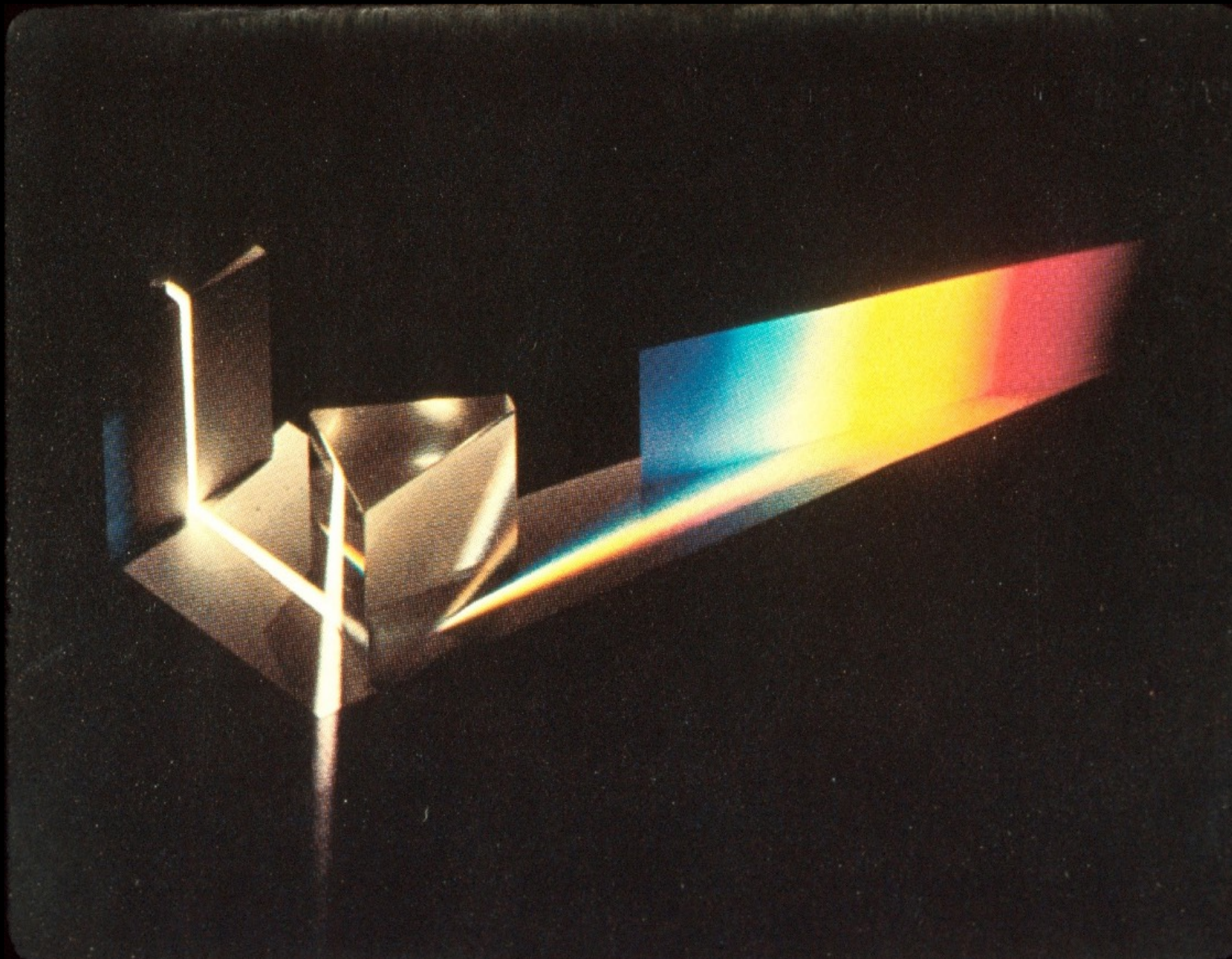
Color Science

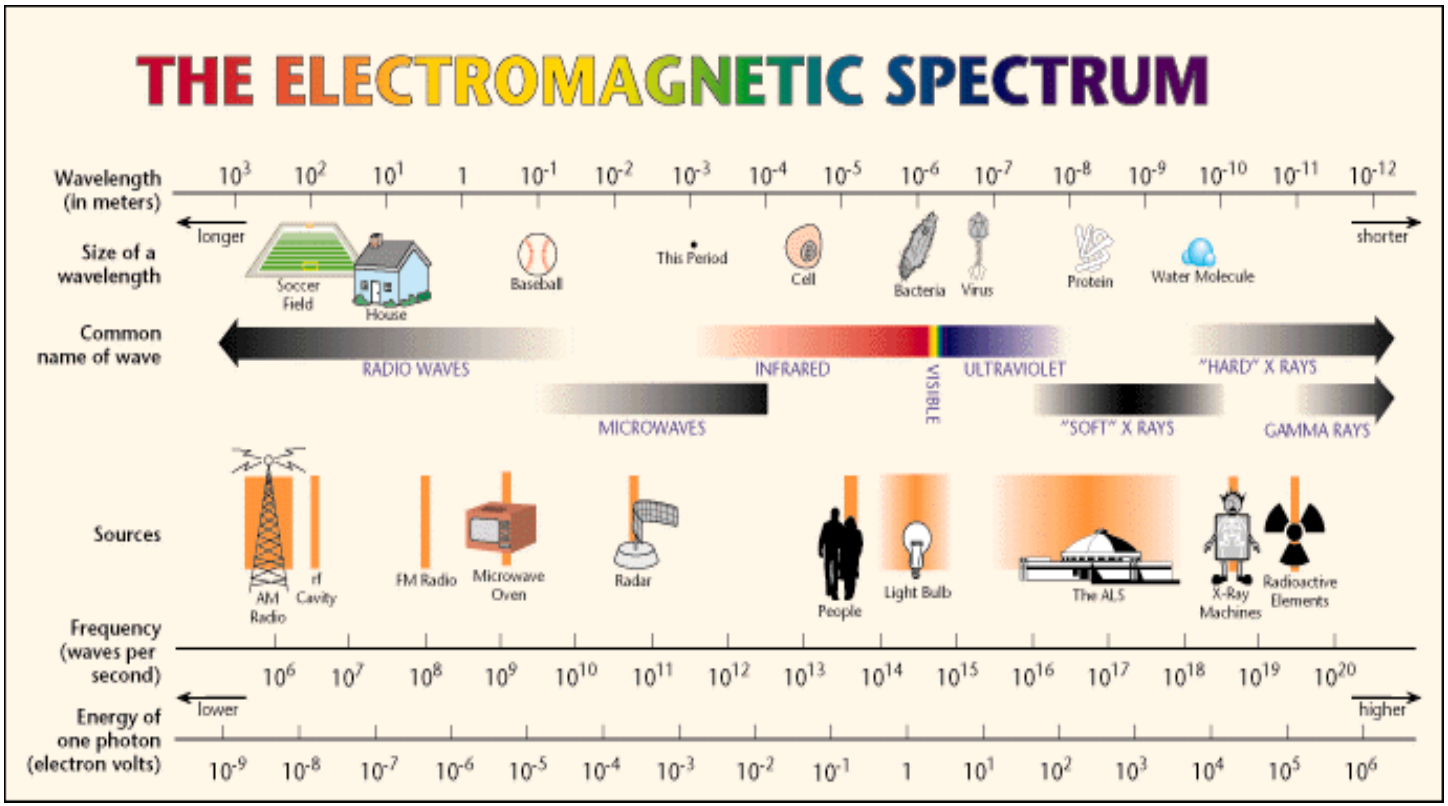
CS 4620 Lecture 24



[source unknown]

What light is

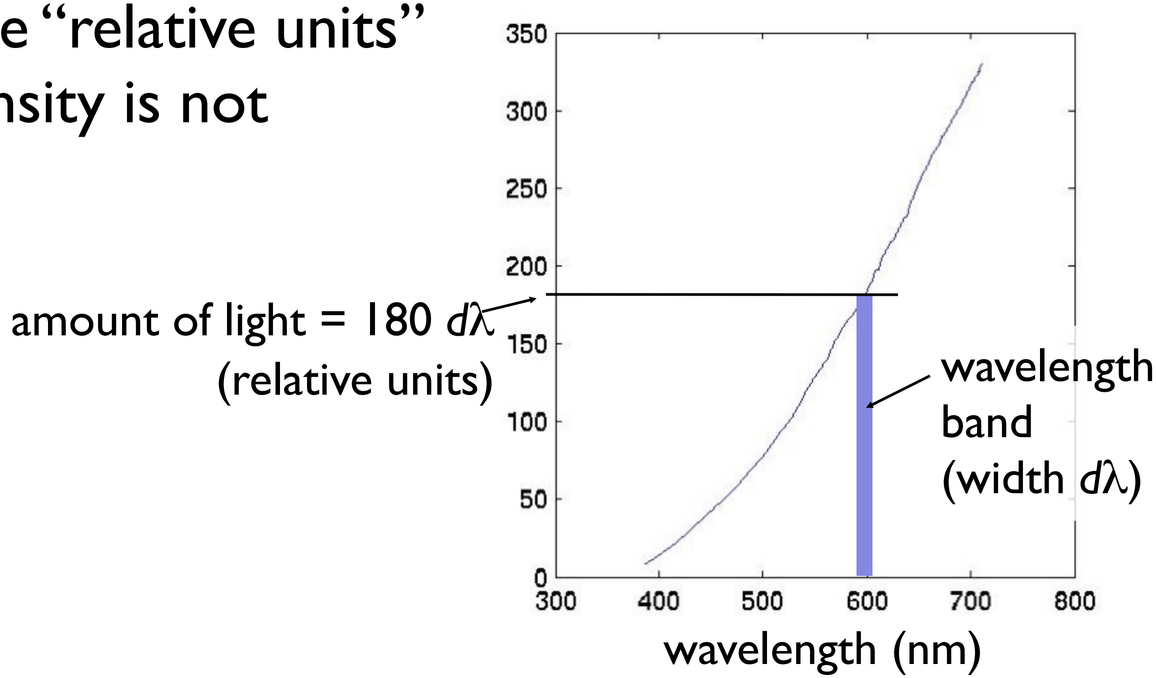
- Light is electromagnetic radiation
  - exists as oscillations of different frequency (or, wavelength)



[Lawrence Berkeley Lab / MicroWorlds]

Measuring light

- Salient property is the *spectral power distribution* (SPD)
  - the amount of light present at each wavelength
  - units: Watts per nanometer (tells you how much power you'll find in a narrow range of wavelengths)
  - for color, often use "relative units" when overall intensity is not important



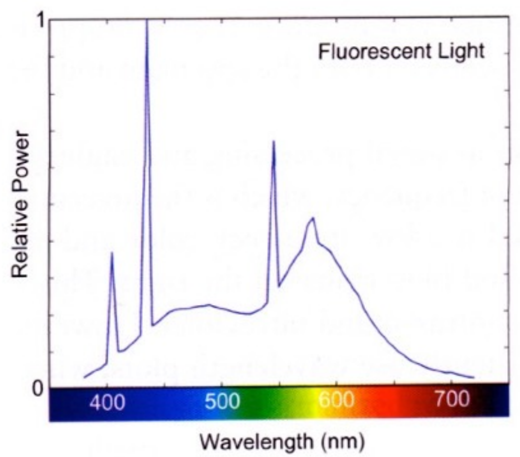
What color is

- Colors are the sensations that arise from light energy of different wavelengths
  - we are sensitive from about 380 to 760 nm—one "octave"
- Color is a phenomenon of human perception; it is **not** a universal property of light
- Roughly speaking, things appear "colored" when they depend on wavelength and "gray" when they do not.

The problem of color science

- Build a model for human color perception
- That is, map a *Physical light description* to a *Perceptual color sensation*

[Stone 2003]



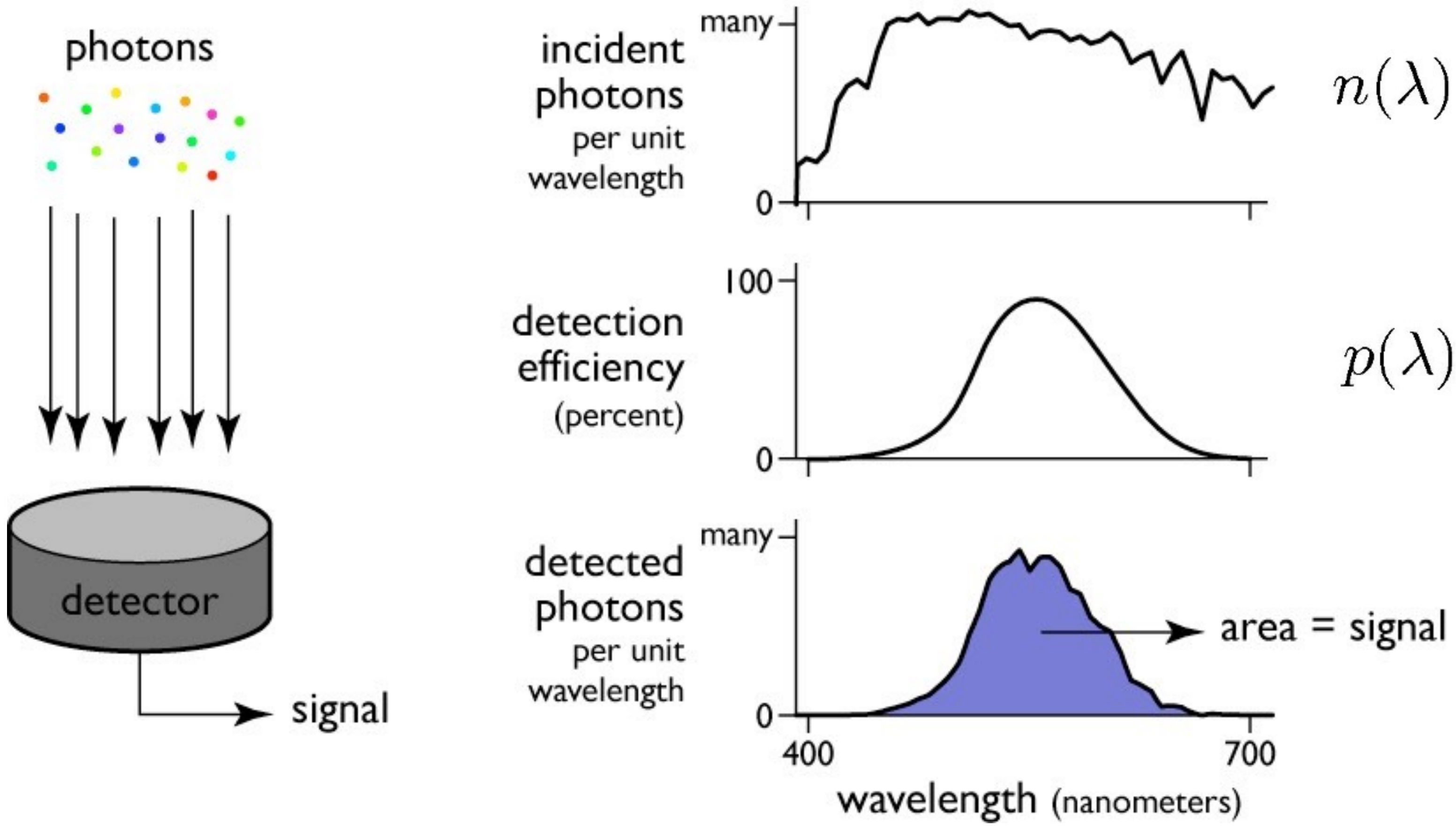
→ ?

Physical *Perceptual*

A simple light detector

- Produces a scalar value (a number) when photons land on it
  - this value depends strictly on the number of photons detected
  - each photon has a probability of being detected that depends on the wavelength
  - there is no way to tell the difference between signals caused by light of different wavelengths: there is just a number
- This model works for many detectors:
  - based on semiconductors (such as in a digital camera)
  - based on visual photopigments (such as in human eyes)

A simple light detector



$$X = \int n(\lambda) p(\lambda)\, d\lambda$$

Light detection math

- Same math carries over to power distributions
  - spectrum entering the detector has its spectral power distribution (SPD), $s(\lambda)$
  - detector has its *spectral sensitivity* or *spectral response*, $r(\lambda)$

$$\underbrace{X}_{\text{measured signal}} = \int \underbrace{s(\lambda)}_{\text{input spectrum}}\underbrace{r(\lambda)}_{\text{detector's sensitivity}}\, d\lambda$$

Light detection math

- If we think of s and r as vectors, this operation is a dot product (aka inner product) or $X = s \cdot r$
  - in fact, the computation is done exactly this way, using sampled representations of the spectra.
    - let λ_i be regularly spaced sample points $\Delta\lambda$ apart; then:

$$X = \int s(\lambda) r(\lambda)\, d\lambda$$

$$\tilde{s}[i] = s(\lambda_i); \tilde{r}[i] = r(\lambda_i)$$

$$\int s(\lambda) r(\lambda)\, d\lambda \approx \sum_i \tilde{s}[i] \tilde{r}[i]\, \Delta\lambda$$

    - this sum is very clearly a dot product

Human observation

- Human eye observes electro-magnetic wavelengths
  - Humans ‘see’ different spectra as different colors
  - Color is a phenomenon of human perception; it is not a universal property of light

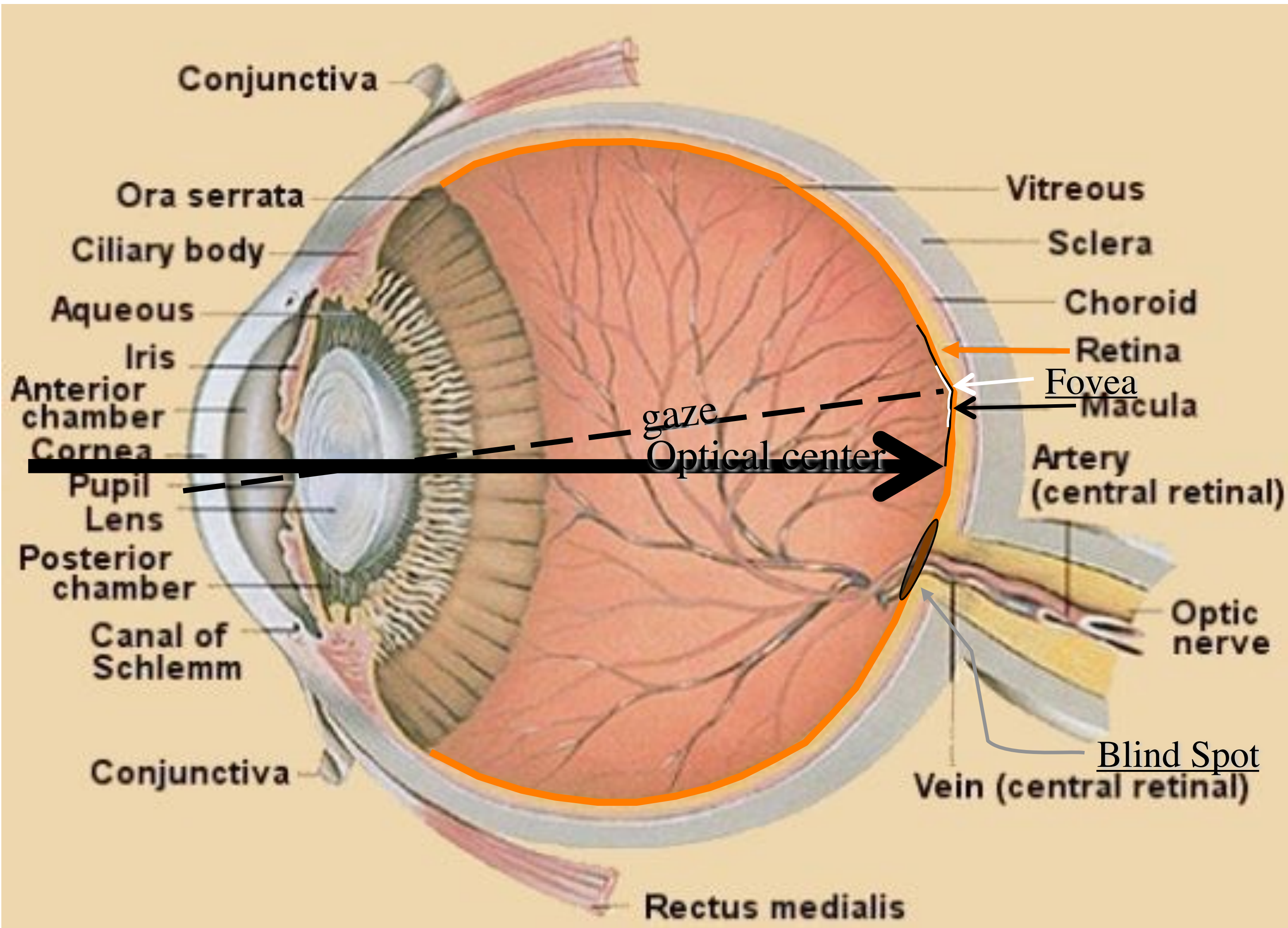
- Other animals observe other wavelengths
  - Bees: 340 – 540 nm
    - (they see no red, but can see ultra-violet)

Insects and color

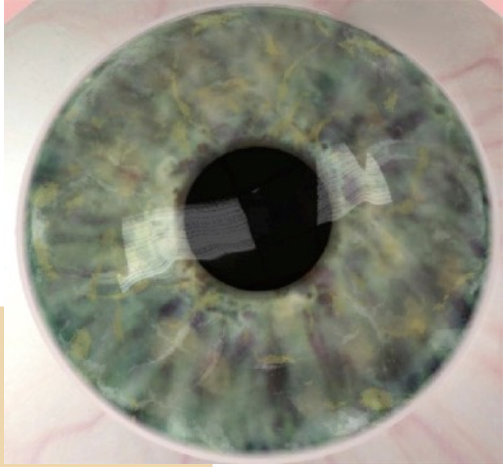
‘Human’

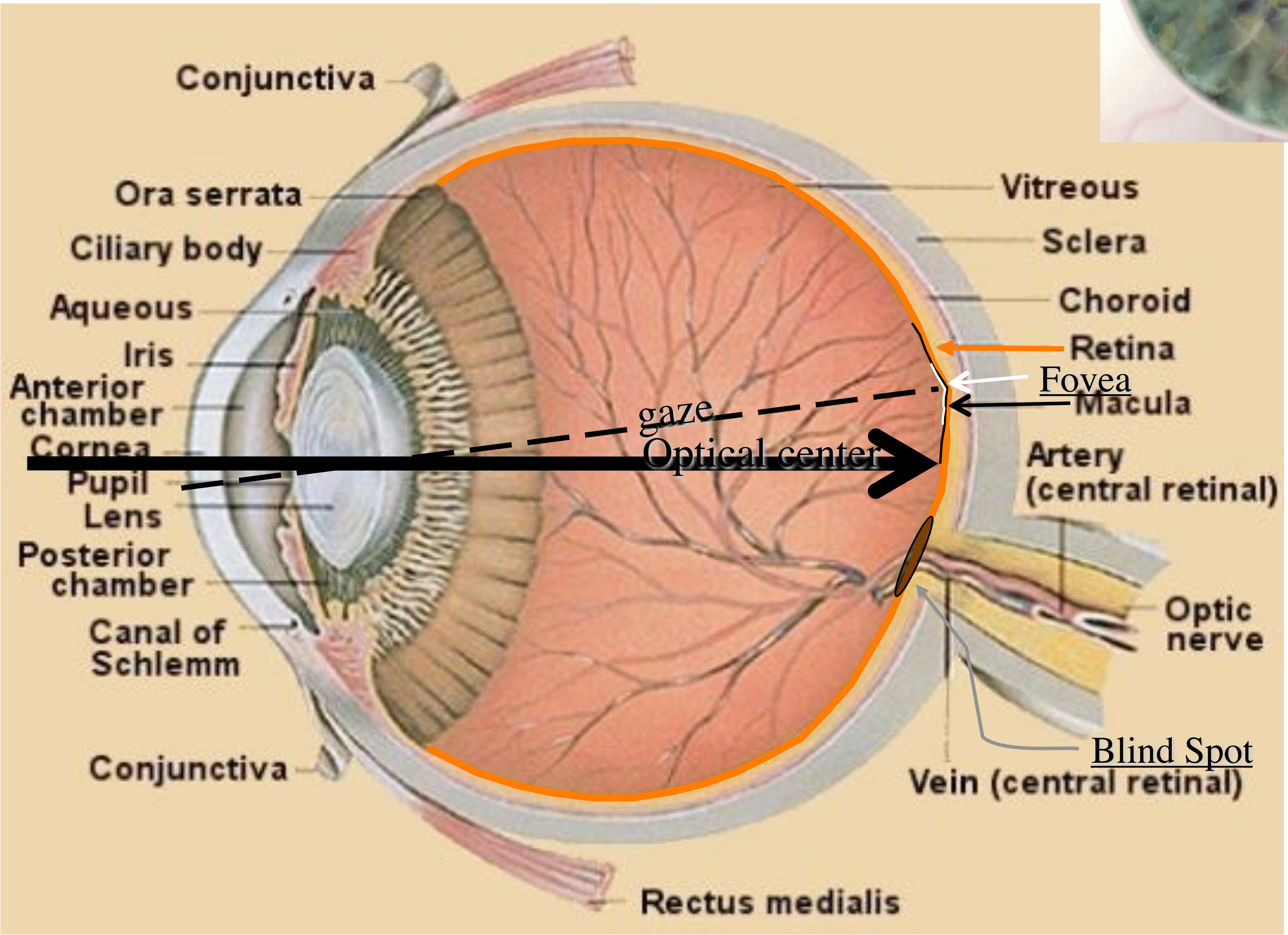
‘Honey Bee’

Human eye


Conjunctiva
Ora serrata
Ciliary body
Aqueous
Iris
Anterior chamber
Cornea
Pupil
Lens
Posterior chamber
Canal of Schlemm
Conjunctiva
Vitreous
Sclera
Choroid
Retina
Fovea
Macula
gaze
Optical center
Artery (central retinal)
Optic nerve
Blind Spot
Vein (central retinal)
Rectus medialis

Human eye

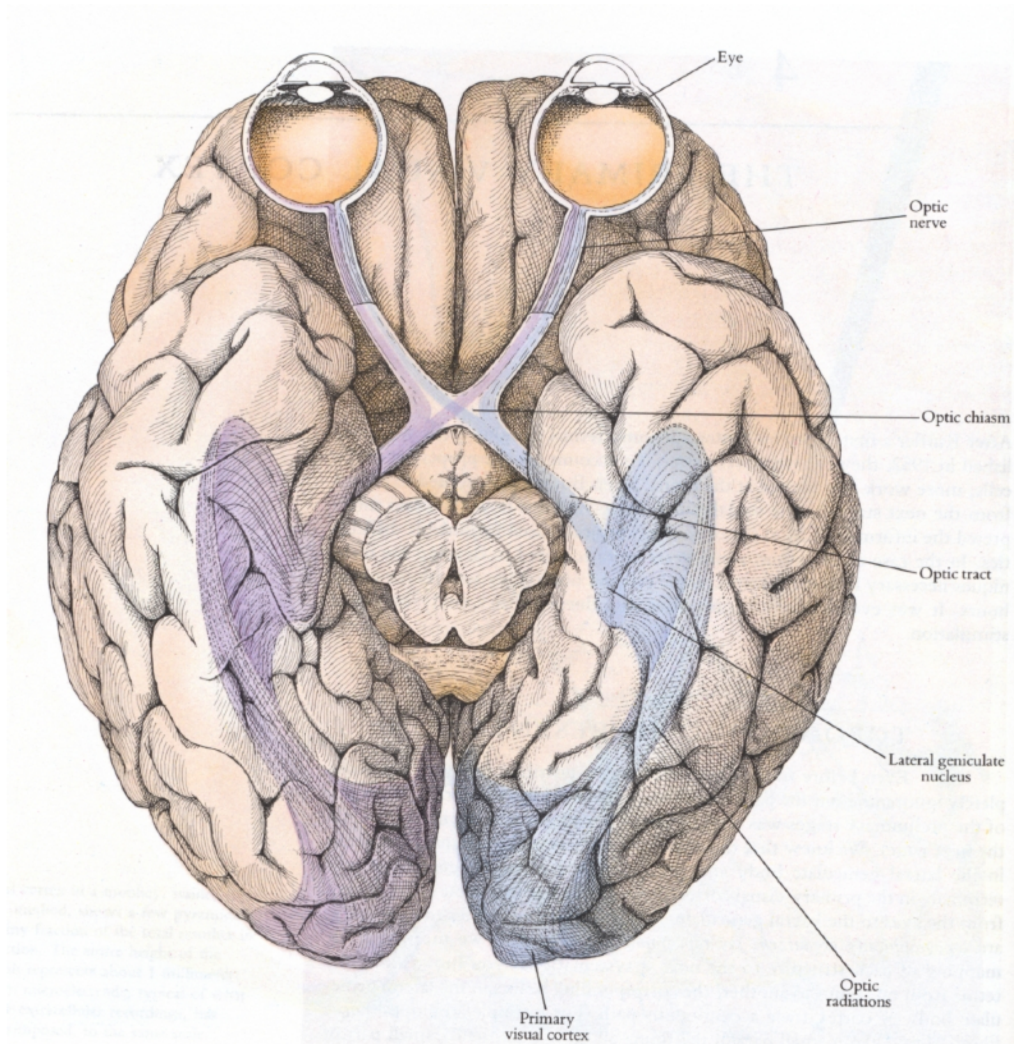




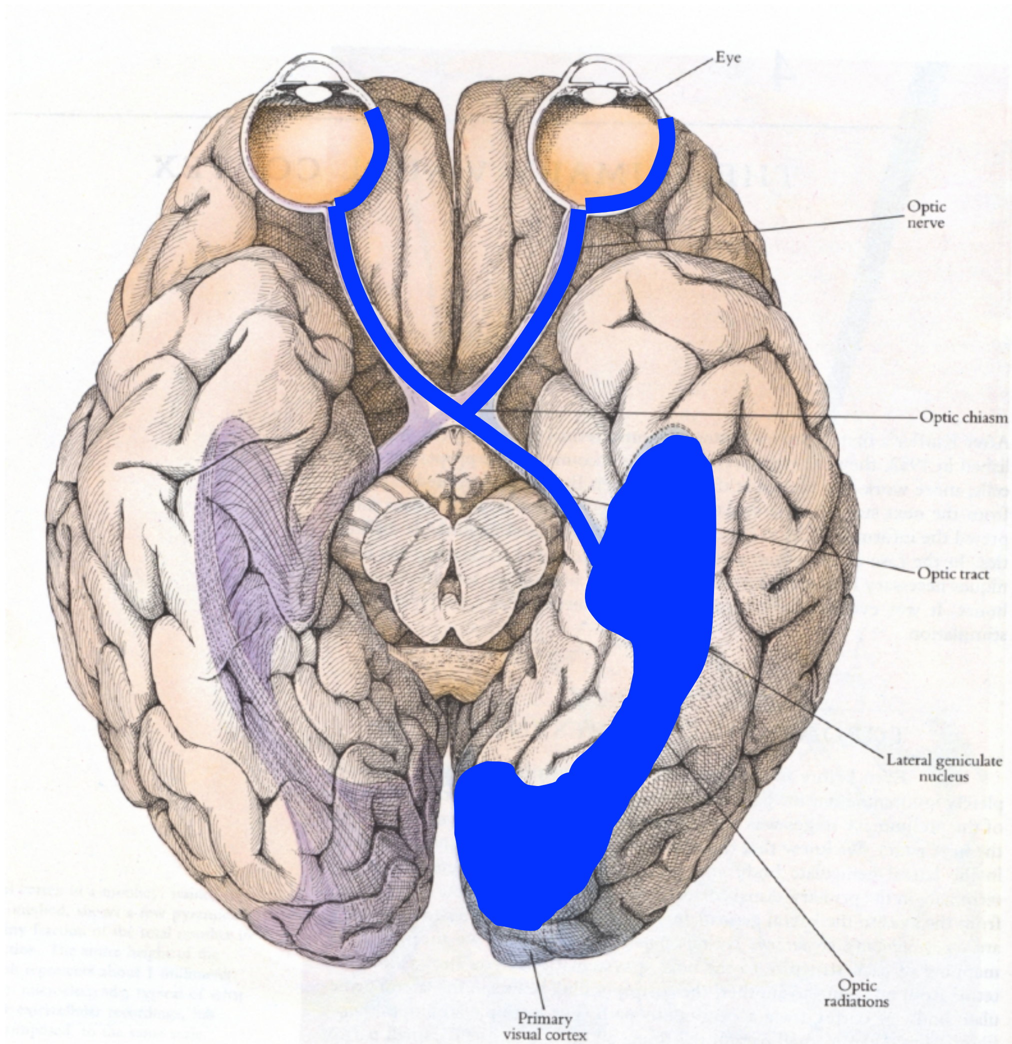
(PCG-Cornell

slide courtesy Pieter Peers

Human Brain


Eye
Optic nerve
Optic chiasm
Optic tract
Lateral geniculate nucleus
Optic radiations
Primary visual cortex

Human Brain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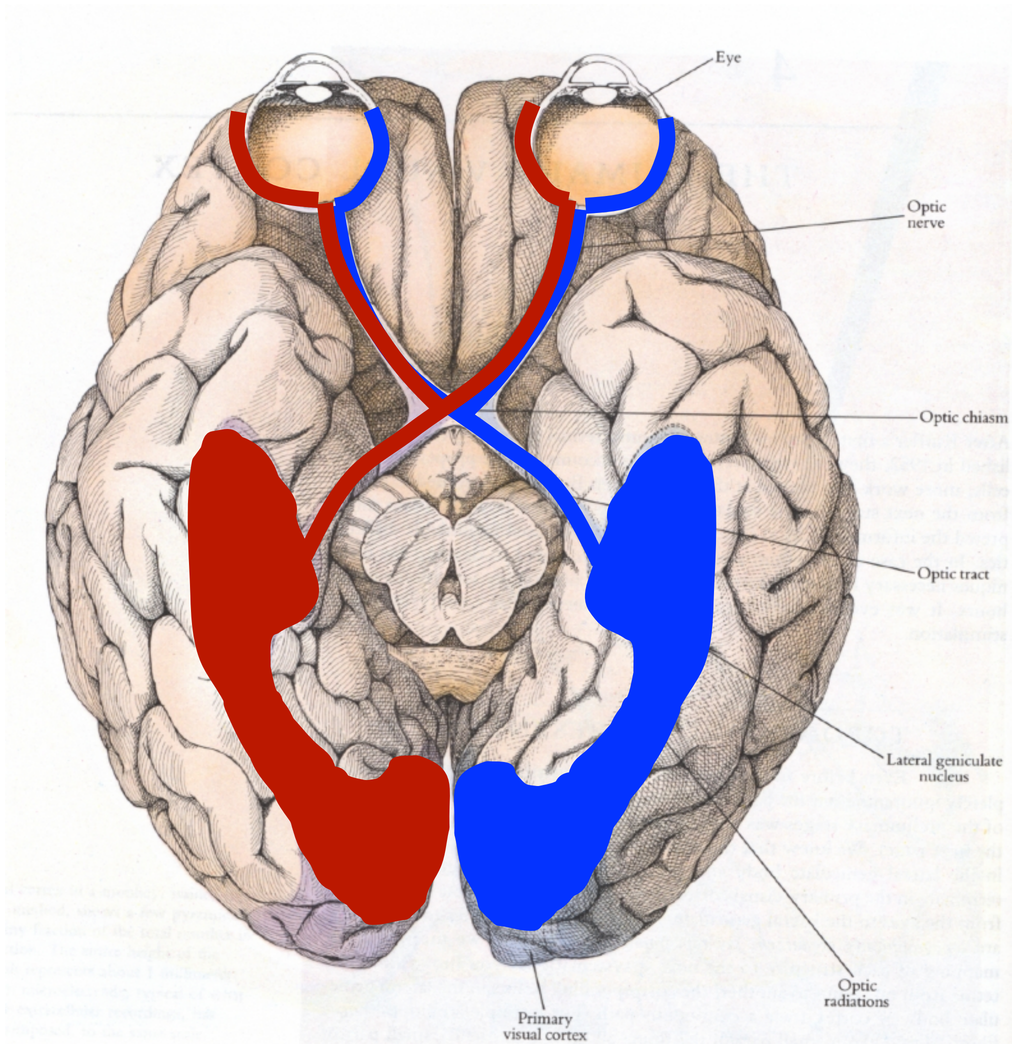
Human Brain



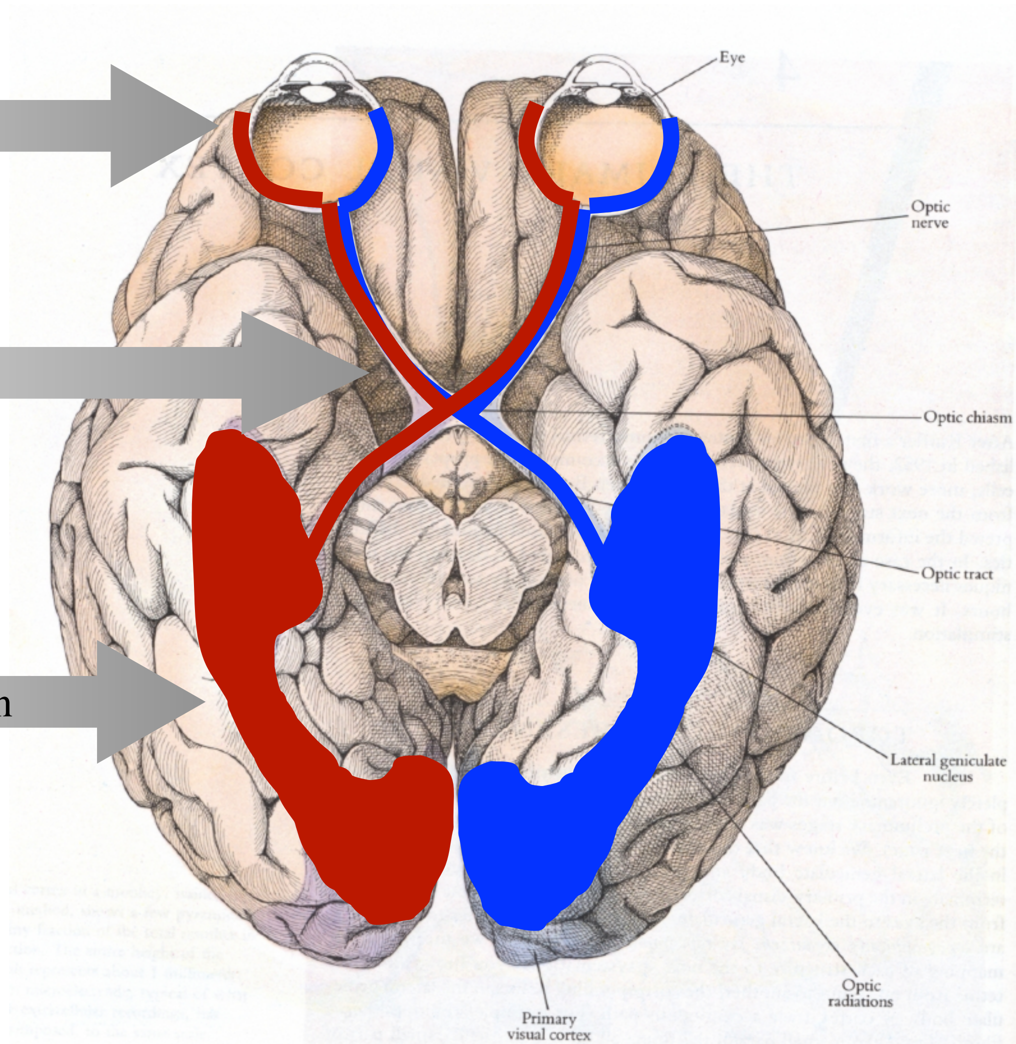
Eye
Optic nerve
Optic chiasm
Optic tract
Lateral geniculate nucleus
Optic radiations
Primary visual cortex

Human Brain

imperfect measuring instrument

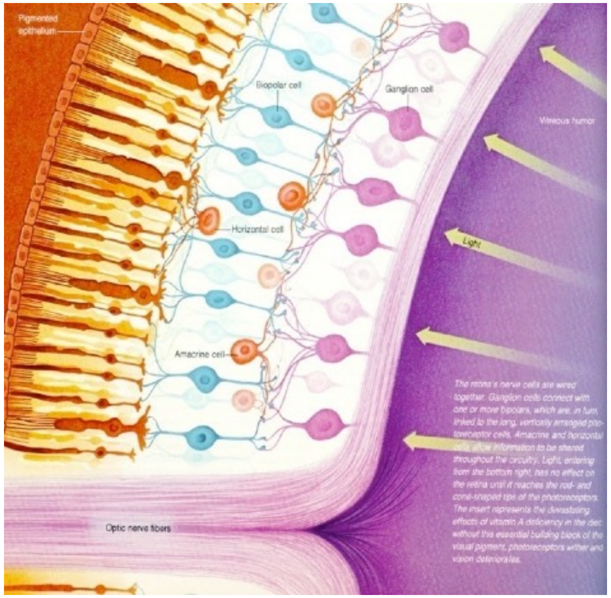
signals

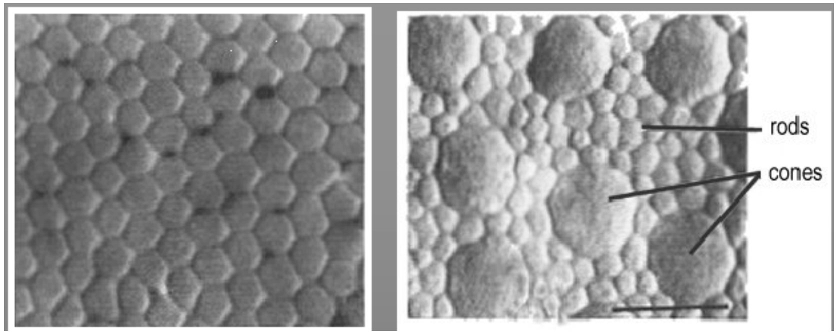
cognitive & visual interpretation

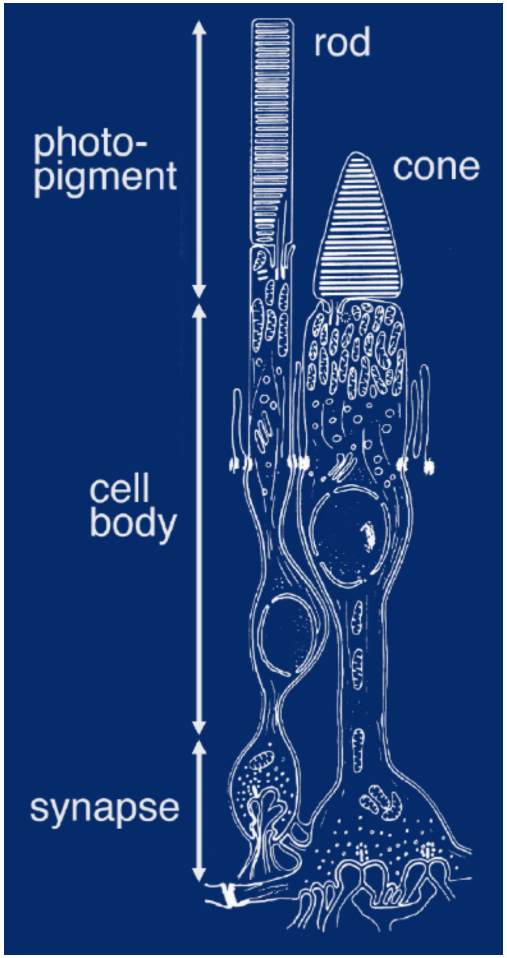


Human eye: retina

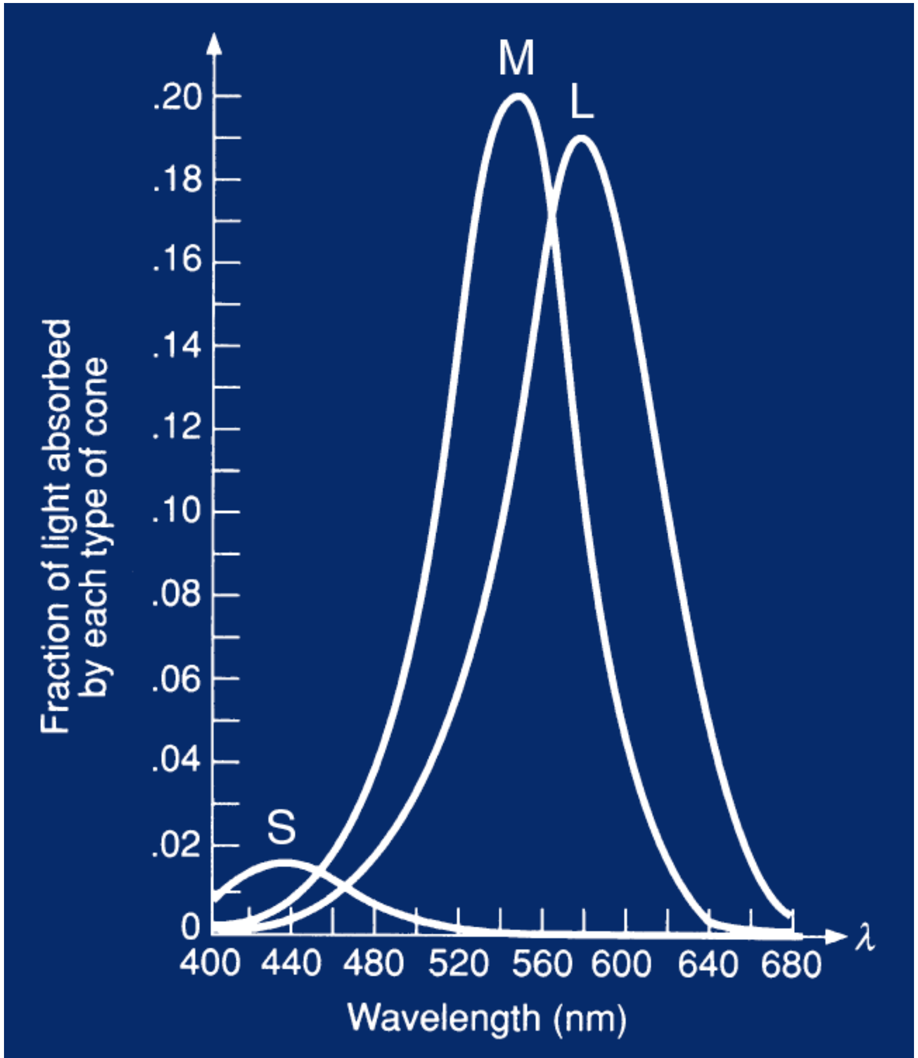
Light passes through blood vessels & retinal layers before reaching the light-sensitive cells
(“rods” & “cones”)





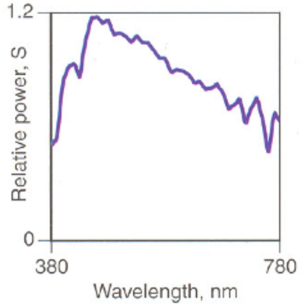


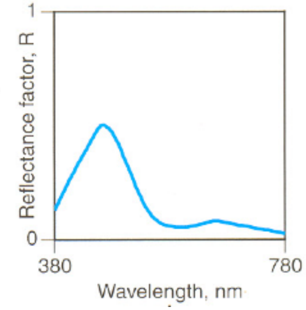
Cone Responses

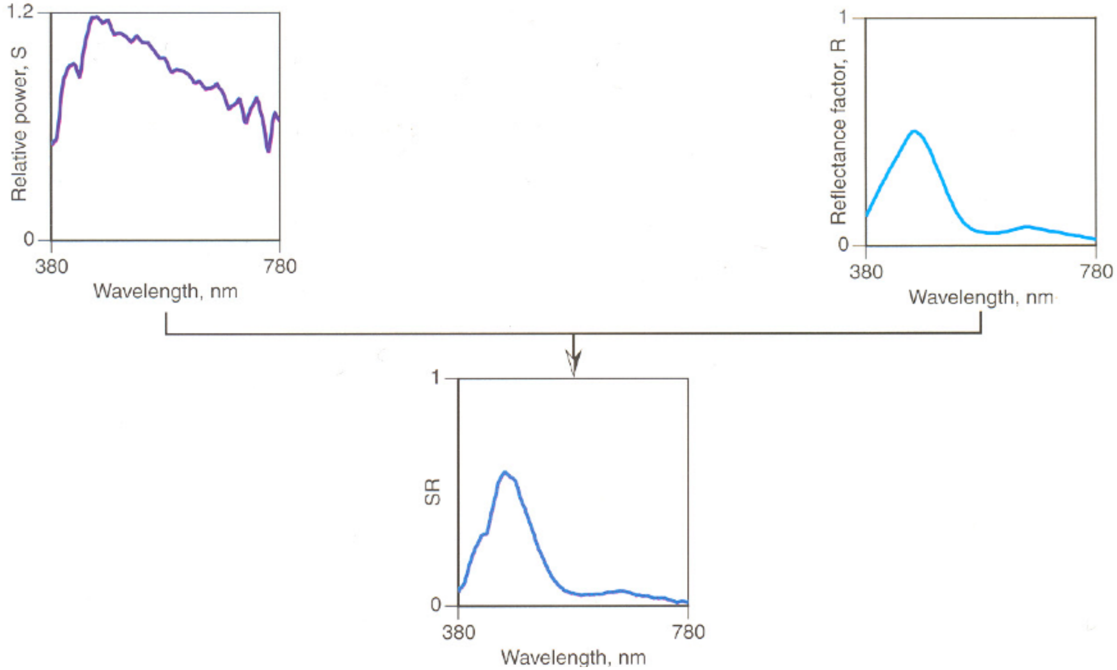


- S,M,L cones have broadband spectral sensitivity
- S,M,L neural response is integrated w.r.t. λ
  - we'll call the response functions r_S, r_M, r_L
- Results in a trichromatic visual system
- S, M, and L are *tristimulus values*

[source unknown]


1.2
0
Relative power, S
380
780
Wavelength, nm


1
0
Reflectance factor, R
380
780
Wavelength, nm


1.2
Relative power, S
0
380
780
Wavelength, nm
1
Reflectance factor, R
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Wavelength, nm
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Wavelength, nm

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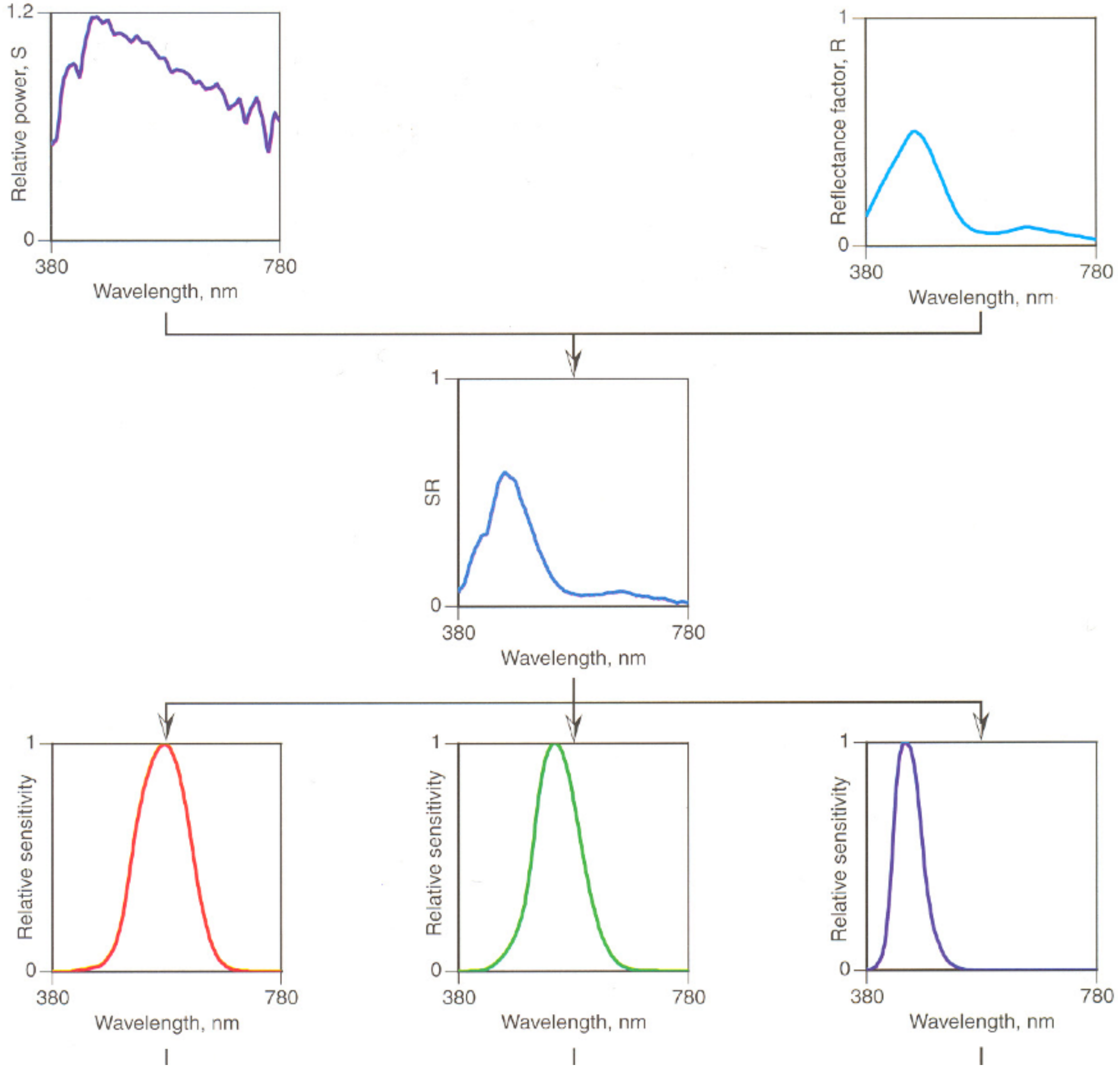
1.2
Relative power, S
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Wavelength, nm
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Reflectance factor, R
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Wavelength, nm
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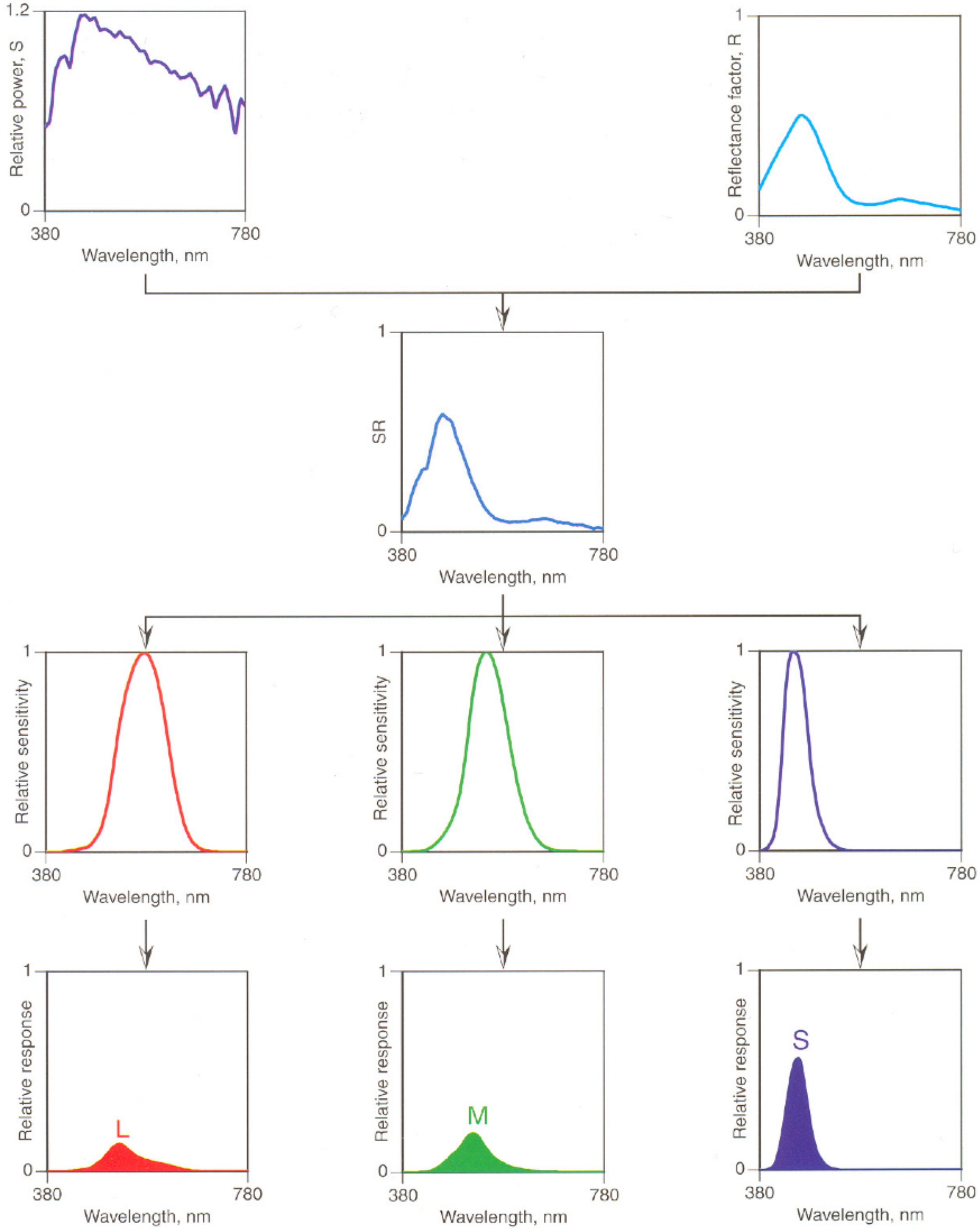
1.2
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Relative power, S
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Wavelength, nm
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Reflectance factor, R
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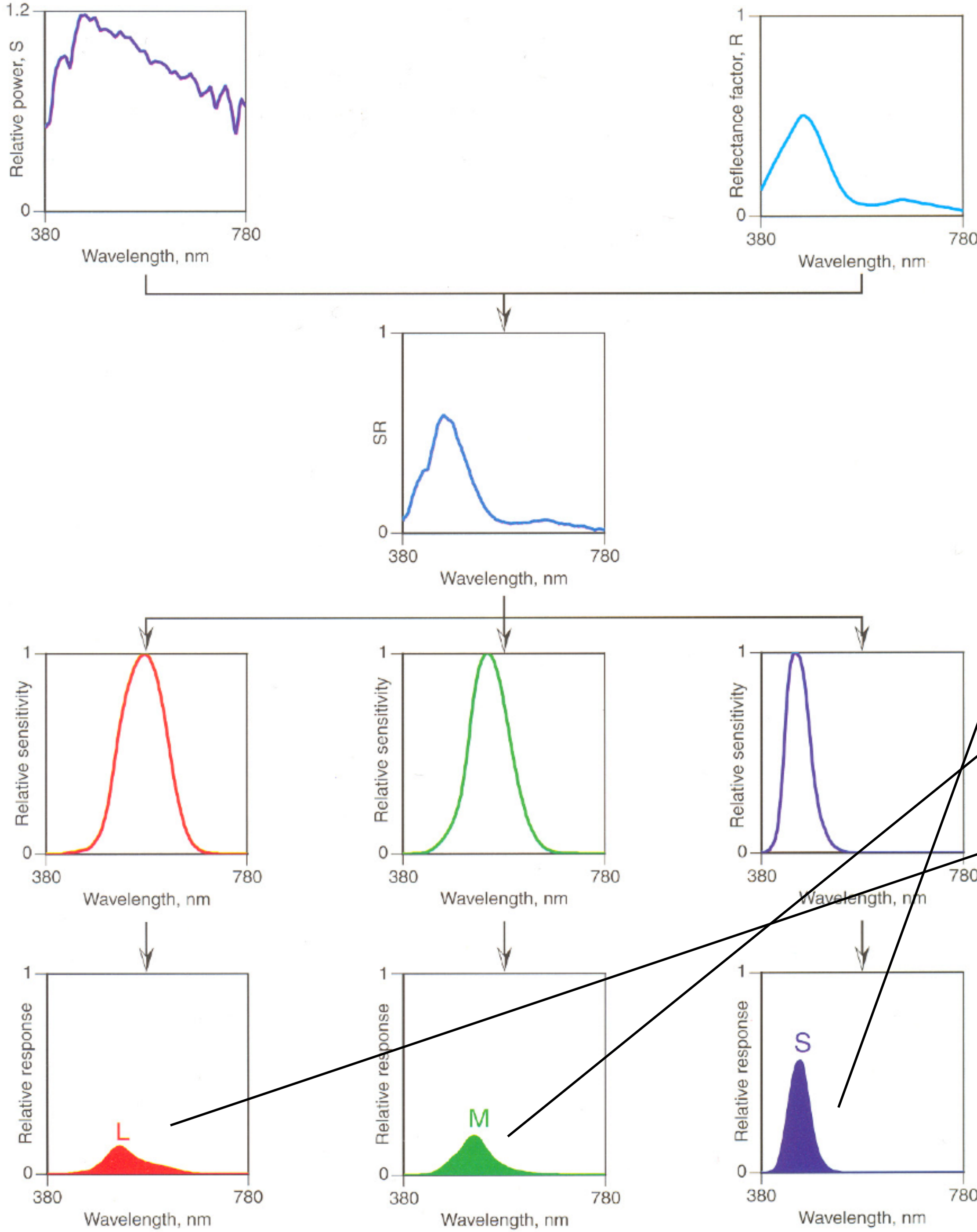
380
780
Wavelength, nm
Relative sensitivity
Relative response
L
M
S

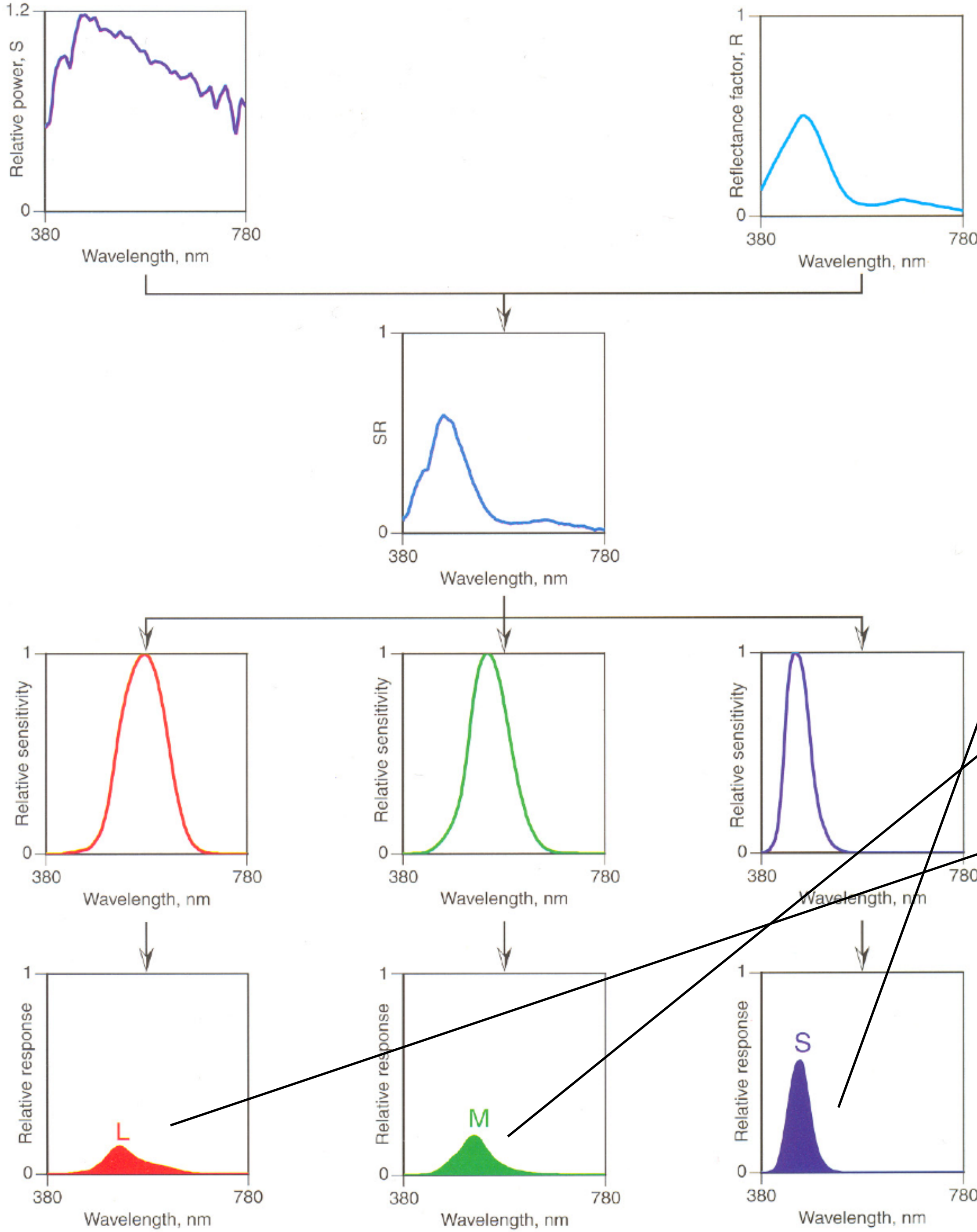
Co



$$S = \int_{380nm}^{800nm} s(\lambda)P(\lambda)d\lambda$$

$$M = \int_{380nm}^{800nm} m(\lambda)P(\lambda)d\lambda$$

$$L = \int_{380nm}^{800nm} l(\lambda)P(\lambda)d\lambda$$



$$S = \int_{380nm}^{800nm} s(\lambda)P(\lambda)d\lambda$$

$$M = \int_{380nm}^{800nm} m(\lambda)P(\lambda)d\lambda$$

$$L = \int_{380nm}^{800nm} l(\lambda)P(\lambda)d\lambda$$

S,M,L responses are what we 'see'

Cone responses to a spectrum *s*

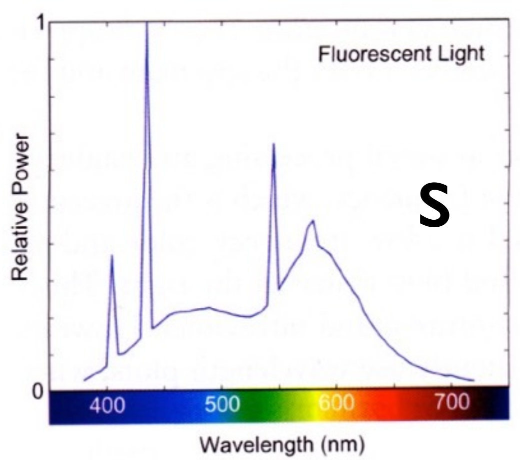
$$S = \int r_S(\lambda) s(\lambda)\, d\lambda = r_S \cdot s$$

$$M = \int r_M(\lambda) s(\lambda)\, d\lambda = r_M \cdot s$$

$$L = \int r_L(\lambda) s(\lambda)\, d\lambda = r_L \cdot s$$

Colorimetry: an answer to the problem

- Wanted to map a *Physical light description* to a *Perceptual color sensation*
- Basic solution was known and standardized by 1930
  - Though not quite in this form—more on that in a bit



Physical

$$S = r_S \cdot s$$
$$M = r_M \cdot s$$
$$L = r_L \cdot s$$

Perceptual

[Stone 2003]

Basic fact of colorimetry

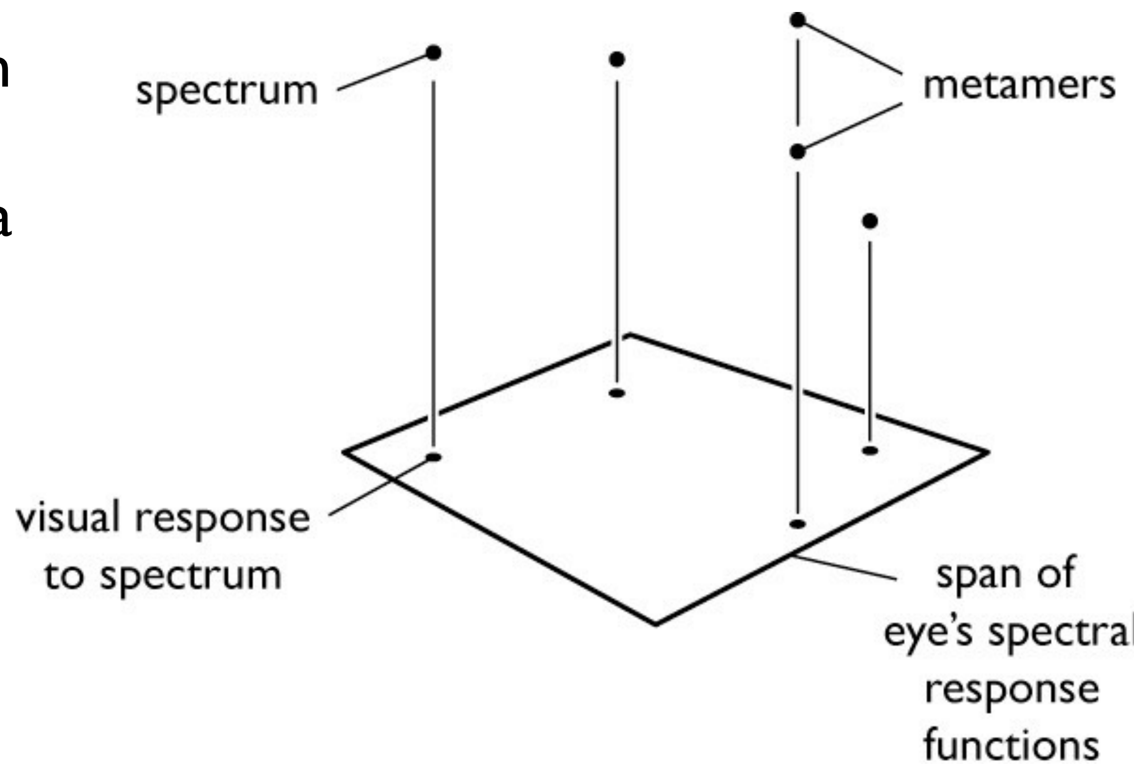
- Take a spectrum (which is a function)
- Eye produces three numbers
- This throws away a lot of information!
  - Quite possible to have two different spectra that have the same S, M, L tristimulus values
  - Two such spectra are *metamers*

Pseudo-geometric interpretation

- A dot product is a projection
- We are projecting a high dimensional vector (a spectrum) onto three vectors
  - differences that are perpendicular to all 3 vectors are not detectable
- For intuition, we can imagine a 3D analog
  - 3D stands in for high-D vectors
  - 2D stands in for 3D
  - Then vision is just projection onto a plane

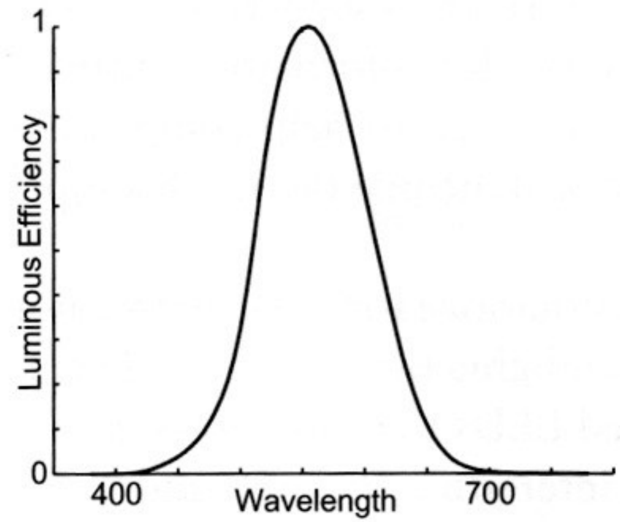
Pseudo-geometric interpretation

- The information available to the visual system about a spectrum is three values
  - this amounts to a loss of information analogous to projection on a pla
- Two spectra that produce the same response are metamers



Basic colorimetric concepts

- Luminance
  - the overall magnitude of the the visual response to a spectrum (independent of its color)
    - corresponds to the everyday concept "brightness"
  - determined by product of SPD with the *luminous efficiency function* V_λ that describes the eye's overall ability to detect light at each wavelength
  - e.g. lamps are optimized to improve their luminous efficiency (tungsten vs. fluorescent vs. sodium vapor)



[Stone 2003]

Luminance, mathematically

- *Y* just has another response curve (like *S*, *M*, and *L*)

$$Y = r_Y \cdot s$$

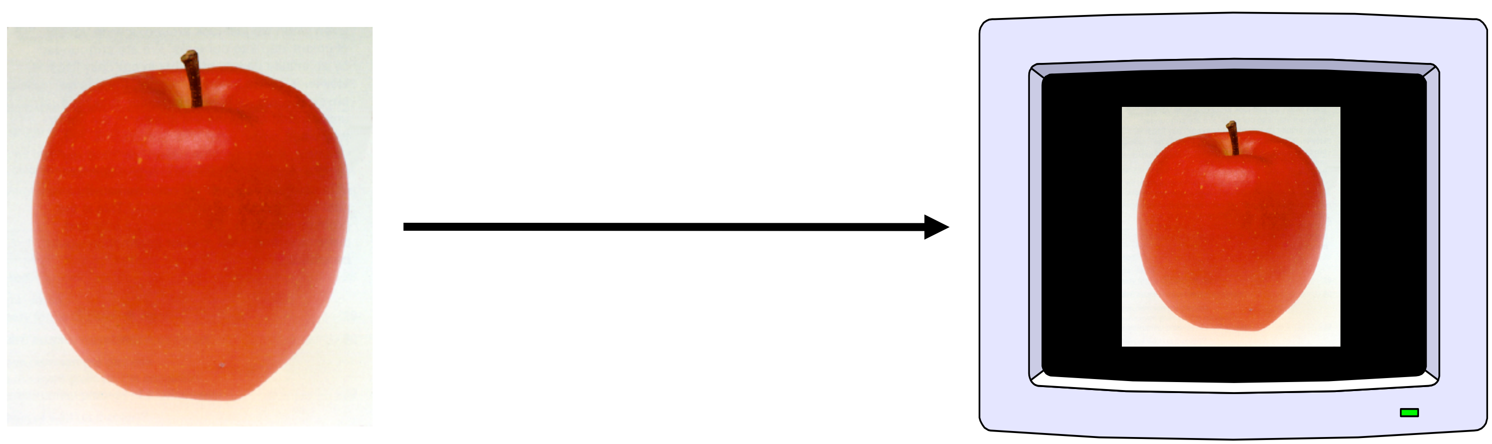
  - r_Y is really called "V_λ"
- V_λ is a linear combination of *S*, *M*, and *L*
  - Has to be, since it's derived from cone outputs

More basic colorimetric concepts

- Chromaticity
  - what's left after luminance is factored out (the color without regard for overall brightness)
  - scaling a spectrum up or down leaves chromaticity alone
- Dominant wavelength
  - many colors can be matched by white plus a spectral color
  - correlates to everyday concept "hue"
- Purity
  - ratio of pure color to white in matching mixture
  - correlates to everyday concept "colorfulness" or "saturation"

Color reproduction

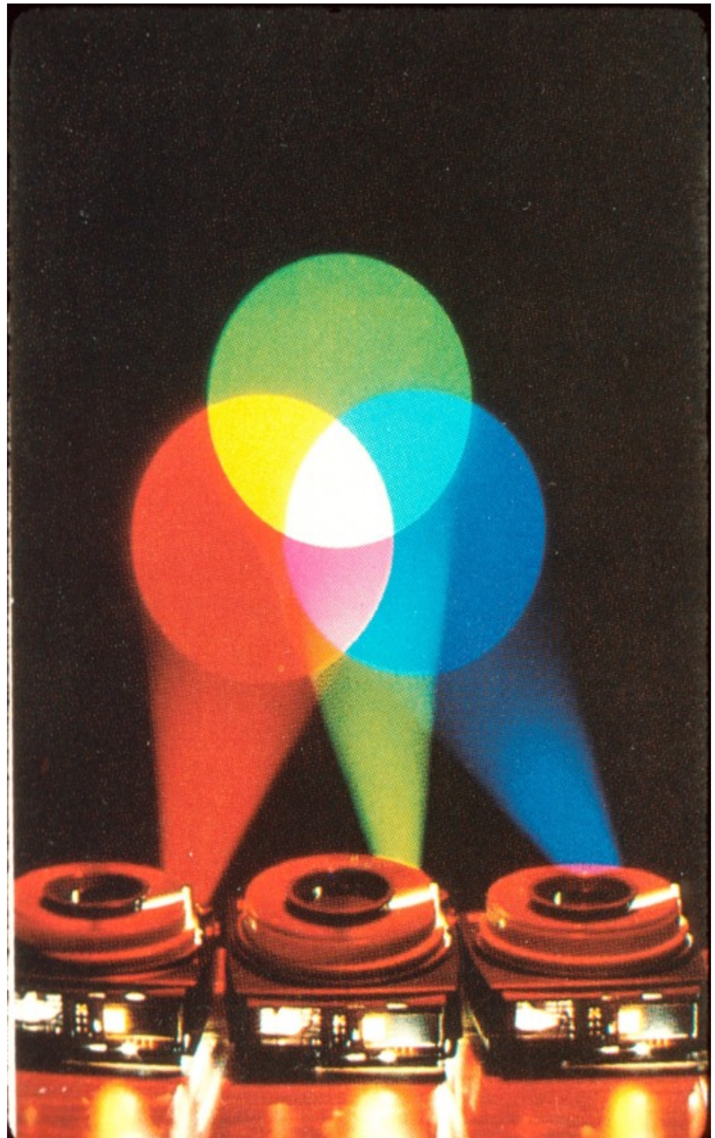
- Have a spectrum *s*; want to match on RGB monitor
  - "match" means it looks the same
  - any spectrum that projects to the same point in the visual color space is a good reproduction
- Must find a spectrum that the monitor *can* produce that is a metamer of *s*



R, G, B?

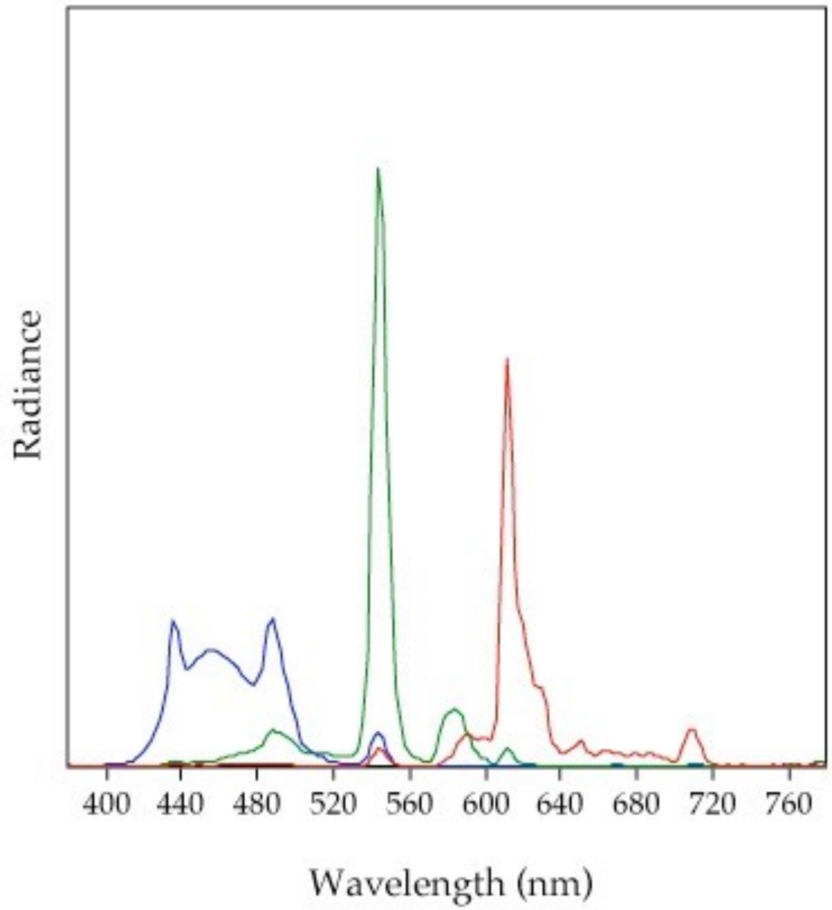
[cs417—Greenberg]

Additive Color



[source unknown]

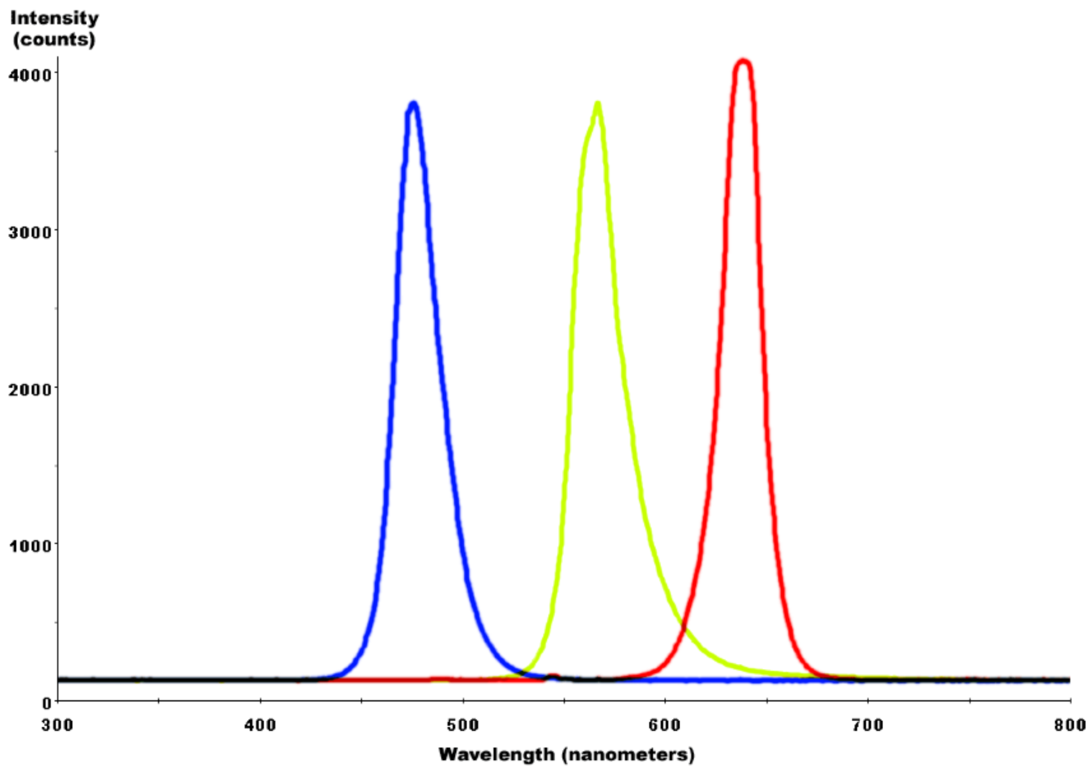
LCD display primaries



[Fairchild 97]

Curves determined by (fluorescent or LED) backlight and filters

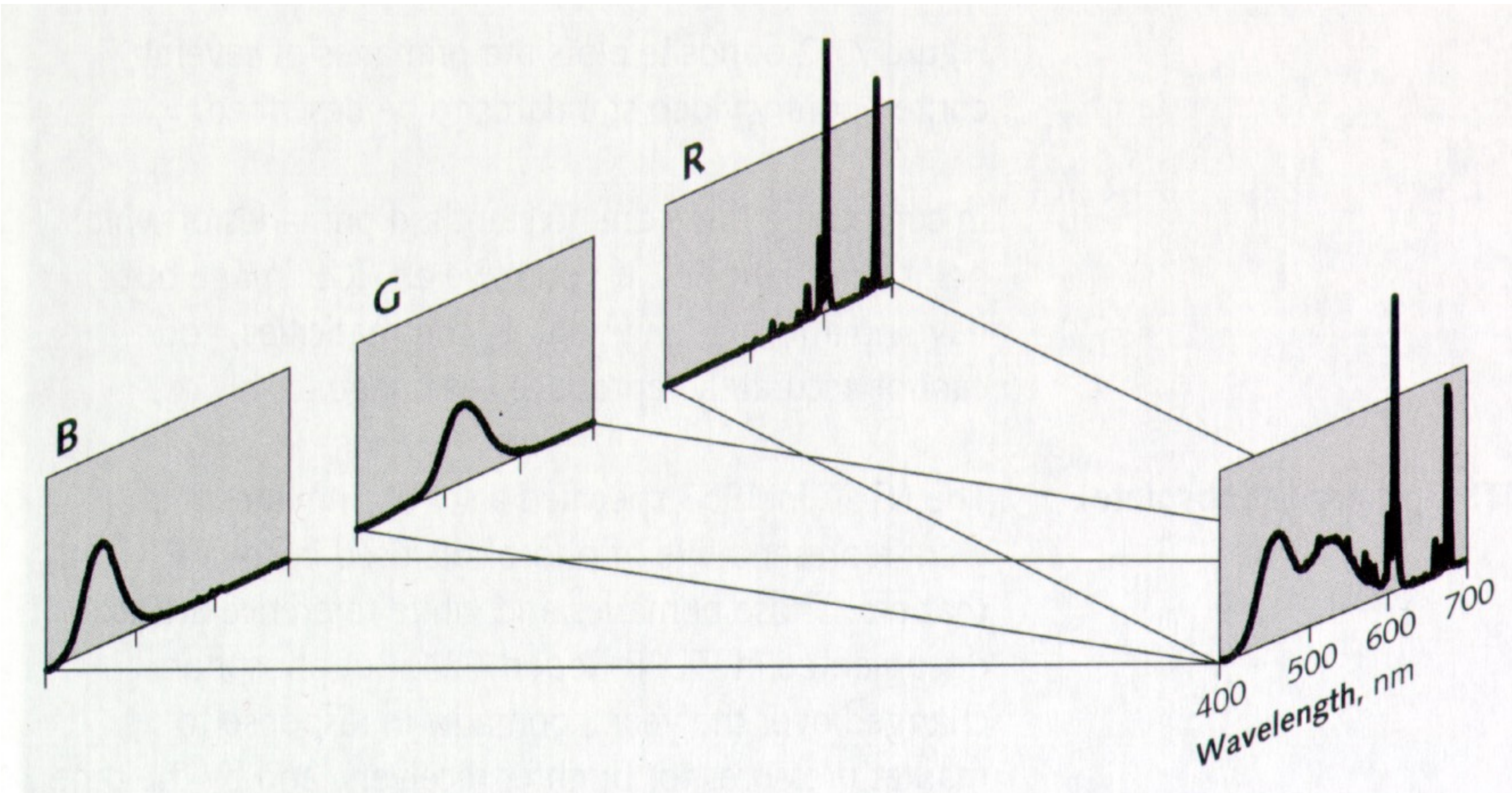
LED display primaries



[wikipedia user Deglr6328]

– Native emission curves of 3 LED types

Combining Monitor Phosphors with Spatial Integration



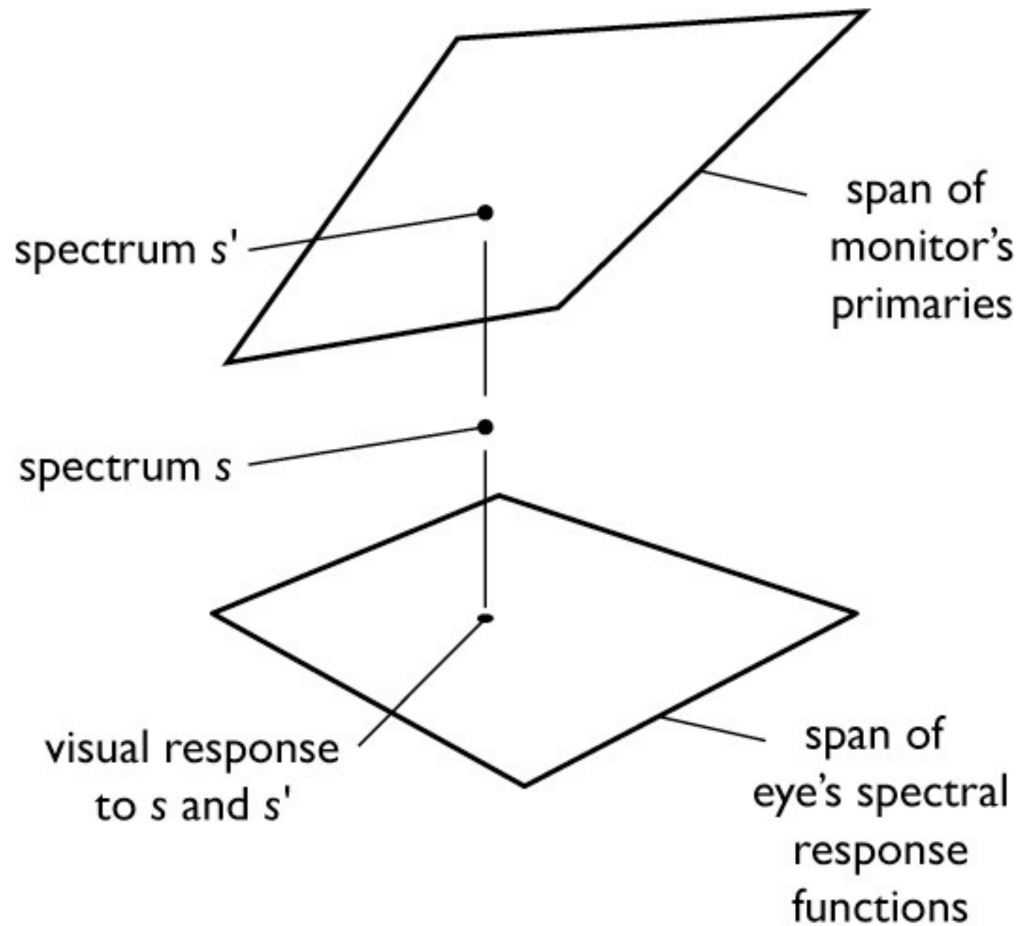
[source unknown]

Color reproduction

- Say we have a spectrum *s* we want to match on an RGB monitor
  - "match" means it looks the same
  - any spectrum that projects to the same point in the visual color space is a good reproduction
- So, we want to find a spectrum that the monitor can produce that matches *s*
  - that is, we want to display a metamer of *s* on the screen

Color reproduction

- We want to compute the combination of r, g, b that will project to the same visual response as *s*.



Color reproduction as linear algebra

- The projection onto the three response functions can be written in matrix form:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} — \; r_S \; — \\ — \; r_M \; — \\ — \; r_L \; — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

or,

$$V = M_{SML}\, s.$$

Color reproduction as linear algebra

- The spectrum that is produced by the monitor for the color signals R, G, and B is:

$$s_a(\lambda) = Rs_r(\lambda) + Gs_g(\lambda) + Bs_b(\lambda).$$

- Again the discrete form can be written as a matrix:

$$\begin{bmatrix} | \\ s_a \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} =$$

or,

$$s_a = M_{RGB}\, C.$$

Color reproduction as linear algebra

- What color do we see when we look at the display?
  - Feed *C* to display
  - Display produces s_a
  - Eye looks at s_a and produces *V*

$$V = M_{SML} M_{RGB} C$$

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color reproduction as linear algebra

- Goal of reproduction: visual response to s and s_a is the same:

$$M_{SML}\,\tilde{s} = M_{SML}\,\tilde{s_a}.$$

- Substituting in the expression for s_a,

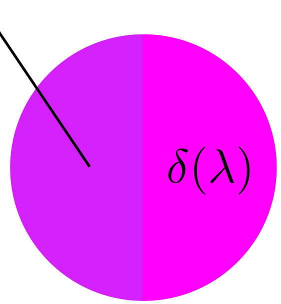
$$M_{SML}\,\tilde{s} = M_{SML} M_{RGB}\,C$$

$$C = \underbrace{(M_{SML} M_{RGB})^{-1} M_{SML}}_{\textit{color matching matrix} \text{ for RGB}}\,\tilde{s}$$

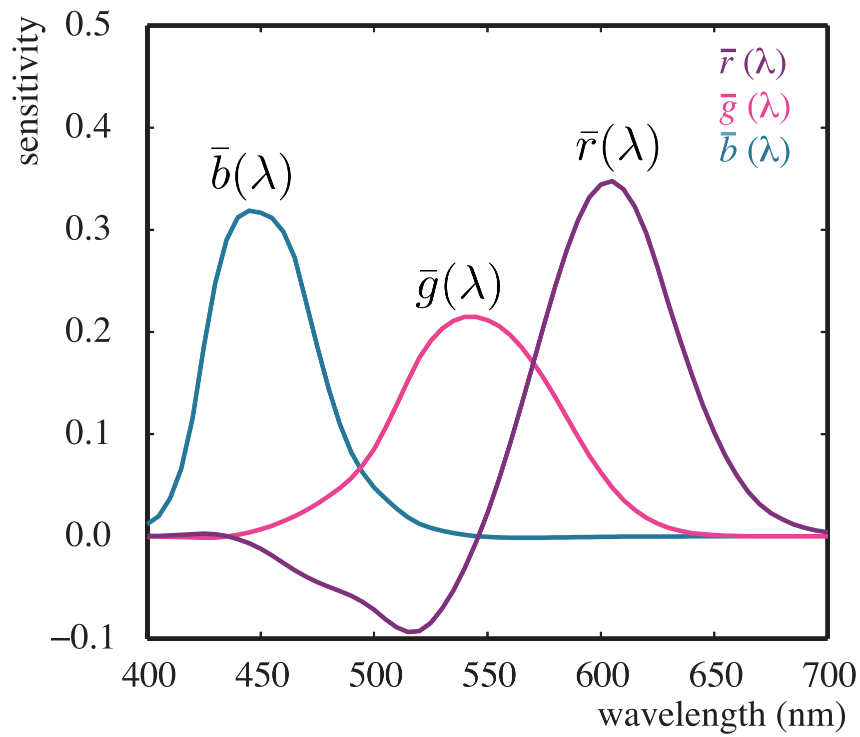
Color matching functions

- For given primaries, how much are needed to match each spectral color?
  - can be determined experimentally without knowing S, M, L
  - experiment:

$$Rs_r(\lambda) + Gs_g(\lambda) + Bs_b(\lambda)$$



single wavelength on right
mixture of 3 primaries on left
(negative weight?
mix in on right)



[Marschner & Shirley ch. 19 by Reinhard & Johnson]

Color spaces

- Need three numbers to specify a color
  - but what three numbers?
  - a color space is an answer to this question
- Common example: monitor RGB
  - define colors by what R, G, B signals will produce them on your monitor
    - (in math, s = $R\mathbf{R} + G\mathbf{G} + B\mathbf{B}$ for some spectra **R**, **G**, **B**)
  - device dependent (depends on gamma, phosphors, gains, …)
    - therefore if I choose RGB by looking at my monitor and send it to you, you may not see the same color
  - also leaves out some colors (limited gamut), e.g. vivid yellow

Standard color spaces

- Standardized RGB (sRGB)
  - makes a particular monitor RGB standard
  - other color devices simulate that monitor by calibration
  - sRGB is usable as an interchange space; widely adopted today
  - gamut is still limited

A universal color space: XYZ

- Standardized by CIE (*Commission Internationale de l'Eclairage*, the standards organization for color science)
- Based on three "imaginary" primaries *X*, *Y*, and *Z*
    - (in math, $s = X\mathbf{X} + Y\mathbf{Y} + Z\mathbf{Z}$)
  - imaginary = only realizable by spectra that are negative at some wavelengths
  - key properties
    - any stimulus can be matched with positive *X*, *Y*, and *Z*
    - separates out luminance: **X**, **Z** have zero luminance, so *Y* tells you the luminance by itself

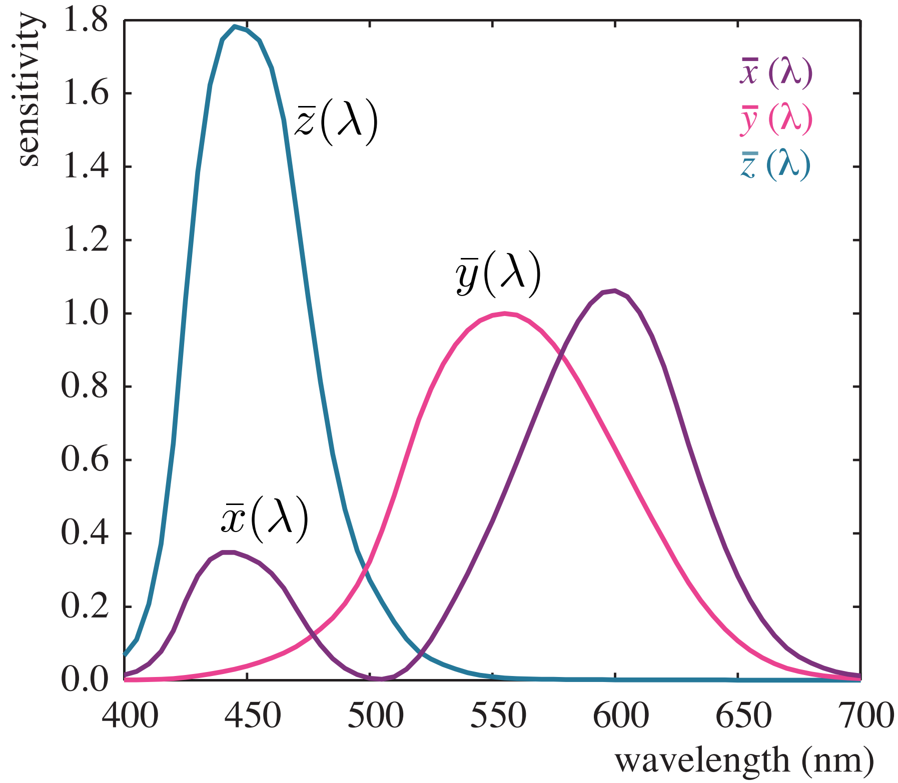
XYZ color matching functions

- XYZ primaries are not specified because they are not needed; instead the color matching functions are in the standard.

$$X = k \int \bar{x}(\lambda) s(\lambda) d\lambda$$

$$Y = k \int \bar{y}(\lambda) s(\lambda) d\lambda$$

$$Z = k \int \bar{z}(\lambda) s(\lambda) d\lambda$$



[Marschner & Shirley ch. 19 by Reinhard & Johnson]

XYZ color matching functions

- They are linear combinations of S, M, L
  - as are all sets of color matching functions for humans
- They are what is standardized
- All other color spaces standardized in terms of XYZ
  - if you have a spectrum to convert to a color, the first step is to integrate it against these functions

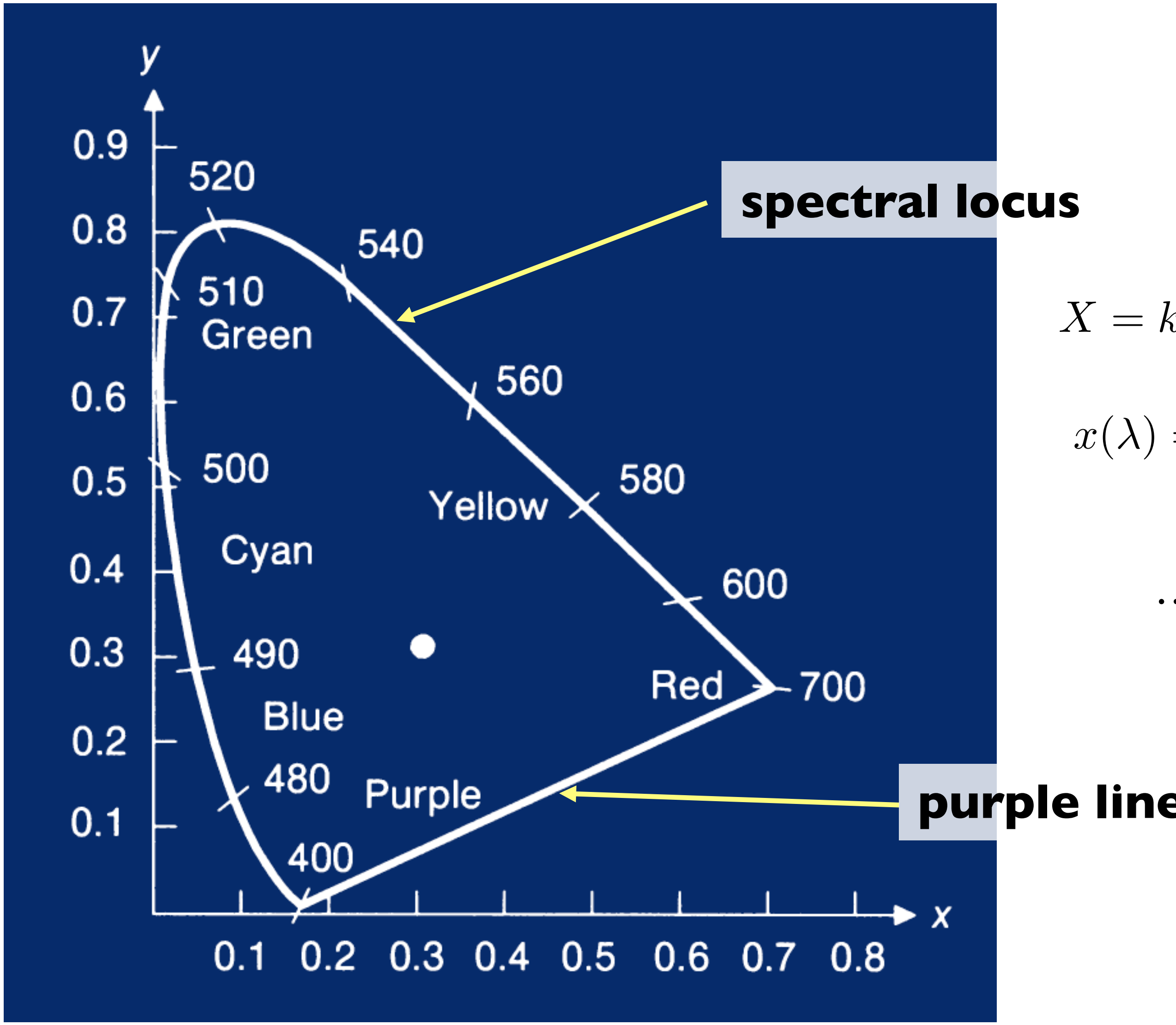
Separating luminance, chromaticity

- Luminance: *Y*
- Chromaticity: *x*, *y*, *z*, defined as

$$x = \frac{X}{X+Y+Z}$$
$$y = \frac{Y}{X+Y+Z}$$
$$z = \frac{Z}{X+Y+Z}$$

  – since $x + y + z = 1$, we only need to record two of the three
    - usually choose *x* and *y*, leading to (x, y, Y) coords

Chromaticity Diagram



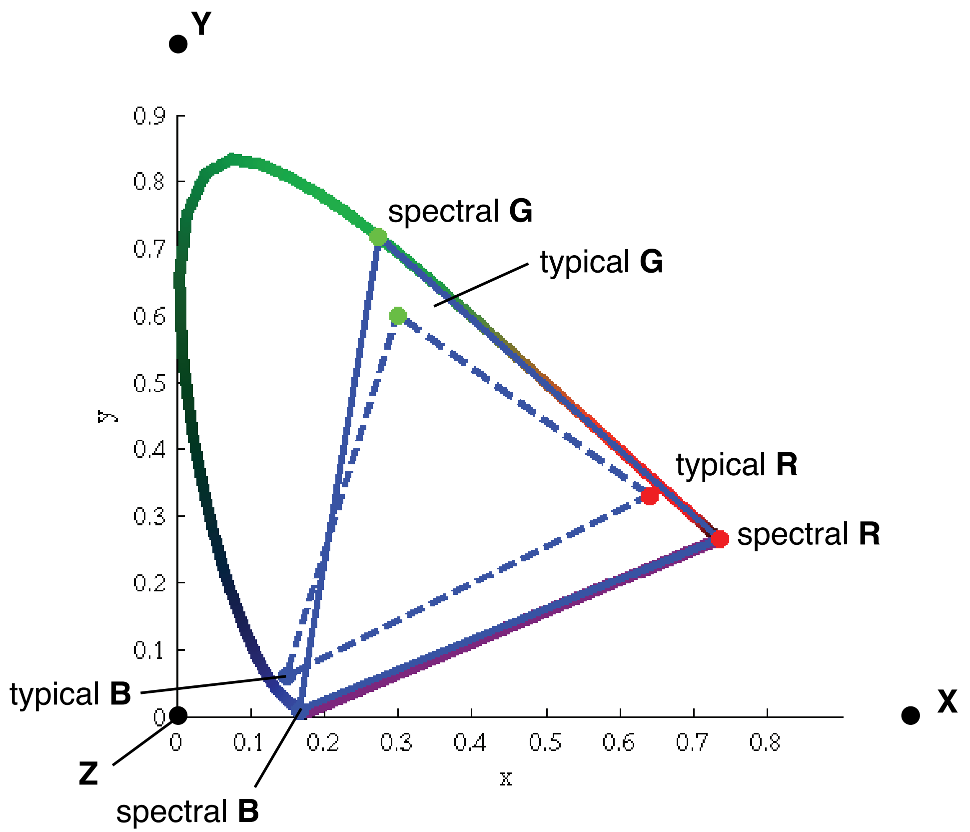
$$X = k \int \bar{x}(\lambda)\delta(\lambda)d\lambda = k\, x(\lambda)$$

$$x(\lambda) = \frac{x(\lambda)}{x(\lambda) + y(\lambda) + z(\lambda)}$$

…and similarly for y(λ)

[source unknown]

Color Gamuts



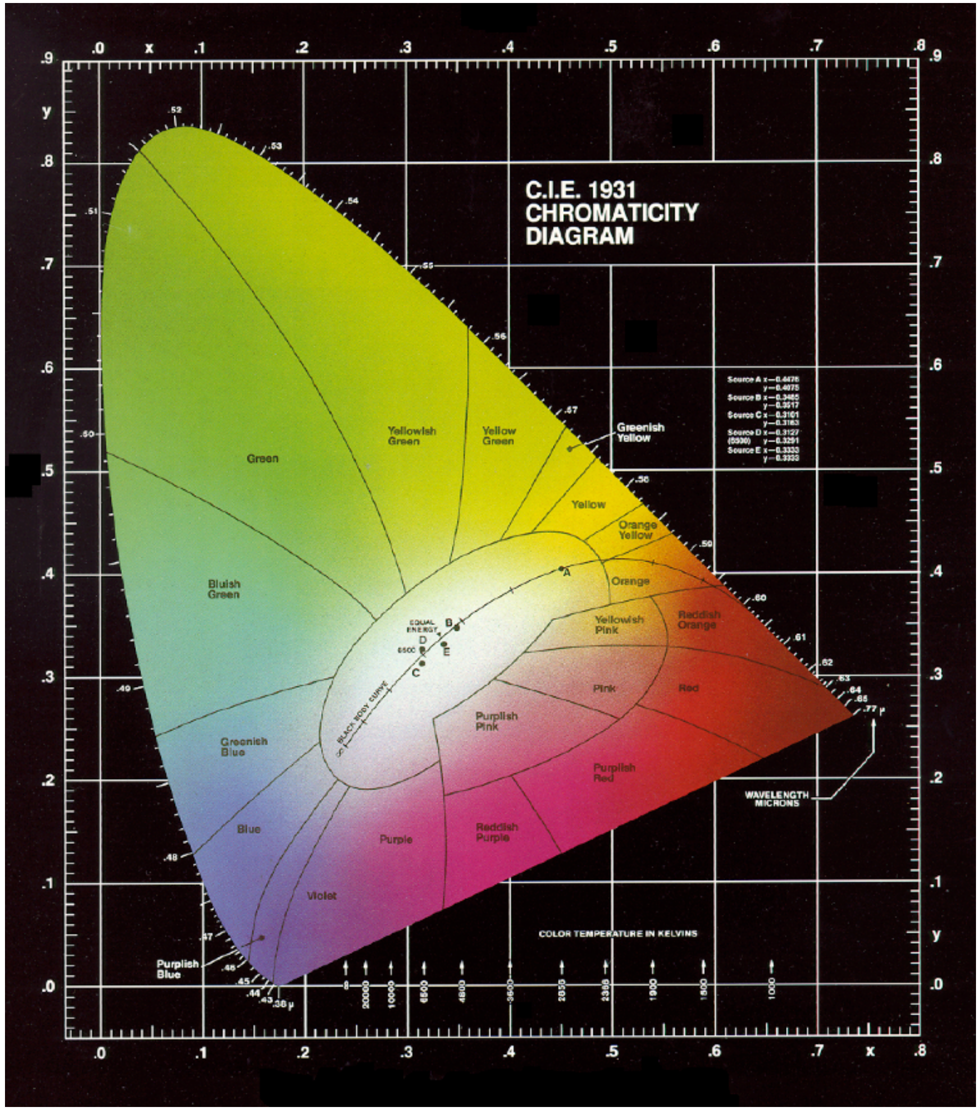
[Marschner & Shirley ch. 19 by Reinhard & Johnson]

Monitors/printers can’t produce all visible colors

Reproduction is limited to a particular domain

For additive color (e.g. monitor) gamut is the triangle defined by the chromaticities of the three primaries.

Chromaticity Diagram



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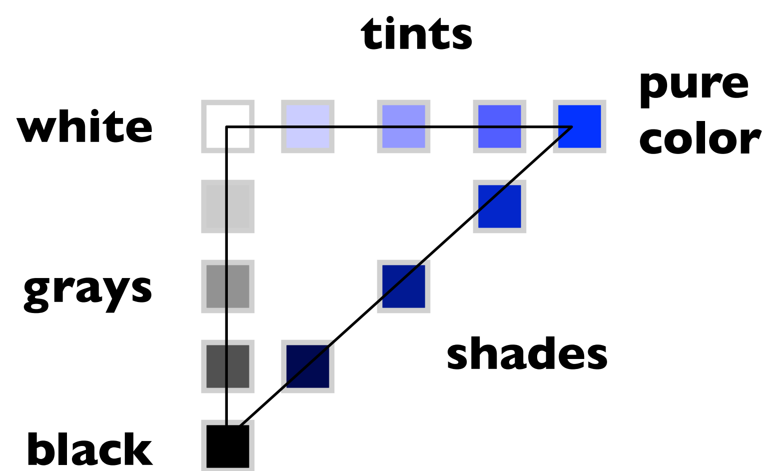
Perceptually organized color spaces

- Artists often refer to colors as *tints*, *shades*, and *tones* of pure pigments
  - tint: mixture with white
  - shade: mixture with black
  - tones: mixture with black and white
  - gray: no color at all (aka. neutral)



[after FvDFH]

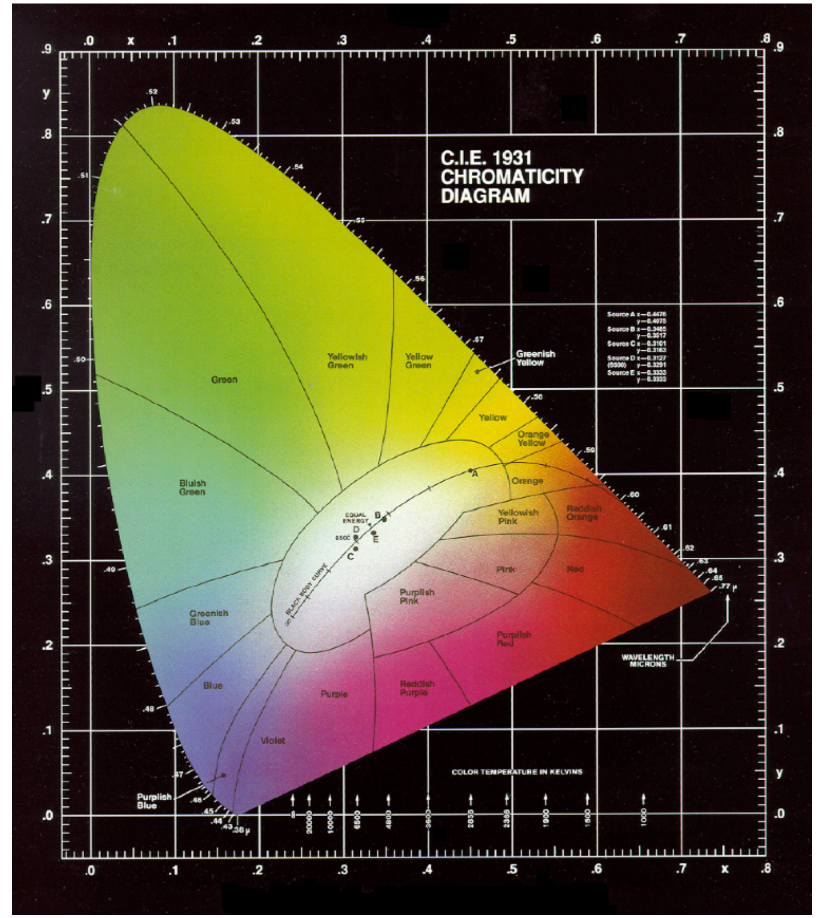
- This seems intuitive
  - tints and shades are inherently related to the pure color
    - "same" color but lighter, darker, paler, etc.

Perceptual dimensions of color

- Hue
  - the "kind" of color, regardless of attributes
  - colorimetric correlate: dominant wavelength
  - artist's correlate: the chosen pigment color
- Saturation
  - the "colorfulness"
  - colorimetric correlate: purity
  - artist's correlate: fraction of paint from the colored tube
- Lightness (or value)
  - the overall amount of light
  - colorimetric correlate: luminance
  - artist's correlate: tints are lighter, shades are darker

Perceptual dimensions: chromaticity

- In *x*, *y*, *Y* (or another luminance/chromaticity space), *Y* corresponds to lightness
- hue and saturation are then like polar coordinates for chromaticity (starting at white, which way did you go and how far?)



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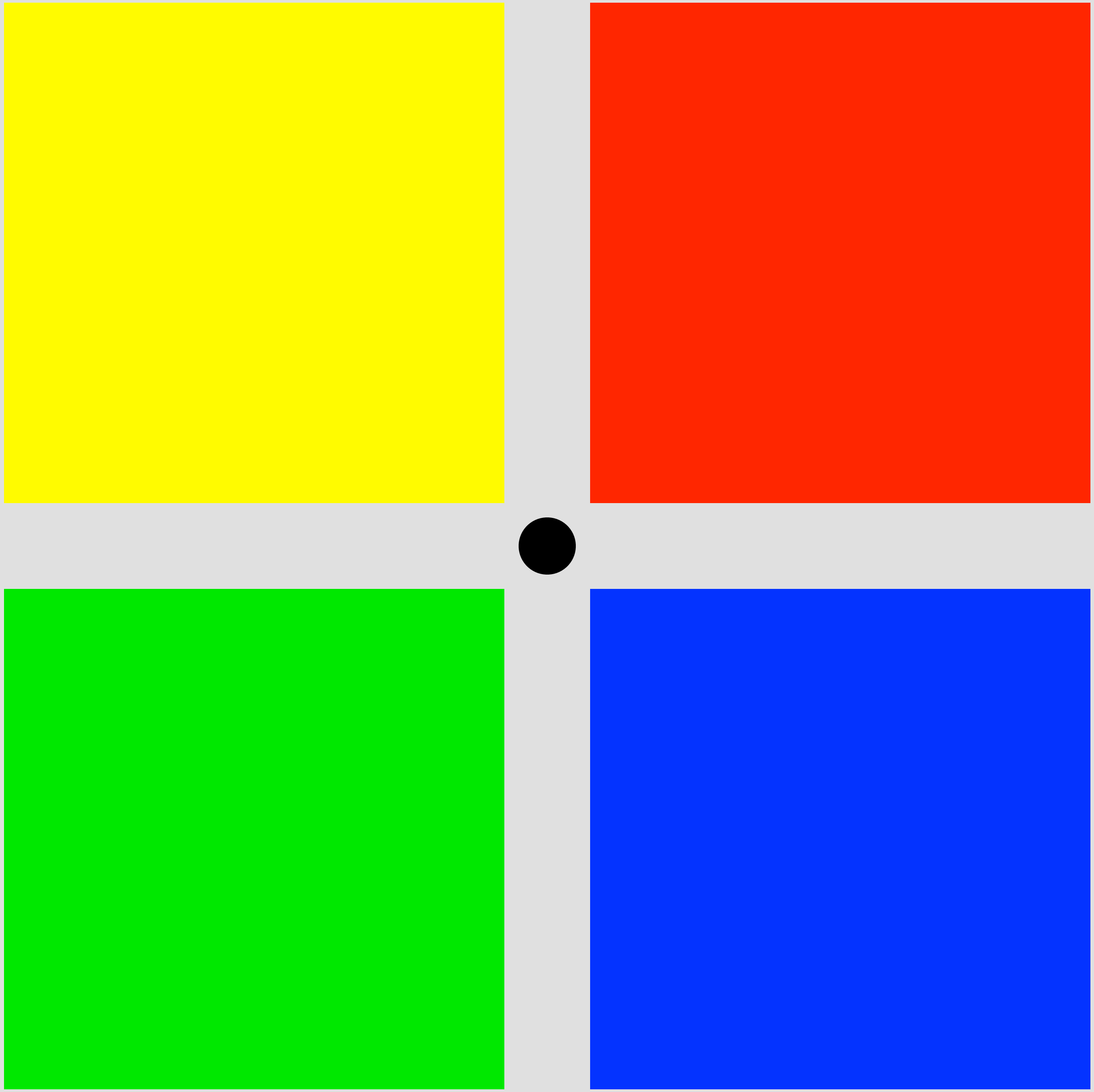
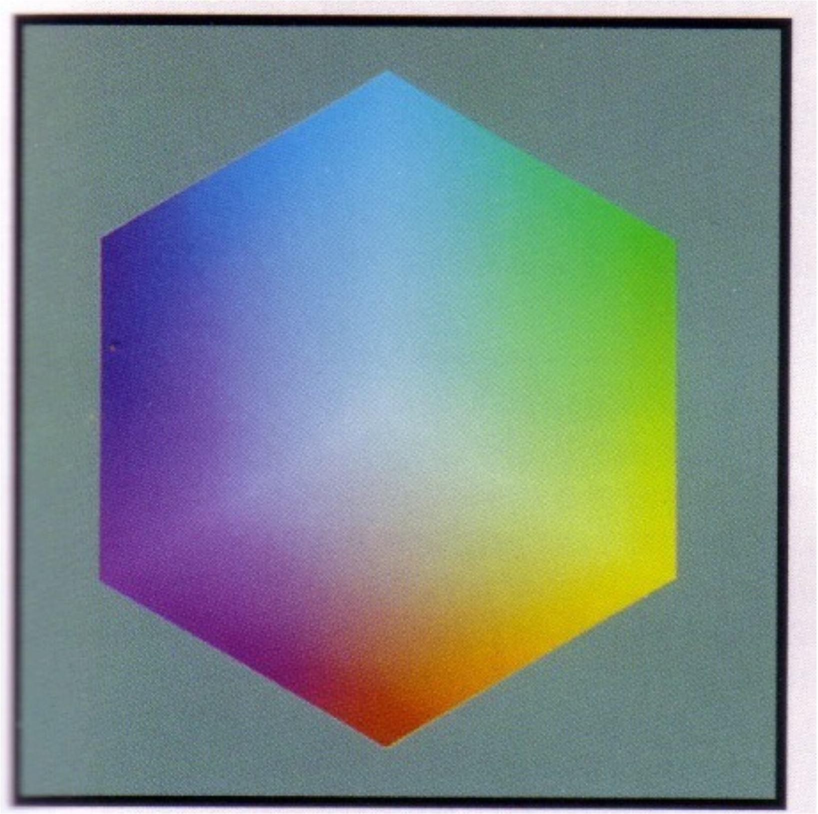
Perceptual dimensions of color

- There's good evidence ("opponent color theory") for a neurological basis for these dimensions
  - the brain seems to encode color early on using three axes:
    - white — black, red — green, yellow — blue
  - the white—black axis is lightness; the others determine hue and saturation
  - one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can't have a reddish green (just doesn't make sense)
    - thus red is the *opponent* to green
  - another piece of evidence: afterimages (next slide)



RGB as a 3D space

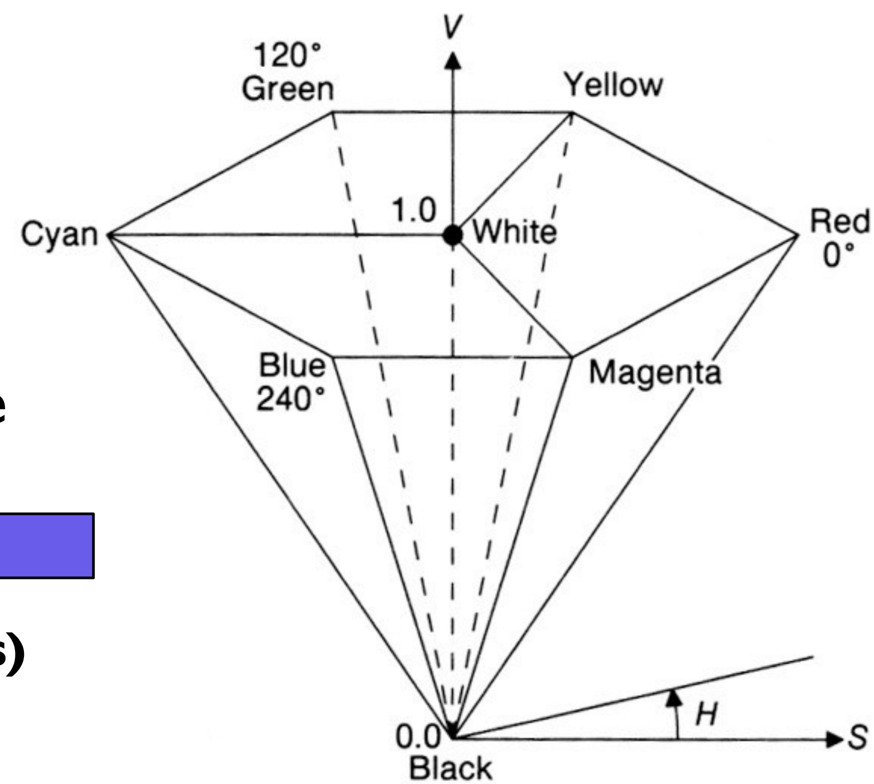
- A cube:



Perceptual organization for RGB: HSV

- Uses hue (an angle, 0 to 360), saturation (0 to 1), and value (0 to 1) as the three coordinates for a color
  - the brightest available RGB colors are those with one of R,G,B equal to 1 (top surface)
  - each horizontal slice is the surface of a sub-cube of the RGB cube

(demo of HSV color pickers)



[FvDFH]

Perceptually uniform spaces

- Two major spaces standardized by CIE
  - designed so that equal differences in coordinates produce equally visible differences in color
  - LUV: earlier, simpler space; L^*, u^*, v^*
  - LAB: more complex but more uniform: L^*, a^*, b^*
  - both separate luminance from chromaticity
  - including a gamma-like nonlinear component is important

