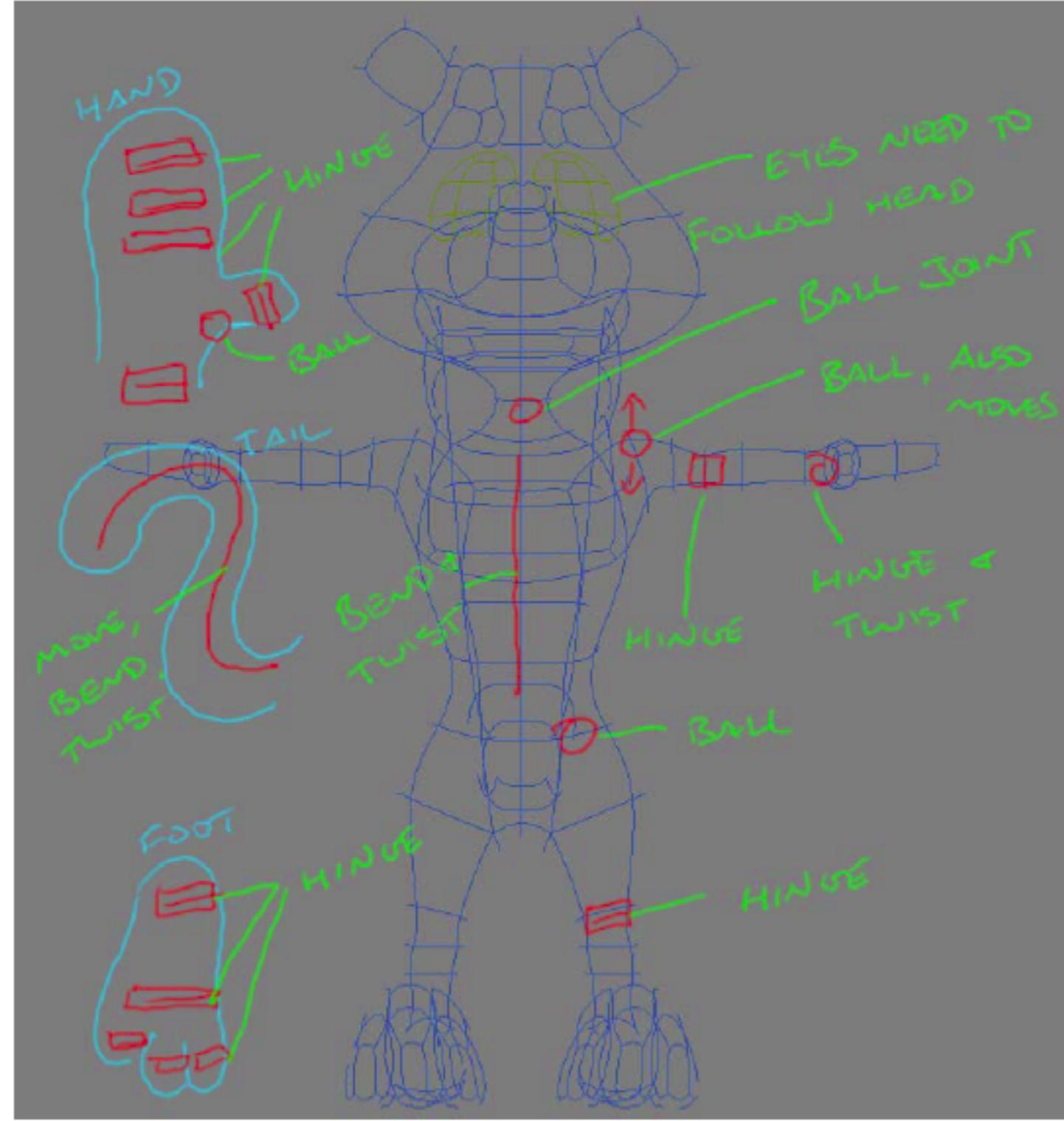
Controlling geometry conveniently

- Manually place every control point at every keyframe?
 - labor intensive
 - hard to get smooth, consistent motion
- Animate using smaller set of meaningful *degrees of freedom*
 - modeling DOFs are inappropriate for animation
 - e.g. “move one square inch of left forearm”
 - animation DOFs need to be higher level
 - e.g. “bend the elbow”

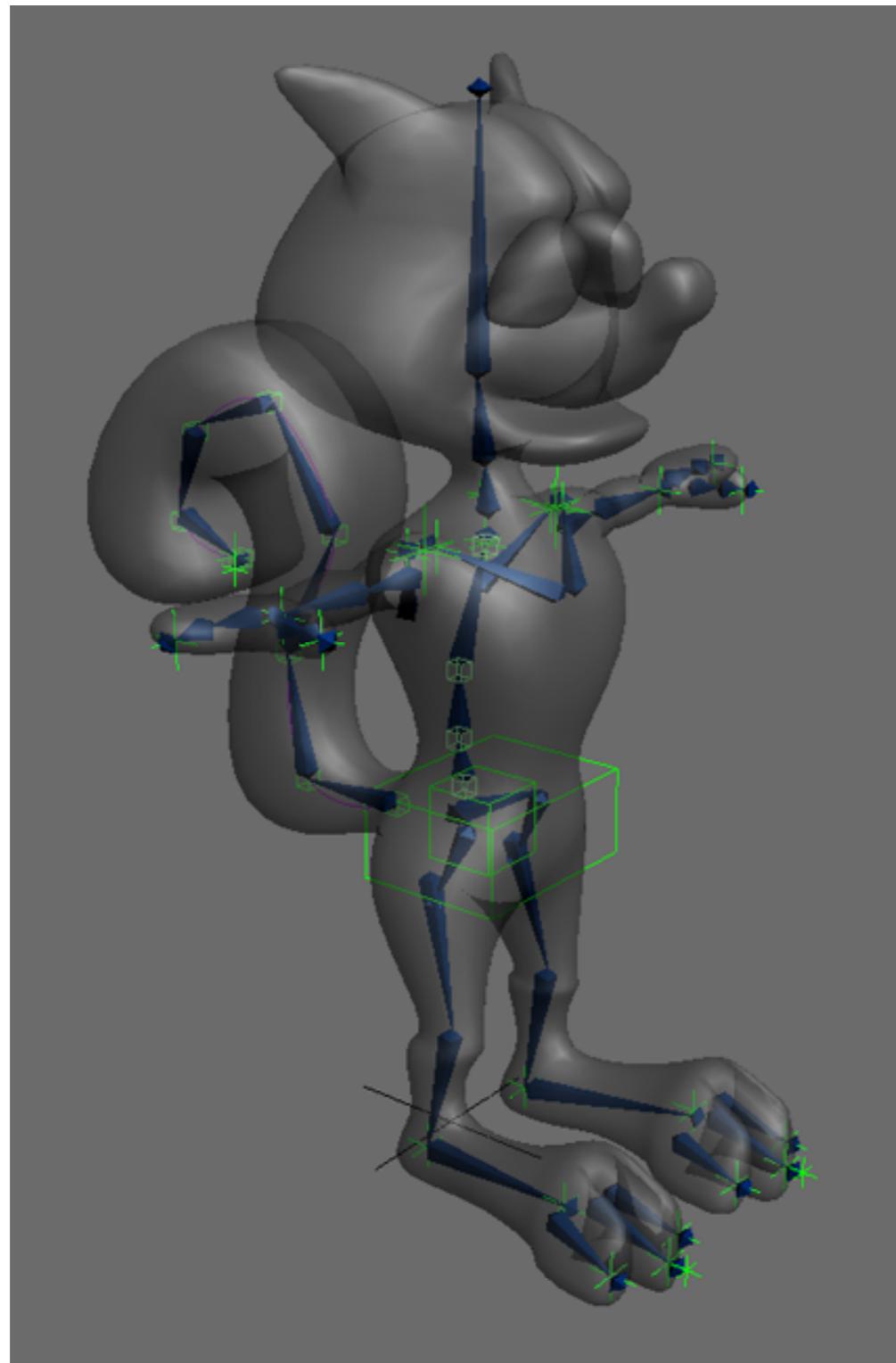
Controlling shape for animation

- Start with *modeling DOFs* (control points)
- *Deformations* control those DOFs at a higher level
 - Example: move first joint of second finger on left hand
- *Animation controls* control those DOFs at a higher level
 - Example: open/close left hand
- Both cases can be handled by the same kinds of deformers

Character with DOFs



Rigged character



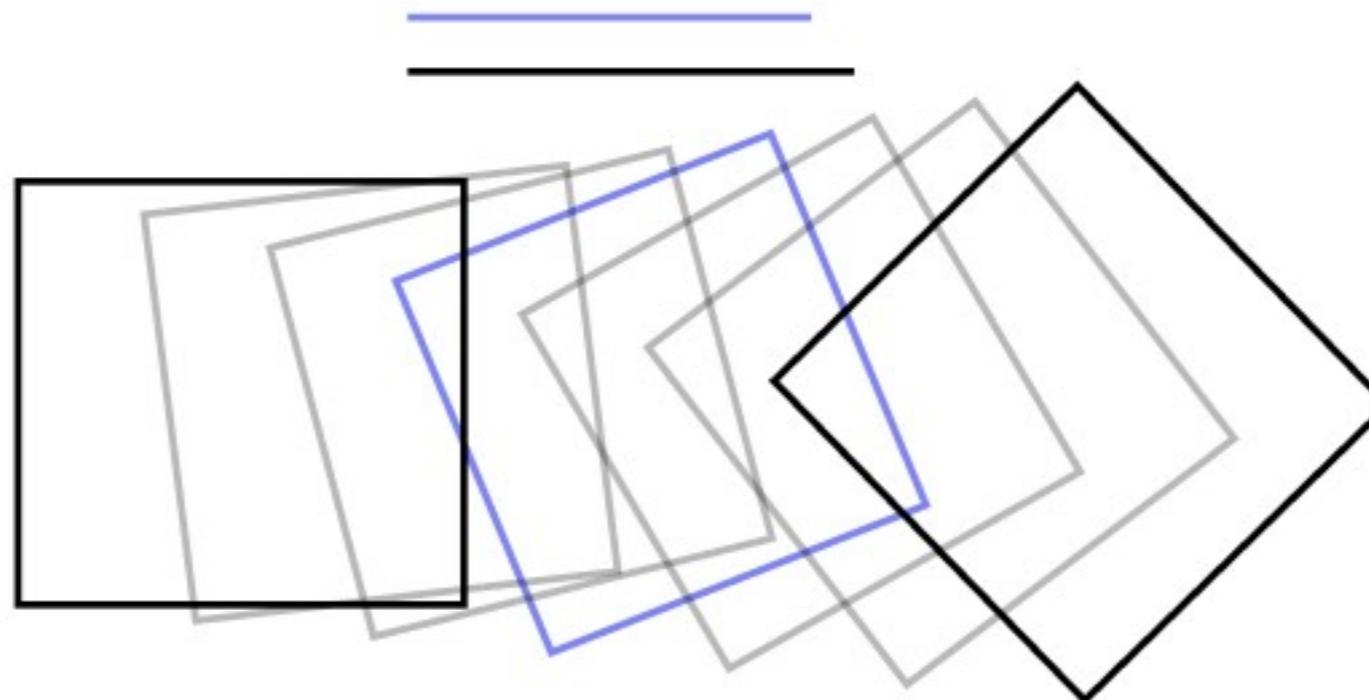
- Surface is deformed by a set of *bones*
- Bones are in turn controlled by a smaller set of *controls*
- The controls are useful, intuitive DOFs for an animator to use

The most basic animation control

- Affine transformations position things in modeling
- Time-varying affine transformations move things around in animation
- A hierarchy of time-varying transformations is the main workhorse of animation
 - and the basic framework within which all the more sophisticated techniques are built

Interpolating transformations

- Move a set of points by applying an affine transformation
- How to animate the transformation over time?
 - interpolate the matrix entries from keyframe to keyframe?
this is fine for translations but bad for rotations



Interpolating transformations

- Linear interpolation of matrices is not effective
 - leads to shrinkage when interpolating rotations
- One approach: always keep transformations in a canonical form (e.g. translate-rotate-scale)
 - then the pieces can be interpolated separately
 - rotations stay rotations, scales stay scales, all is good
- But you might be faced with just a matrix. What then?

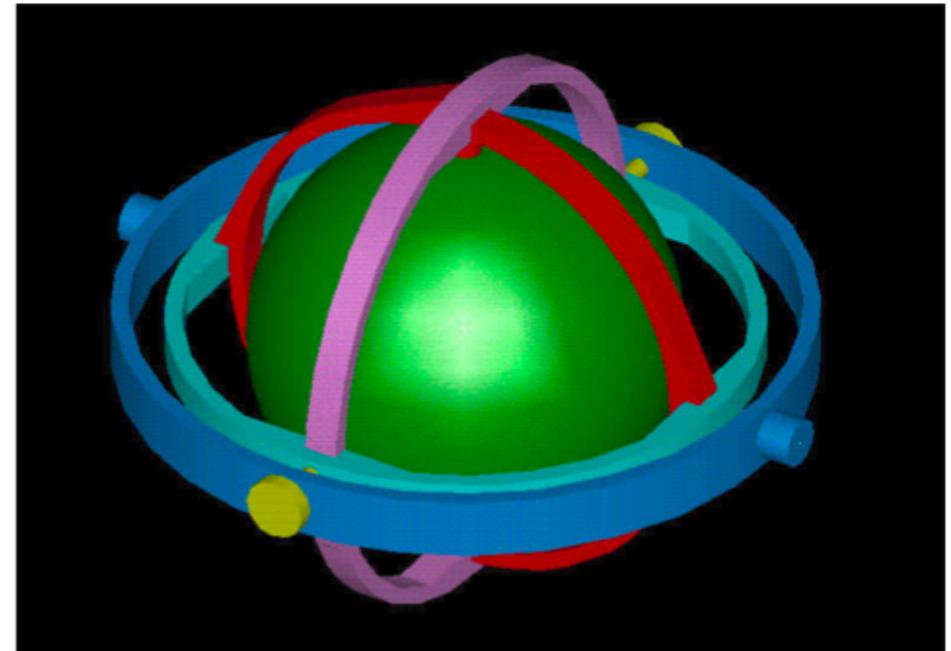
Decomposing transformations

- A product $M = TRS$ is not hard to take apart
 - translation sits in the top right
 - RS still has three orthogonal columns (prove?)
 - scale factors are the lengths of the columns
- But that doesn't cover everything
 - count DOFs: $3 + 3 + 3 < 12$
- If we allow S to be a scale along arbitrary axes then it does work— $M = TRS$ where
 - T is a translation
 - R is a rotation
 - S is a symmetric matrix (positive definite if no reflection)
 - linear algebra name: polar decomposition (at least the $A = RS$ part)

Parameterizing rotations

- Euler angles
 - rotate around x, then y, then z
 - nice and simple

$$R(\theta_x, \theta_y, \theta_z) = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$$

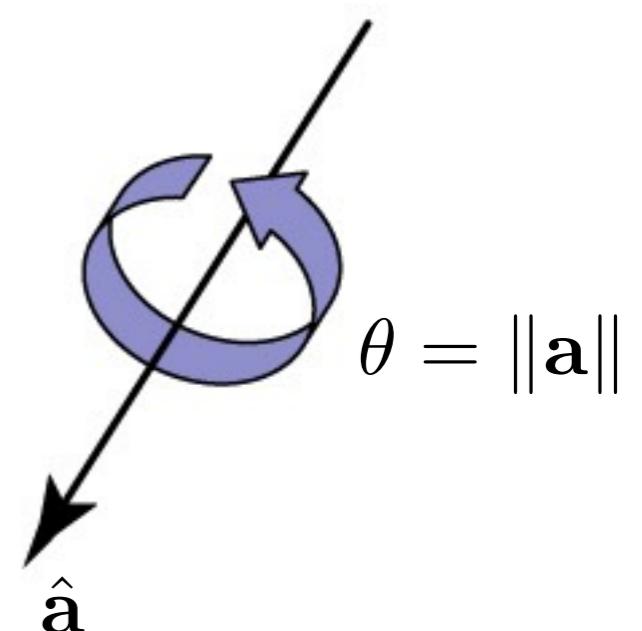


- Axis/angle
 - specify axis to rotate around, then angle by which to rotate
 - multiply axis and angle to get a more compact form

$$R(\hat{\mathbf{a}}, \theta) = F_{\hat{\mathbf{a}}} R_x(\theta) F_{\hat{\mathbf{a}}}^{-1}$$

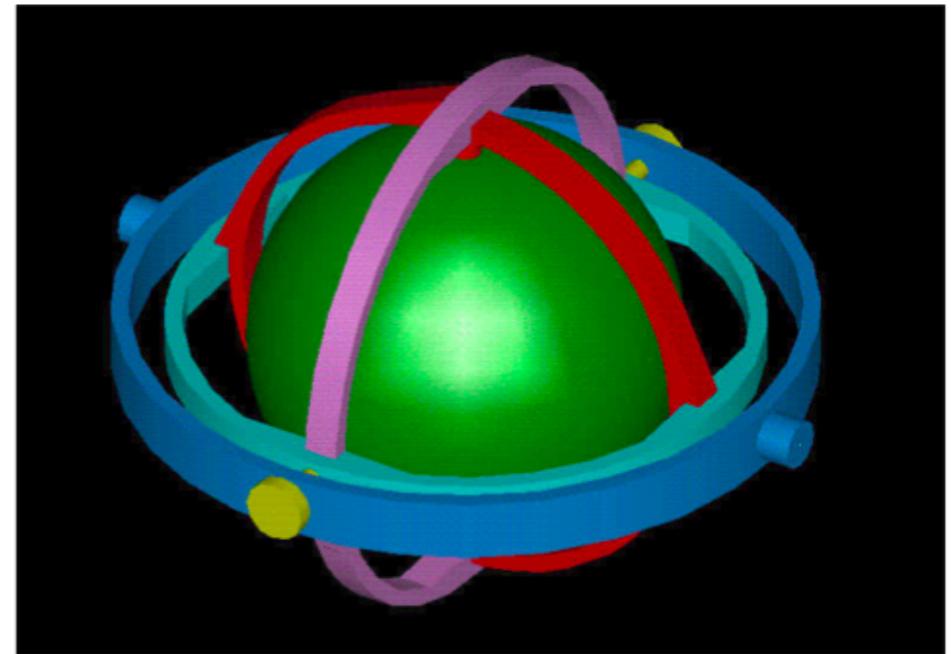
$F_{\hat{\mathbf{a}}}$ is a frame matrix with \mathbf{a} as its first column.

$$R(\mathbf{a}) = R(\hat{\mathbf{a}}, \|\mathbf{a}\|)$$



Parameterizing rotations

- Euler angles
 - Rotate around x, then y, then z
 - Problem: gimbal lock
If two axes coincide, you lose one DOF
- Unit quaternions
 - A 4D representation (like 3D unit vectors for 2D sphere)
 - Good choice for interpolating rotations
- These are first examples of motion control
 - Matrix = deformation
 - Angles/quaternion = animation controls



Problems

- Euler angles
 - gimbal lock (saw this before)
 - some rotations have many representations
- Axis/angle
 - multiple representations for identity rotation
 - even with combined rotation angle, making small changes near 180 degree rotations requires larger changes to parameters
- These resemble the problems with polar coordinates on the sphere
 - as with choosing poles, choosing the reference orientation for an object changes how the representation works

What is a rotation?

- Think of the set of possible orientations of a 3D object
 - you get from one orientation to another by rotating
 - if we agree on some starting orientation, rotations and orientations are pretty much the same thing
- It is a smoothly connected three-dimensional space
 - how can you tell? For any orientation, I can make a small rotation around any axis (pick axis = 2D, pick angle = 1D)
- This set is a subset of linear transformations called $\text{SO}(3)$
 - **O** for orthogonal, **S** for “special” (determinant +1), **3** for 3D

Calculating with rotations

- Representing rotations with numbers requires a function
$$f : \mathbb{R}^n \rightarrow SO(3)$$
- The situation is analogous to representing directions in 3-space
 - there we are dealing with the set S^2 , the two-dimensional sphere (I mean the sphere is a 2D surface)
 - like $SO(3)$ it is very symmetric; no directions are specially distinguished

Warm-up: spherical coordinates

- We can use latitude and longitude to parameterize the 2-sphere (aka. directions in 3D), but with some annoyances
 - the poles are special, and are represented many times
 - if you are at the pole, going East does nothing
 - near the pole you have to change longitude a lot to get anywhere
 - traveling along straight lines in (latitude, longitude) leads to some pretty weird paths on the globe

you are standing one mile from the pole, facing towards it; to get to the point 2 miles ahead of you the map tells you to turn right and walk 3.14 miles along a latitude line...
 - Conclusion: use unit vectors instead

Warm-up: unit vectors

- When we want to represent directions we use unit vectors: points that are literally on the unit sphere in \mathbb{R}^3
 - now no points are special
 - every point has a unique representation
 - equal sized changes in coordinates are equal sized changes in direction
- Down side: one too many coordinates
 - have to maintain normalization
 - but normalize() is a simple and easy operation

Warm-up: interpolating directions

- Interpolating in the space of 3D vectors is well behaved
- Simple computation: interpolate linearly and normalize
 - this is what we do all the time, e.g. with normals for fragment shading
$$\hat{\mathbf{v}}(t) = \text{normalize}((1 - t)\mathbf{v}_0 + t\mathbf{v}_1)$$
 - but for far-apart endpoints the speed is uneven (faster towards the middle)
- For constant speed: spherical linear interpolation
 - build basis $\{\mathbf{v}_0, \mathbf{w}\}$ from \mathbf{v}_0 and \mathbf{v}_1
 - interpolate angle from 0 to θ
 - (slicker way in a few slides)
$$\begin{aligned}\mathbf{w} &= \hat{\mathbf{v}}_1 - (\hat{\mathbf{v}}_0 \cdot \hat{\mathbf{v}}_1)\hat{\mathbf{v}}_0 \\ \hat{\mathbf{w}} &= \mathbf{w}/\|\mathbf{w}\| \\ \theta &= \text{acos}(\hat{\mathbf{v}}_0 \cdot \hat{\mathbf{v}}_1) \\ \hat{\mathbf{v}}(t) &= (\cos t\theta) \hat{\mathbf{v}}_0 + (\sin t\theta) \hat{\mathbf{w}}\end{aligned}$$

Warm-up: rays vs. lines

- The set of directions (unit vectors) describes the set of rays leaving a point
- The set of lines through a point is a bit different
 - no notion of “forward” vs. “backward”
- Would probably still represent using unit vectors
 - but every line has exactly two representations, \mathbf{v} and $-\mathbf{v}$

Quaternions

- A quaternion is a 4-vector

$$q = (w, x, y, z) \in \mathbf{H}$$

- We tend to think of it as a scalar and a 3-vector

$$q = (s, \mathbf{v}) \text{ where } s = w \text{ and } \mathbf{v} = (x, y, z)$$

- alternate notation: $q = s + \mathbf{v}$

- Unit quaternions are unit 4-vectors:

$$w^2 + x^2 + y^2 + z^2 = 1, \quad \text{or} \quad s^2 + \|\mathbf{v}\|^2 = 1$$

- think of s and $\|\mathbf{v}\|$ as sin and cos of an angle: $q = \cos \psi + \hat{\mathbf{v}} \sin \psi$

- They can represent rotations in an axis-angle-like way

- direction of \mathbf{v} tells you the axis of rotation
 - s and $\|\mathbf{v}\|$ tell you the angle of rotation

Quaternions and rotations

There is a natural association between the unit quaternion

$$\cos \psi + \hat{\mathbf{v}} \sin \psi \in S^3 \subset \mathbf{H}$$

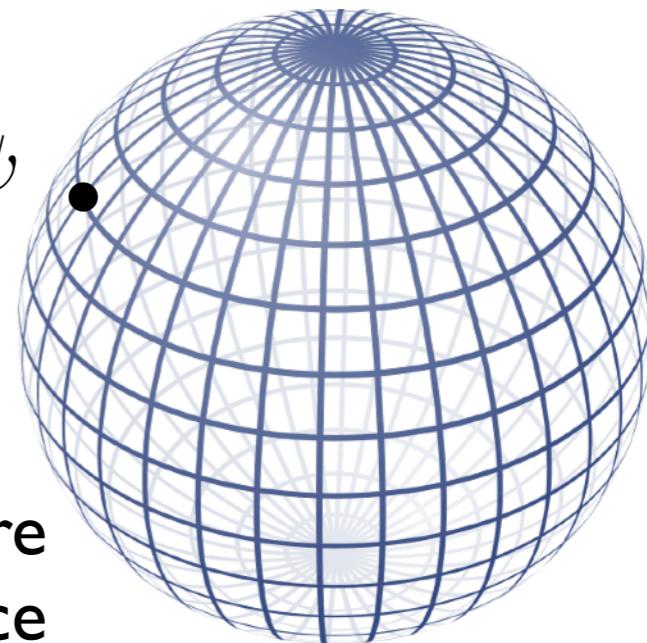
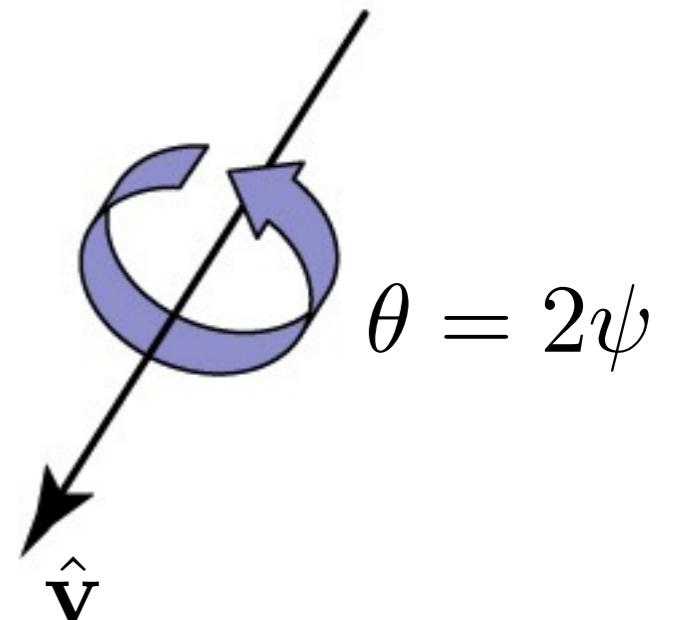
and the 3D axis-angle rotation

$$R_{\hat{\mathbf{v}}}(\theta) \in SO(3)$$

where $\theta = 2\psi$.

$$\cos \psi + \hat{\mathbf{v}} \sin \psi$$

unit 3-sphere
in 4D space



Unit quaternions and axis/angle

- We can write down a parameterization of 3D rotations using unit quaternions (points on the 3-sphere)

$$f : S^3 \subset \mathbf{H} \rightarrow SO(3)$$

$$: \cos \psi + \hat{\mathbf{v}} \sin \psi \mapsto R_{\hat{\mathbf{v}}} (2\psi)$$

$$: (w, x, y, z) \mapsto \begin{bmatrix} w^2 + x^2 - y^2 - z^2 & 2(xy - wz) & 2(xz + wy) \\ 2(xy + wz) & w^2 - x^2 + y^2 - z^2 & 2(yz - wx) \\ 2(xz - wy) & 2(yz + wx) & w^2 - x^2 - y^2 + z^2 \end{bmatrix}$$

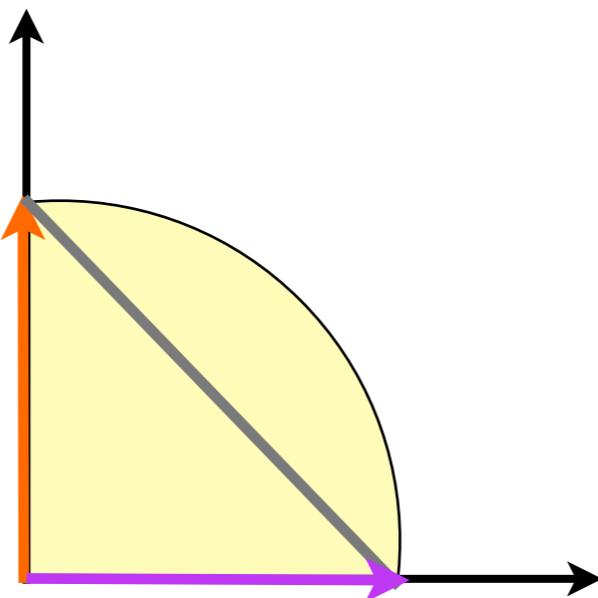
- This mapping is wonderfully uniform:
 - is exactly 2-to-1 everywhere
 - has constant speed in all directions
 - has constant Jacobian (does not distort “volume”)
 - maps shortest paths to shortest paths
 - and... it comes with a multiplication operation (not mentioned today)

Why Quaternions?

- Fast, few operations, not redundant
- Numerically stable for incremental changes
- Composes rotations nicely
- Convert to matrices at the end
- Biggest reason: spherical interpolation

Interpolating between quaternions

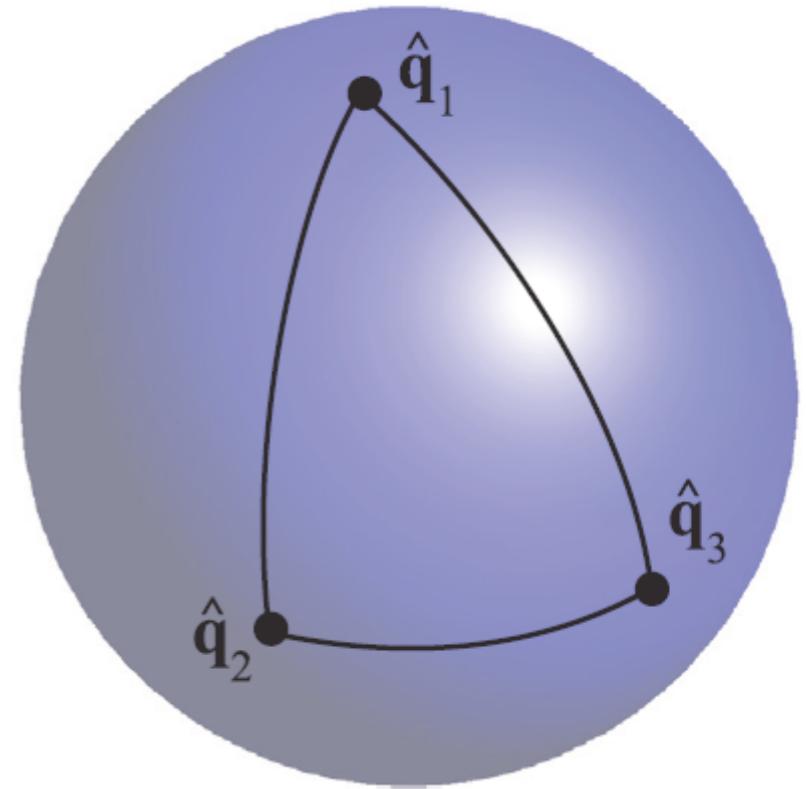
- Why not linear interpolation?
 - Need to be normalized
 - Does not have constant rate of rotation



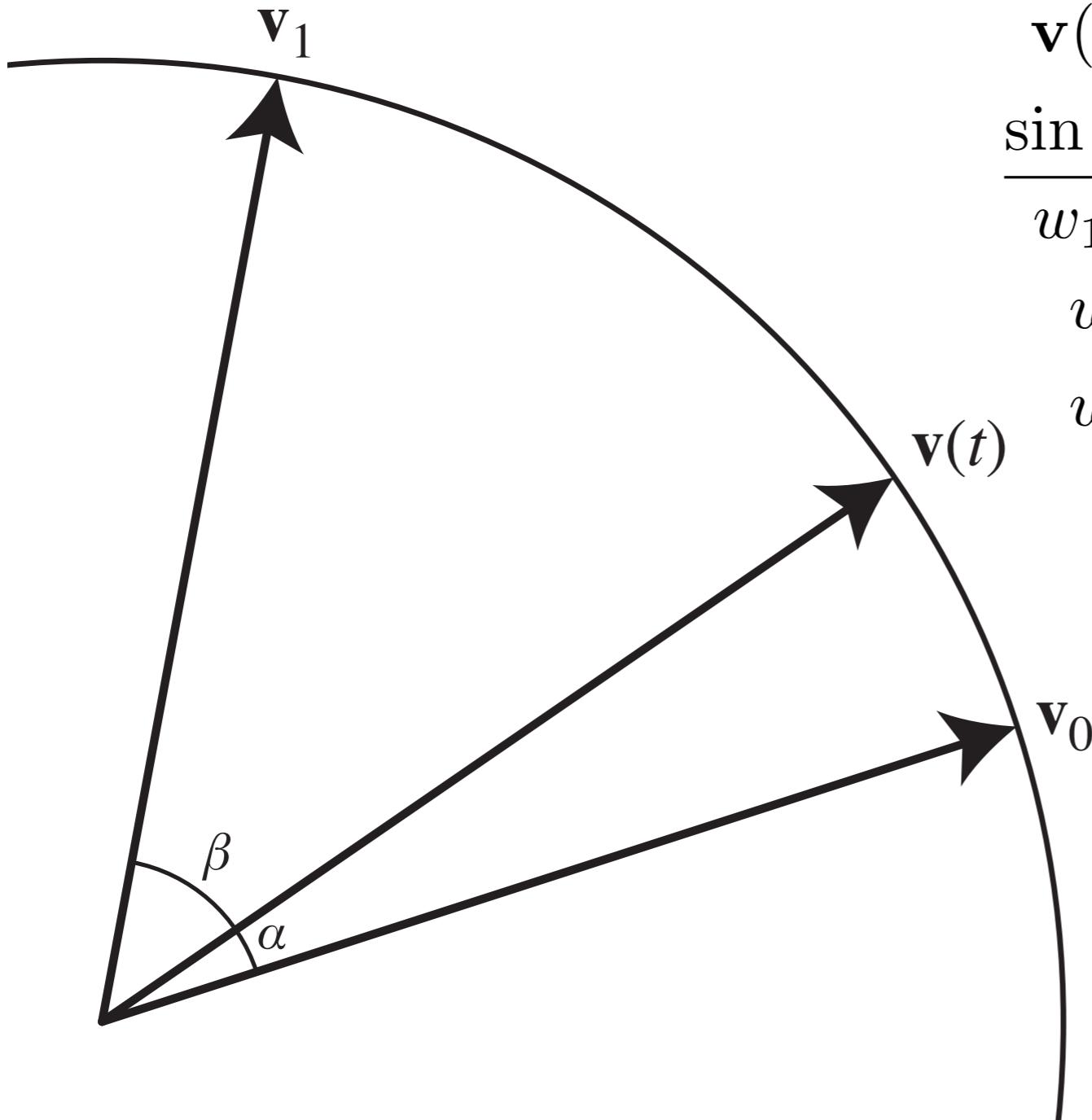
$$\frac{(1 - \alpha)x + \alpha y}{\|(1 - \alpha)x + \alpha y\|}$$

Spherical Linear Interpolation

- Intuitive interpolation between different orientations
 - Nicely represented through quaternions
 - Useful for animation
 - Given two quaternions, interpolate between them
 - Shortest path between two points on sphere
Geodesic, on Great Circle



Spherical linear interpolation (“slerp”)



$$\alpha + \beta = \psi$$

$$\mathbf{v}(t) = w_0 \mathbf{v}_0 + w_1 \mathbf{v}_1$$

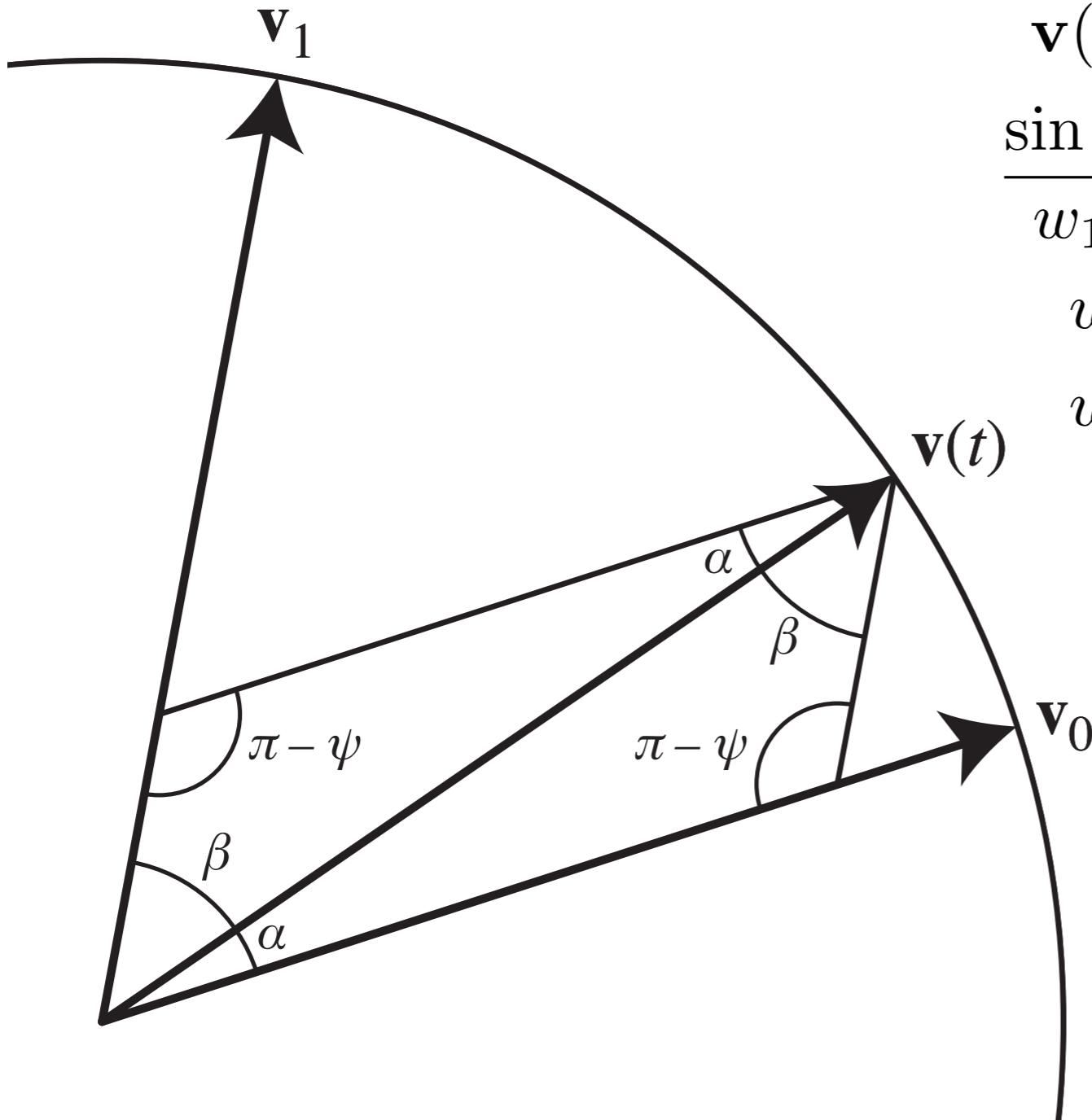
$$\frac{\sin \alpha}{w_1} = \frac{\sin \beta}{w_0} = \frac{\sin(\pi - \psi)}{1} = \sin \psi$$

$$w_0 = \sin \beta / \sin \psi$$

$$w_1 = \sin \alpha / \sin \psi$$

$$\psi = \cos^{-1}(\mathbf{v}_0 \cdot \mathbf{v}_1)$$

Spherical linear interpolation (“slerp”)



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$$\mathbf{v}(t) = w_0 \mathbf{v}_0 + w_1 \mathbf{v}_1$$

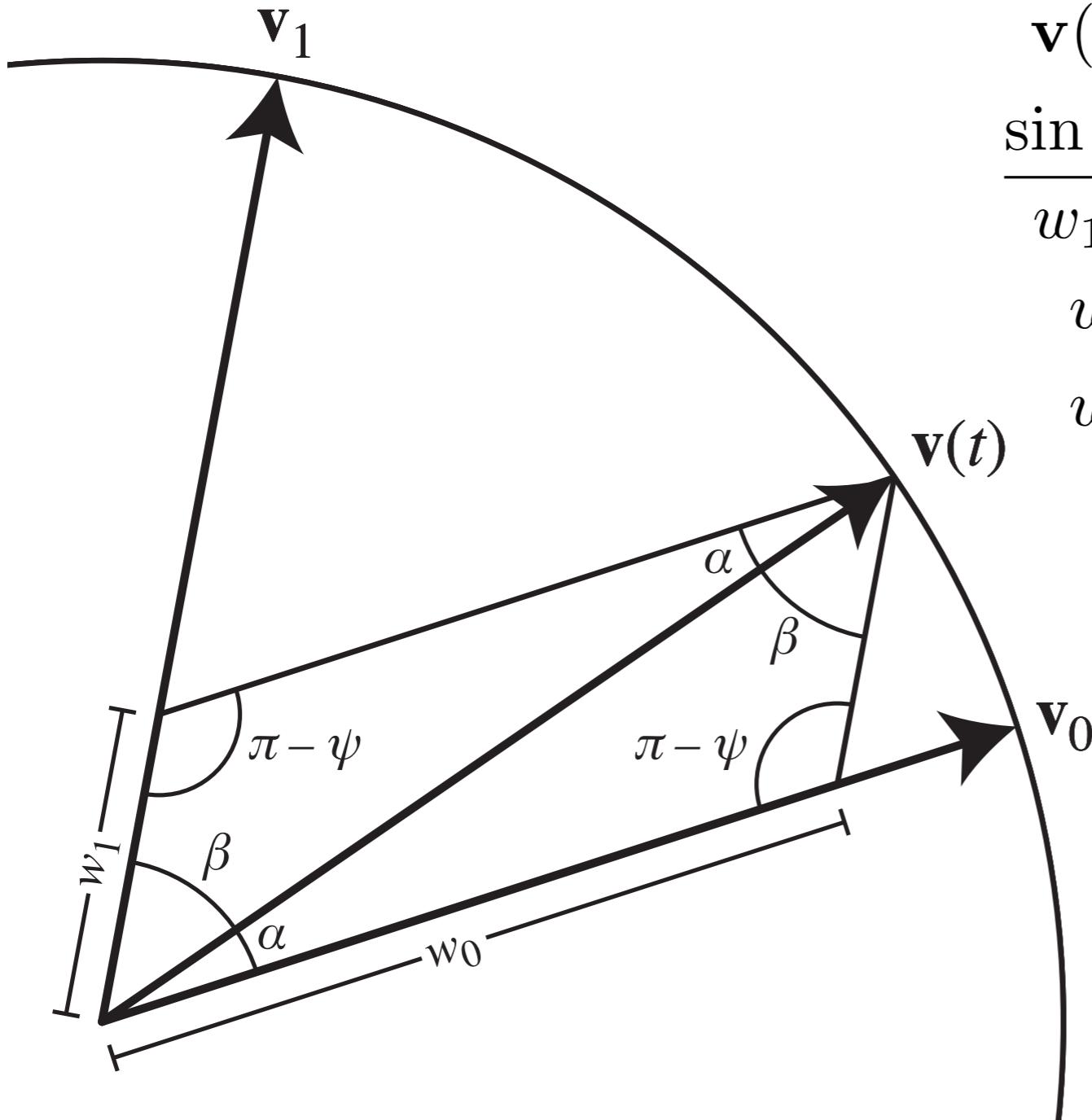
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Quaternion Interpolation

- Spherical linear interpolation naturally works in any dimension
- Traverses a great arc on the sphere of unit quaternions
 - Uniform angular rotation velocity about a fixed axis

$$\psi = \cos^{-1}(q_0 \cdot q_1)$$

$$q(t) = \frac{q_0 \sin(1-t)\psi + q_1 \sin t\psi}{\sin \psi}$$

Practical issues

- When angle gets close to zero, estimation of Ψ is inaccurate
 - slerp naturally approaches linear interpolation for small Ψ
 - so switch to linear interpolation when $q_0 \approx q_1$.
- q is same rotation as $-q$
 - if $q_0 \cdot q_1 > 0$, slerp between them
 - else, slerp between q_0 and $-q_1$

Hierarchies and articulated figures

- Luxo as an example
 - small number of animation controls control many transformations
 - constraint: the joints hold together
- Some operations are tricky with hierarchies
 - how to ensure lampshade touches ball?
- In mechanics, the relationship between DOFs and 3D pose is *kinematics*
- Robotics as source of math. Methods
 - robots are transformation hierarchies
 - forward kinematics
 - inverse kinematics



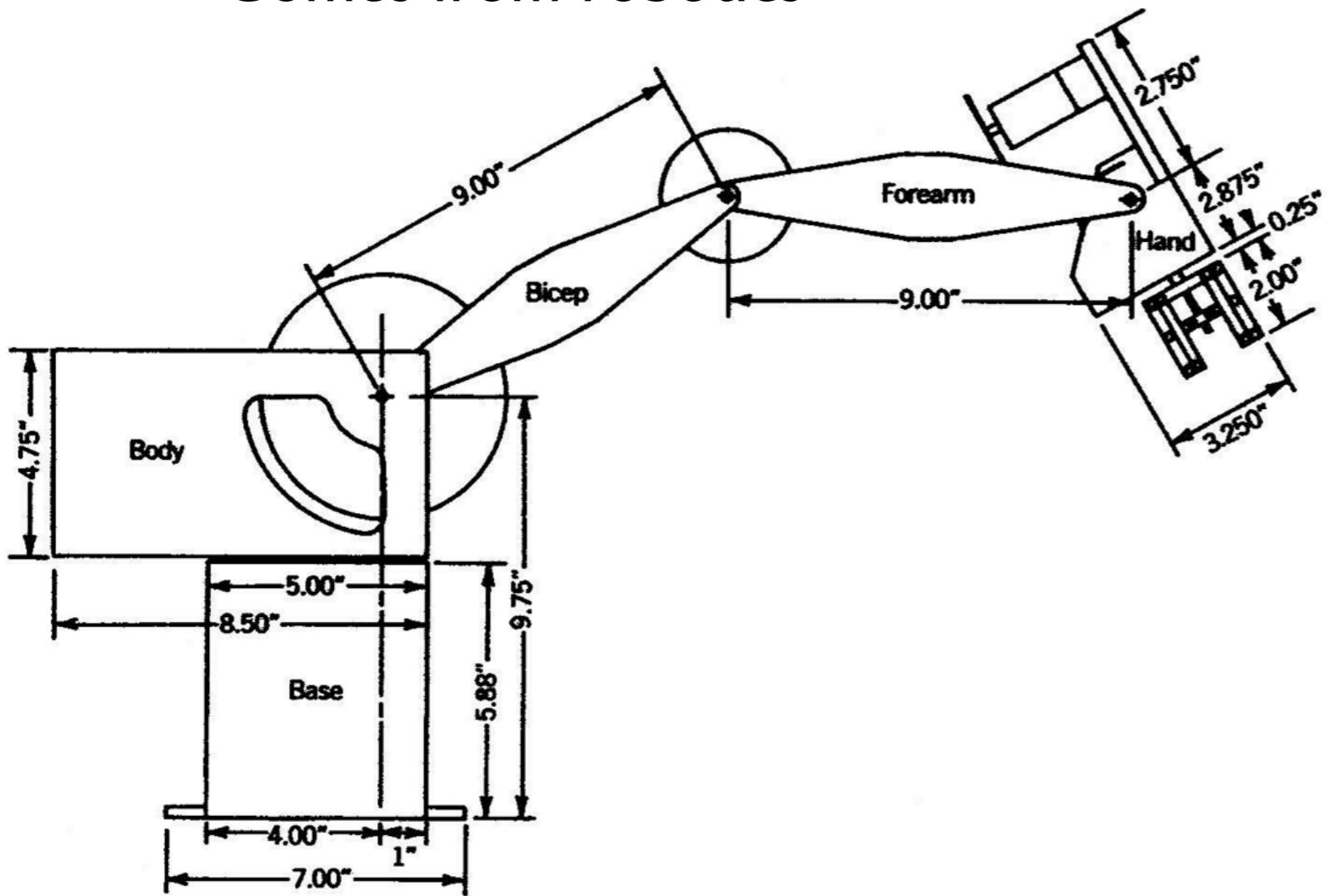
Forward Kinematics



Inverse Kinematics

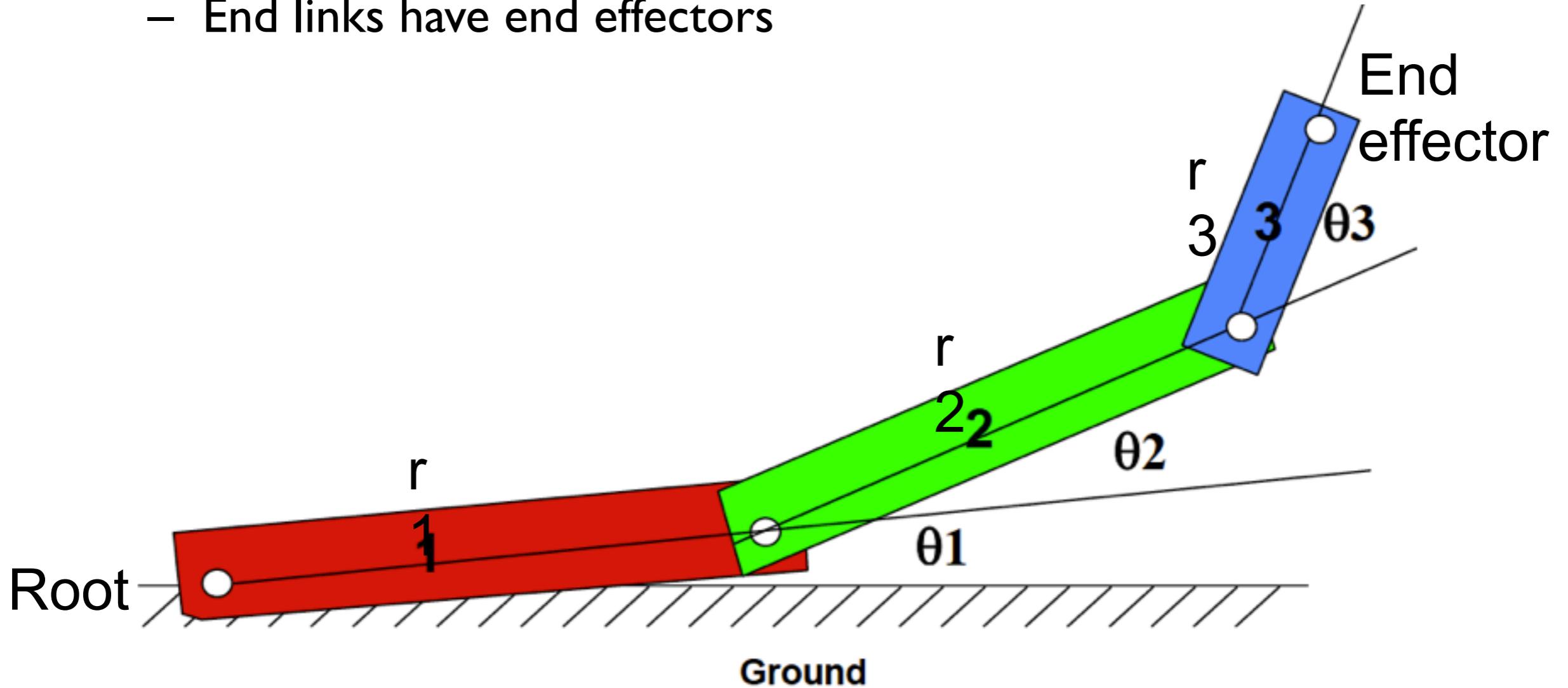
Forward Kinematics

- Articulated body
 - Hierarchical transforms
 - Comes from robotics

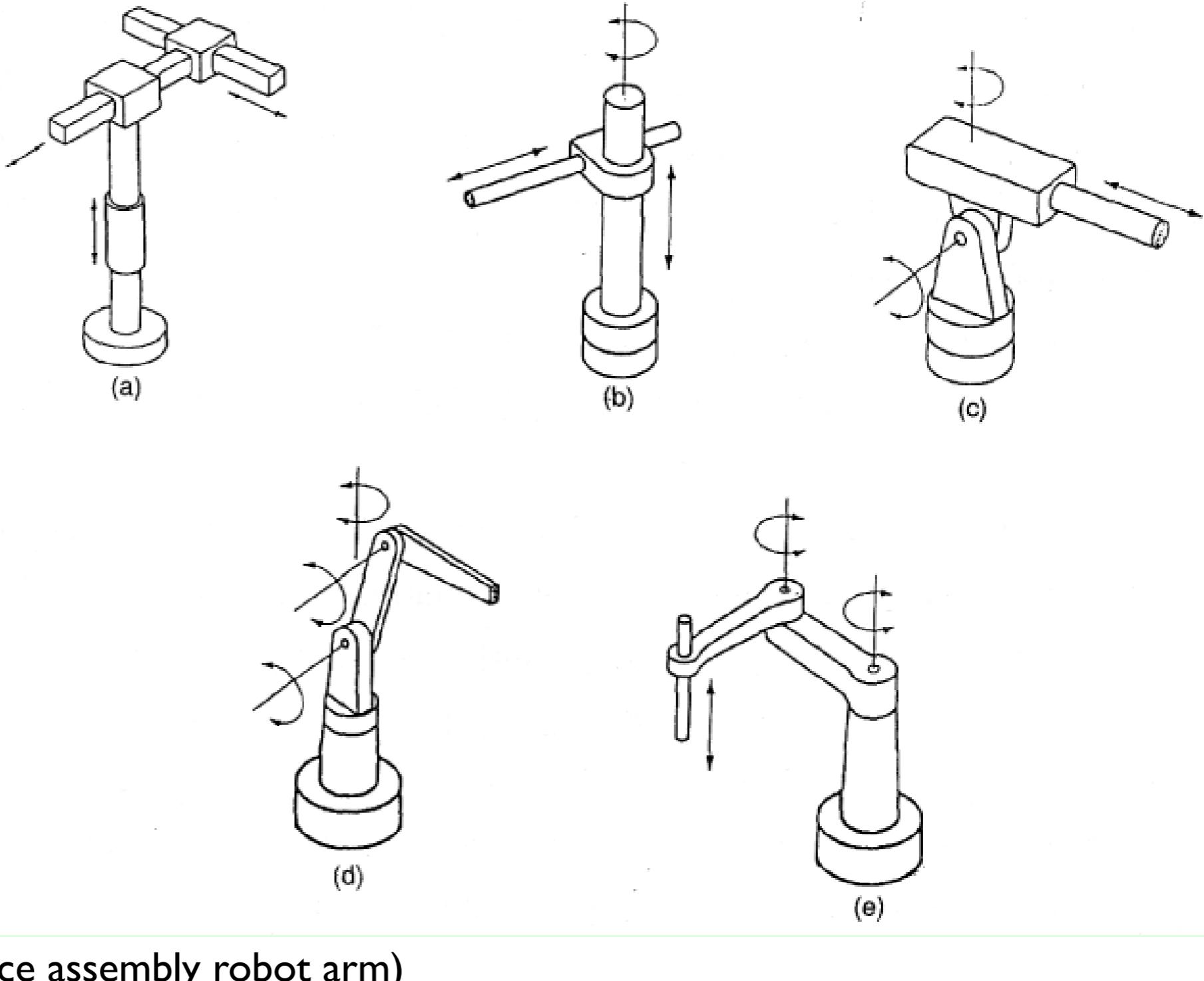


Rigid Links and Joint Structure

- Links connected by joints
 - Joints are purely rotational (single DOF)
 - Links form a tree (no loops)
 - End links have end effectors



Articulation in robotics



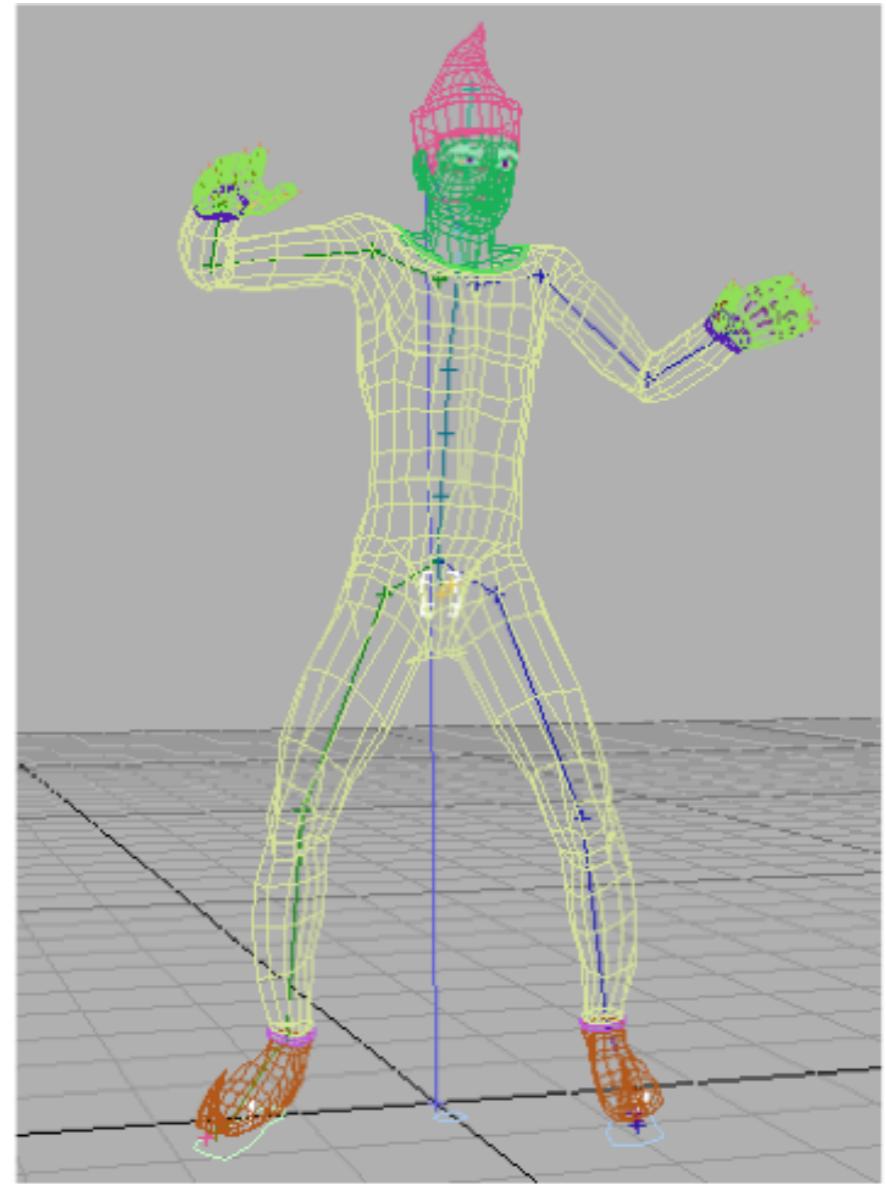
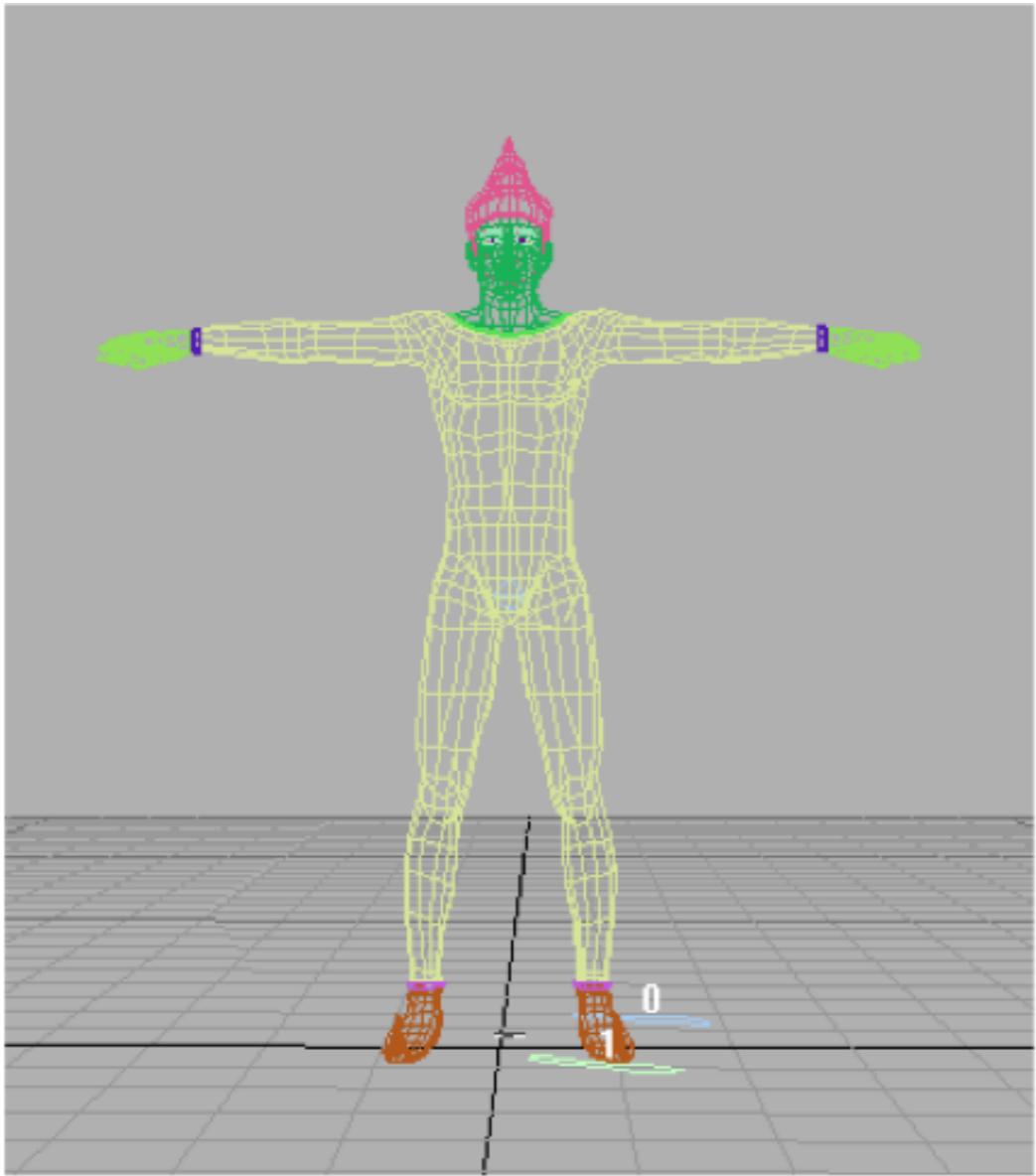
- a. rectangular or cartesian
- b. cylindrical or post-type
- c. spherical or polar
- d. joint-arm or articulated
- e. SCARA (selective compliance assembly robot arm)

Basic surface deformation methods

- Mesh skinning: deform a mesh based on an underlying skeleton
- Blend shapes: make a mesh by combining several meshes
- Both use simple linear algebra
 - Easy to implement—first thing to try
 - Fast to run—used in games
- The simplest tools in the offline animation toolbox

Mesh skinning

- A simple way to deform a surface to follow a skeleton



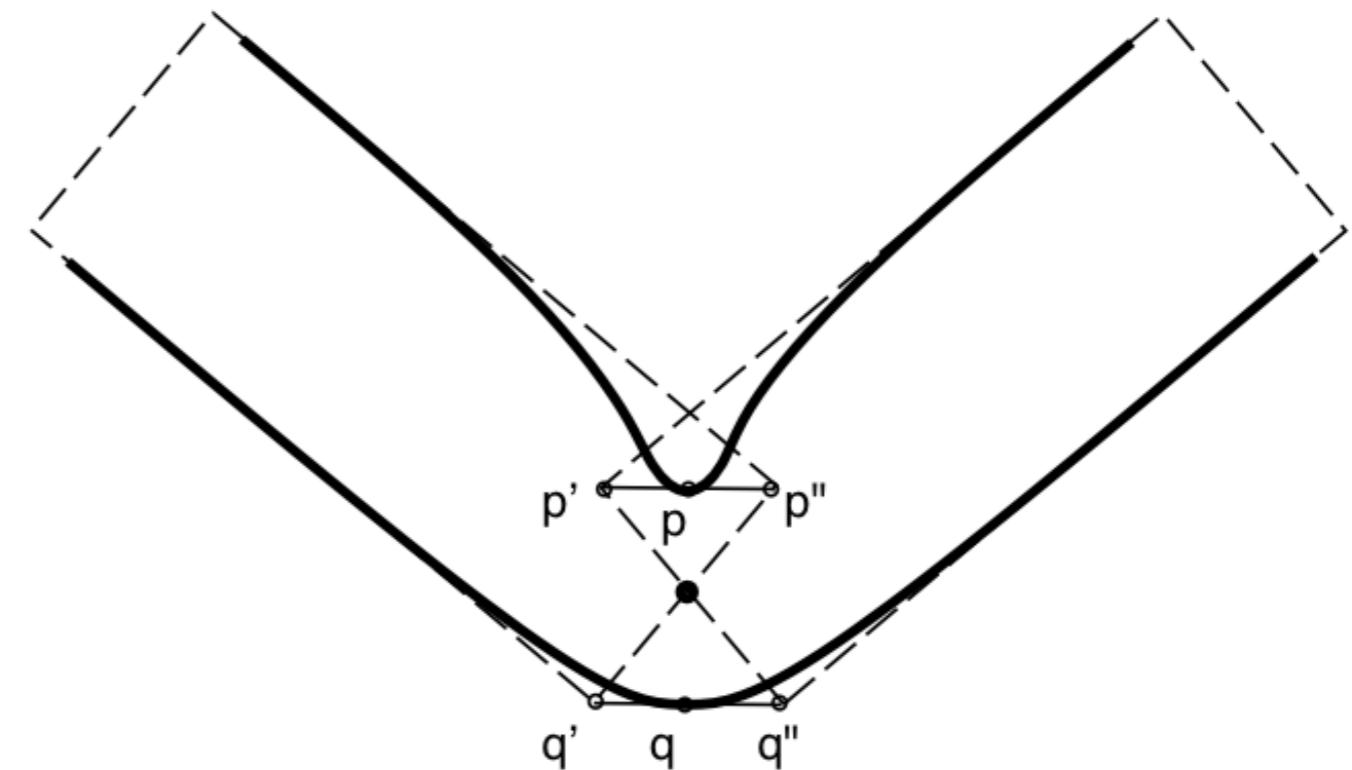
Mesh skinning math: setup

- Surface has control points \mathbf{p}_i
 - Triangle vertices, spline control points, subdiv base vertices
- Each bone has a transformation matrix M_j
 - Normally a rigid motion
- Every point–bone pair has a weight w_{ij}
 - In practice only nonzero for small # of nearby bones
 - The weights are provided by the user

Mesh skinning math

- Deformed position of a point is a weighted sum
 - of the positions determined by each bone's transform alone
 - weighted by that vertex's weight for that bone

$$\mathbf{p}'_i = \sum_j w_{ij} M_j \mathbf{p}_i$$



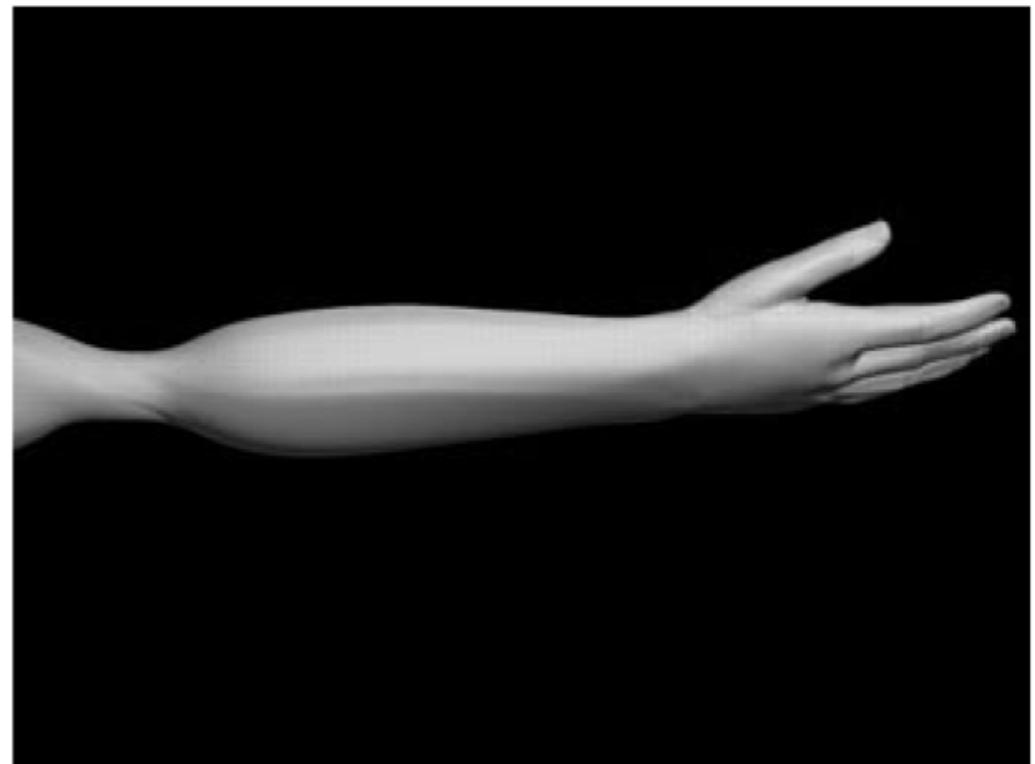
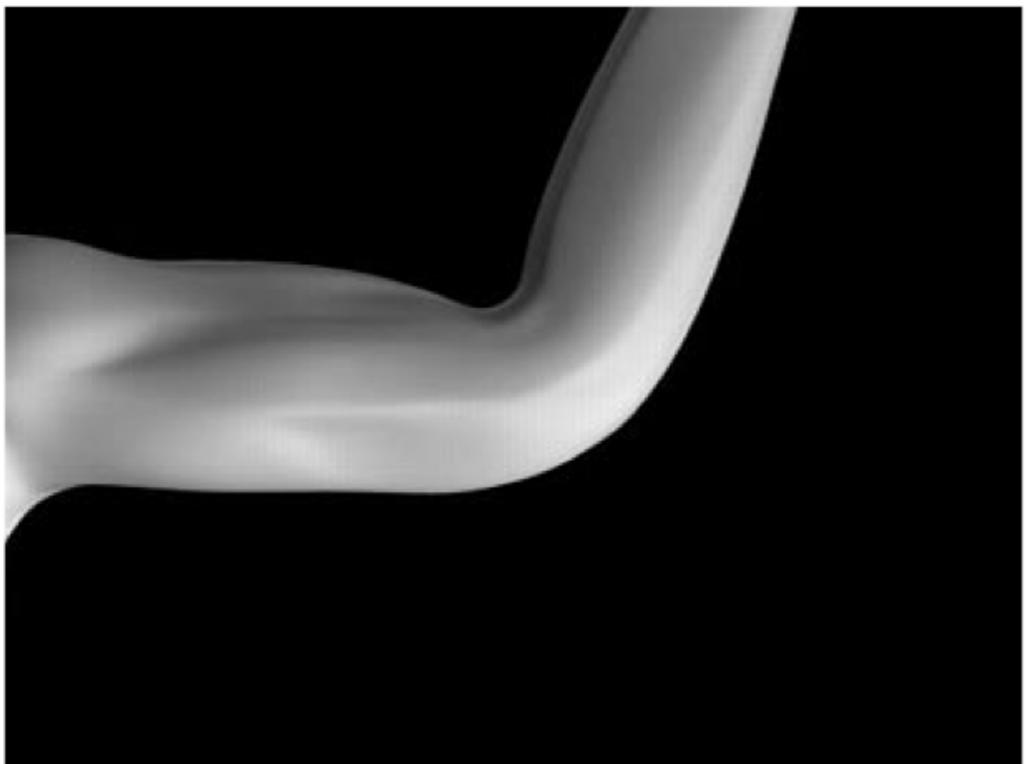
[Lewis et al. SIGGRAPH 2000]

Mesh skinning

- Simple and fast to compute
 - Can even compute in the vertex stage of a graphics pipeline
- Used heavily in games
- One piece of the toolbox for offline animation
 - Many other deformers also available

Mesh skinning: classic problems

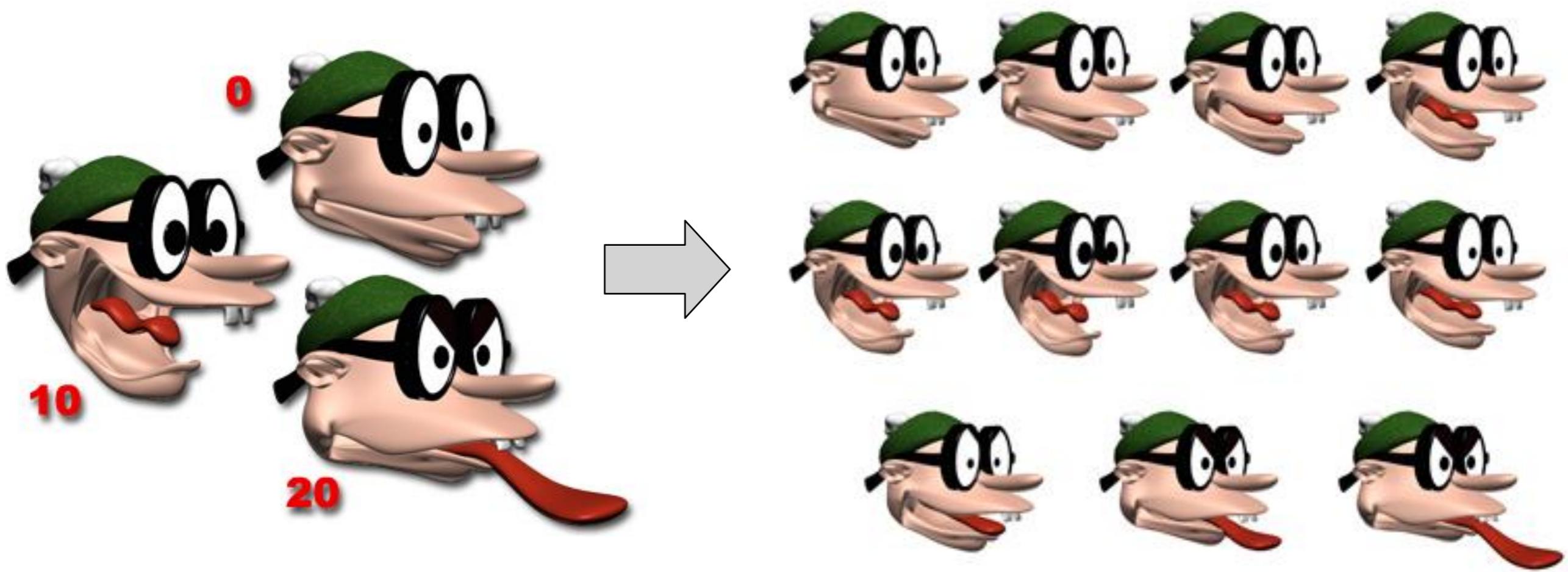
- Surface collapses on the inside of bends and in the presence of strong twists
 - Average of two rotations is not a rotation!
 - Add more bones to keep adjacent bones from being too different, or change the blending rules.



[Lewis et al. SIGGRAPH 2000]

Blend shapes

- Another very simple surface control scheme
- Based on interpolating among several key poses
 - Aka. blend shapes or morph targets



[3D Studio Max example]

Blend shapes math

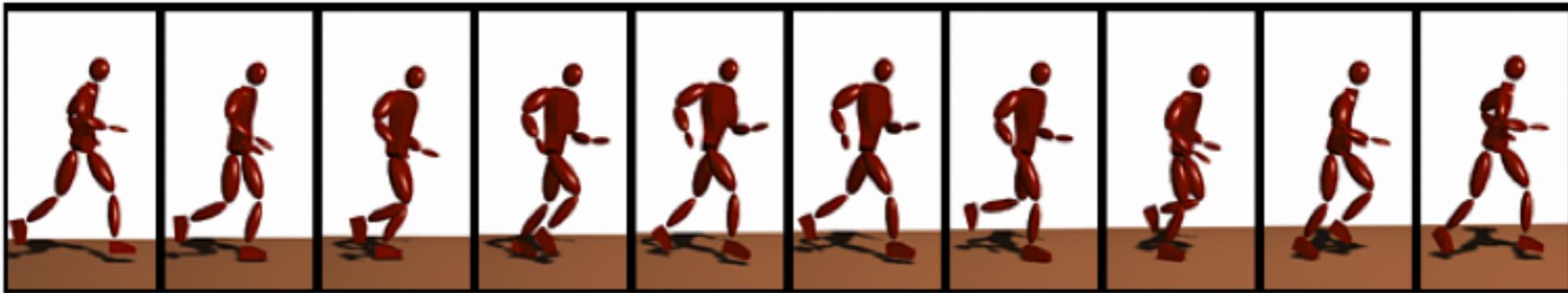
- Simple setup
 - User provides key shapes—that is, a position for every control point in every shape: \mathbf{p}_{ij} for point i , shape j
 - Per frame: user provides a weight w_j for each key shape
Must sum to 1.0

- Computation of deformed shape

$$\mathbf{p}'_i = \sum_j w_j \mathbf{p}_{ij}$$

- Works well for relatively small motions
 - Often used for facial animation
 - Runs in real time; popular for games

Motion capture



- A method for creating complex motion quickly: measure it from the real world

[thanks to Zoran Popović for many visuals]

Motion capture in movies



[Final Fantasy]

Motion capture in movies



[*The Two Towers* | New Line Productions]

Motion capture in games



Magnetic motion capture

- Tethered
- Nearby metal objects cause distortions
- Low freq. (60Hz)



Mechanical motion capture

- Measures joint angles directly
- Works in any environment
- Restricts motion

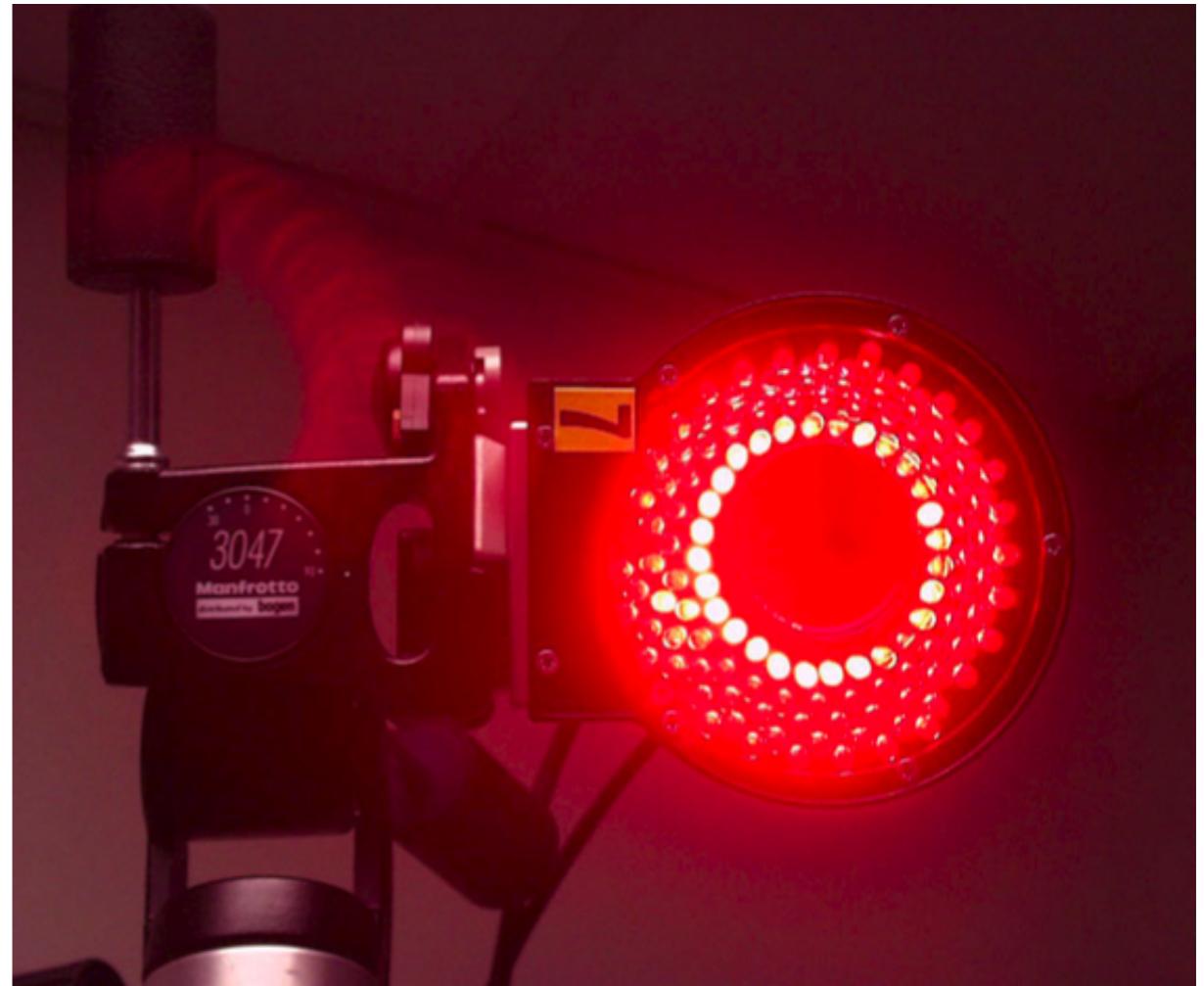


Optical motion capture

- Passive markers on subject



Retroreflective markers



Cameras with IR illuminators

- Markers observed by cameras
 - Positions via triangulation

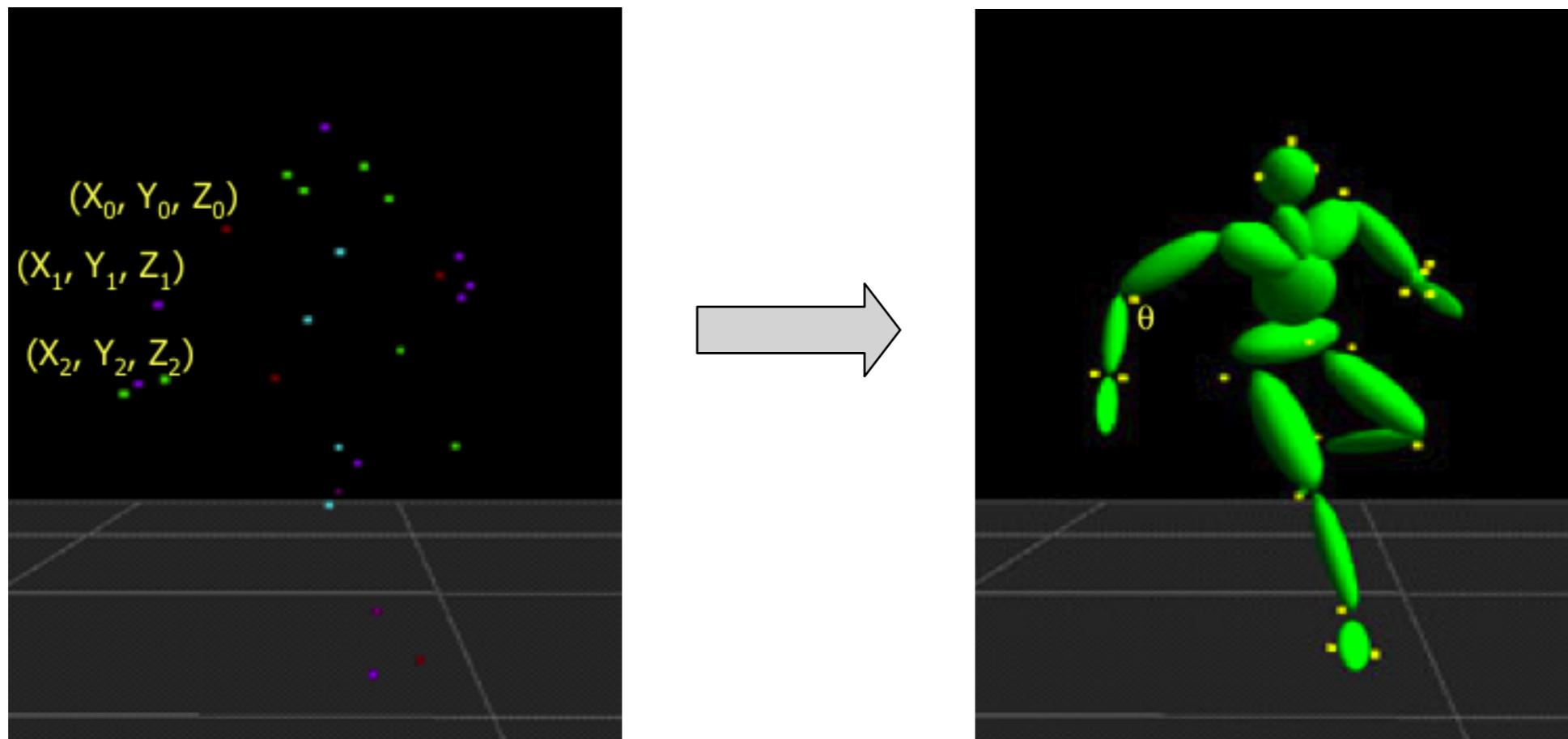
Optical motion capture

- 8 or more cameras
- Restricted volume
- High frequency (240Hz)
- Occlusions are troublesome



From marker data to usable motion

- Motion capture system gives inconvenient raw data
 - Optical is “least information” case: accurate position but:
Which marker is which?
Where are the markers relative to the skeleton?



Motion capture data processing

- Marker identification: which marker is which
 - Start with standard rest pose
 - Track forward through time (but watch for markers dropping out due to occlusion!)
- Calibration: match skeleton, find offsets to markers
 - Use a short sequence that exercises all DOFs of the subject
 - A nonlinear minimization problem
- Computing joint angles: explain data using skeleton DOFs
 - A inverse kinematics problem per frame!

Motion capture in context

- Mocap data is very realistic
 - Timing matches performance exactly
 - Dimensions are exact
- But it is not enough for good character animation
 - Too few DOFs
 - Noise, errors from nonrigid marker mounting
 - Contains no exaggeration
 - Only applies to human-shaped characters
- Therefore mocap data is generally a starting point for skilled animators to create the final product