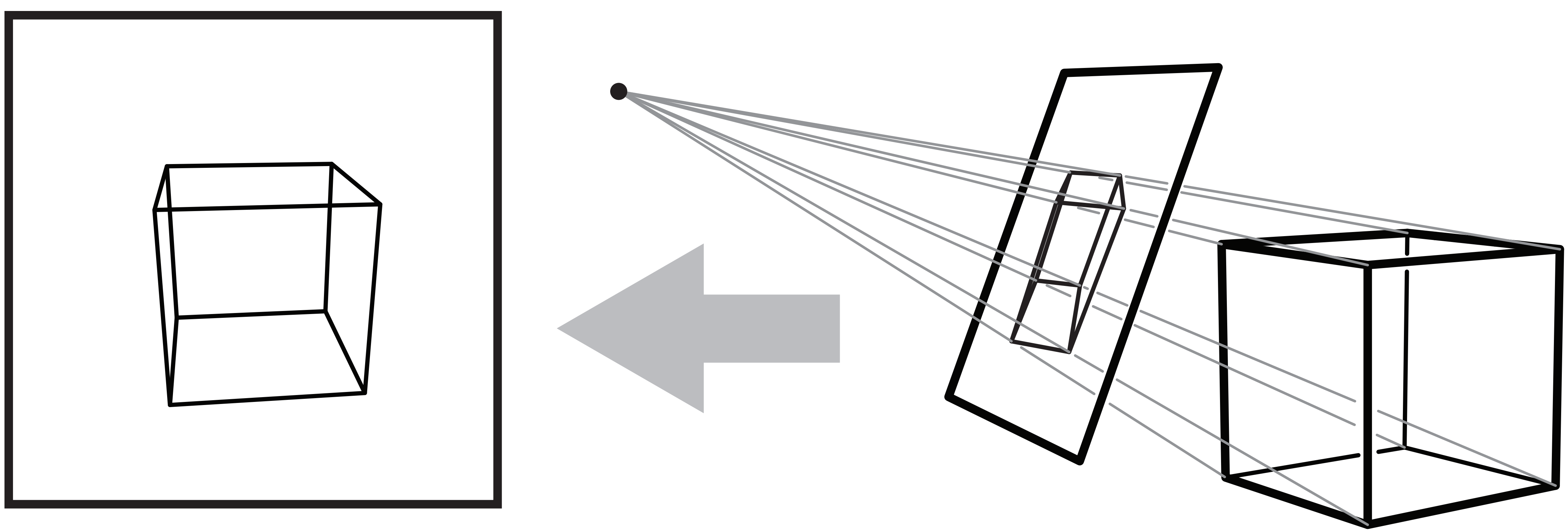
Viewing and Ray Tracing

CS 4620 Lecture 4

Projection

- **To render an image of a 3D scene, we *project* it onto a plane**
- **Most common projection type is *perspective projection***



Two approaches to rendering

```
for each object in the scene {
   for each pixel in the image {
      if (object affects pixel) {
         do something
      }
   }
}
```

object order
or
rasterization

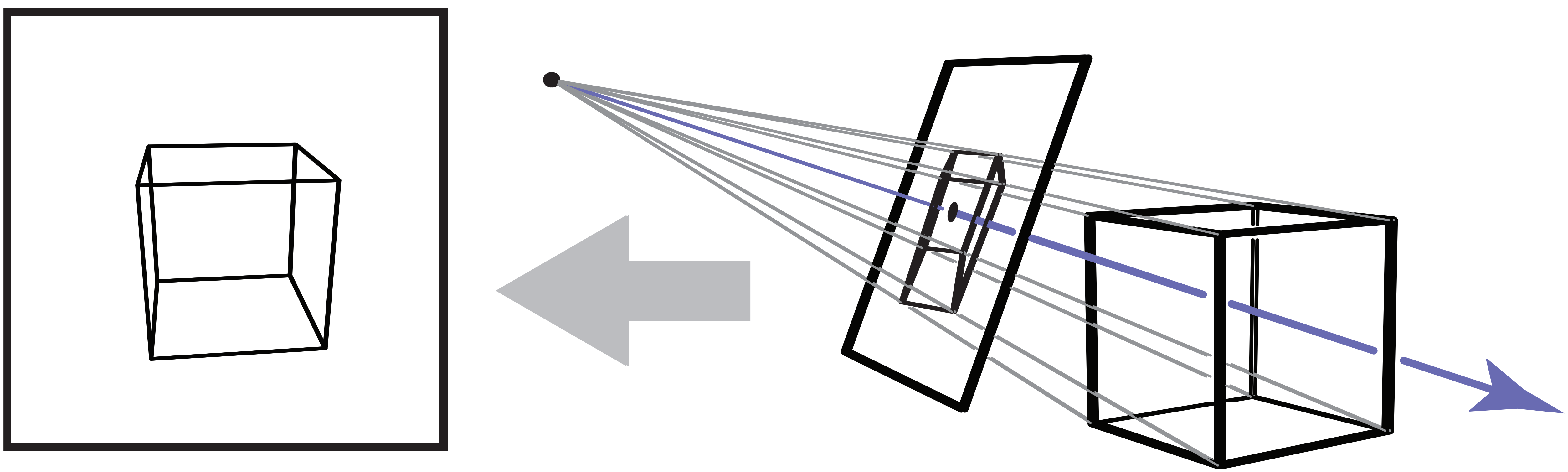
```
for each pixel in the image {
   for each object in the scene {
      if (object affects pixel) {
         do something
      }
   }
}
```

We will do this first

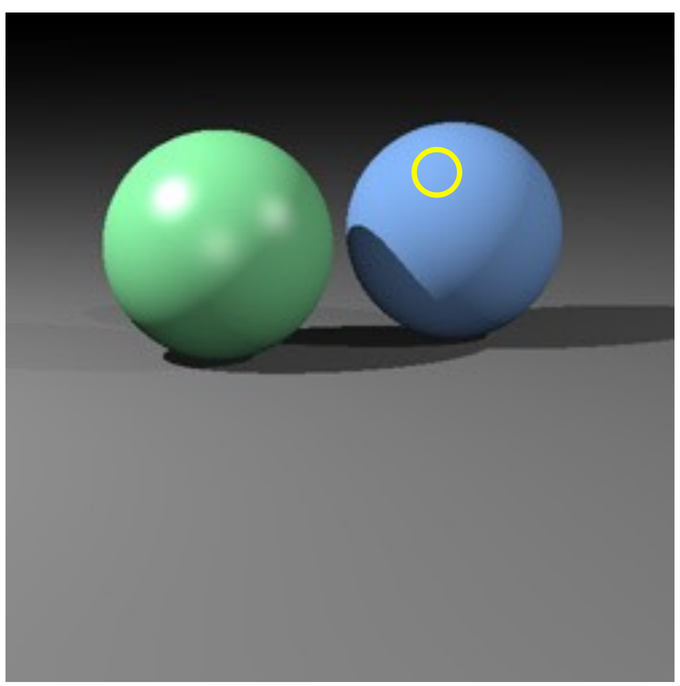
image order
or
ray tracing

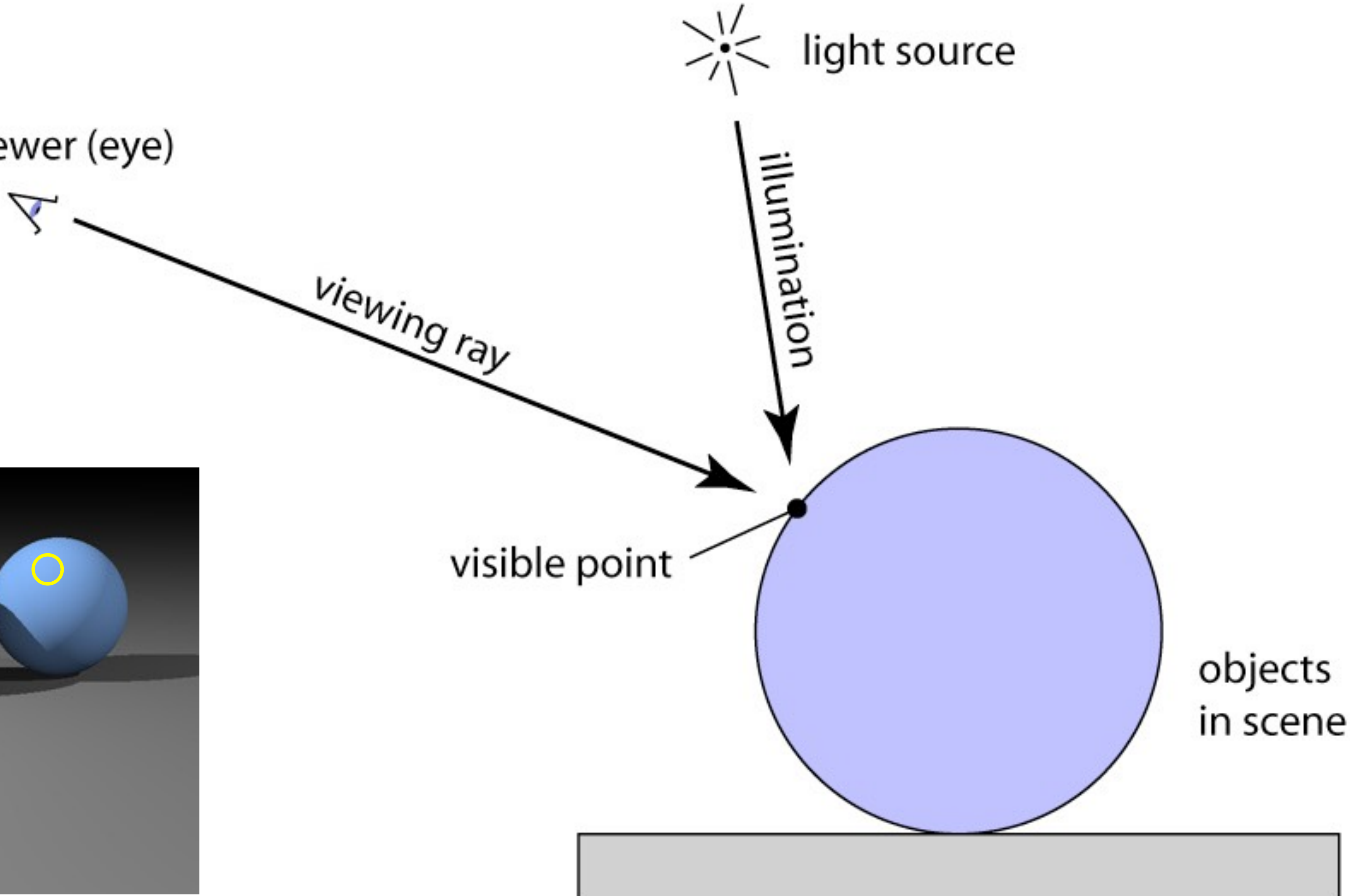
Ray tracing idea

- **Start with a pixel—what belongs at that pixel?**
- **Set of points that project to a point in the image: a ray**

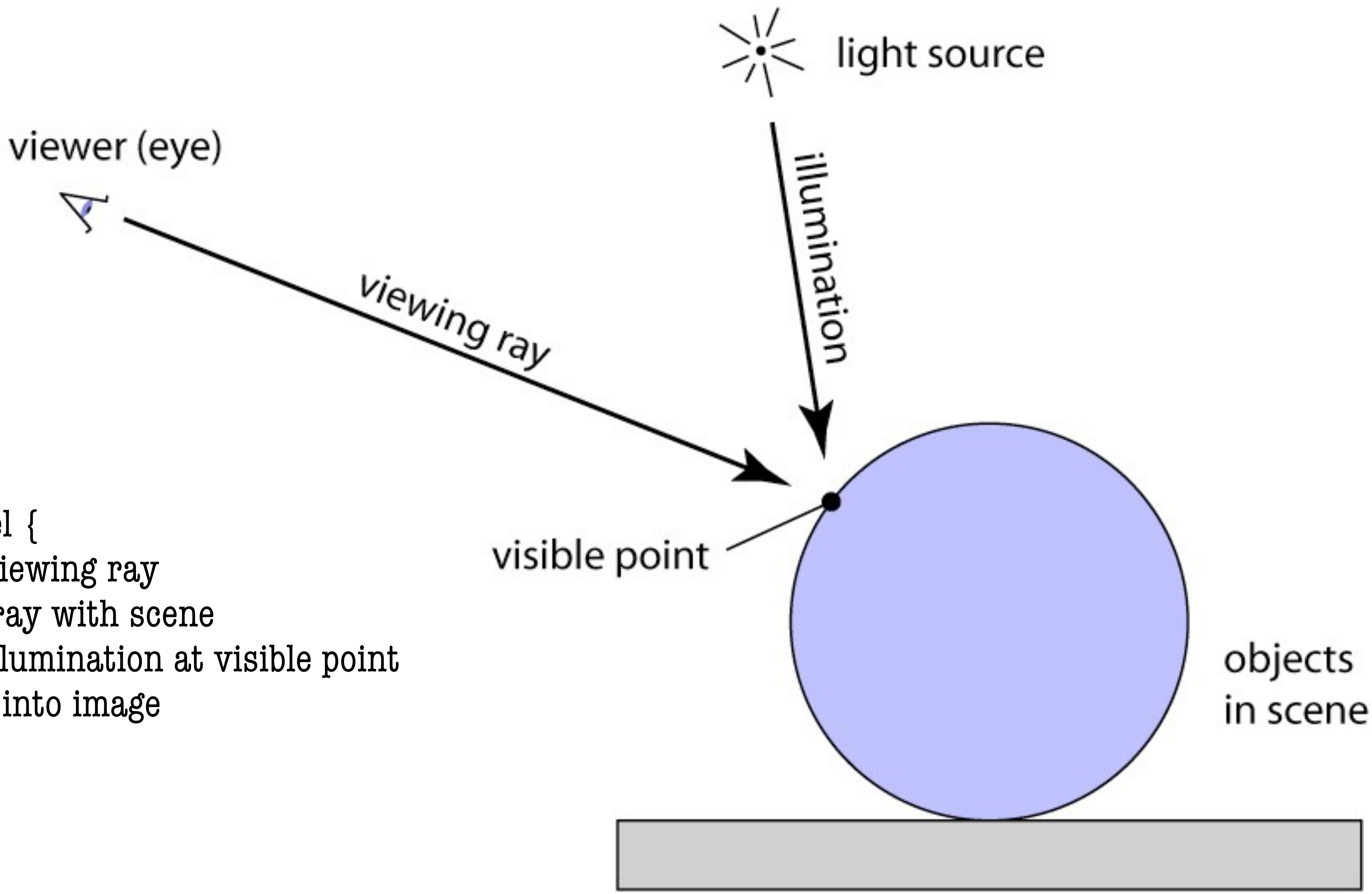


Ray tracing idea





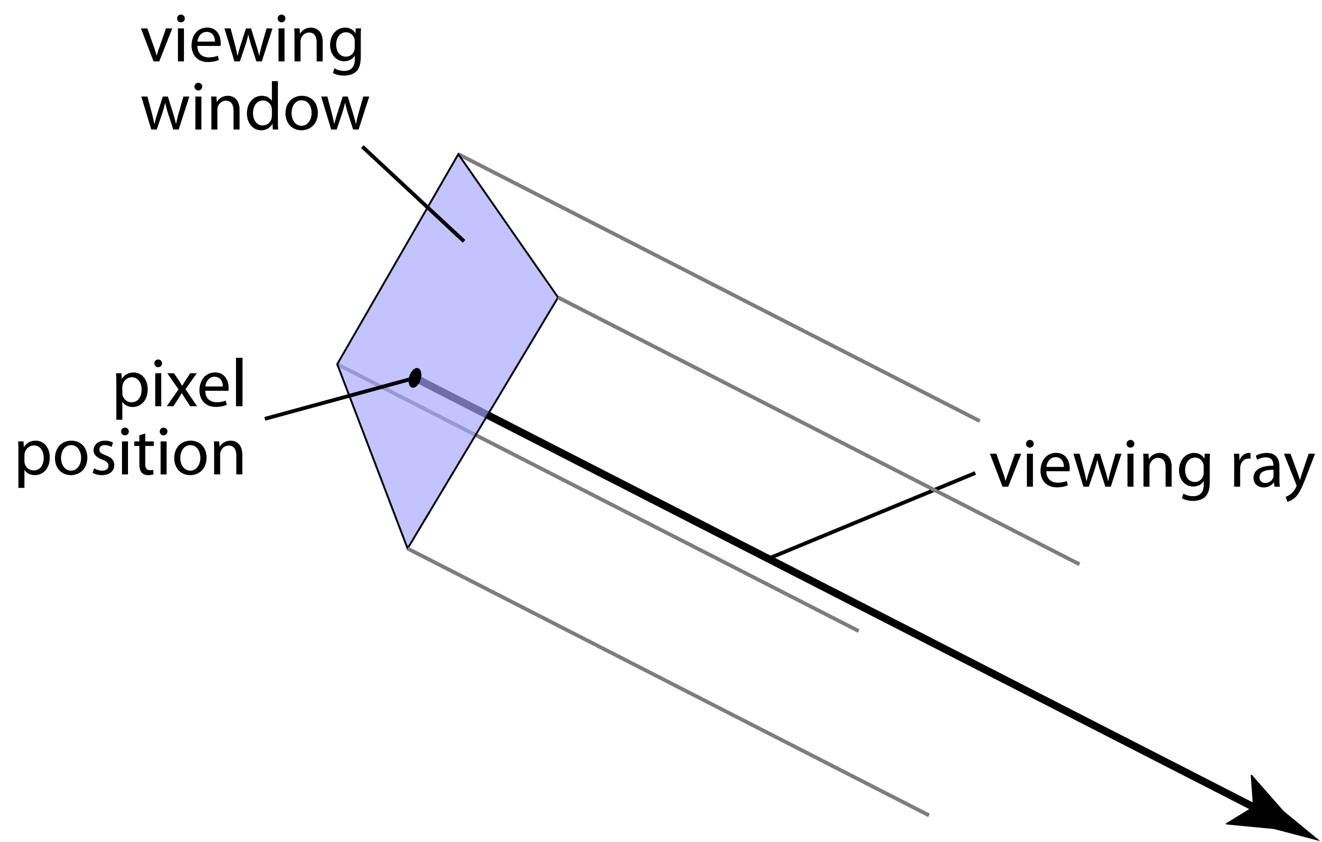
Ray tracing algorithm




```
for each pixel {
    compute viewing ray
    intersect ray with scene
    compute illumination at visible point
    put result into image
}
```

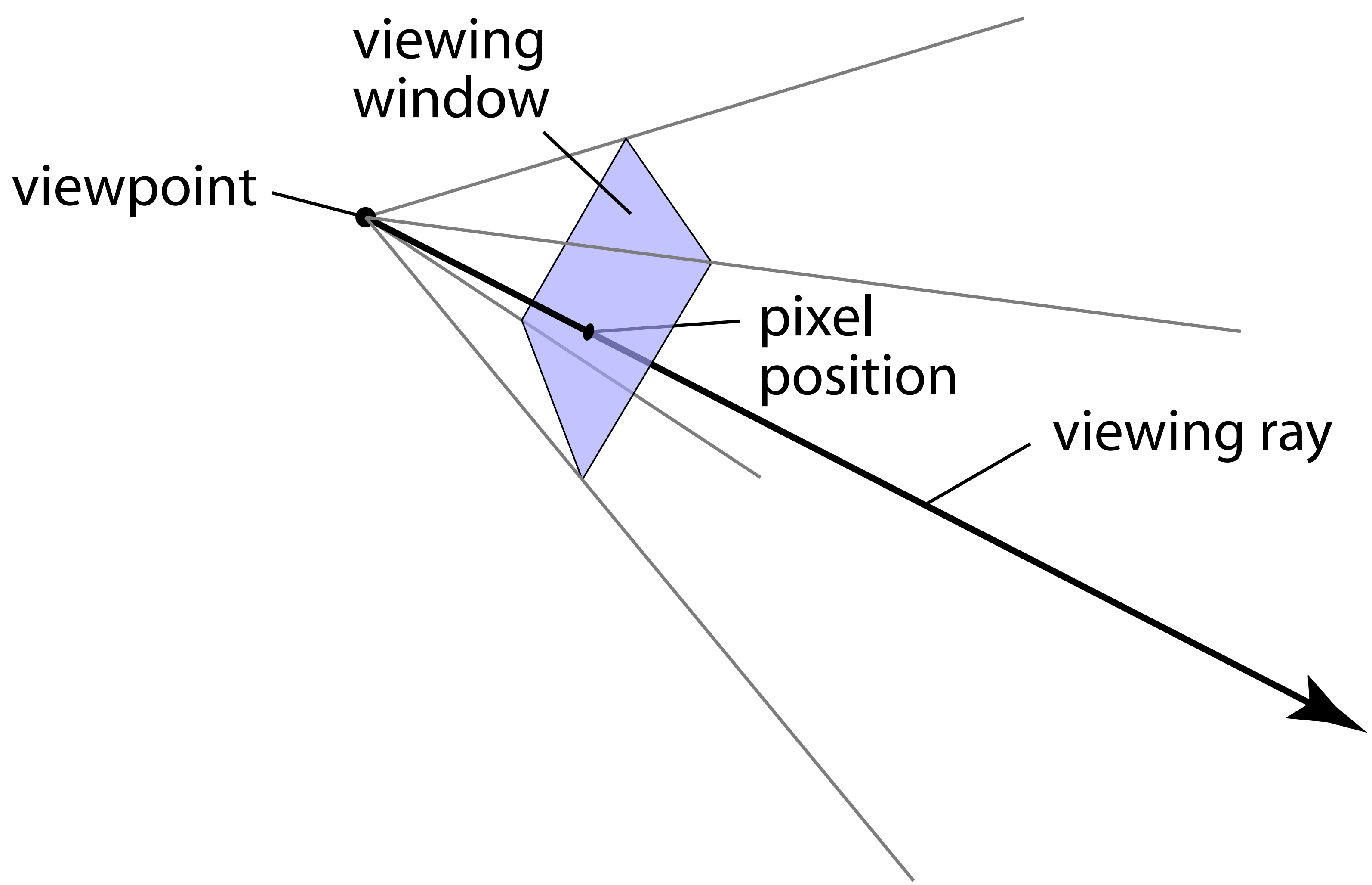

Generating eye rays—planar projection

- **Ray origin (varying):** pixel position on viewing window
- **Ray direction (constant):** view direction



Generating eye rays—perspective

- **Ray origin (constant):** viewpoint
- **Ray direction (varying):** toward pixel position on viewing window



Software interface for cameras

- **Key operation: generate ray for image position**

```
class Camera {
   ...
  Ray generateRay(int col, int row);
}
```

args go from 0, 0
to width – 1, height – 1

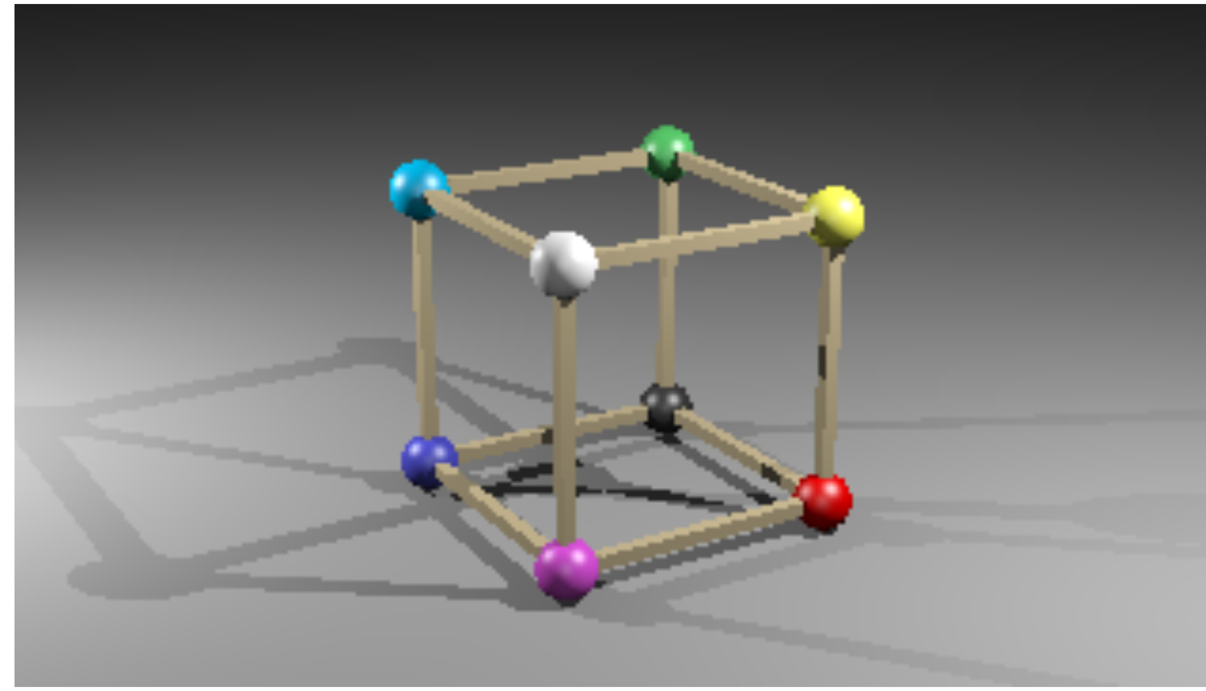
- **Modularity problem: Camera shouldn't have to worry about image resolution**
  - better solution: normalized coordinates

```
class Camera {
   ...
  Ray generateRay(float u, float v);
}
```

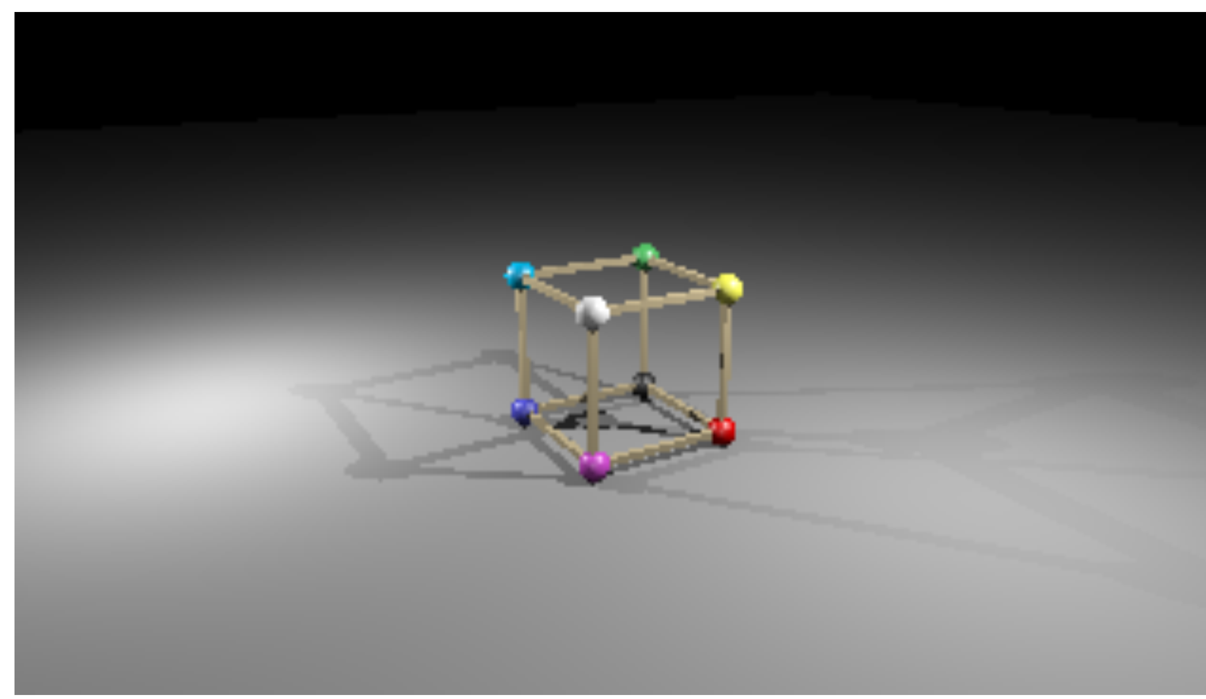
args go from 0, 0 to 1, 1

Specifying views in Ray 1

```
<camera type="PerspectiveCamera">
  <viewPoint>10 4.2 6</viewPoint>
  <viewDir>-5 -2.1 -3</viewDir>
  <viewUp>0 1 0</viewUp>
  <projDistance>6</projDistance>
  <viewWidth>4</viewWidth>
  <viewHeight>2.25</viewHeight>
</camera>
```

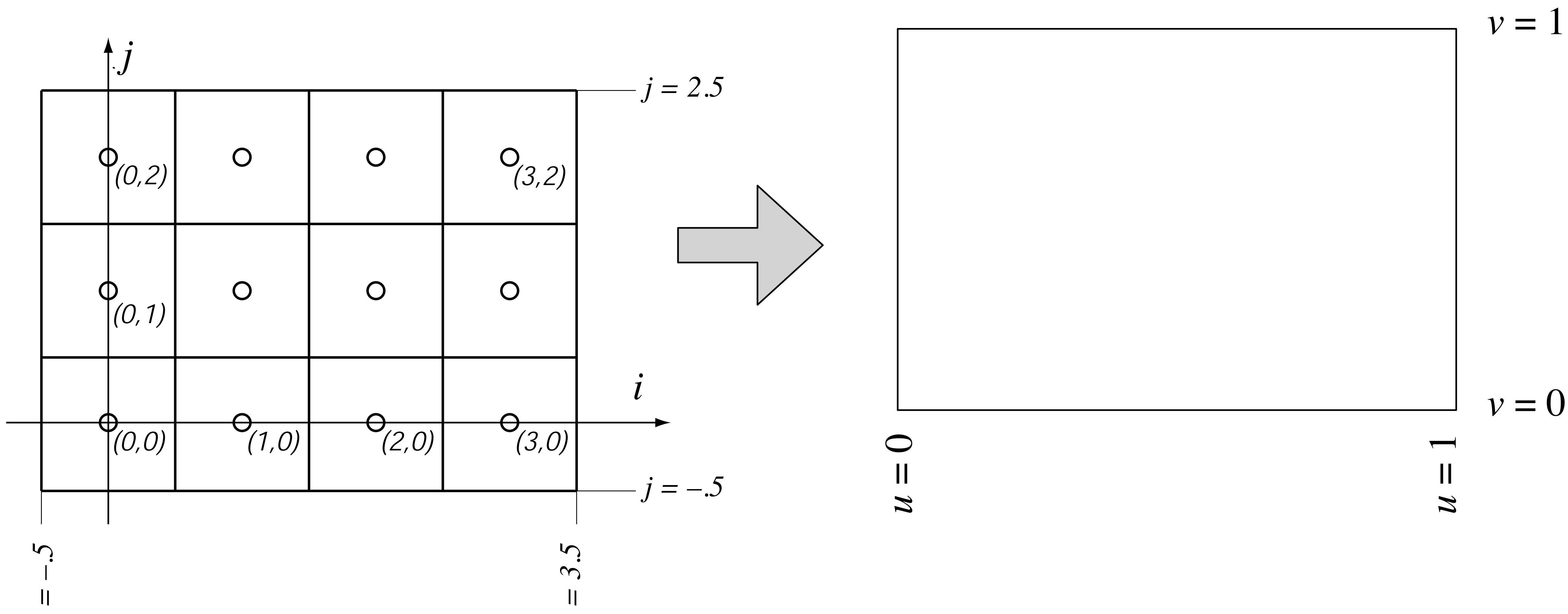


```
<camera type="PerspectiveCamera">
  <viewPoint>10 4.2 6</viewPoint>
  <viewDir>-5 -2.1 -3</viewDir>
  <viewUp>0 1 0</viewUp>
  <projDistance>3</projDistance>
  <viewWidth>4</viewWidth>
  <viewHeight>2.25</viewHeight>
</camera>
```



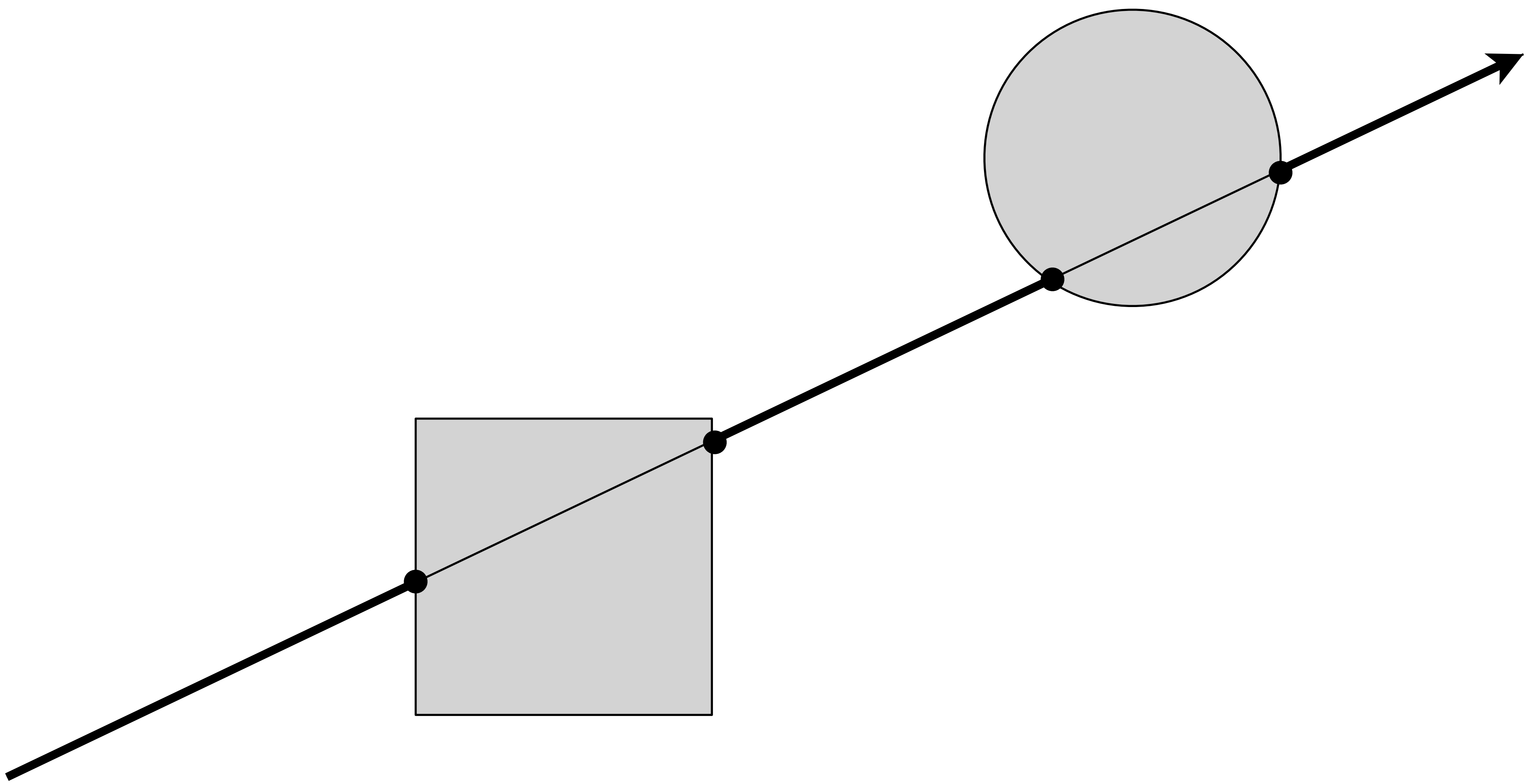
Pixel-to-image mapping

- **One last detail: exactly where are pixels located?**



$$u = (i + 0.5)/n_x$$
$$v = (j + 0.5)/n_y$$

Ray intersection

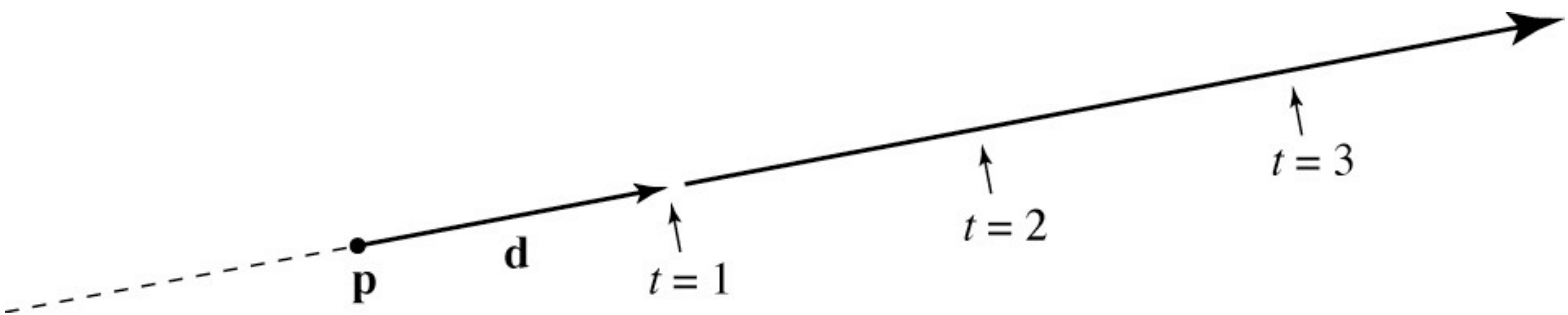


Ray: a half line

- **Standard representation: point p and direction d**

  $$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

  – this is a ***parametric equation*** for the line
  – lets us directly generate the points on the line
  – if we restrict to $t > 0$ then we have a ray
  – note replacing **d** with **αd** doesn't change ray ($\alpha > 0$)



Ray-sphere intersection: algebraic

- **Condition 1: point is on ray**

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

- **Condition 2: point is on sphere**
  - assume unit sphere; see book or notes for general

$$\|\mathbf{x}\| = 1 \Leftrightarrow \|\mathbf{x}\|^2 = 1$$

$$f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{x} - 1 = 0$$

- **Substitute:**

$$(\mathbf{p} + t\mathbf{d}) \cdot (\mathbf{p} + t\mathbf{d}) - 1 = 0$$

  - this is a quadratic equation in ***t***

Ray-sphere intersection: algebraic

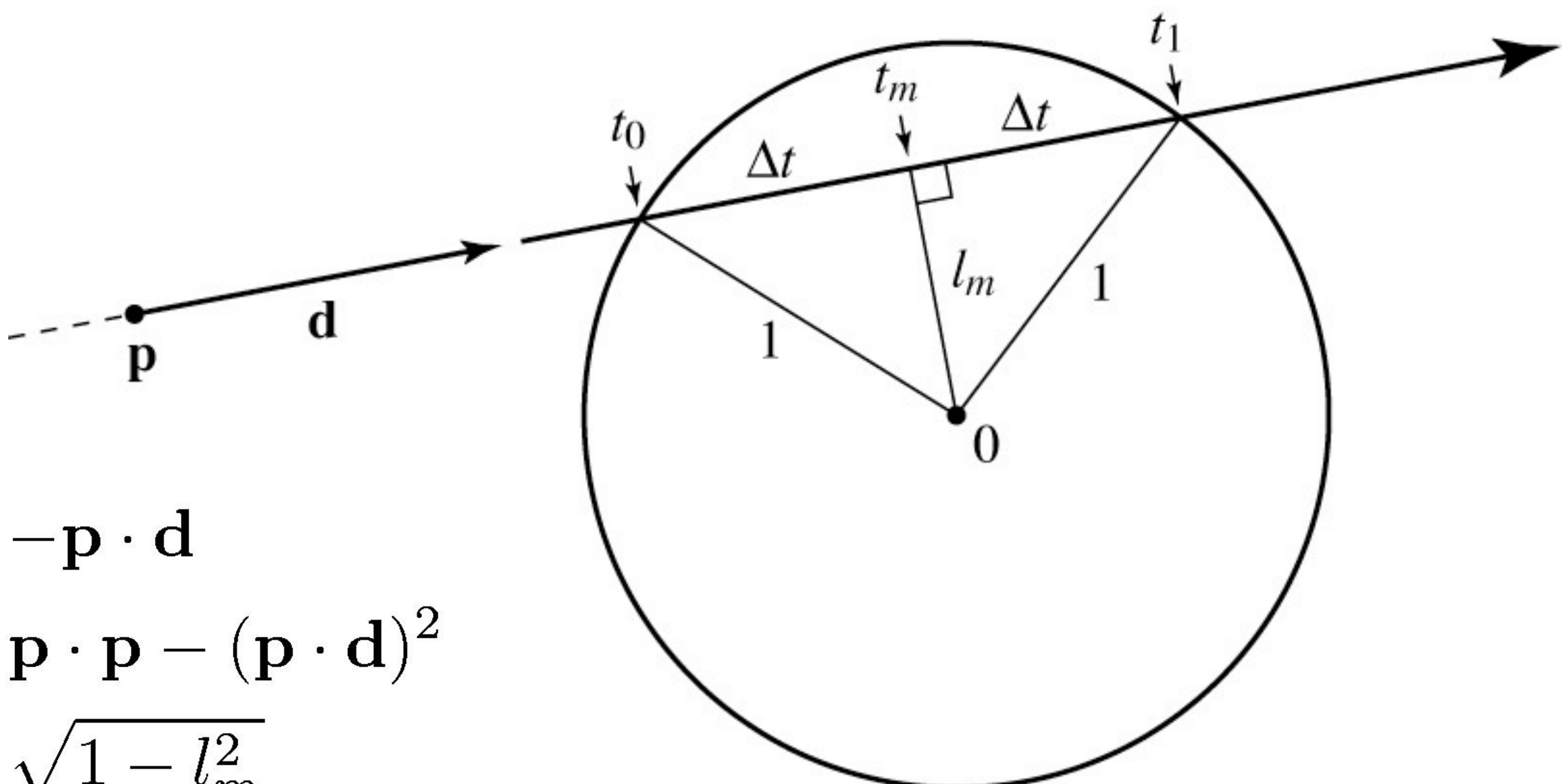
- **Solution for t by quadratic formula:**

$$t = \frac{-\mathbf{d} \cdot \mathbf{p} \pm \sqrt{(\mathbf{d} \cdot \mathbf{p})^2 - (\mathbf{d} \cdot \mathbf{d})(\mathbf{p} \cdot \mathbf{p} - 1)}}{\mathbf{d} \cdot \mathbf{d}}$$

$$t = -\mathbf{d} \cdot \mathbf{p} \pm \sqrt{(\mathbf{d} \cdot \mathbf{p})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

  – simpler form holds when **d** is a unit vector
    but we won’t assume this in practice (reason later)
  – I’ll use the unit-vector form to make the geometric interpretation

Ray-sphere intersection: geometric



$$t_m = -\mathbf{p} \cdot \mathbf{d}$$

$$l_m^2 = \mathbf{p} \cdot \mathbf{p} - (\mathbf{p} \cdot \mathbf{d})^2$$

$$\Delta t = \sqrt{1 - l_m^2}$$

$$= \sqrt{(\mathbf{p} \cdot \mathbf{d})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

$$t_{0,1} = t_m \pm \Delta t = -\mathbf{p} \cdot \mathbf{d} \pm \sqrt{(\mathbf{p} \cdot \mathbf{d})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

Ray-triangle intersection

- **Condition 1: point is on ray**

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

- **Condition 2: point is on plane**

$$(\mathbf{x} - \mathbf{a}) \cdot \mathbf{n} = 0$$

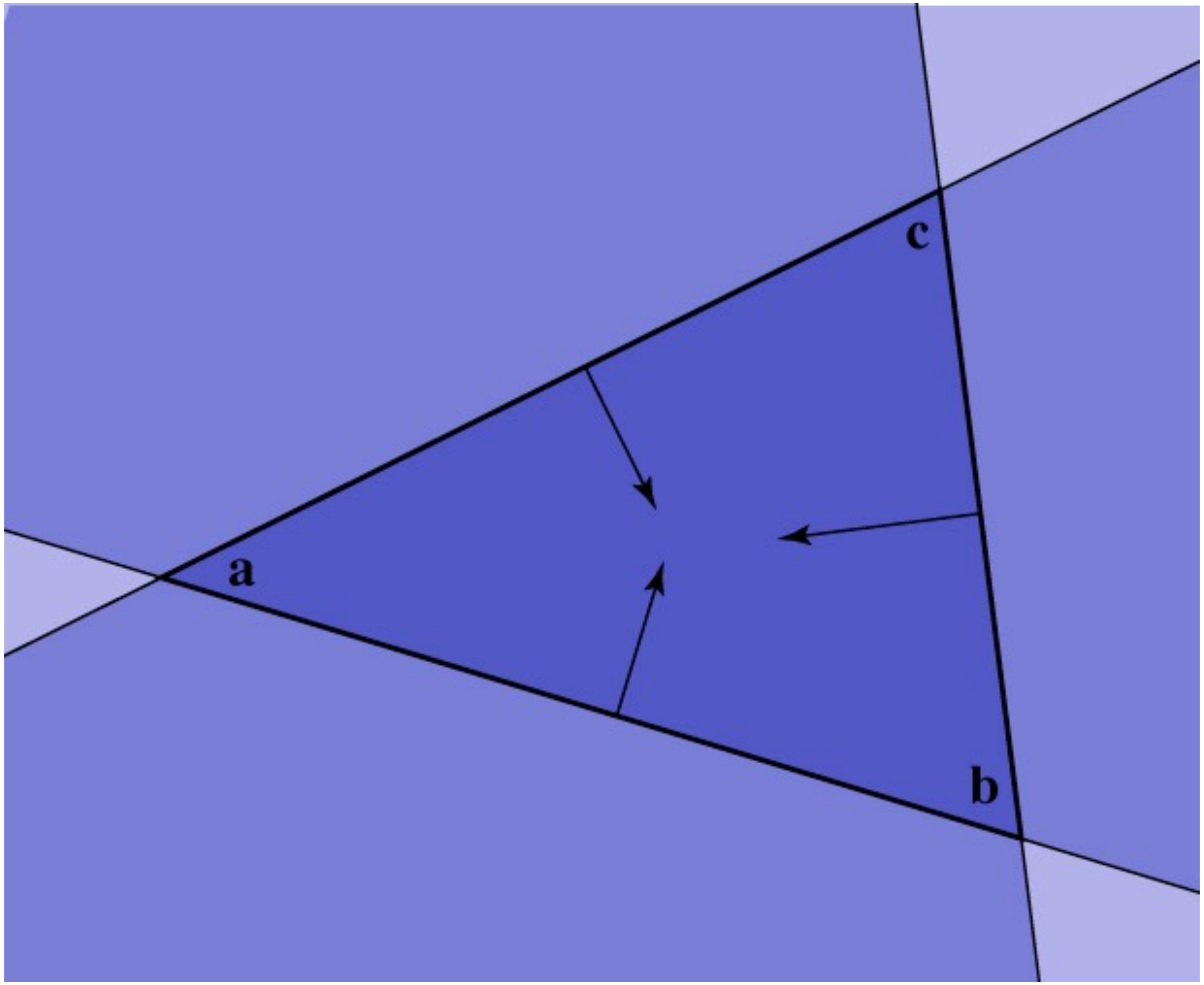
- **Condition 3: point is on the inside of all three edges**
- **First solve 1&2 (ray–plane intersection)**
  - substitute and solve for ***t***:

$$(\mathbf{p} + t\mathbf{d} - \mathbf{a}) \cdot \mathbf{n} = 0$$

$$t = \frac{(\mathbf{a} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$

Ray-triangle intersection

- **In plane, triangle is the intersection of 3 half spaces**



Deciding about insideness

- **Need to check whether hit point is inside 3 edges**
  - easiest to do in 2D coordinates on the plane
- **Will also need to know where we are in the triangle**
  - for textures, shading, etc. … next couple of lectures
- **Efficient solution: transform to coordinates aligned to the triangle**

Barycentric coordinates

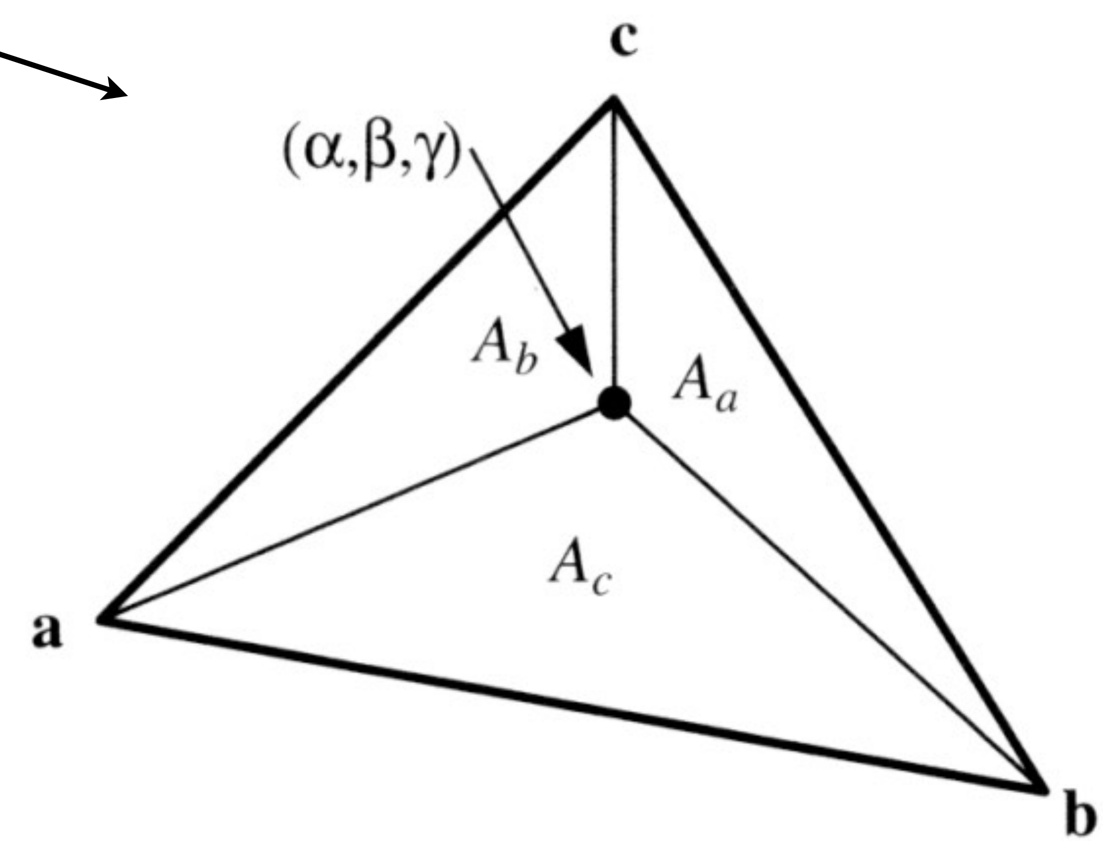
- **A coordinate system for triangles**
  - algebraic viewpoint:

$$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

$$\alpha + \beta + \gamma = 1$$

  - geometric viewpoint (areas):
- **Triangle interior test:**

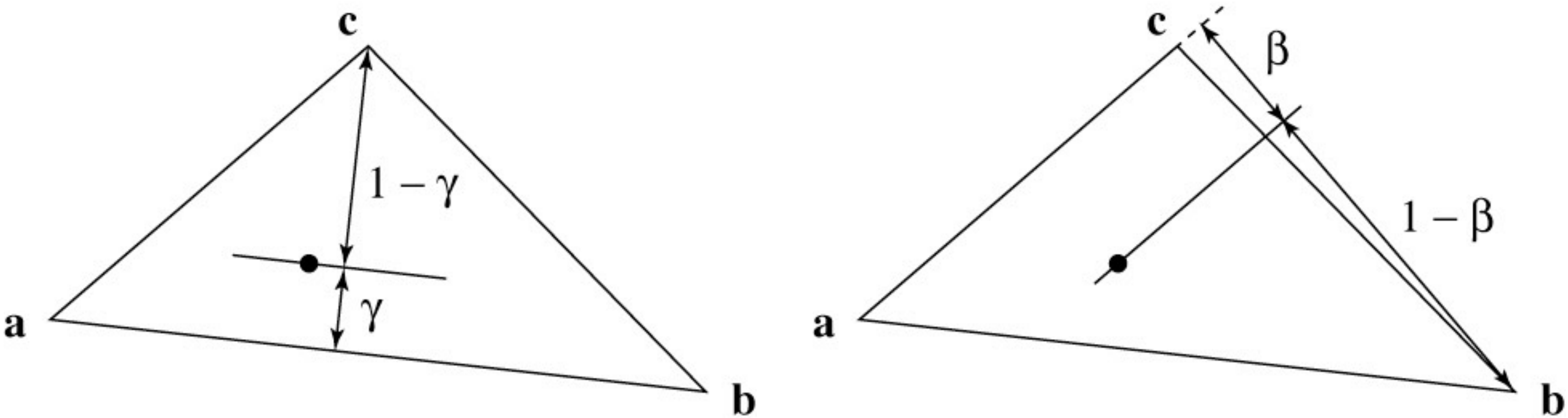
$$\alpha > 0; \quad \beta > 0; \quad \gamma > 0$$



[Shirley 2000]

Barycentric coordinates

- **A coordinate system for triangles**
  - geometric viewpoint: distances

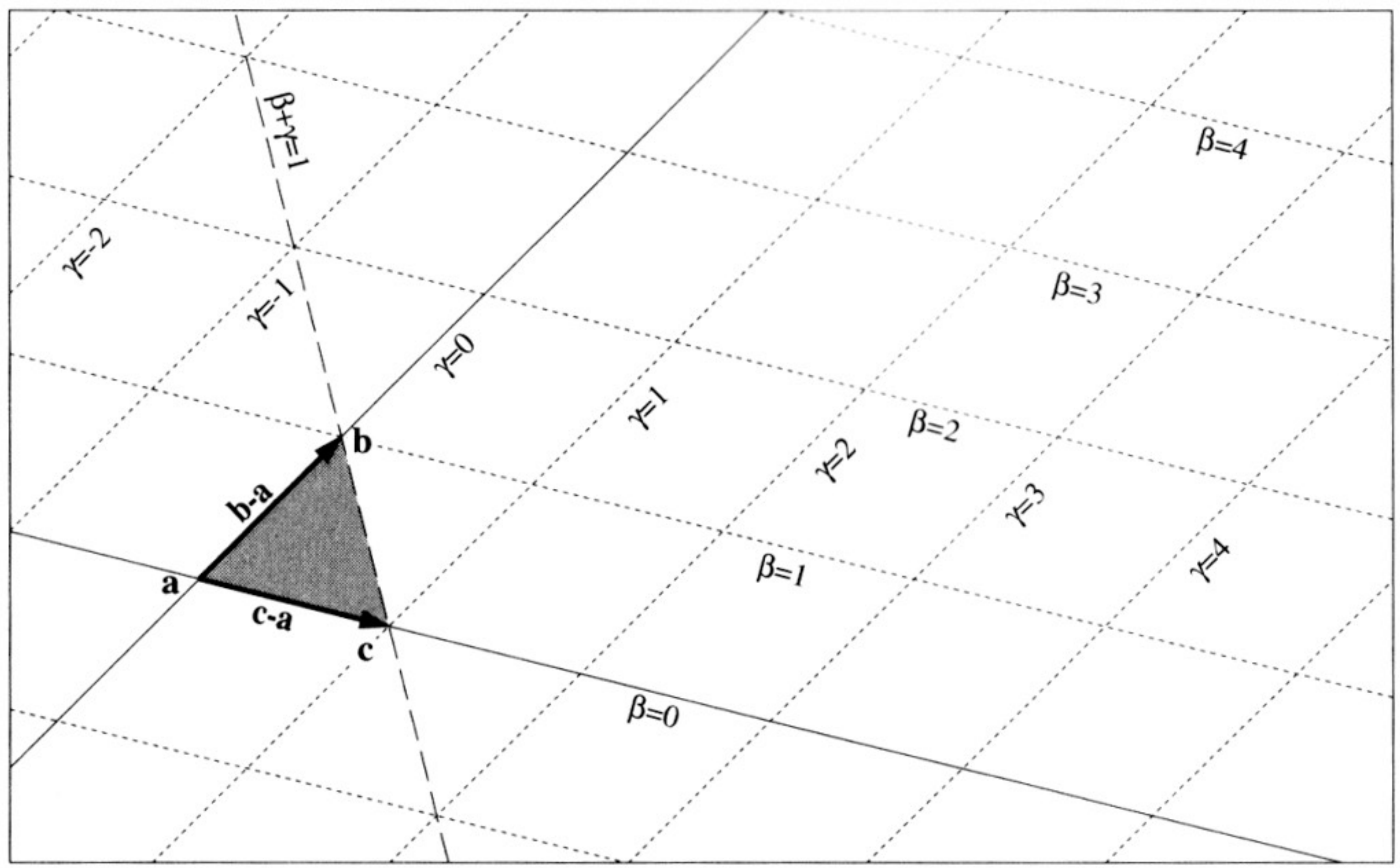


  - linear viewpoint: basis of edges

$$\alpha = 1 - \beta - \gamma$$
$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

Barycentric coordinates

- **Linear viewpoint: basis for the plane**



[Shirley 2000]

  – in this view, the triangle interior test is just

$$\beta > 0; \quad \gamma > 0; \quad \beta + \gamma < 1$$

Barycentric ray-triangle intersection

- **Every point on the plane can be written in the form:**

  $$\mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

  for some numbers $\boldsymbol{\beta}$ and γ.
- **If the point is also on the ray then it is**

  $$\mathbf{p} + t\mathbf{d}$$

  for some number $\boldsymbol{t}$.
- **Set them equal: 3 linear equations in 3 variables**

  $$\mathbf{p} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

  …solve them to get $\boldsymbol{t}$, $\boldsymbol{\beta}$, and γ all at once!

Barycentric ray-triangle intersection

$$\mathbf{p} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

$$\beta(\mathbf{a} - \mathbf{b}) + \gamma(\mathbf{a} - \mathbf{c}) + t\mathbf{d} = \mathbf{a} - \mathbf{p}$$

$$\begin{bmatrix} \mathbf{a} - \mathbf{b} & \mathbf{a} - \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} \mathbf{a} - \mathbf{p} \end{bmatrix}$$

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_p \\ y_a - y_p \\ z_a - z_p \end{bmatrix}$$

Cramer’s rule is a good fast way to solve this system

(see text Ch. 2 and Ch. 4 for details)

Ray intersection in software

- **All surfaces need to be able to intersect rays with themselves.**

ray to be intersected

```
class Surface {
  ...
  abstract boolean intersect(IntersectionRecord result, Ray r);
}
```

was there an intersection?

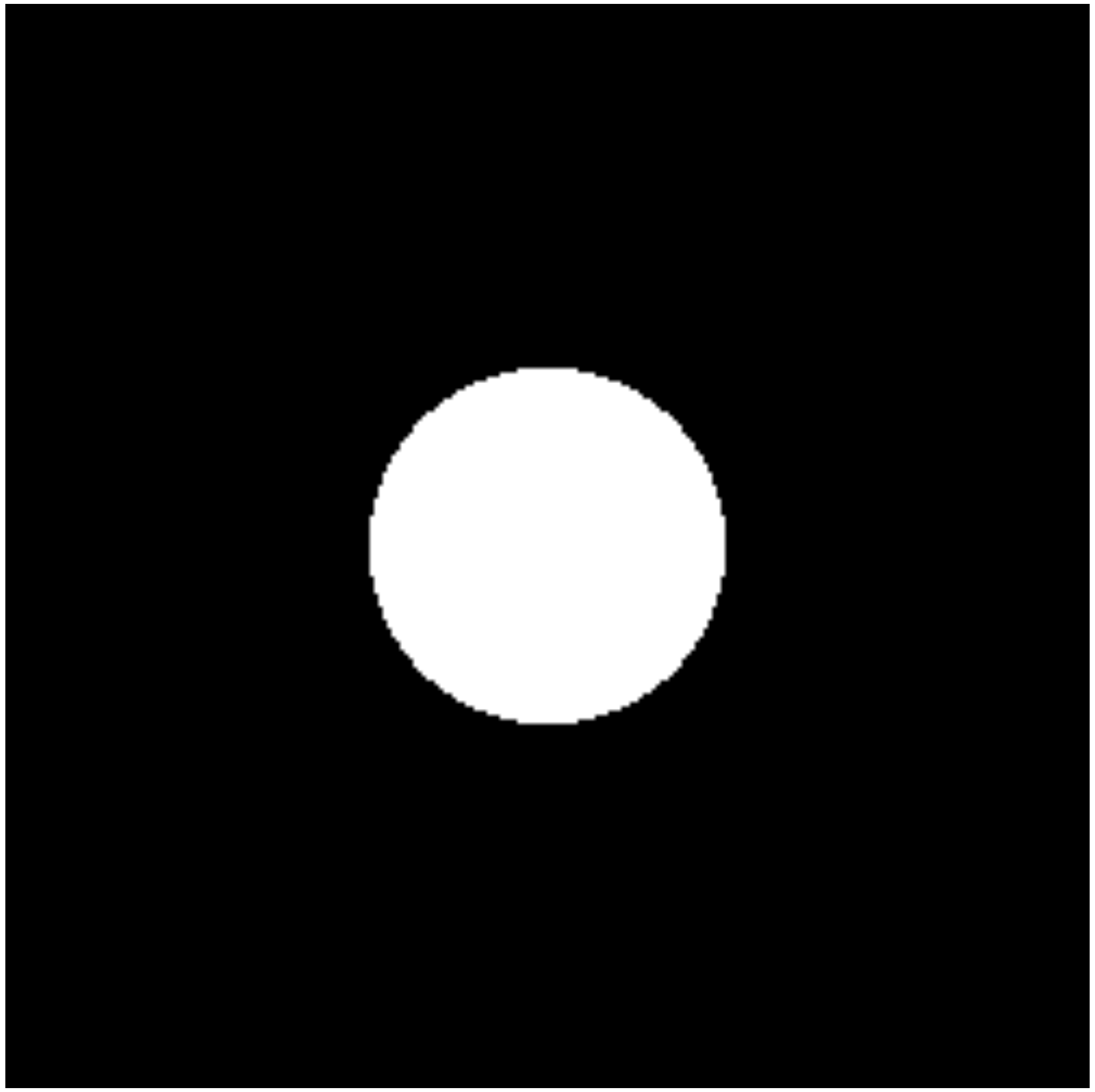
information about first intersection

```
class IntersectionRecord {
   float t;
   Vector3 hitLocation;
   Vector3 normal;
   ...
}
```


Image so far

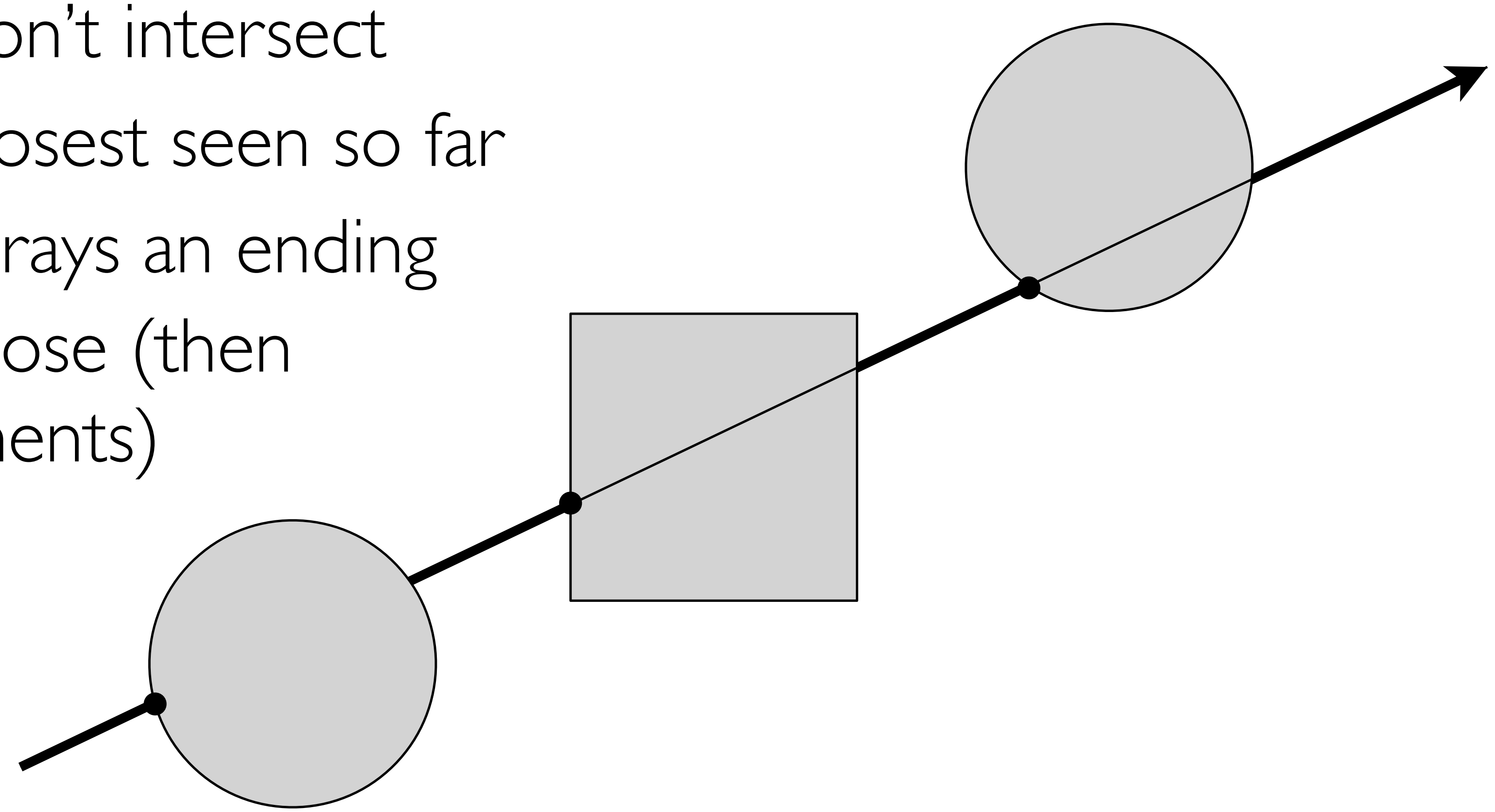
- **With eye ray generation and sphere intersection**

```
Surface s = new Sphere((0.0, 0.0, 0.0), 1.0);
for 0 <= iy < ny
    for 0 <= ix < nx {
        ray = camera.getRay(ix, iy);
        hitSurface, t = s.intersect(ray, 0, +inf)
        if hitSurface is not null
            image.set(ix, iy, white);
    }
```



Ray intersection in software

- **Scenes usually have many objects**
- **Need to find the first intersection along the ray**
  - that is, the one with the smallest positive t value
- **Loop over objects**
  - ignore those that don't intersect
  - keep track of the closest seen so far
  - Convenient to give rays an ending t value for this purpose (then they are really segments)



Intersection against many shapes

- **The basic idea is:**

```
intersect (ray, tMin, tMax) {
    tBest = +inf; firstSurface = null;
    for surface in surfaceList {
        hitSurface, t = surface.intersect(ray, tMin, tBest);
        if hitSurface is not null {
            tBest = t;
            firstSurface = hitSurface;
        }
    }
    return hitSurface, tBest;
}
```

  – this is linear in the number of shapes
  – real applications use sublinear methods (acceleration structures) which we will see later

