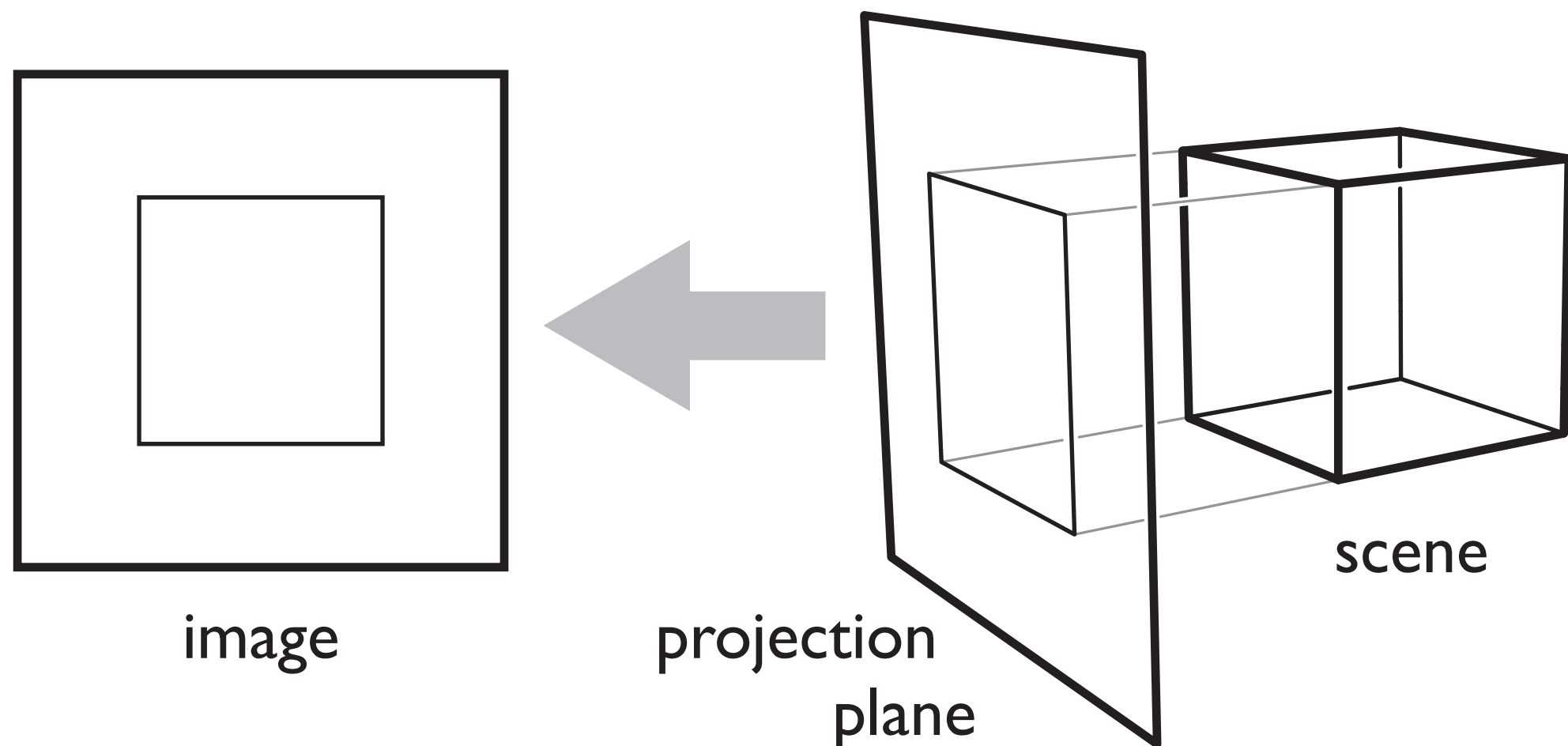


# Perspective

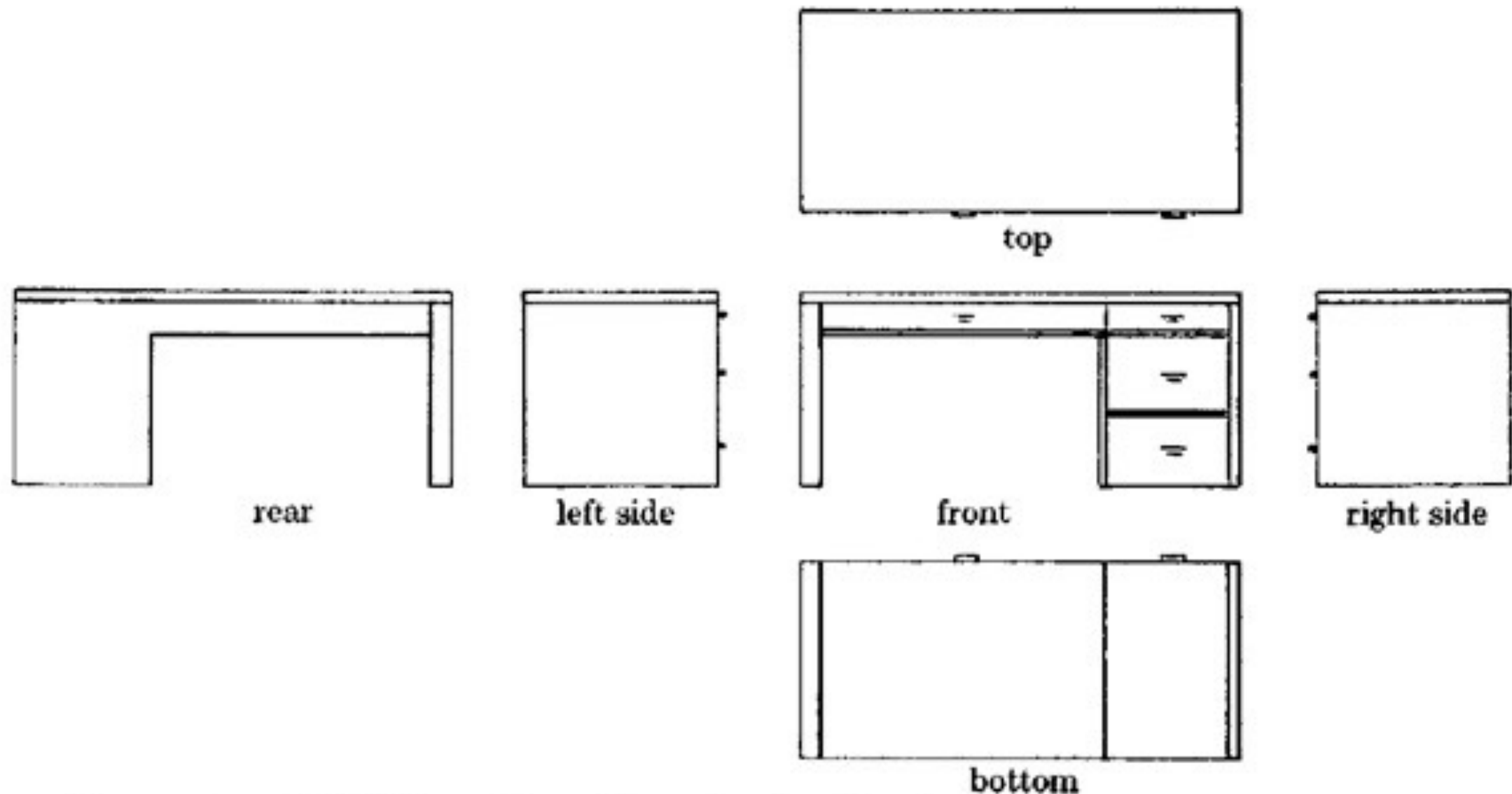
## **CS 4620 Lecture 5**

# Parallel projection

- To render an image of a 3D scene, we *project* it onto a plane
- Simplest kind of projection is *parallel projection*



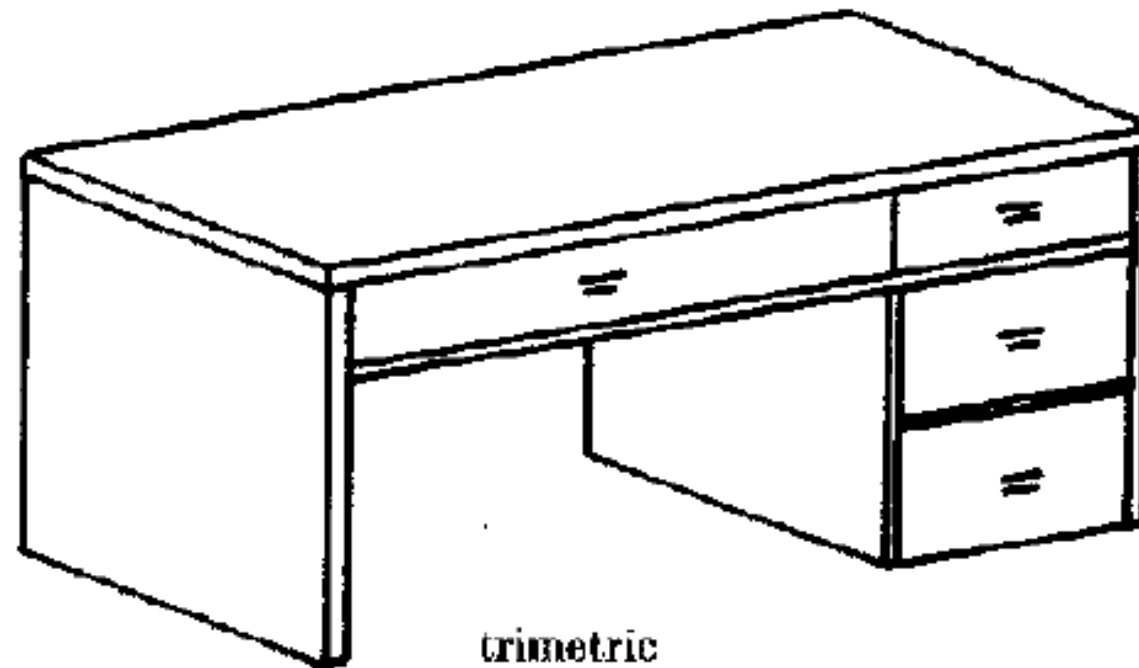
# Orthographic in traditional drawing



[Carl bom & Paciorek 78]

- projection plane parallel to a coordinate plane
- projection direction perpendicular to projection plane

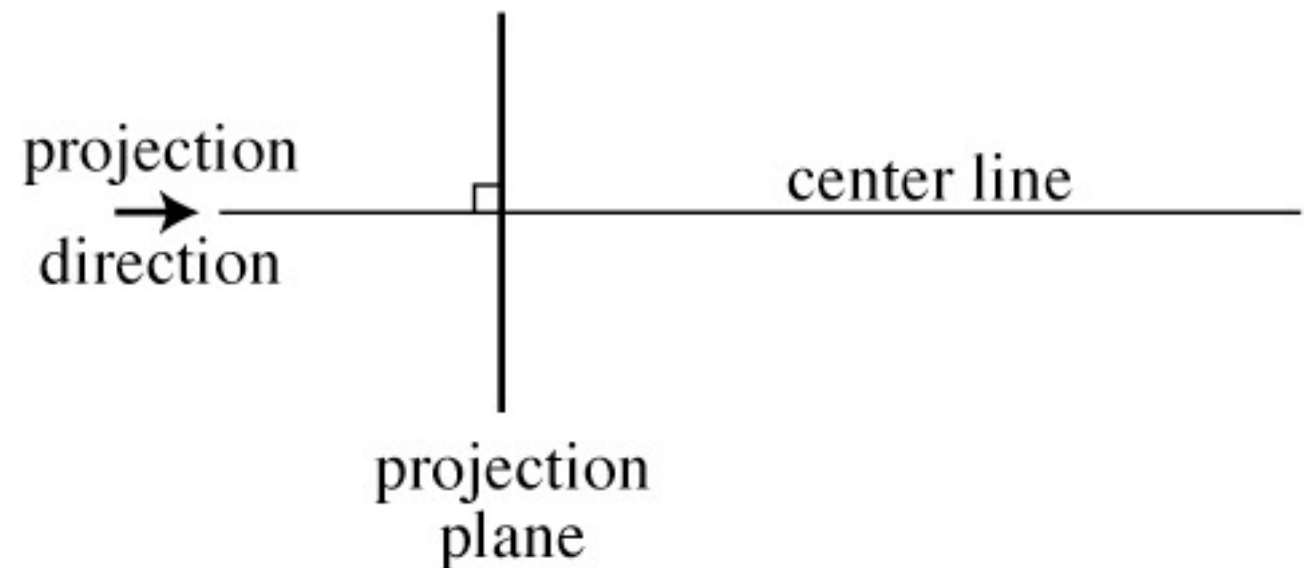
# Other parallel in traditional drawing



**axonometric:** projection plane perpendicular to projection direction but not parallel to coordinate planes

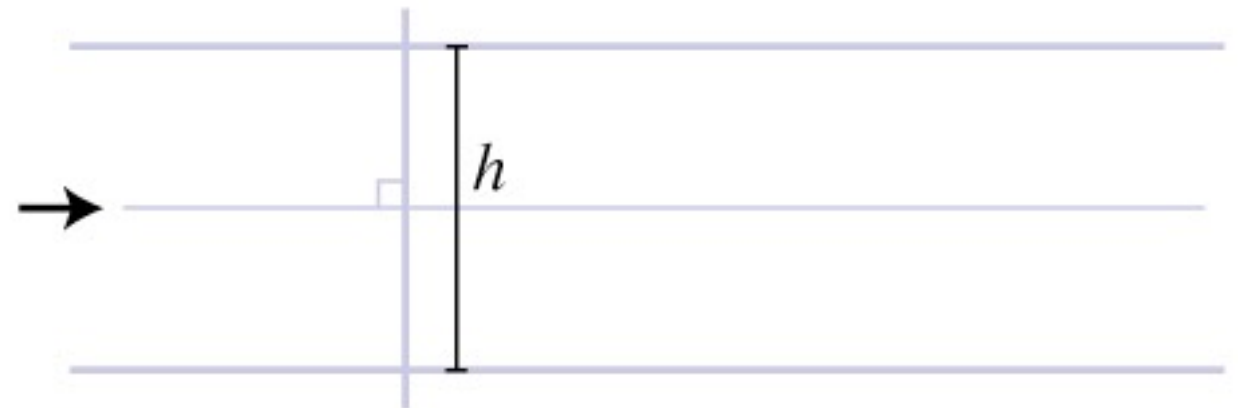
# “Orthographic” projection

- **In graphics usually we lump axonometric with orthographic**
  - projection plane perpendicular to projection direction
  - image height determines size of objects in image



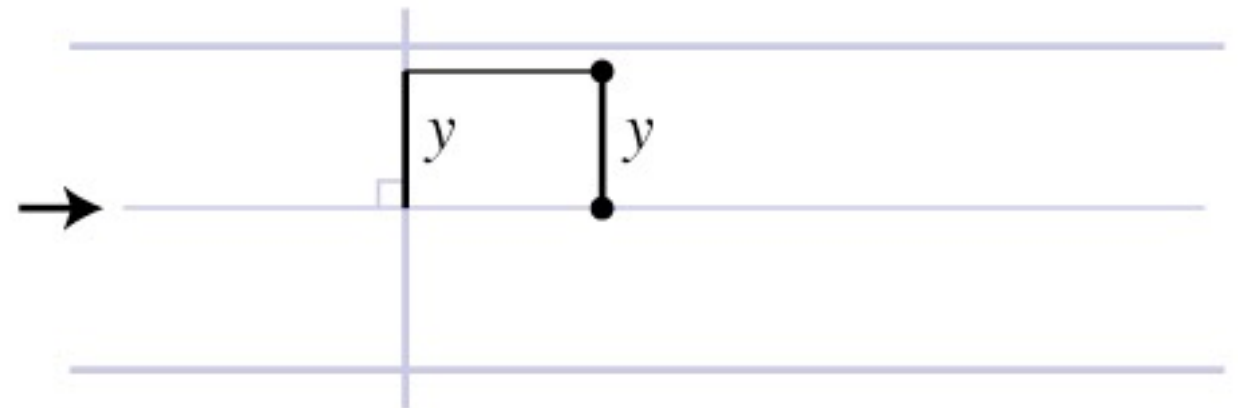
# “Orthographic” projection

- **In graphics usually we lump axonometric with orthographic**
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  - image height determines size of objects in image



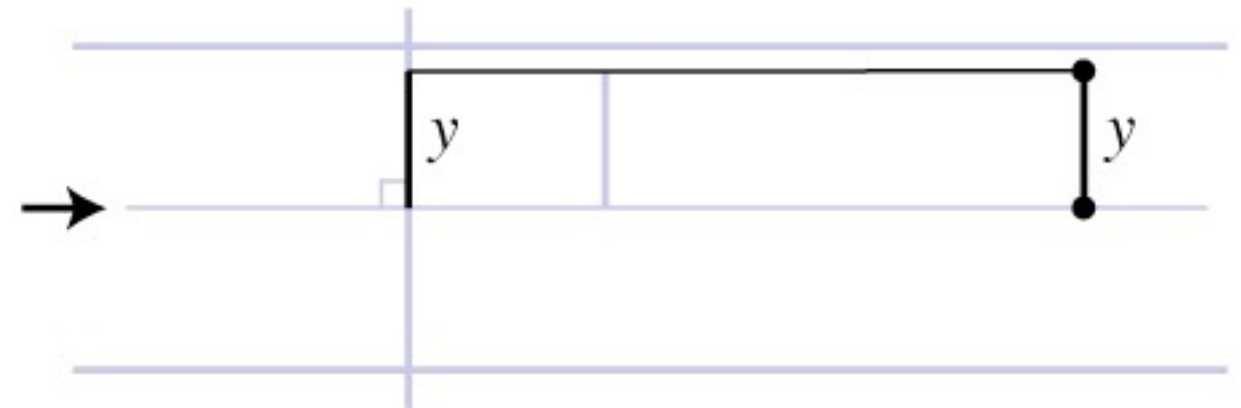
# “Orthographic” projection

- **In graphics usually we lump axonometric with orthographic**
  - projection plane perpendicular to projection direction
  - image height determines size of objects in image



# “Orthographic” projection

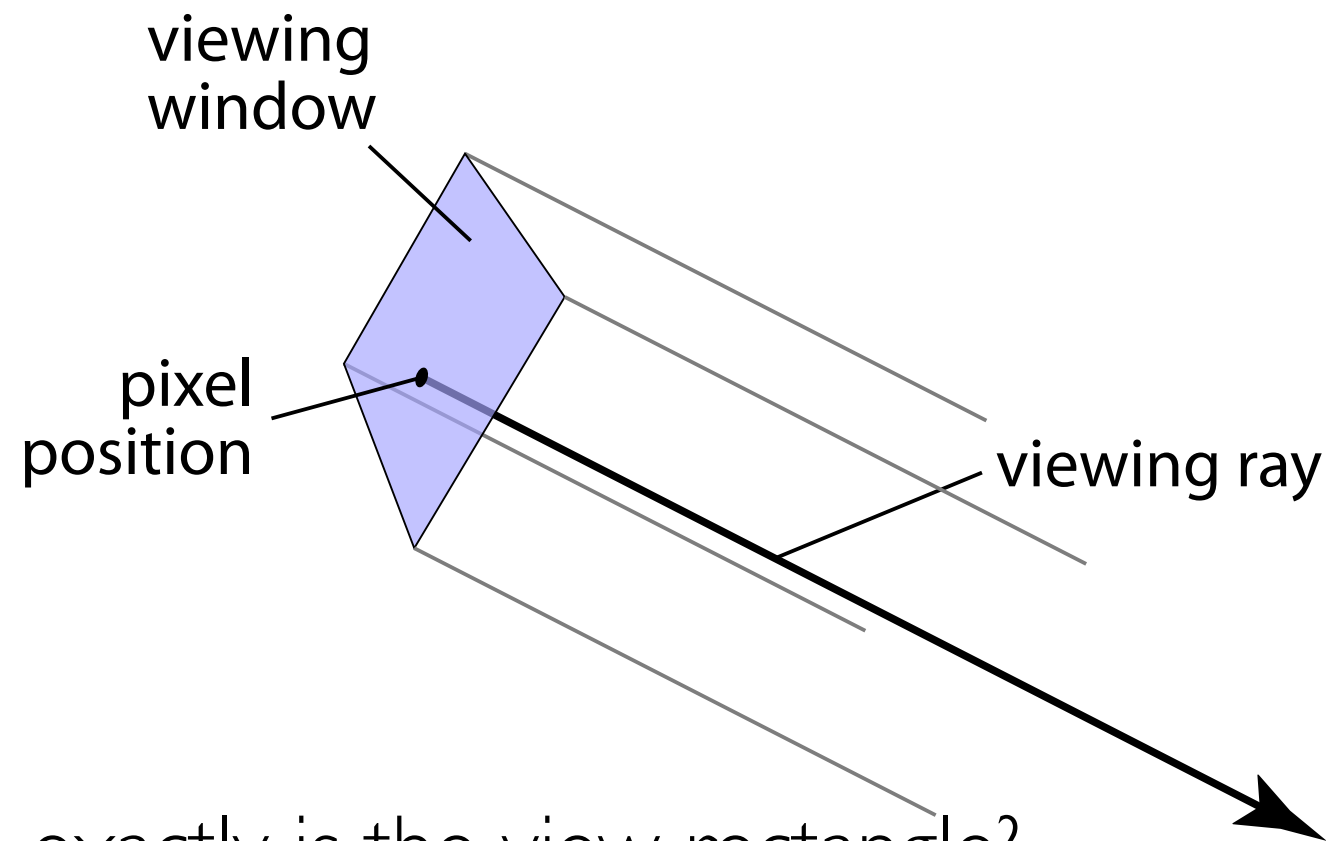
- **In graphics usually we lump axonometric with orthographic**
  - projection plane perpendicular to projection direction
  - image height determines size of objects in image





# Generating eye rays—orthographic

- **Ray origin (varying): pixel position on viewing window**
- **Ray direction (constant): view direction**



— but where exactly is the view rectangle?

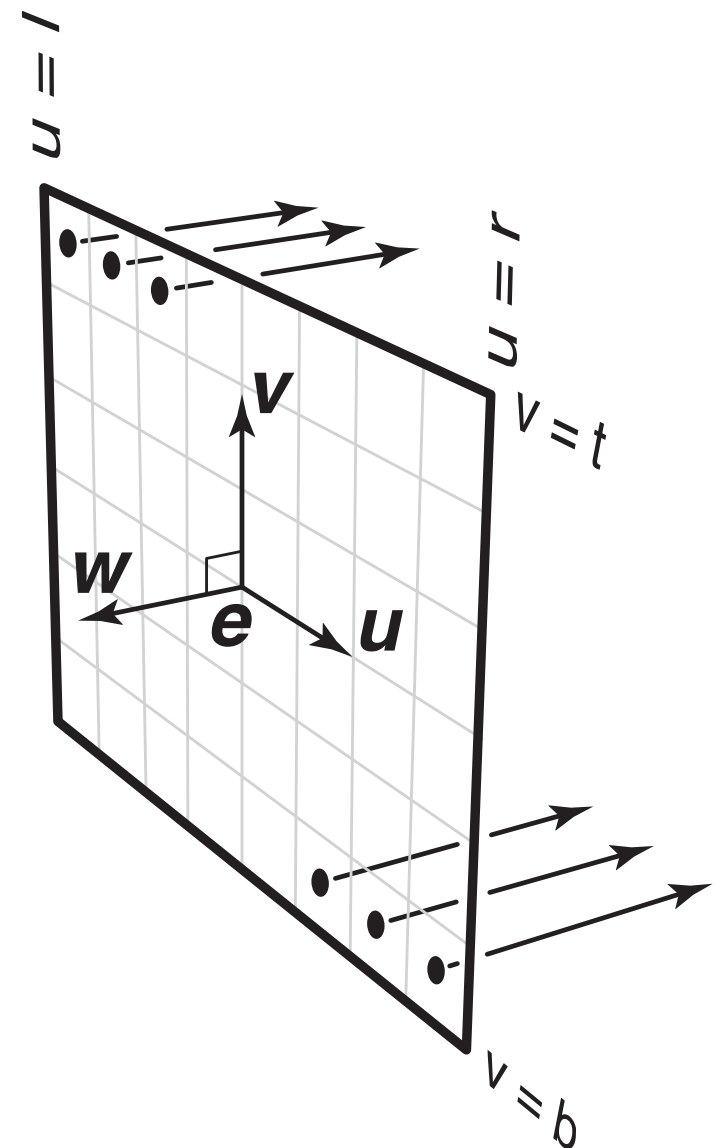
# Generating eye rays—orthographic

- **Positioning the view rectangle**

- establish three vectors to be *camera basis*: **u**, **v**, **w**
- view rectangle is in **u–v** plane, specified by  $l, r, t, b$   
(often  $l = -r$  and  $b = -t$ )

- **Generating rays**

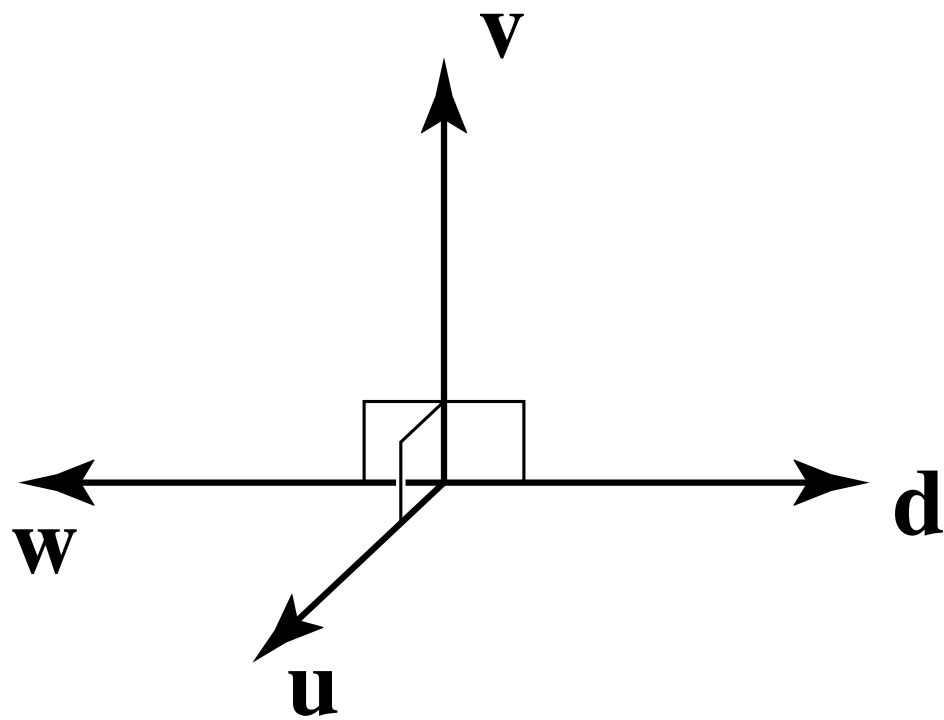
- for  $(u, v)$  in  $[l, r] \times [b, t]$
- $\text{ray.origin} = \mathbf{e} + u \mathbf{u} + v \mathbf{v}$
- $\text{ray.direction} = -\mathbf{w}$



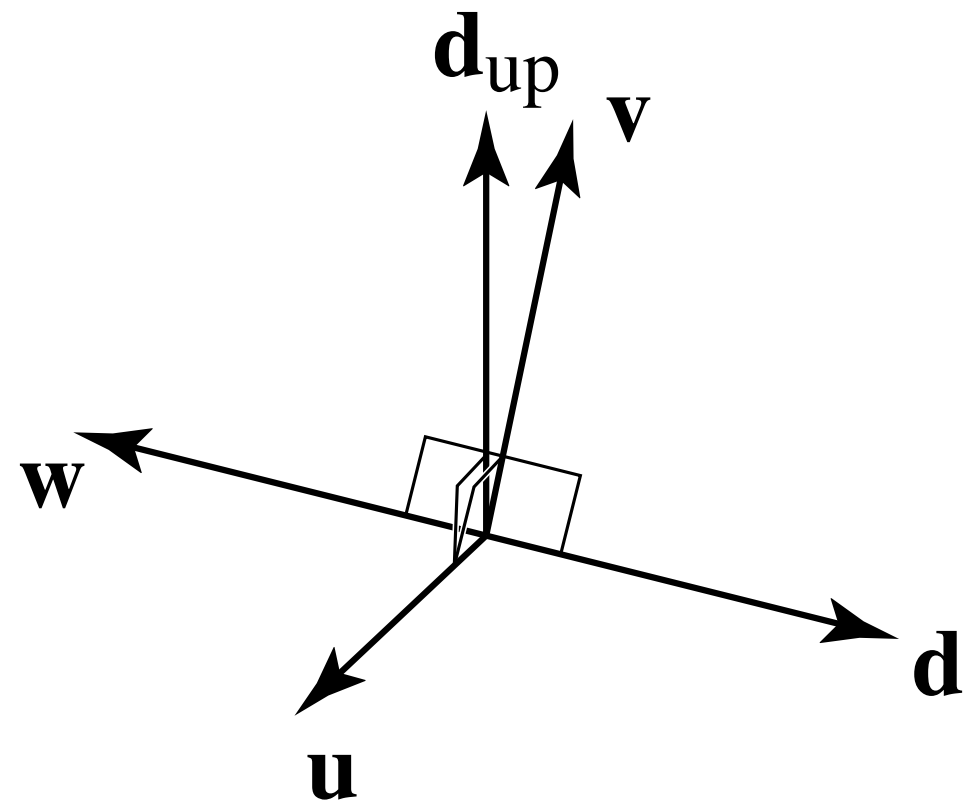
# Establishing the camera basis

- **Could require user to provide  $\mathbf{e}$ ,  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$** 
  - but this is error prone and unintuitive
- **Instead, calculate basis from things the user cares about**
  - viewpoint: where the camera is  $\rightarrow \mathbf{e}$
  - view direction: which way the camera is looking  $\rightarrow \mathbf{d}$
  - view plane normal (normally same as view direction)
  - up vector: how the camera is oriented
- **This is enough to calculate  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$** 
  - set  $\mathbf{w}$  parallel to v.p. normal, facing away from  $\mathbf{d}$
  - set  $\mathbf{u}$  perpendicular to  $\mathbf{w}$  and perpendicular to up-vector
  - set  $\mathbf{v}$  perpendicular to  $\mathbf{w}$  and  $\mathbf{u}$  to form a right-handed ONB

# Camera basis



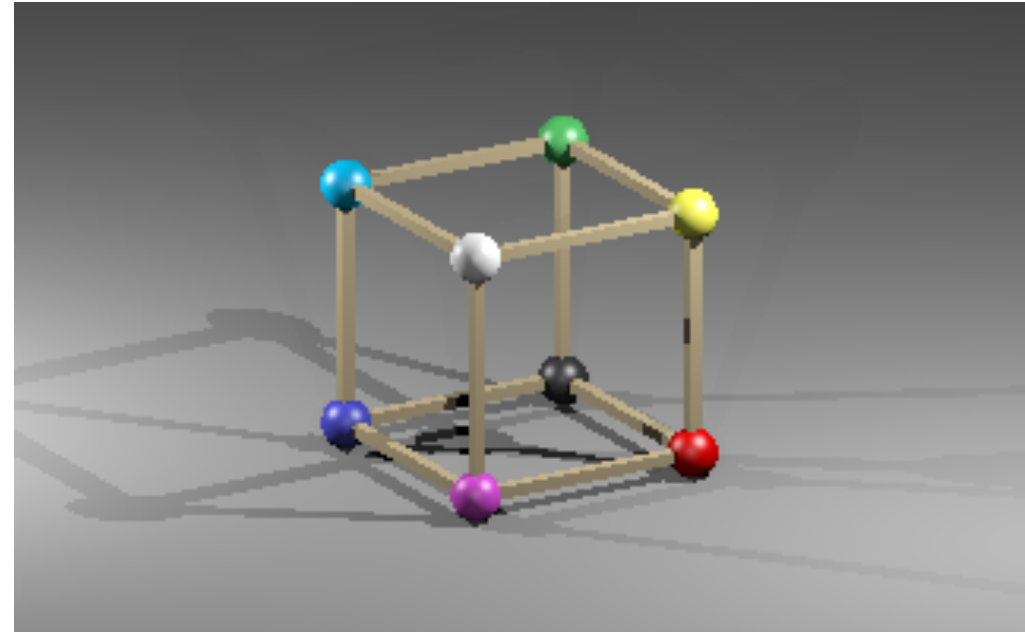
forming the basis  
with  $\mathbf{d}$  and  $\mathbf{v}$  given



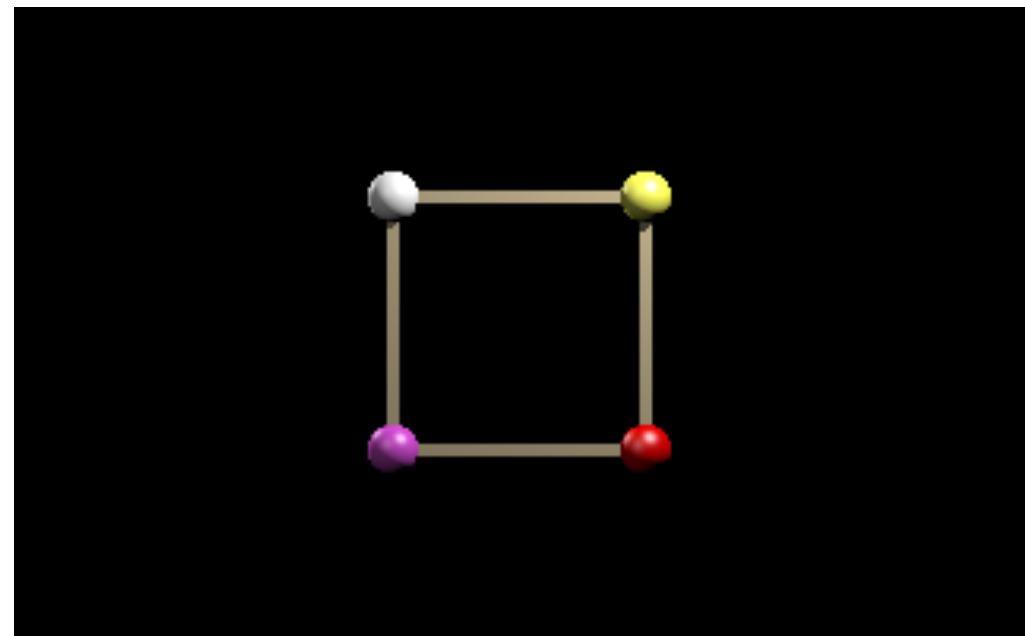
forming the basis  
with  $\mathbf{d}$  and up vector given

# Orthographic views in Ray 1

```
<camera type="OrthographicCamera">  
  <viewPoint>10 4.2 6</viewPoint>  
  <viewDir>-5 -2.1 -3</viewDir>  
  <viewUp>0 1 0</viewUp>  
  <viewWidth>8</viewWidth>  
  <viewHeight>5</viewHeight>  
</camera>
```



```
<camera type="OrthographicCamera">  
  <viewPoint>10 0 0</viewPoint>  
  <viewDir>-1 0 0</viewDir>  
  <viewUp>0 1 0</viewUp>  
  <viewWidth>8</viewWidth>  
  <viewHeight>5</viewHeight>  
</camera>
```



# History of projection

- **Ancient times: Greeks wrote about laws of perspective**
- **Renaissance: perspective is adopted by artists**

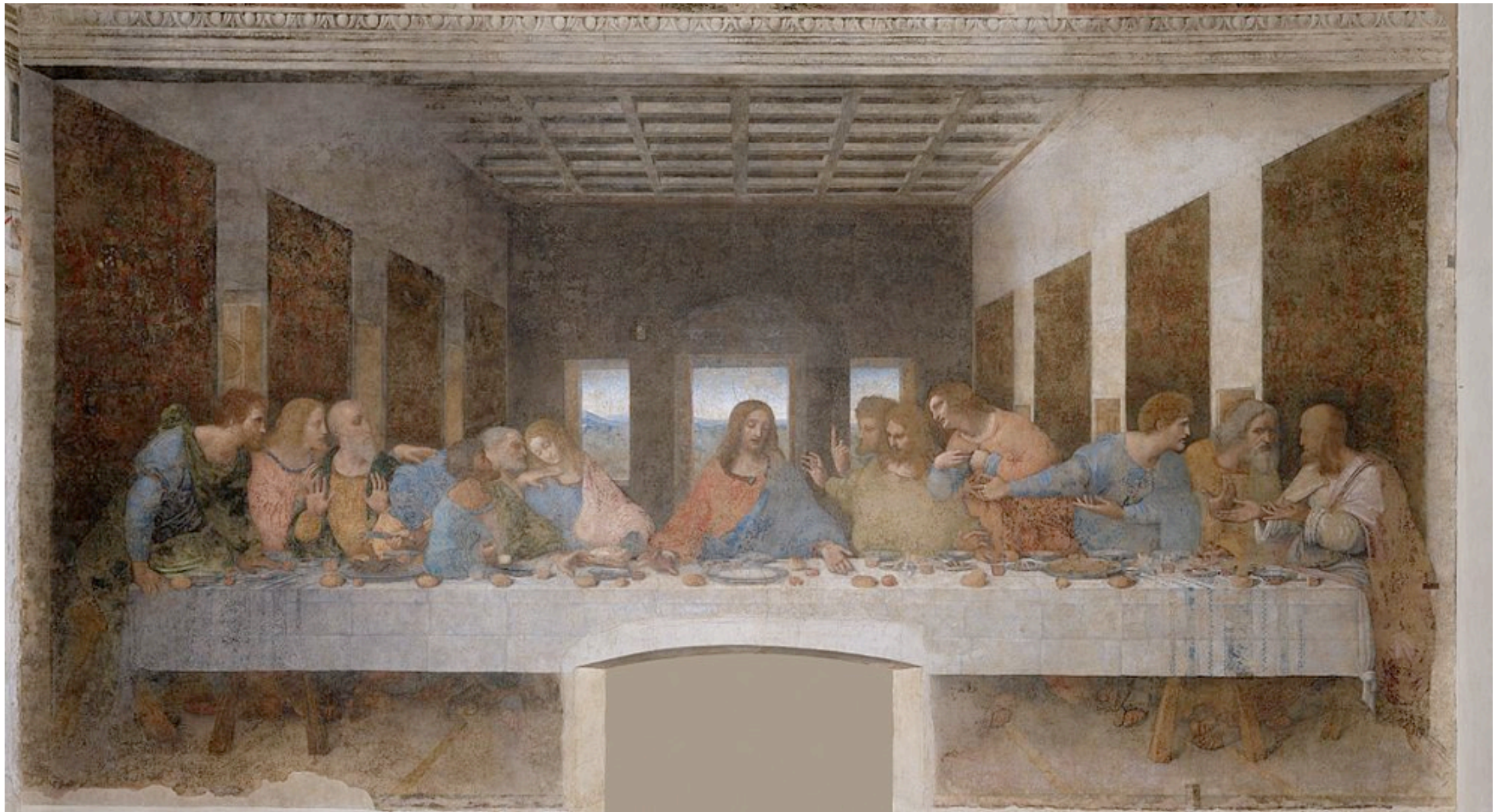


Duccio c. 1308



# History of projection

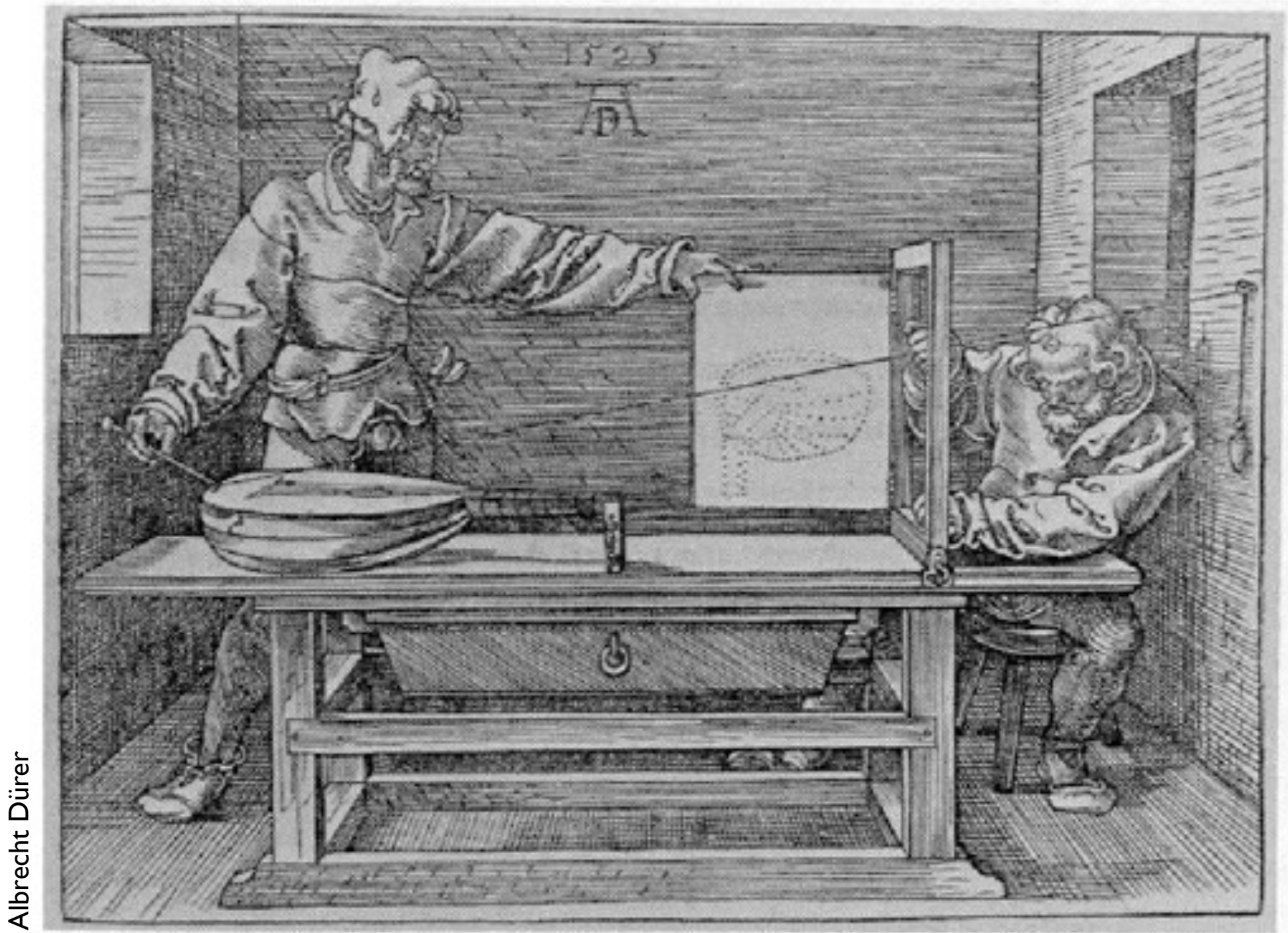
- **Later Renaissance: perspective formalized precisely**



da Vinci c. 1498



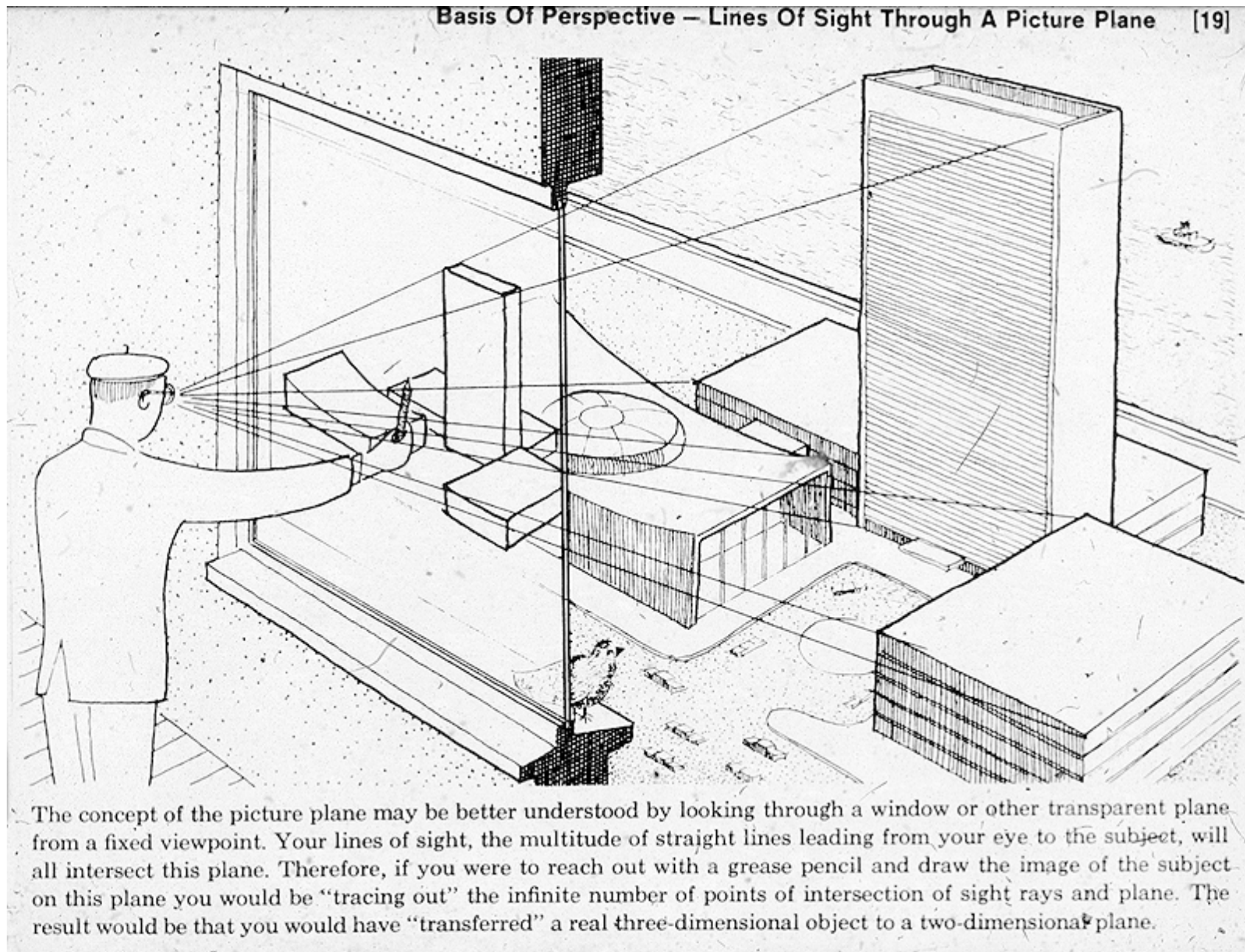
# Plane projection in drawing



Albrecht Dürer



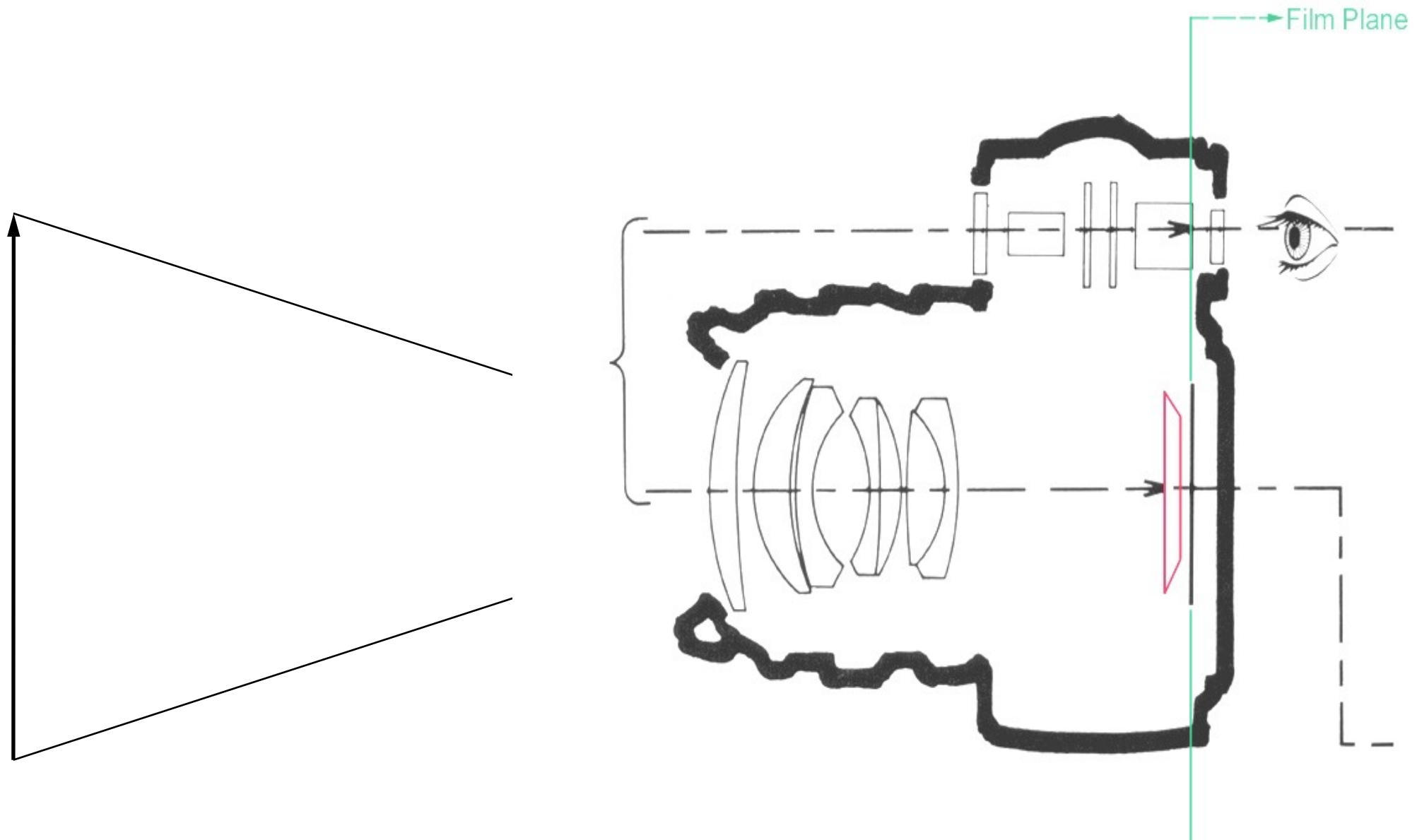
# Plane projection in drawing



source unknown

# Plane projection in photography

- **This is another model for what we are doing**
  - applies more directly in realistic rendering



[Source unknown]

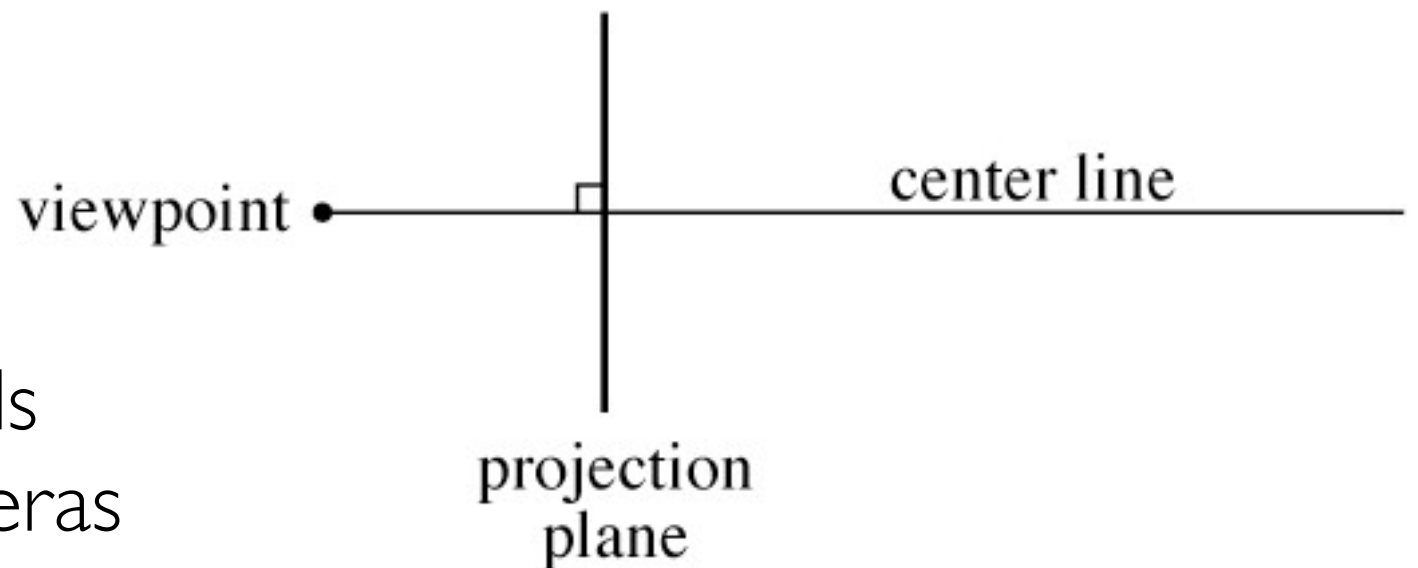
# Plane projection in photography



[Richard Zakia]

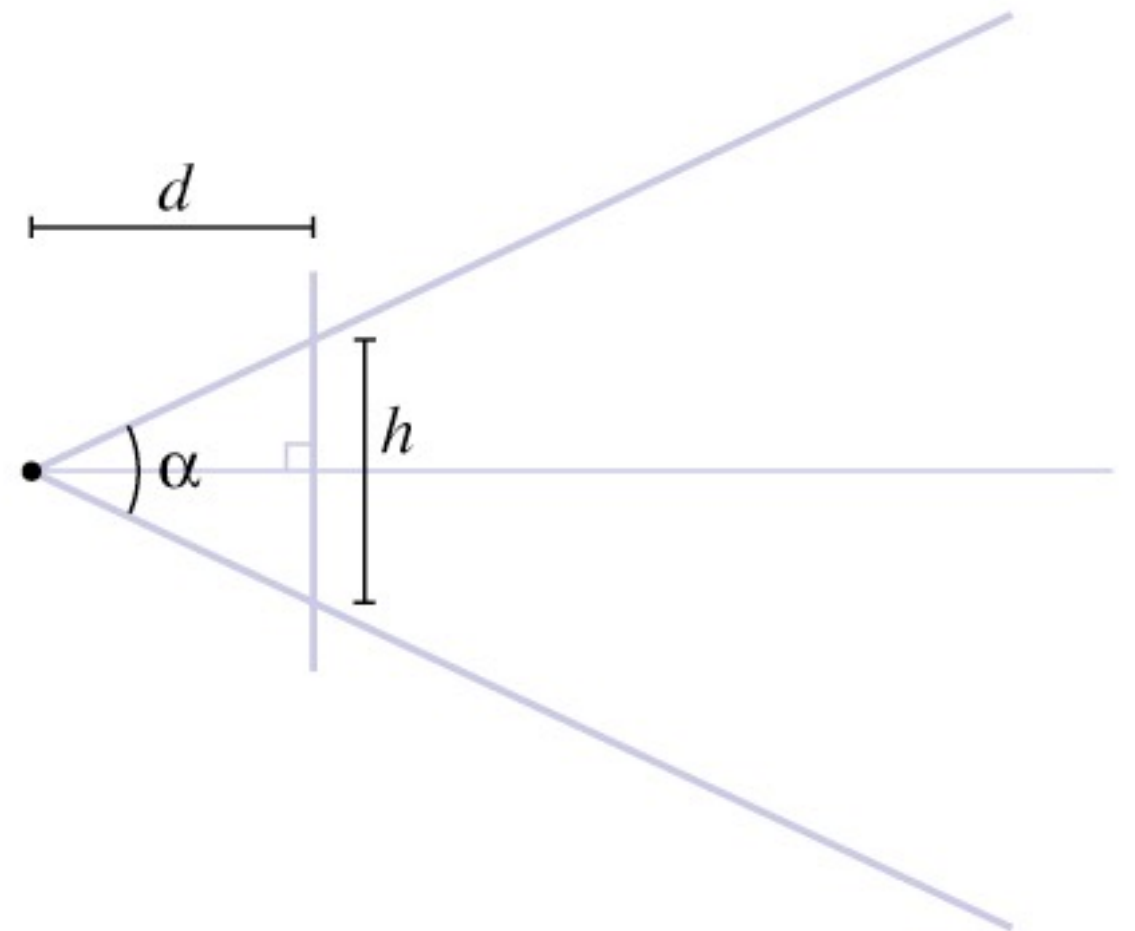
# Perspective projection (normal)

- **Perspective is projection by lines through a point; “normal” = plane perpendicular to view direction**
  - magnification determined by:
    - image height
    - object depth
    - image plane distance
  - f.o.v.  $\alpha = 2 \operatorname{atan}(h/(2d))$
  - $y' = d y / z$
  - “normal” case corresponds to common types of cameras



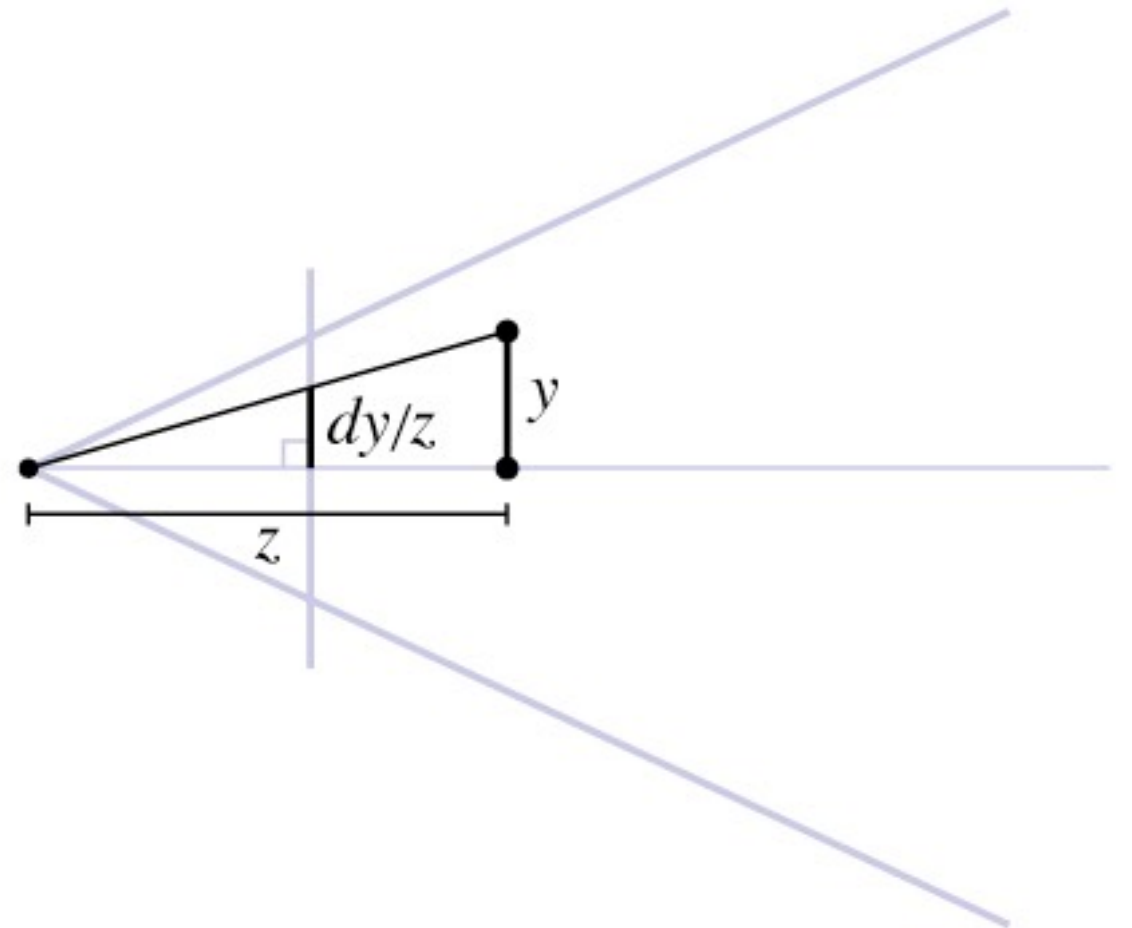
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# Perspective projection (normal)

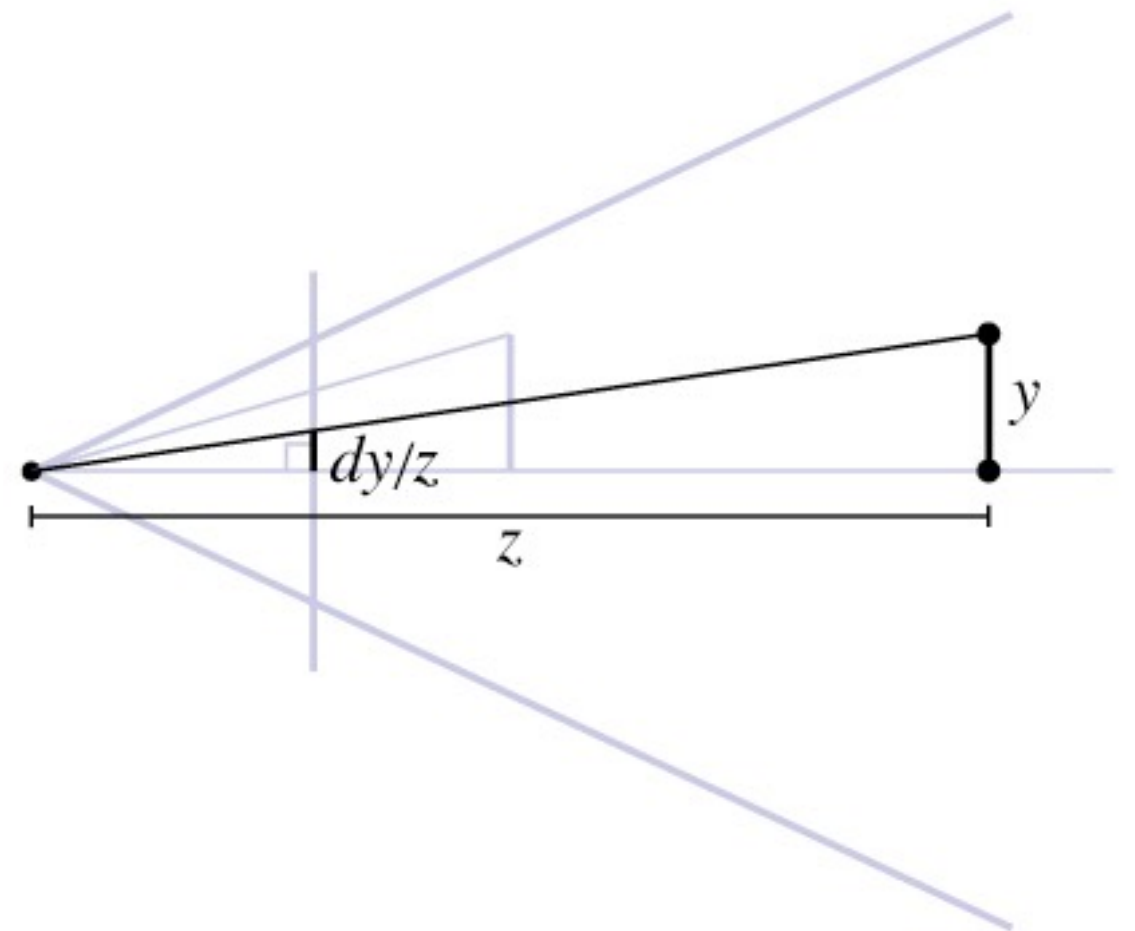
- **Perspective is projection by lines through a point; “normal” = plane perpendicular to view direction**
  - magnification determined by:
    - image height
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    - image plane distance
  - f.o.v.  $\alpha = 2 \operatorname{atan}(h/(2d))$
  - $y' = d y / z$
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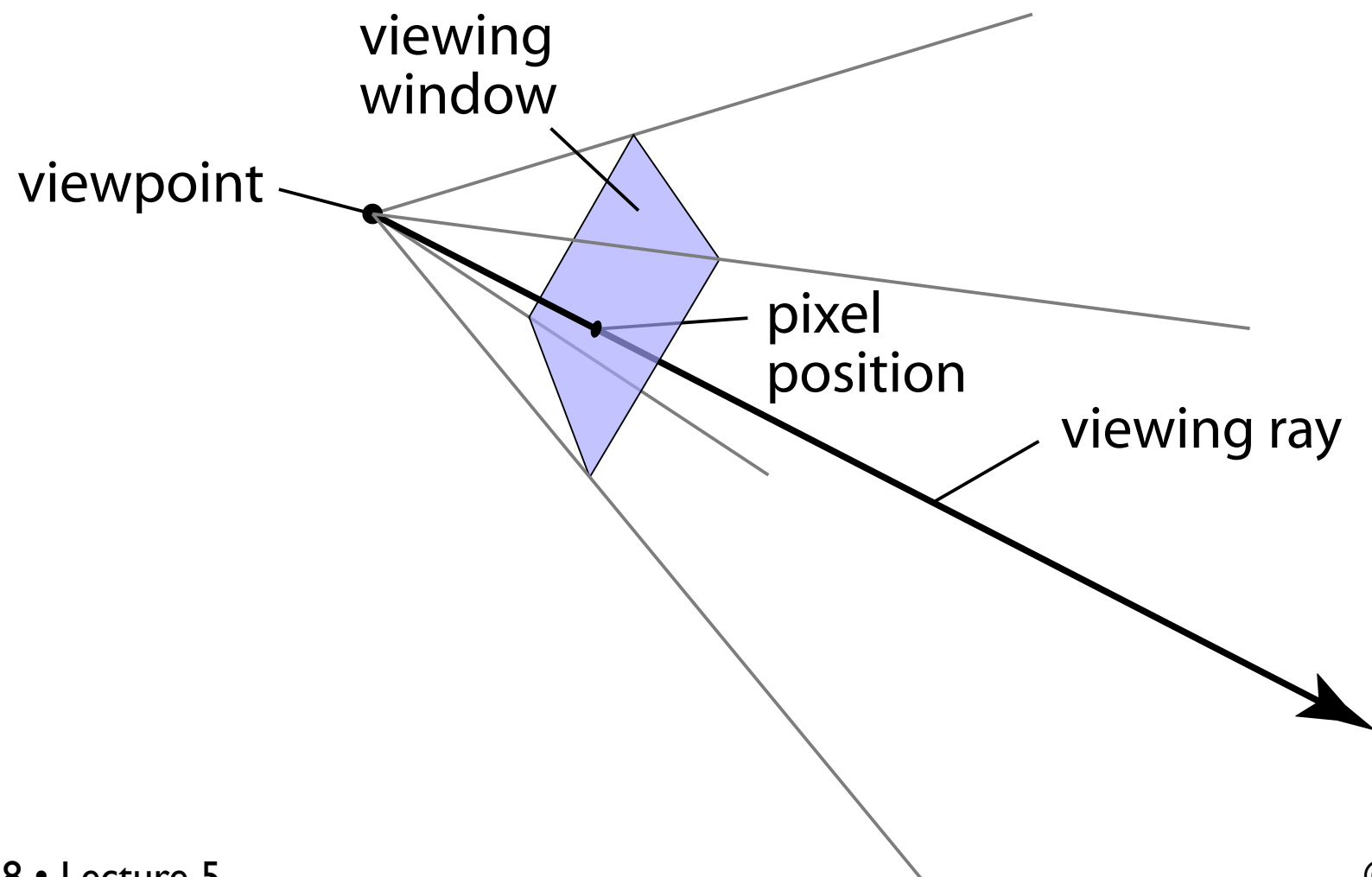
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    - object depth
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  - f.o.v.  $\alpha = 2 \operatorname{atan}(h/(2d))$
  - $y' = d y / z$
  - “normal” case corresponds to common types of cameras



# Generating eye rays—perspective

- **Use window analogy directly**
- **Ray origin (constant): viewpoint**
- **Ray direction (varying): toward pixel position on viewing window**





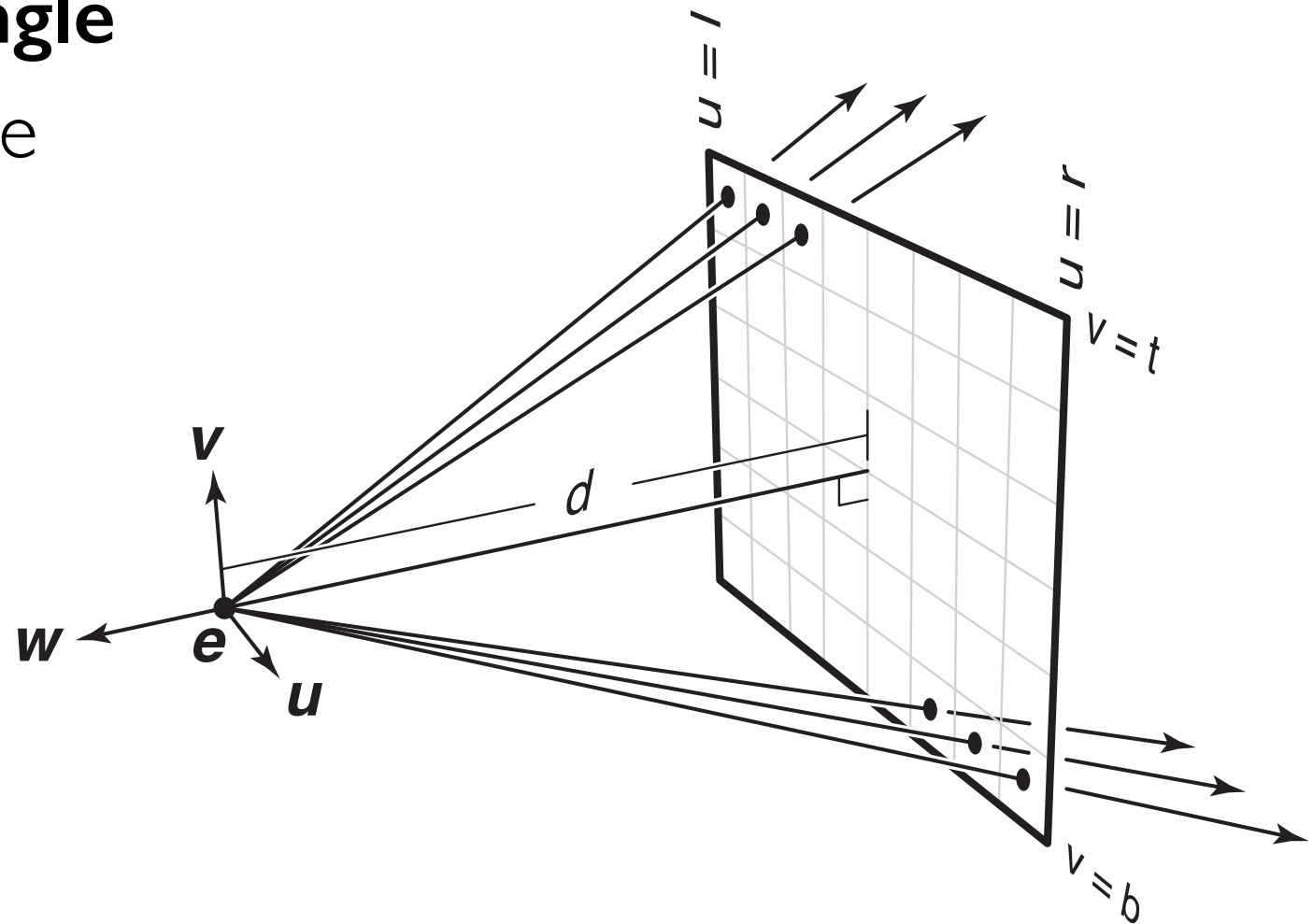
# Generating eye rays—perspective

- **Positioning the view rectangle**

- establish three vectors to be *camera basis*: **u**, **v**, **w**
- view rectangle is parallel to **u**–**v** plane, at  $w = -d$ , specified by  $l, r, t, b$

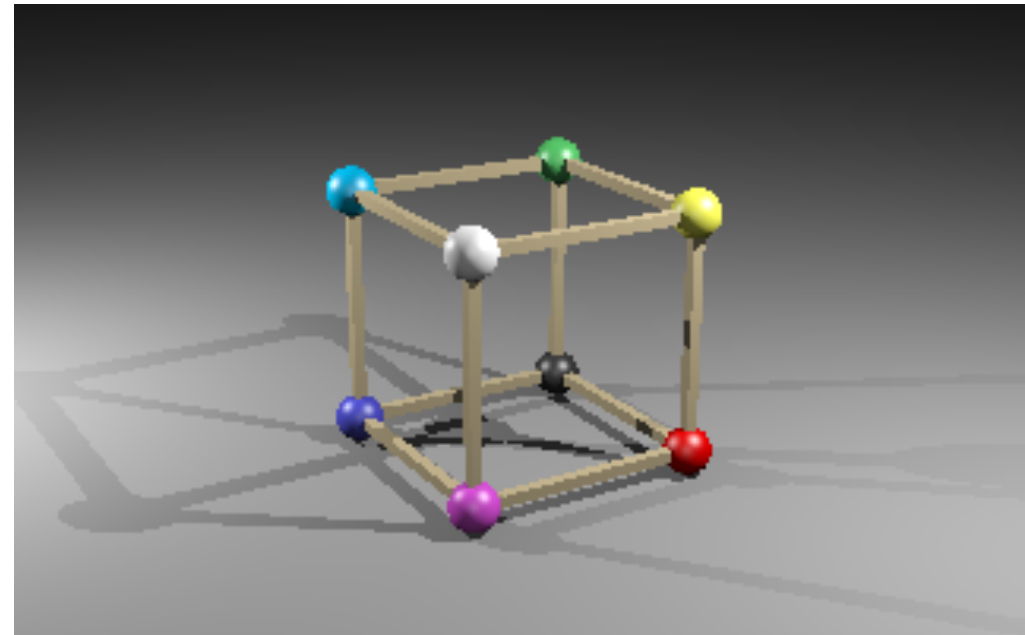
- **Generating rays**

- for  $(u, v)$  in  $[l, r] \times [b, t]$
- ray.origin = **e**
- ray.direction =  $-d \mathbf{w} + u \mathbf{u} + v \mathbf{v}$

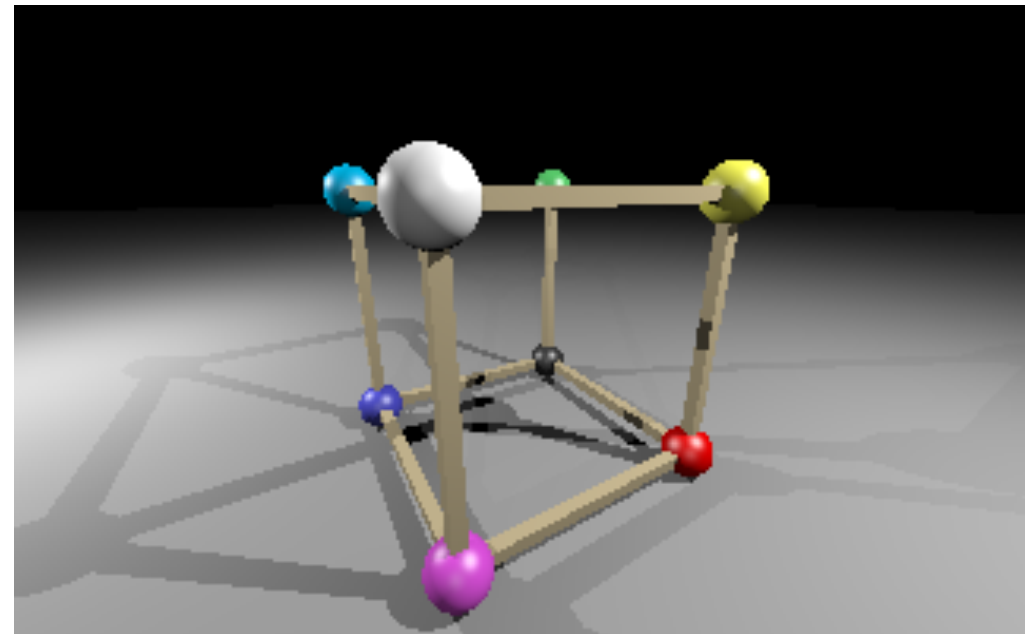


# Perspective views in Ray I

```
<camera type="PerspectiveCamera">  
  <viewPoint>10 4.2 6</viewPoint>  
  <viewDir>-5 -2.1 -3</viewDir>  
  <viewUp>0 1 0</viewUp>  
  <projDistance>12</projDistance>  
  <viewWidth>8</viewWidth>  
  <viewHeight>5</viewHeight>  
</camera>
```



```
<camera type="PerspectiveCamera">  
  <viewPoint>2.5 1.05 1.5</viewPoint>  
  <viewDir>-5 -2.1 -3</viewDir>  
  <viewUp>0 1 0</viewUp>  
  <projDistance>3</projDistance>  
  <viewWidth>8</viewWidth>  
  <viewHeight>5</viewHeight>  
</camera>
```



# Field of view (or f.o.v.)

- **The angle between the rays corresponding to opposite edges of a perspective image**
  - simpler to compute for “normal” perspective
  - have to decide to measure vert., horiz., or diag.
- **In cameras, determined by focal length**
  - confusing because of many image sizes
  - for 35mm format (36mm by 24mm image)
    - 18mm =  $67^\circ$  v.f.o.v. — super-wide angle
    - 28mm =  $46^\circ$  v.f.o.v. — wide angle
    - 50mm =  $27^\circ$  v.f.o.v. — “normal”
    - 100mm =  $14^\circ$  v.f.o.v. — narrow angle (“telephoto”)

# Field of view

- Determines “strength” of perspective effects



close viewpoint  
wide angle  
prominent foreshortening



far viewpoint  
narrow angle  
little foreshortening

[Ansel Adams]

# Choice of field of view

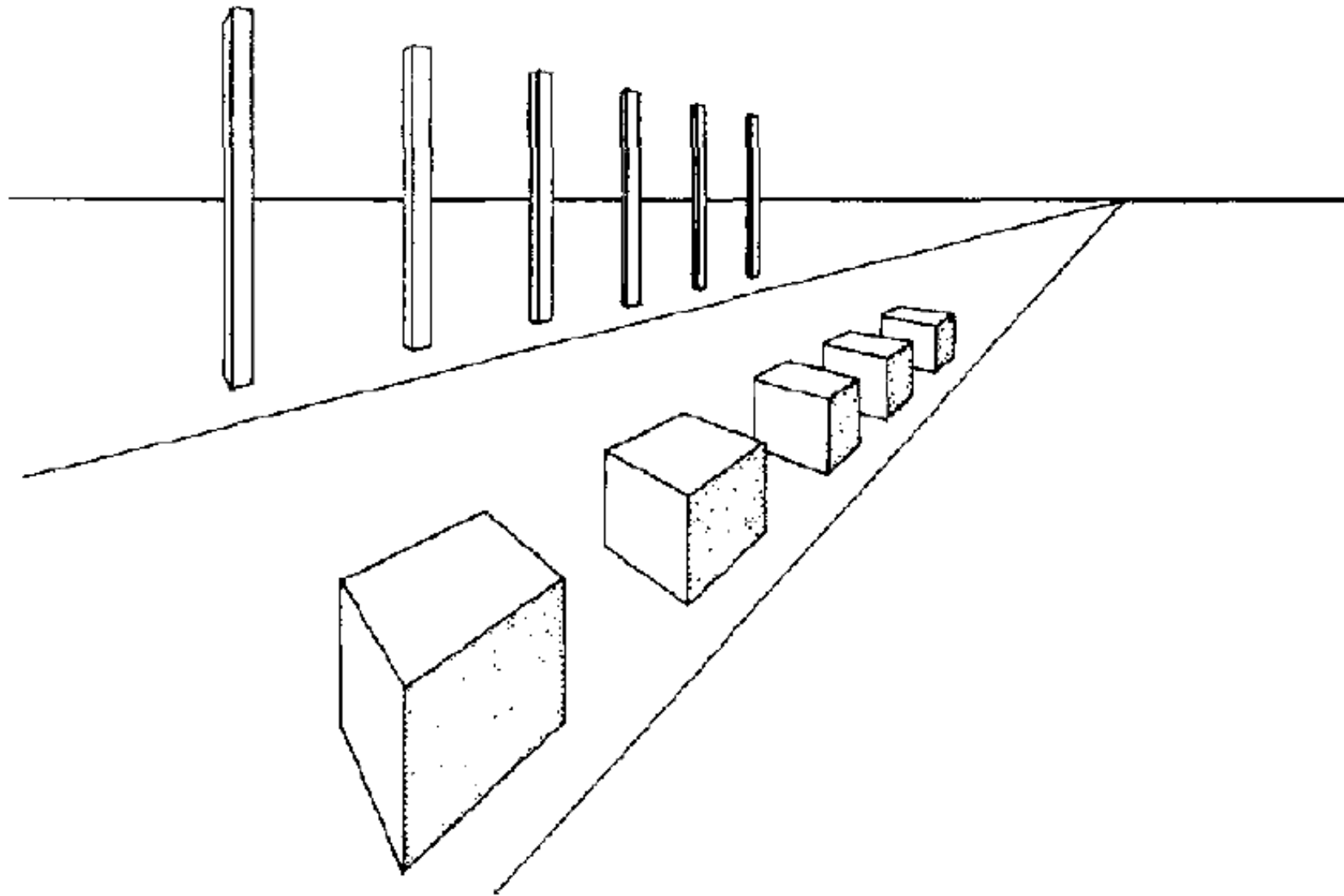
- **In photography, wide angle lenses are specialty tools**
  - “hard to work with”
  - easy to create weird-looking perspective effects
- **In graphics, you can type in whatever f.o.v. you want**
  - and people often type in big numbers!



[Ken Perlin]

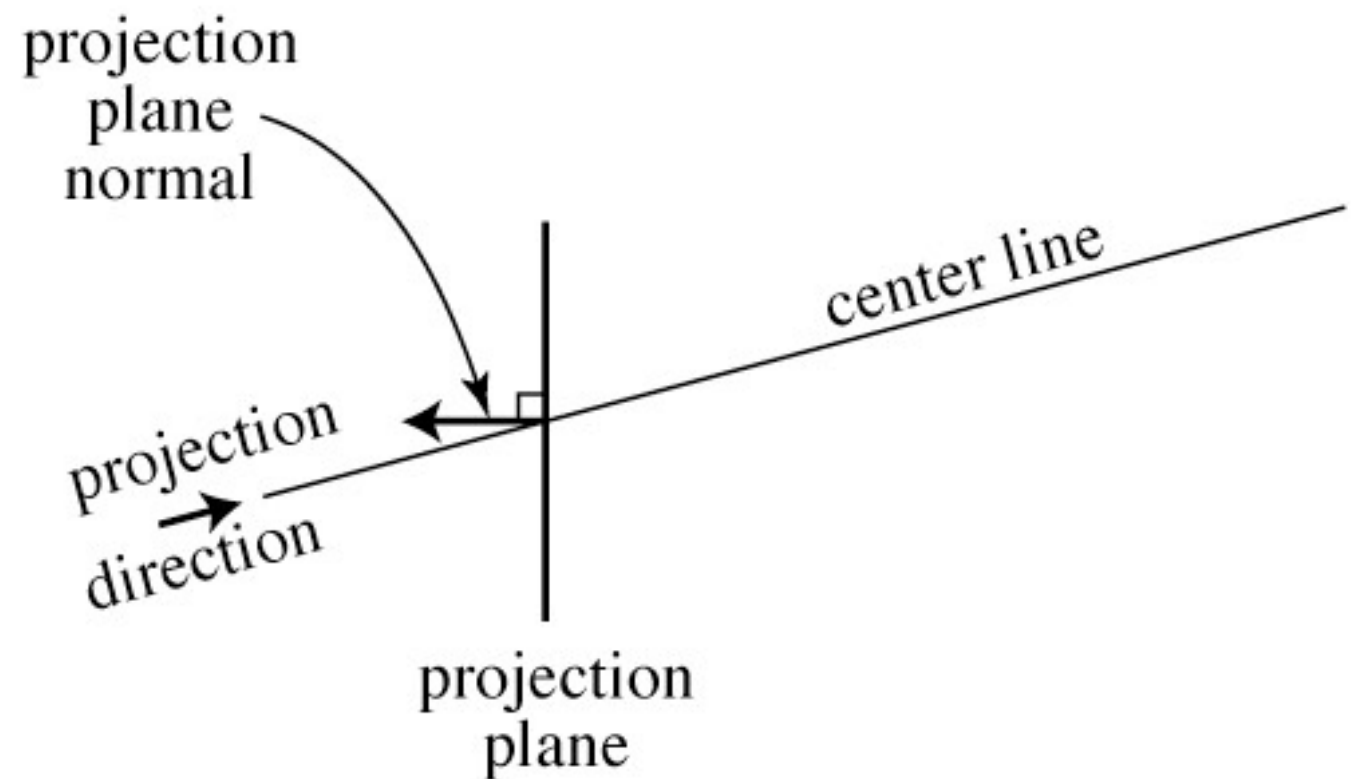
# Perspective distortions

- **Lengths, length ratios**



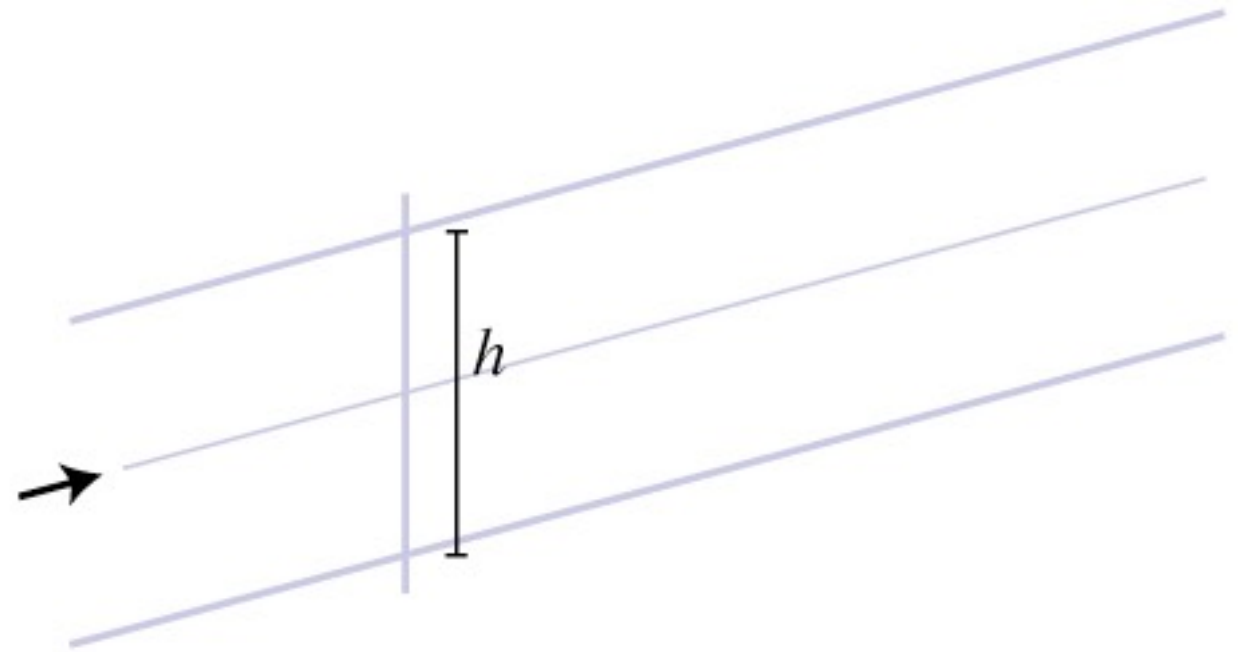
# Oblique projection

- **View direction no longer coincides with projection plane normal (one more parameter)**
  - objects at different distances still same size
  - objects are shifted in the image depending on their depth



# Oblique projection

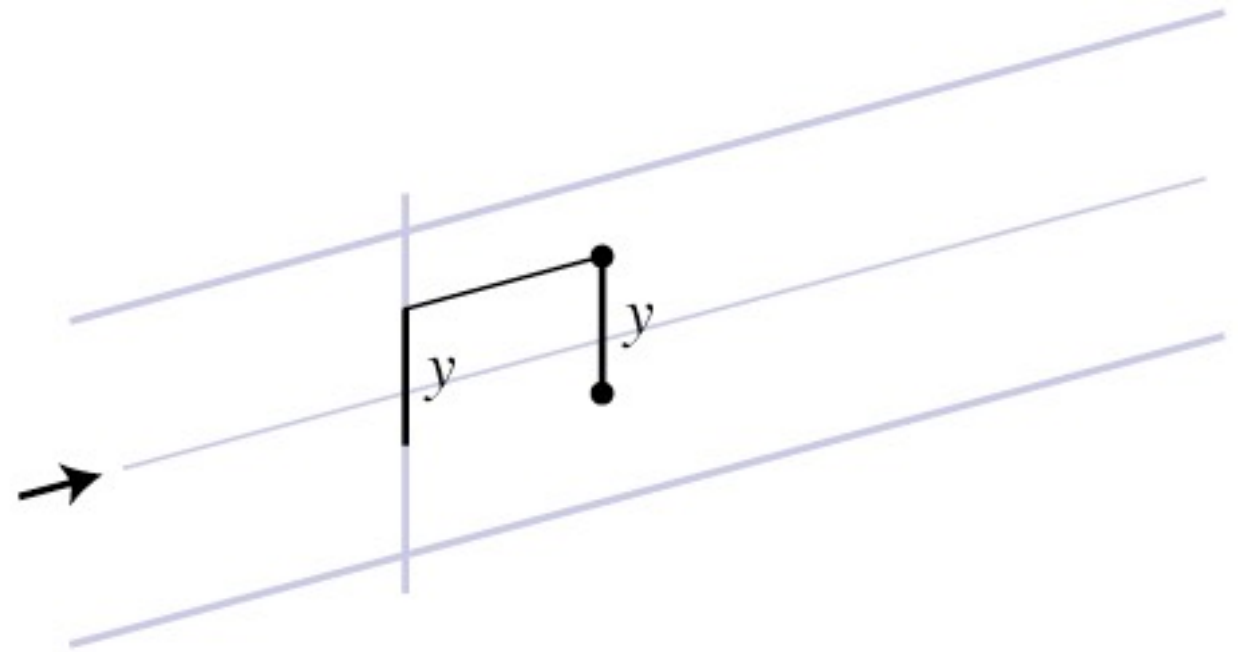
- **View direction no longer coincides with projection plane normal (one more parameter)**
  - objects at different distances still same size
  - objects are shifted in the image depending on their depth





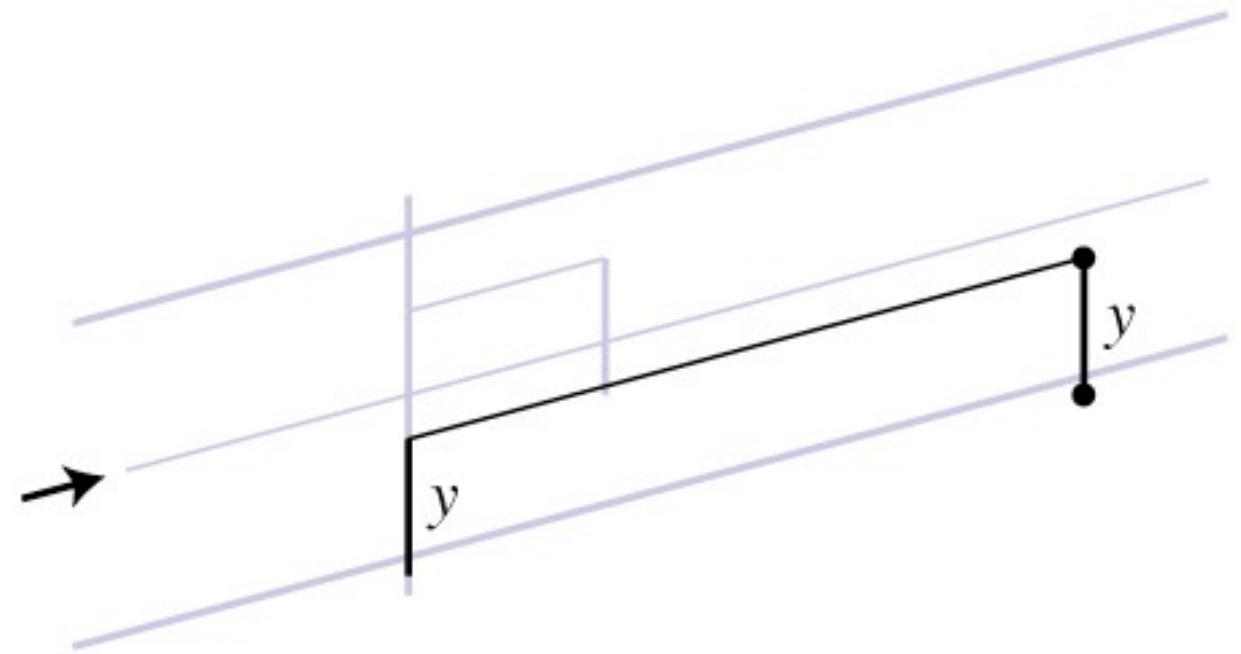
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  - objects at different distances still same size
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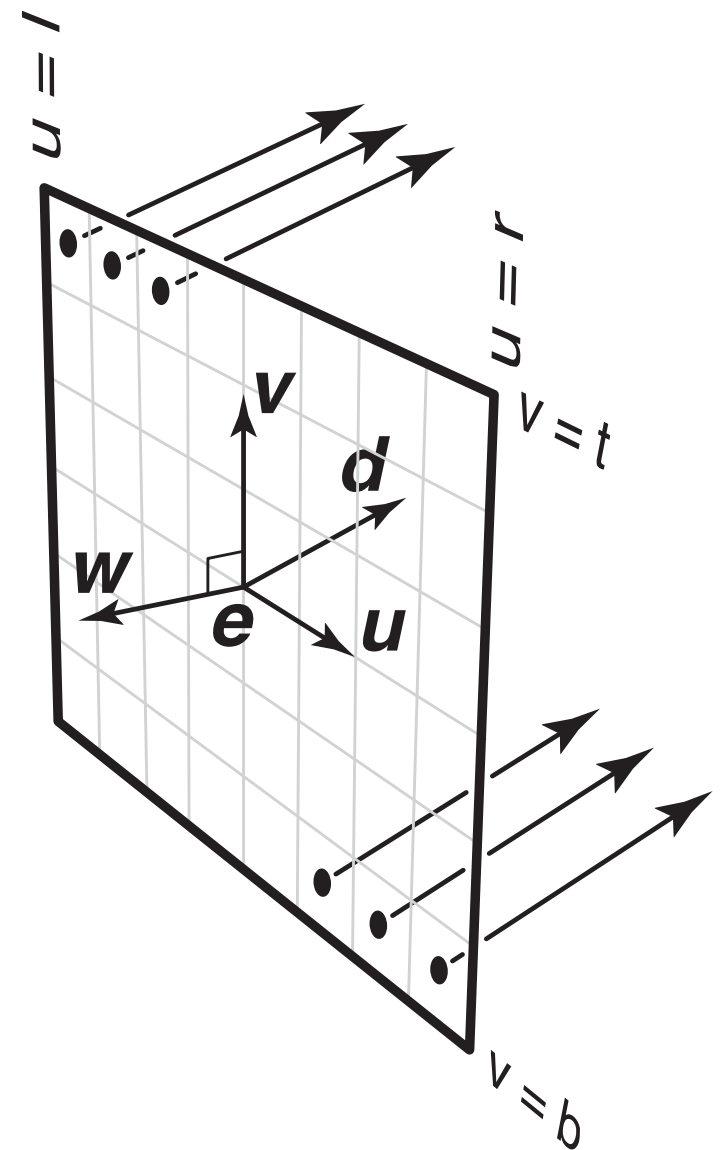
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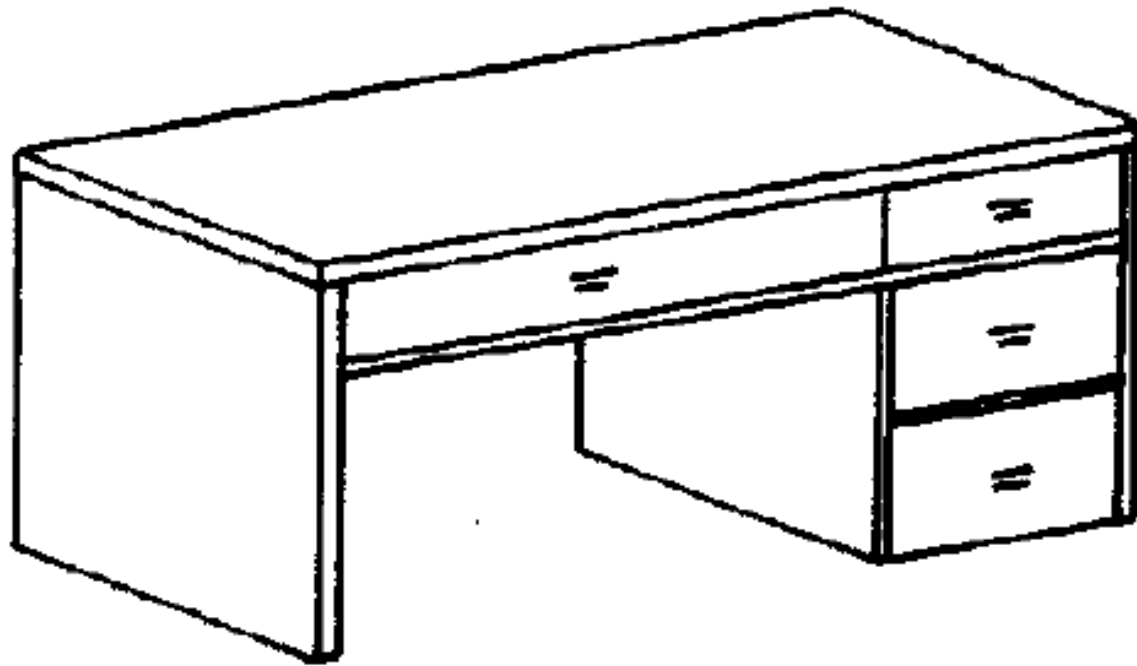


# Oblique parallel views

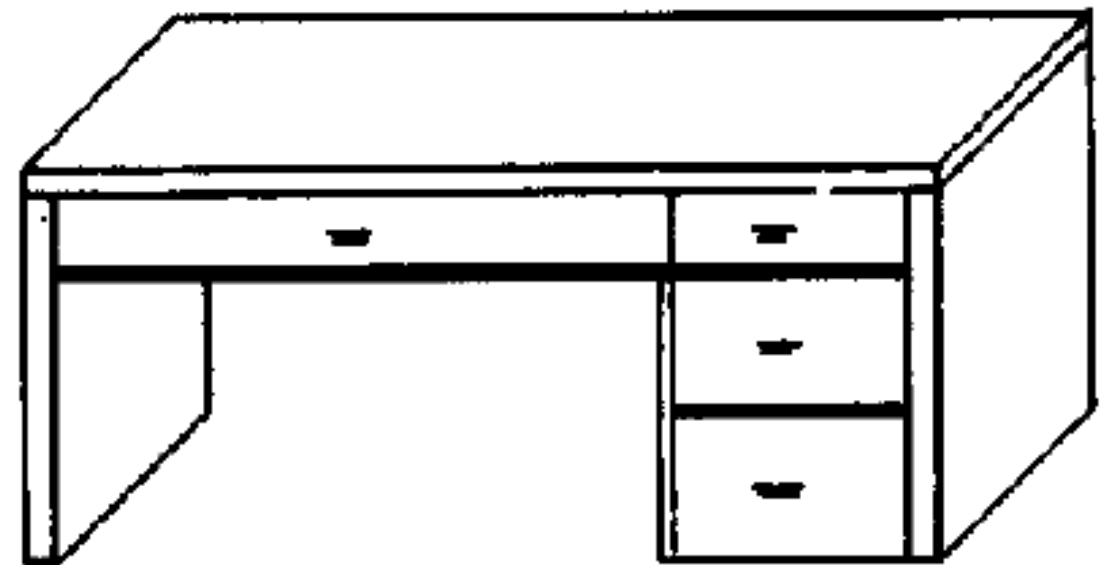
- **View rectangle is the same**
  - ray origins identical to orthographic
  - view direction **d** differs from **-w**
- **Generating rays**
  - for  $(u, v)$  in  $[l, r] \times [b, t]$
  - ray.origin = **e** +  $u$  **u** +  $v$  **v**
  - ray.direction = **d**



# Off-axis parallel



**axonometric:** projection plane perpendicular to projection direction but not parallel to coordinate planes

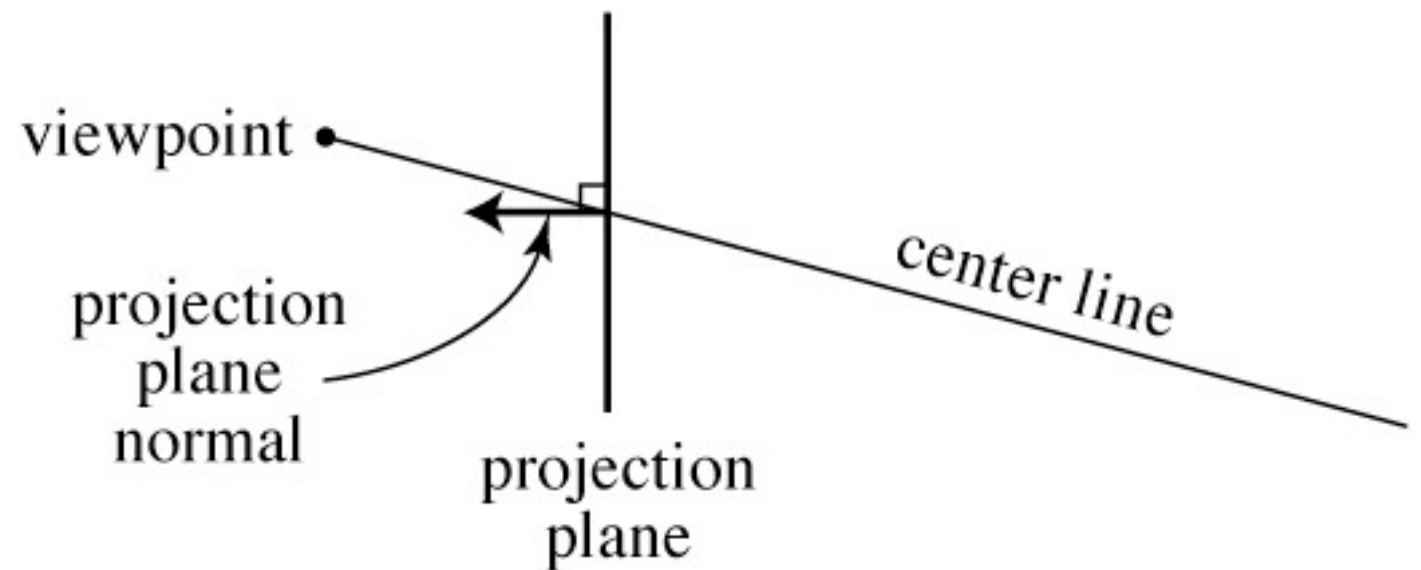


**oblique:** projection plane parallel to a coordinate plane but not perpendicular to projection direction.

[Carl bom & Paciorek 78]

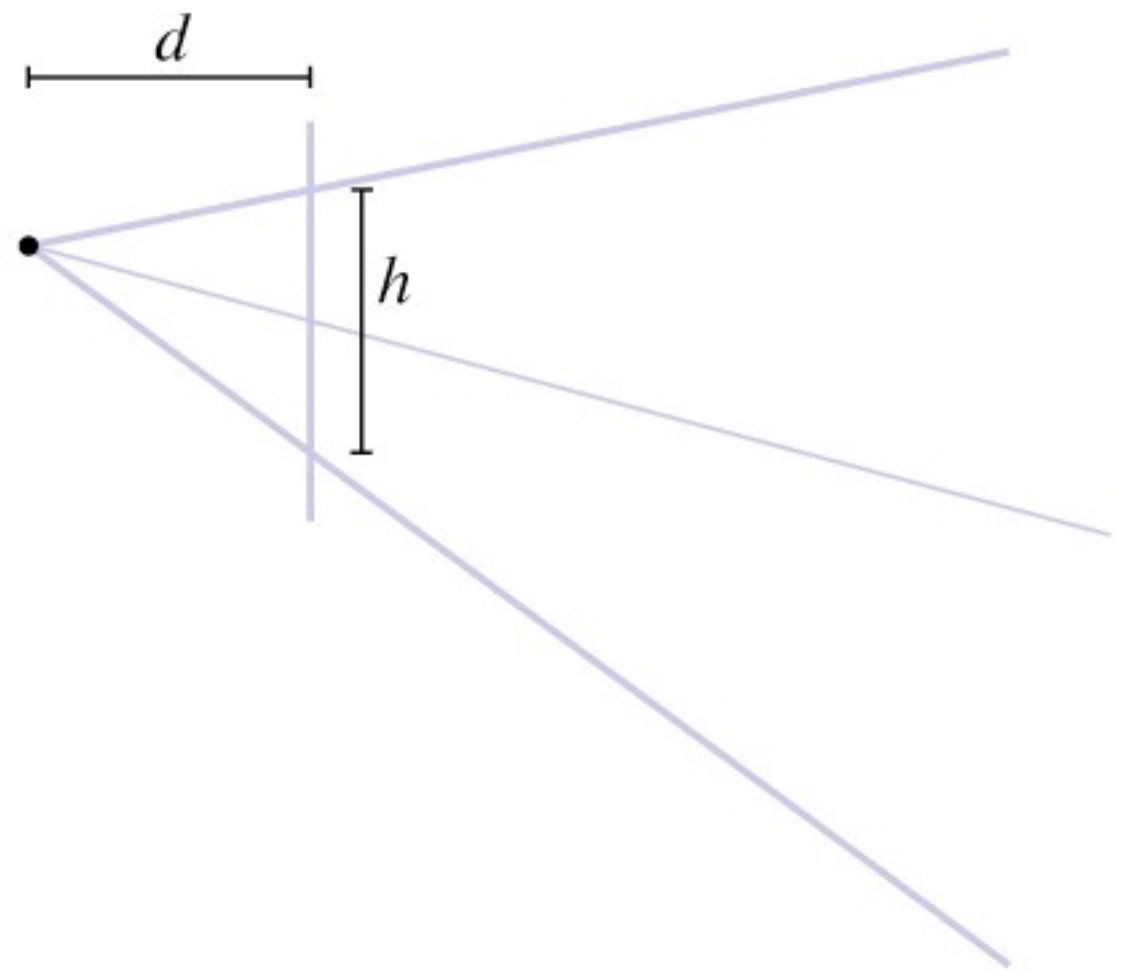
# Shifted perspective projection

- **Perspective but with projection plane not perpendicular to view direction**
  - additional parameter:  
projection plane normal
  - exactly equivalent to  
cropping out an off-center  
rectangle from a larger  
“normal” perspective
  - corresponds to  
***view camera***  
in photography



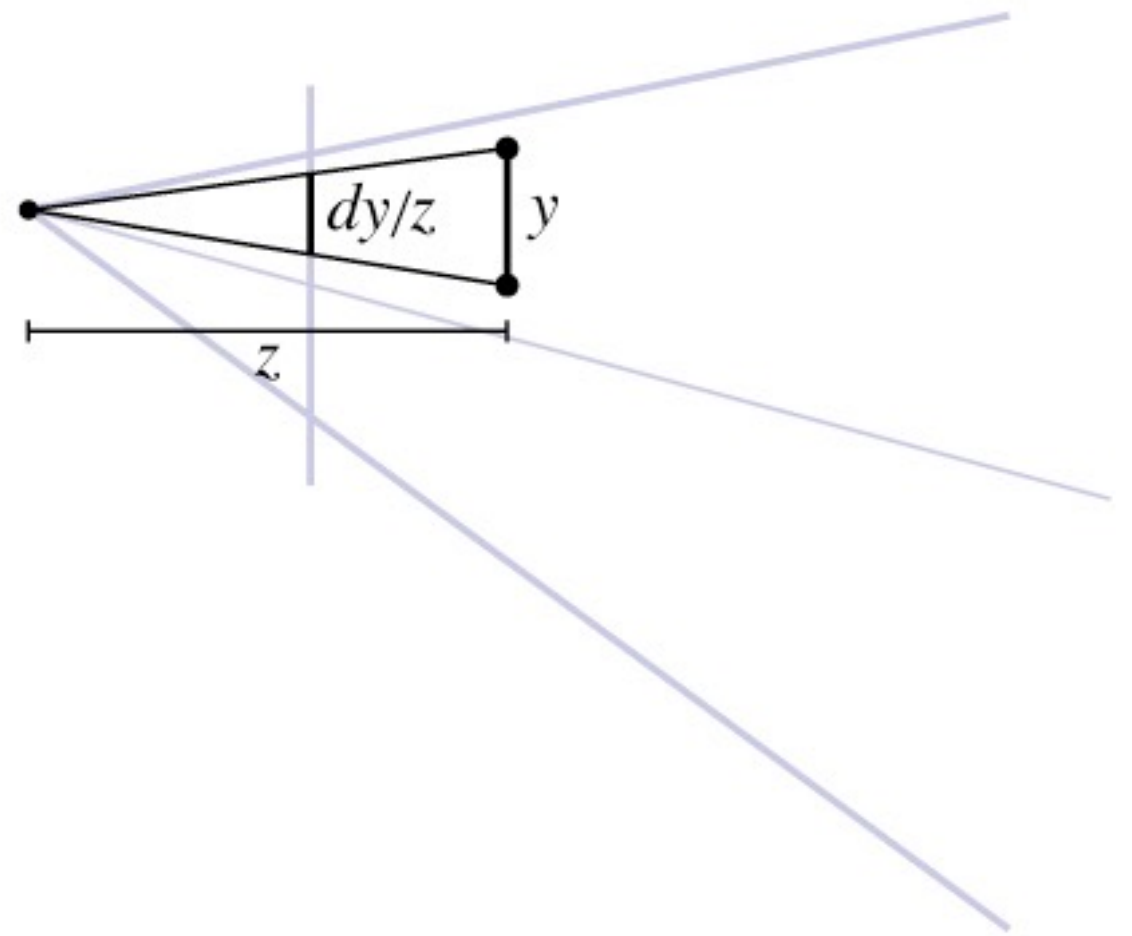
# Shifted perspective projection

- **Perspective but with projection plane not perpendicular to view direction**
  - additional parameter: projection plane normal
  - exactly equivalent to cropping out an off-center rectangle from a larger “normal” perspective
  - corresponds to **view camera** in photography



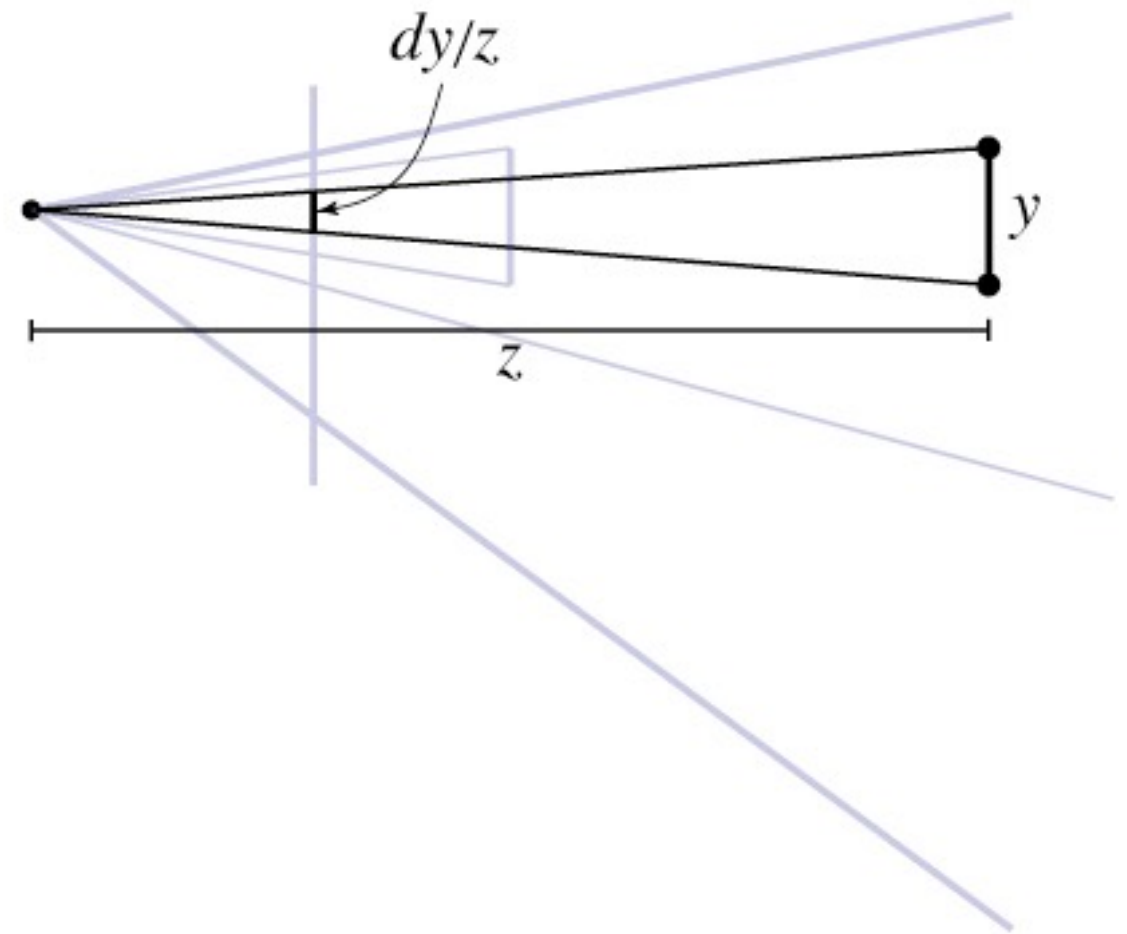
# Shifted perspective projection

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  - additional parameter:  
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  - exactly equivalent to  
cropping out an off-center  
rectangle from a larger  
“normal” perspective
  - corresponds to  
***view camera***  
in photography



# Shifted perspective projection

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  - additional parameter: projection plane normal
  - exactly equivalent to cropping out an off-center rectangle from a larger “normal” perspective
  - corresponds to **view camera** in photography





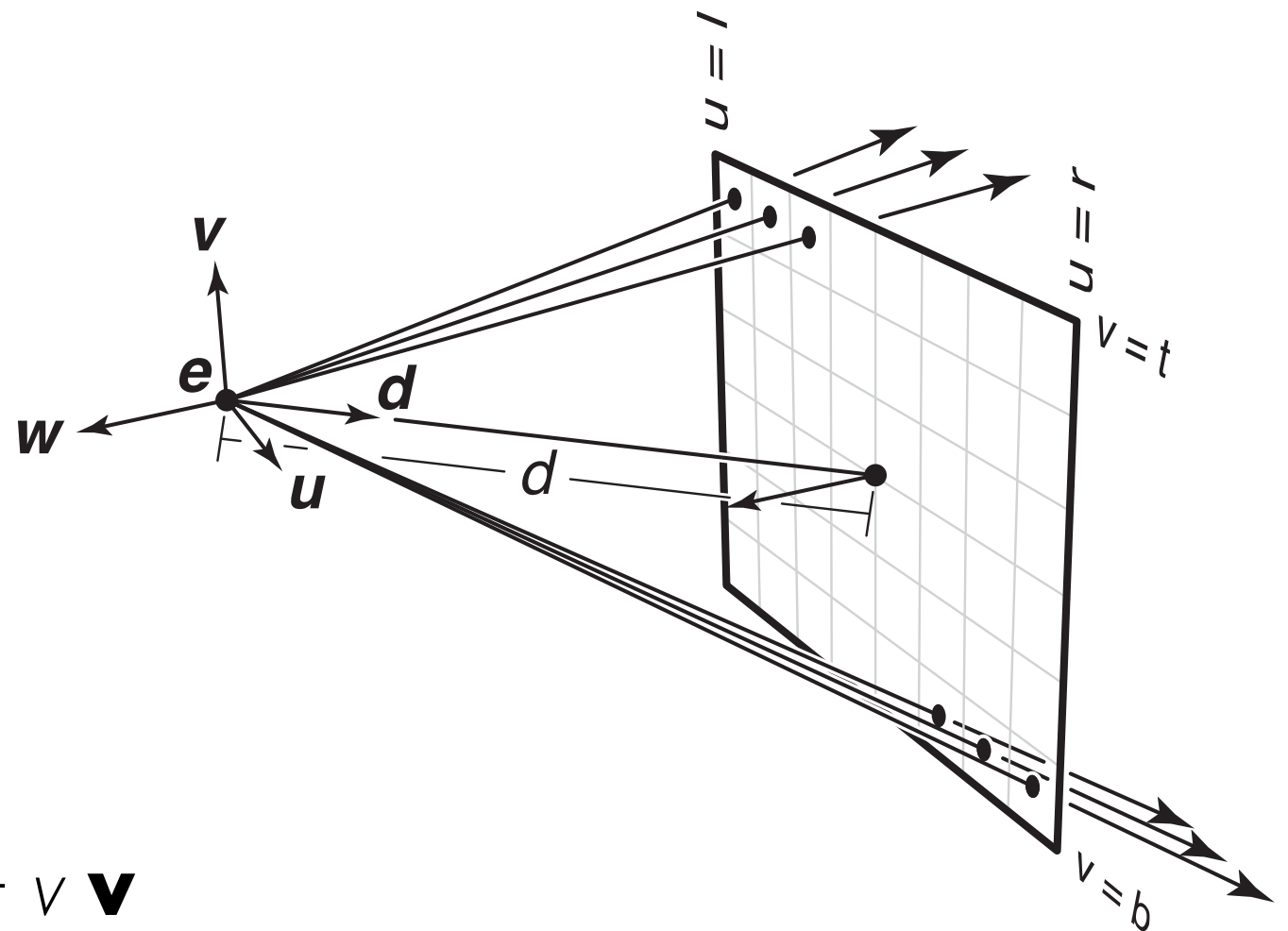
# Oblique perspective views

- **Positioning the view rectangle**

- establish three vectors to be *camera basis*:  **$\mathbf{u}$** ,  **$\mathbf{v}$** ,  **$\mathbf{w}$**
- view rectangle is the same, but shifted so that the center is in the direction  **$\mathbf{d}$**  from  **$\mathbf{e}$**

- **Generating rays**

- for  $(u, v)$  in  $[l, r] \times [b, t]$
- ray.origin =  **$\mathbf{e}$**
- ray.direction =  $d \mathbf{d} + u \mathbf{u} + v \mathbf{v}$



# Why shifted perspective?

- **Control convergence of parallel lines**
- **Standard example: architecture**
  - buildings are taller than you, so you look up
  - top of building is farther away, so it looks smaller
- **Solution: make projection plane parallel to facade**
  - top of building is the same distance *from the projection plane*
- **Same perspective effects can be achieved using post-processing**
  - (though not the focus effects)
  - choice of *which* rays vs. arrangement of rays in image



camera tilted up: converging vertical lines





lens shifted up: parallel vertical lines