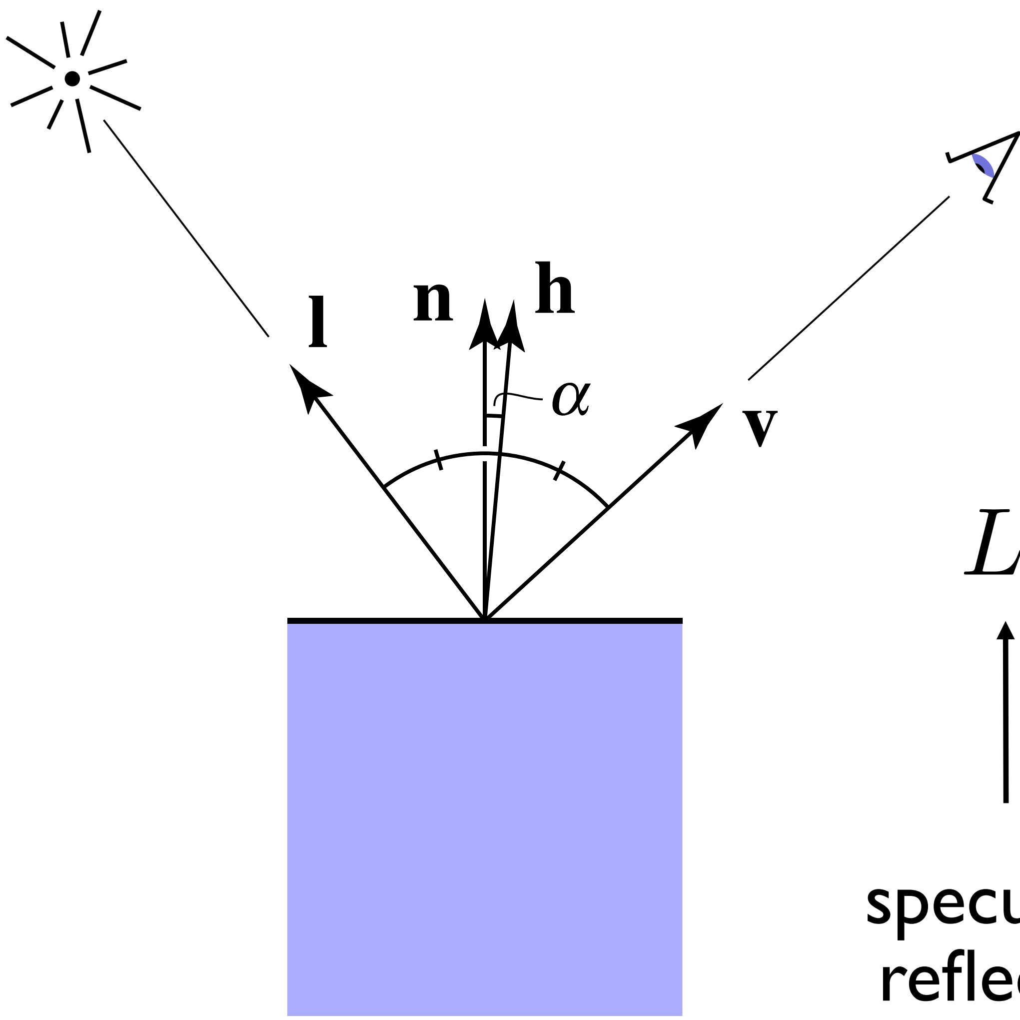
Surface Reflection

CS 4620 Lecture 19

General shading (BRDF)



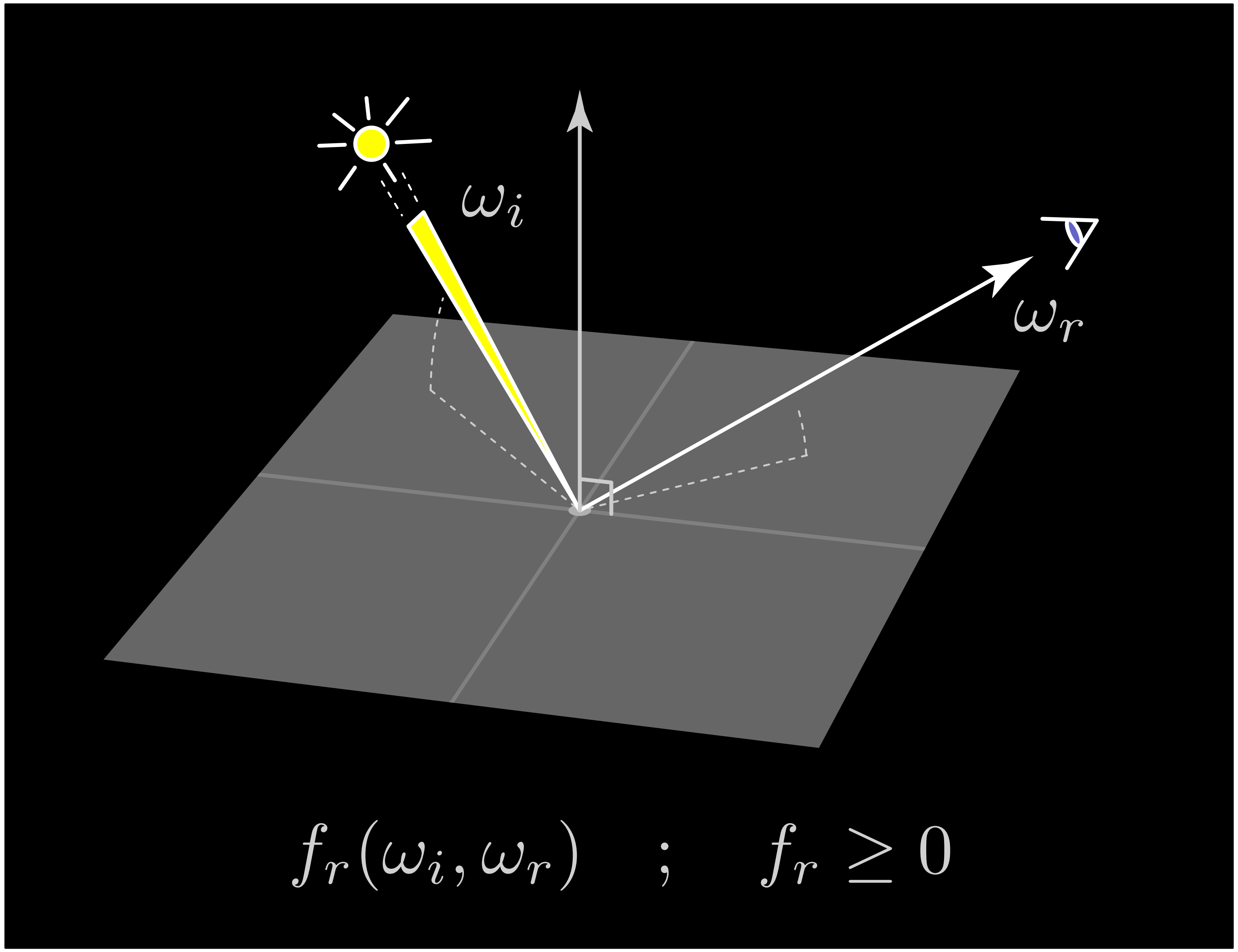
irradiance from source

$$L_s = f_r(\mathbf{n}, \mathbf{l}, \mathbf{v}) \, \overbrace{\frac{\max(0, \mathbf{n} \cdot \mathbf{l})}{r^2} I}$$

specularly reflected radiance

specular BRDF value

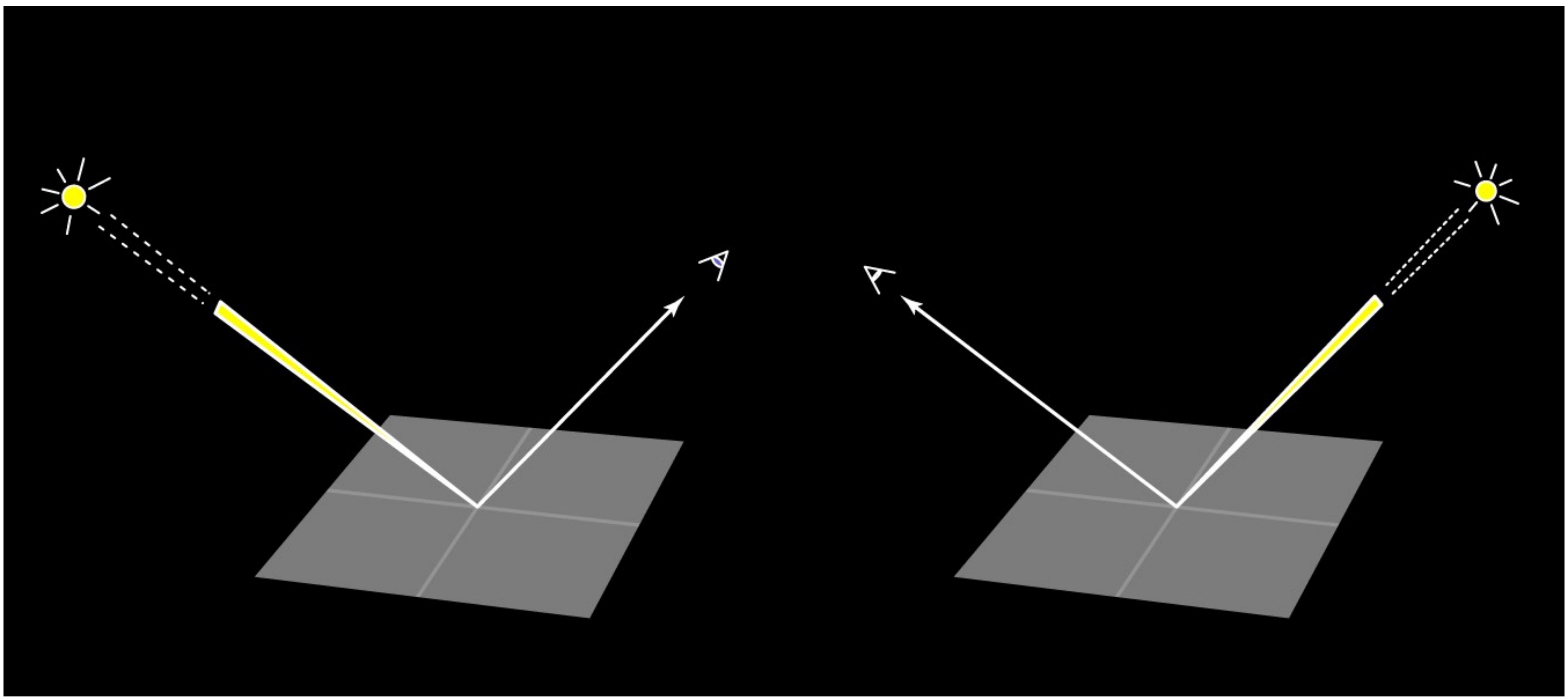
BRDF



Bidirectional Reflectance Distribution Function

Reciprocity

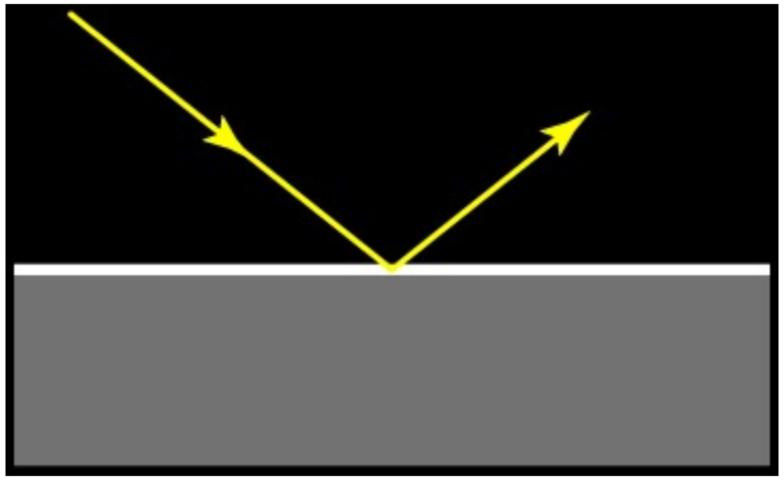
- **Interchanging arguments**
- **Physical requirement**

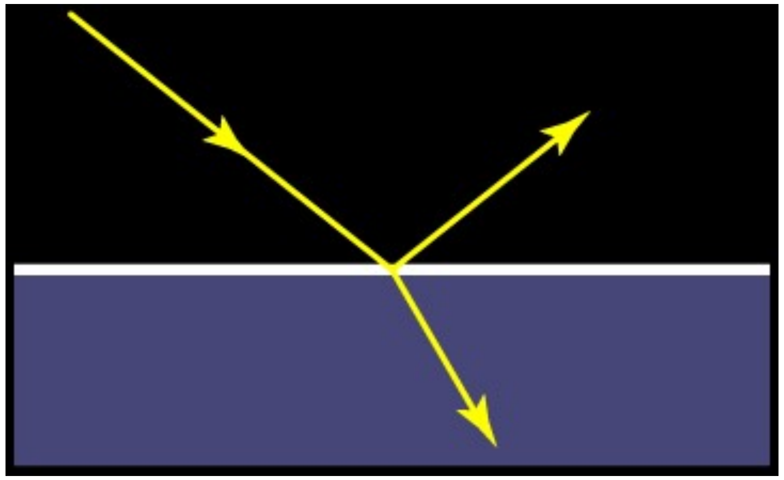


Smooth surfaces

metal

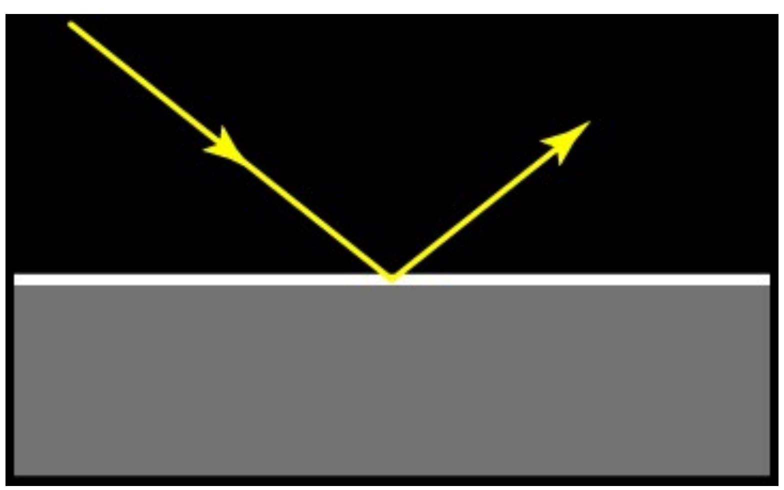
dielectric



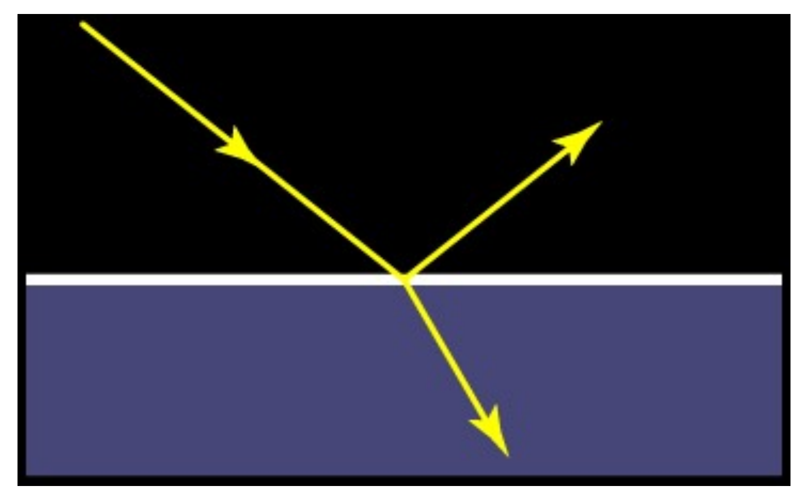


Ideal specular reflection

- **Smooth surfaces of pure materials have ideal specular reflection**
  – Metals (conductors) and dielectrics (insulators) behave differently
- **Reflectance (fraction of light reflected) depends on angle**

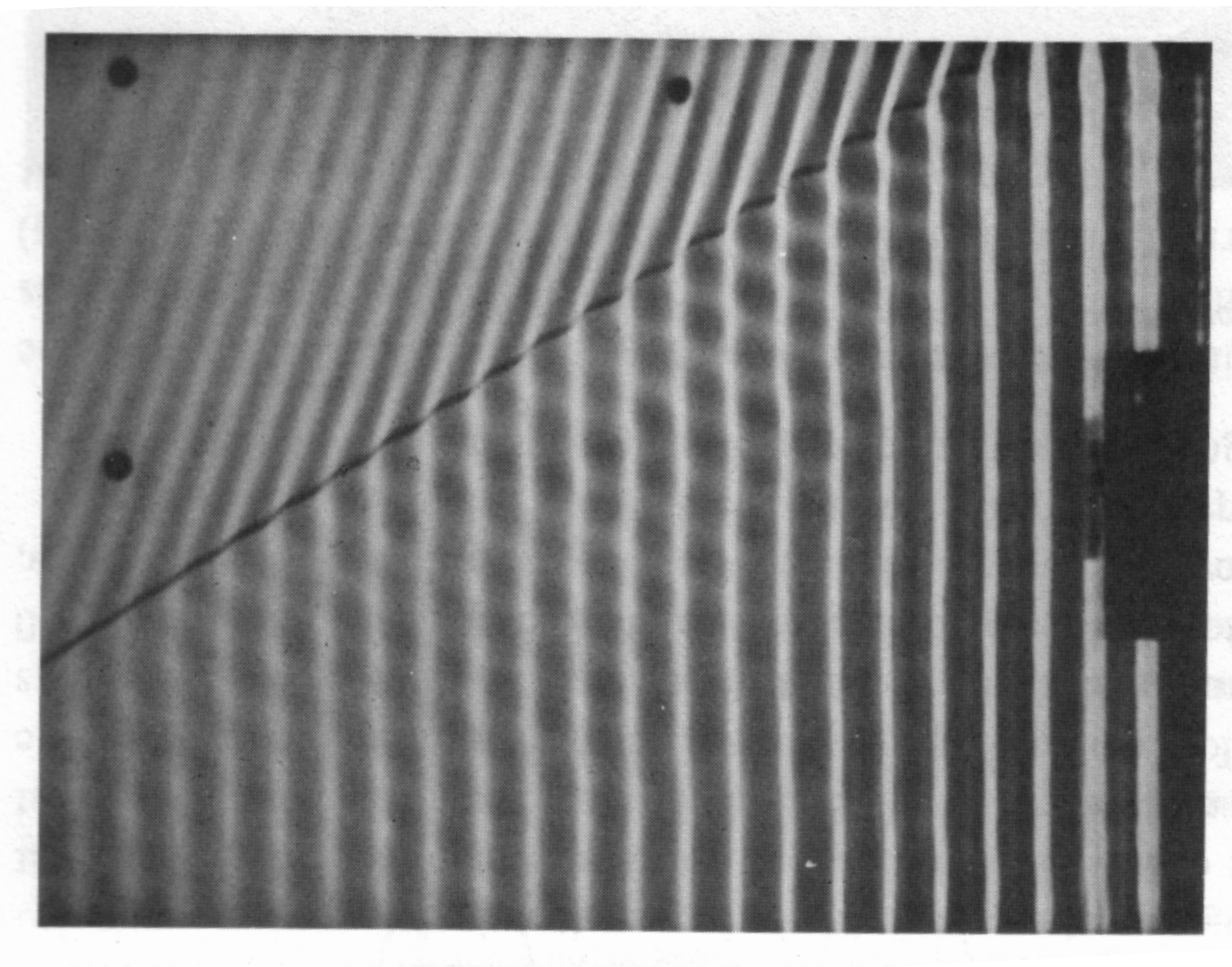


metal



dielectric

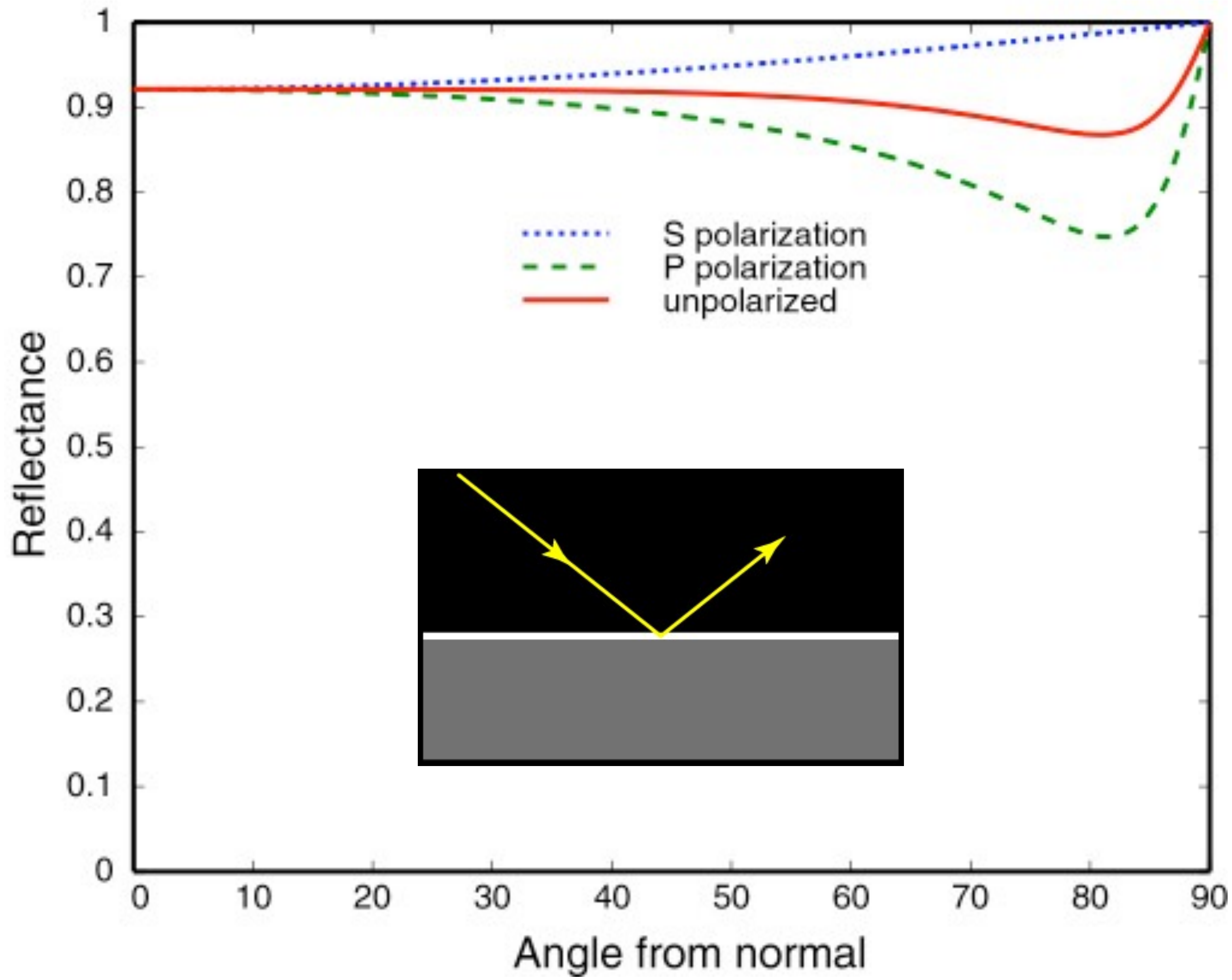
Refraction at boundary of media



Specular reflection from metal

- **Reflectance does depend on angle**
  - but not much
  - safely ignored in basic rendering

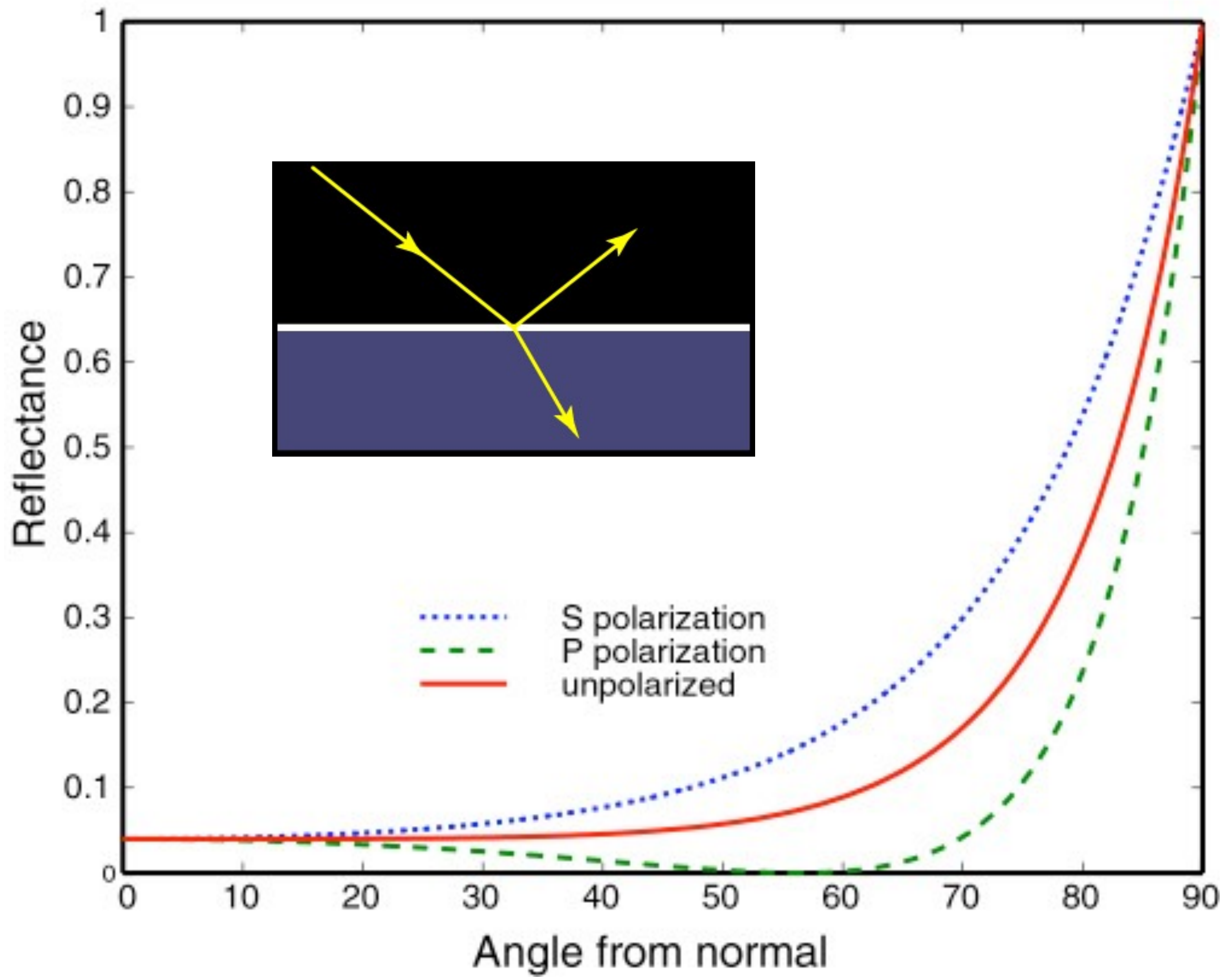
Aluminum



Specular reflection from glass/water

- **Dependence on angle is dramatic!**
  - about 4% at normal incidence
  - always 100% at grazing
  - remaining light is transmitted
- **This is important for proper appearance**

Glass



Fresnel's formulas

- **They predict how much light reflects from a smooth interface between two materials**
  - usually one material is empty space

$$F_p = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$F_s = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_2}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2}$$

$$R = \frac{1}{2}\left(F_p^2 + F_s^2\right)$$

where

$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

note: the formula in the notes and assignment is different but equivalent.

  - R is the fraction that is reflected
  - $(1 - R)$ is the fraction that is transmitted

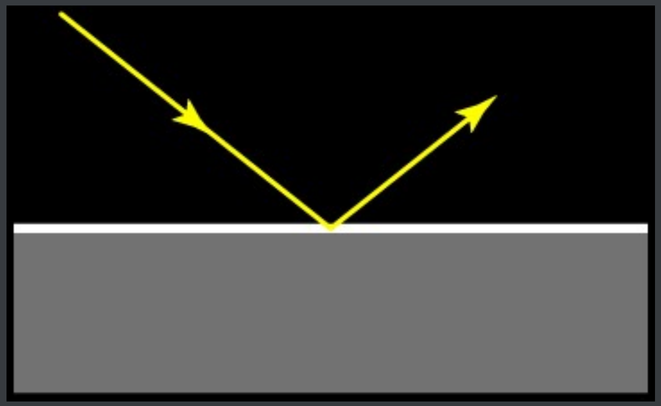
Fresnel reflection

[Mike Hill & Gaain Kwan | Stanford cs348 competition 2001]

Simple kinds of scattering

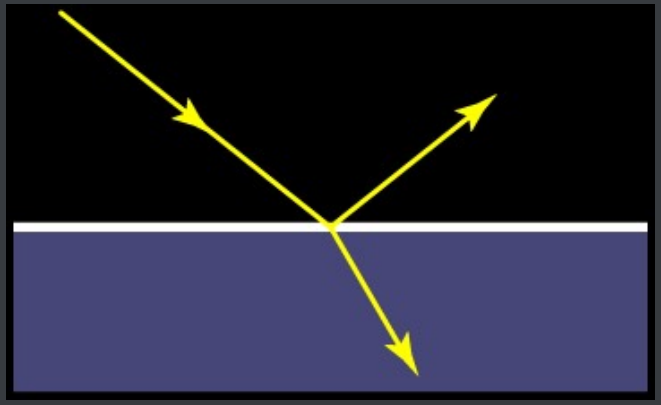
Ideal specular reflection

- incoming ray reflected to a single direction
- mirror-like behavior
- arises at smooth surfaces



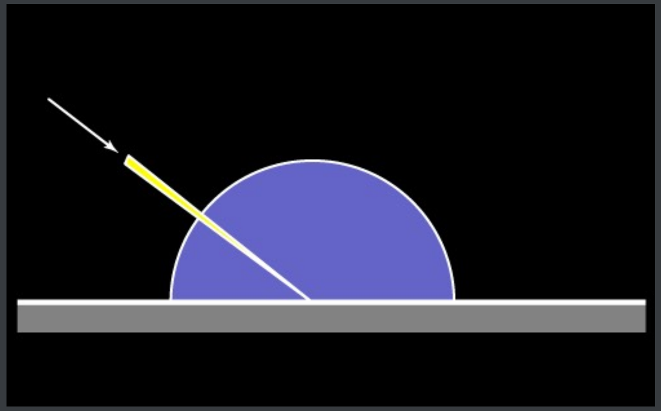
Ideal specular transmission

- incoming ray refracted to a single direction
- glass-like behavior
- arises at smooth dielectric (nonmetal) surfaces

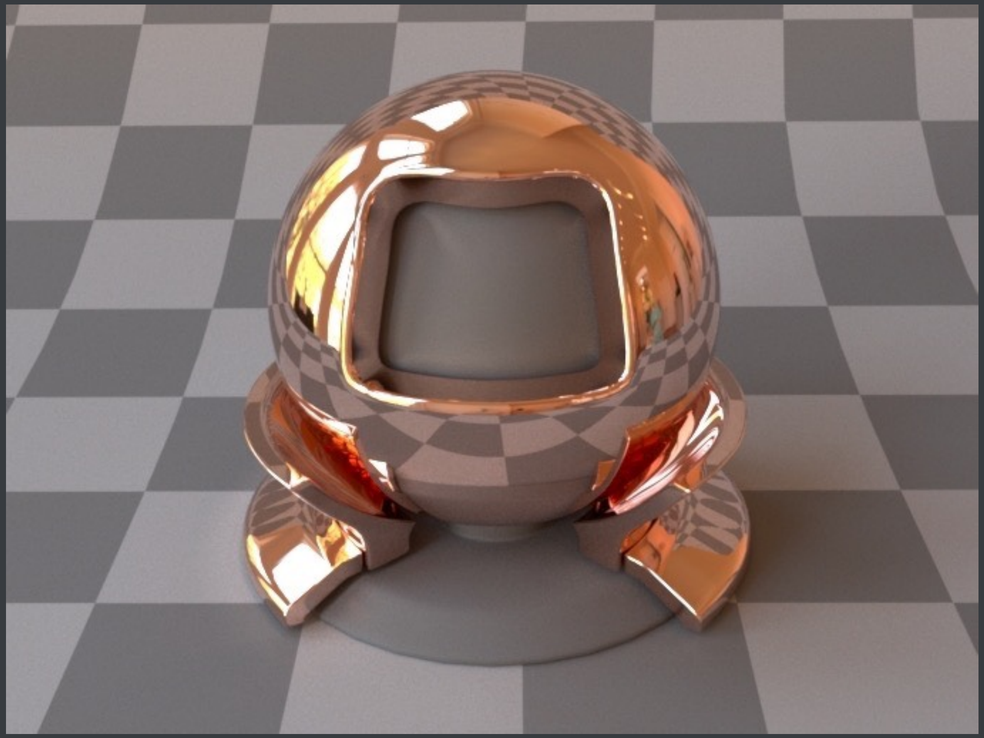


Ideal diffuse reflection or transmission

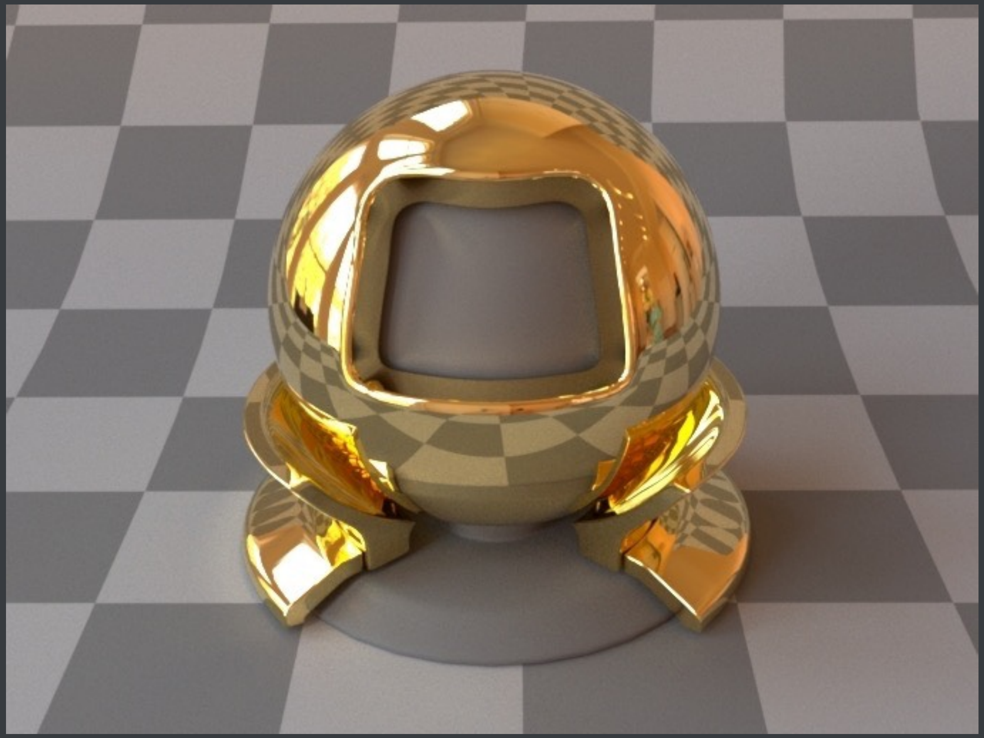
- outgoing radiance independent of direction
- arises from subsurface multiple scattering



Ideal specular reflection from metals



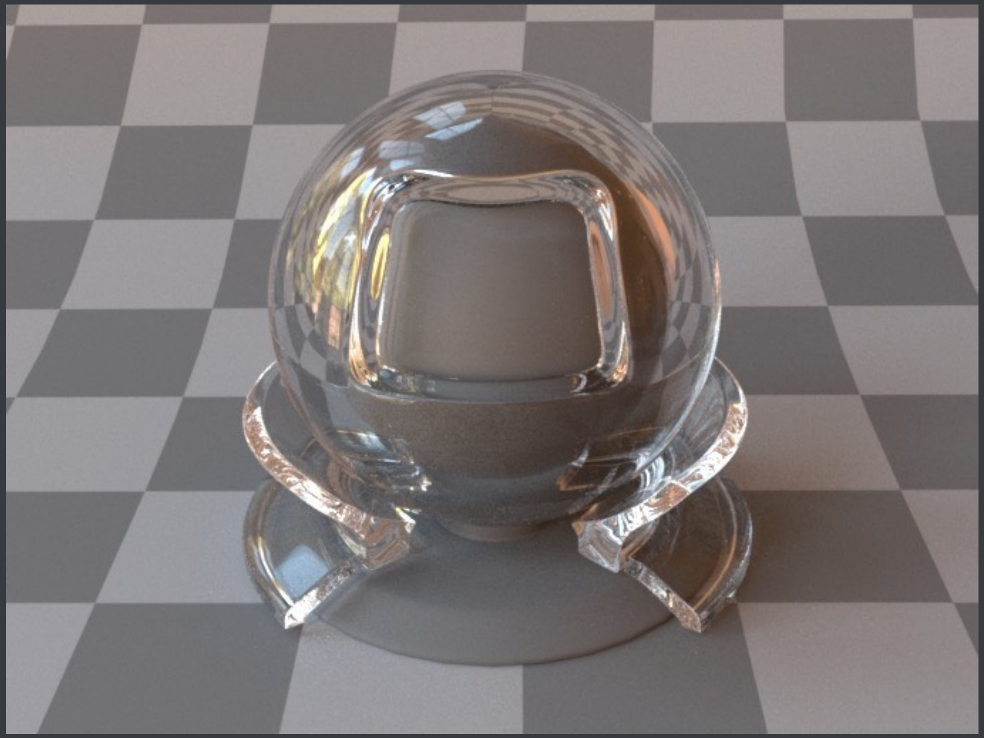
Cu



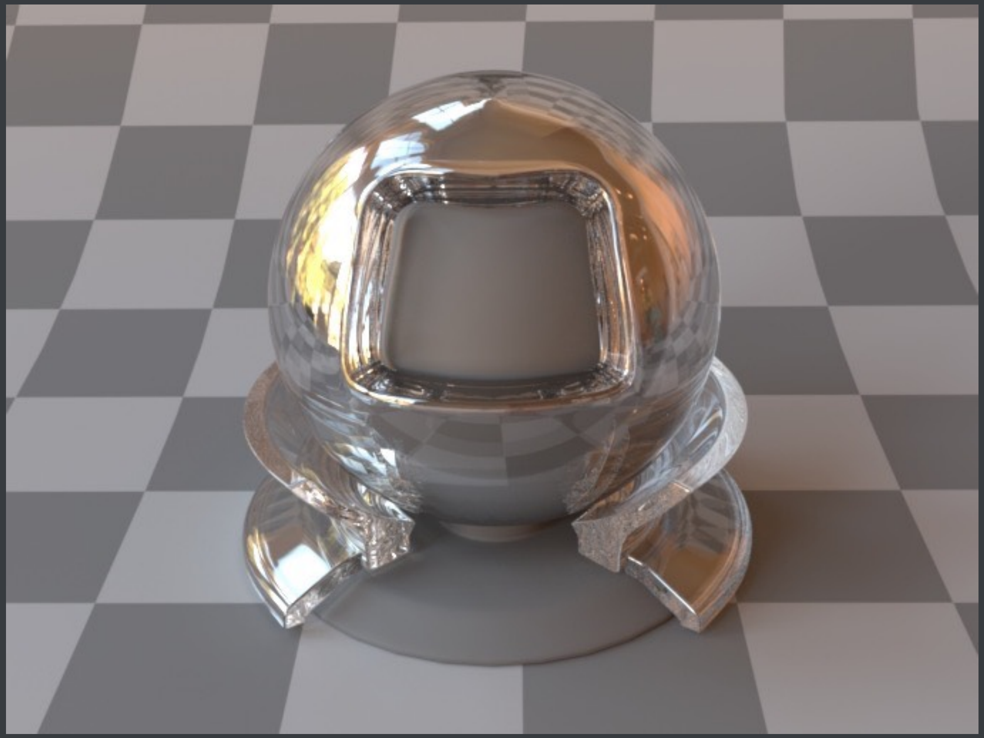
Au

Wenzel Jakob / Mistuba

Ideal reflection and transmission from smooth dielectrics



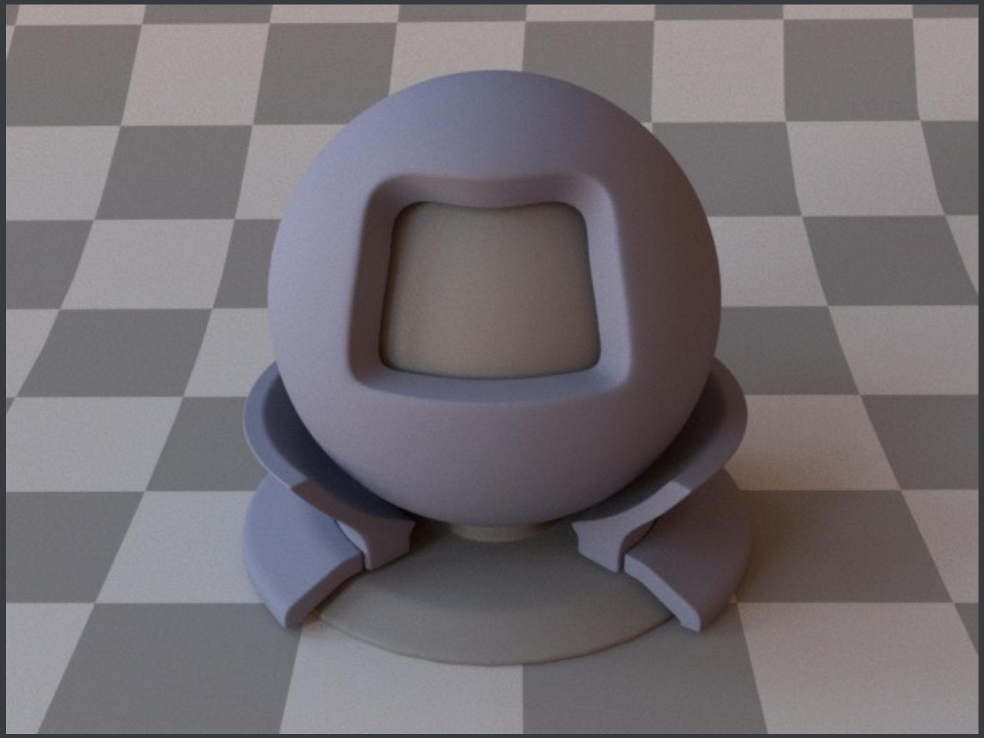
Water (ior = 1.33)

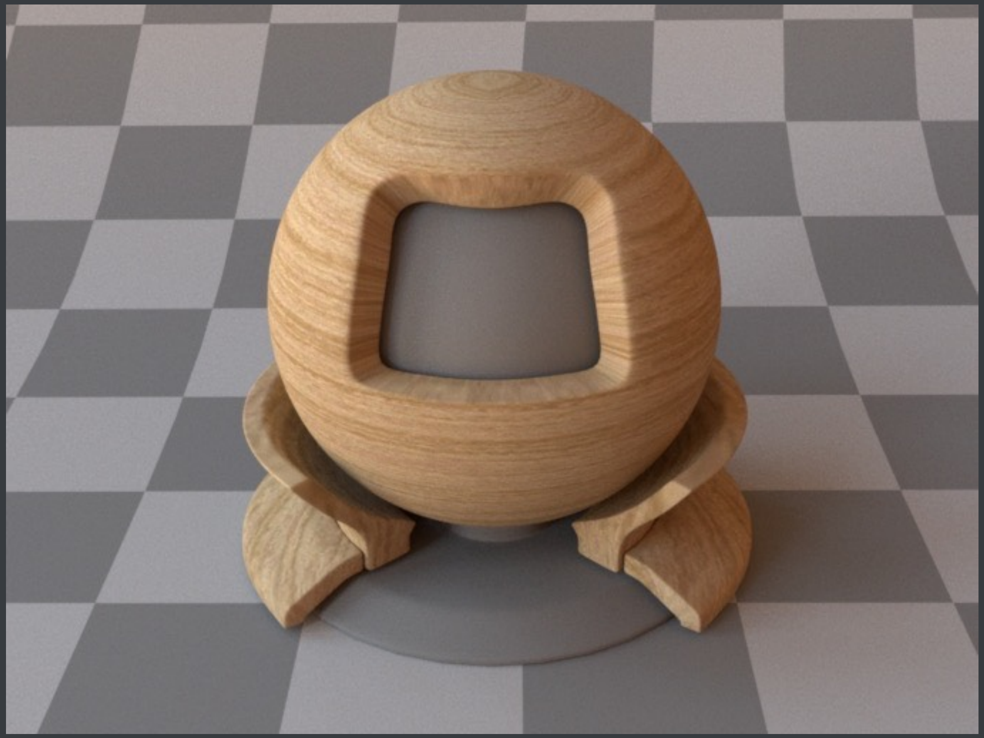


Diamond (ior = 2.4)

Wenzel Jakob / Mistuba

Two diffuse surfaces





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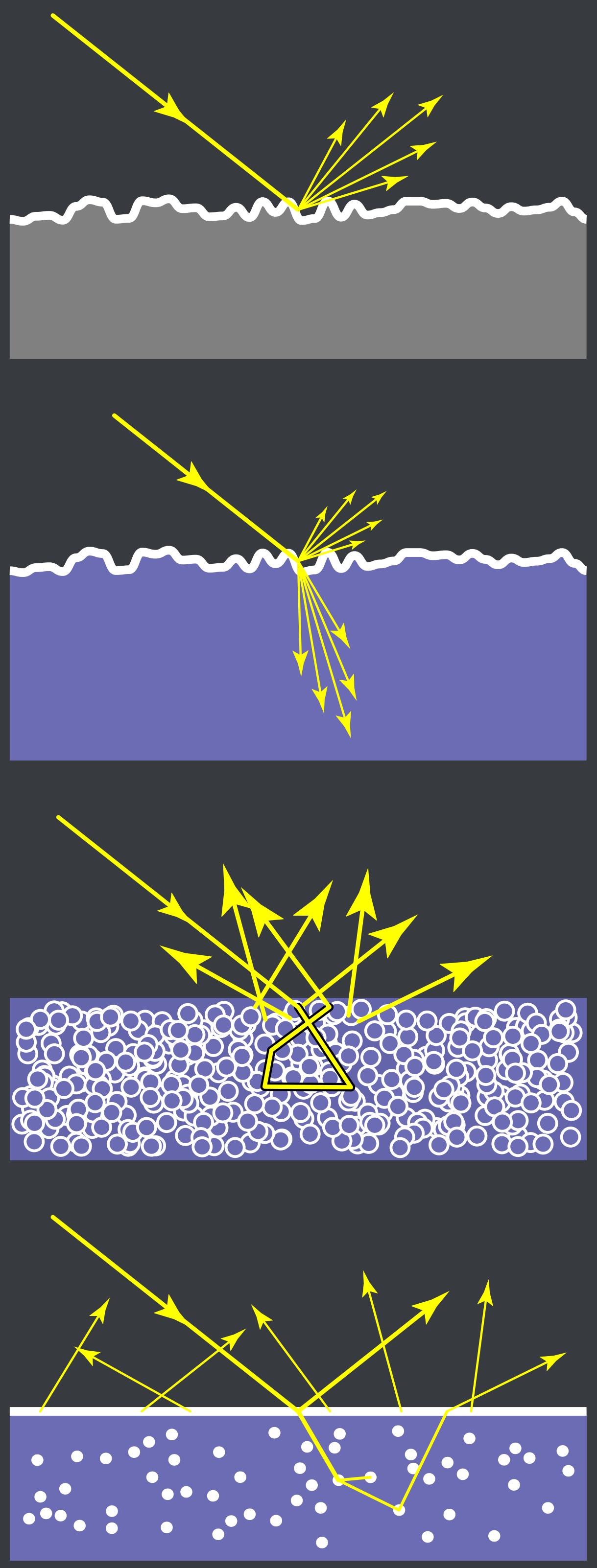
More complex scattering



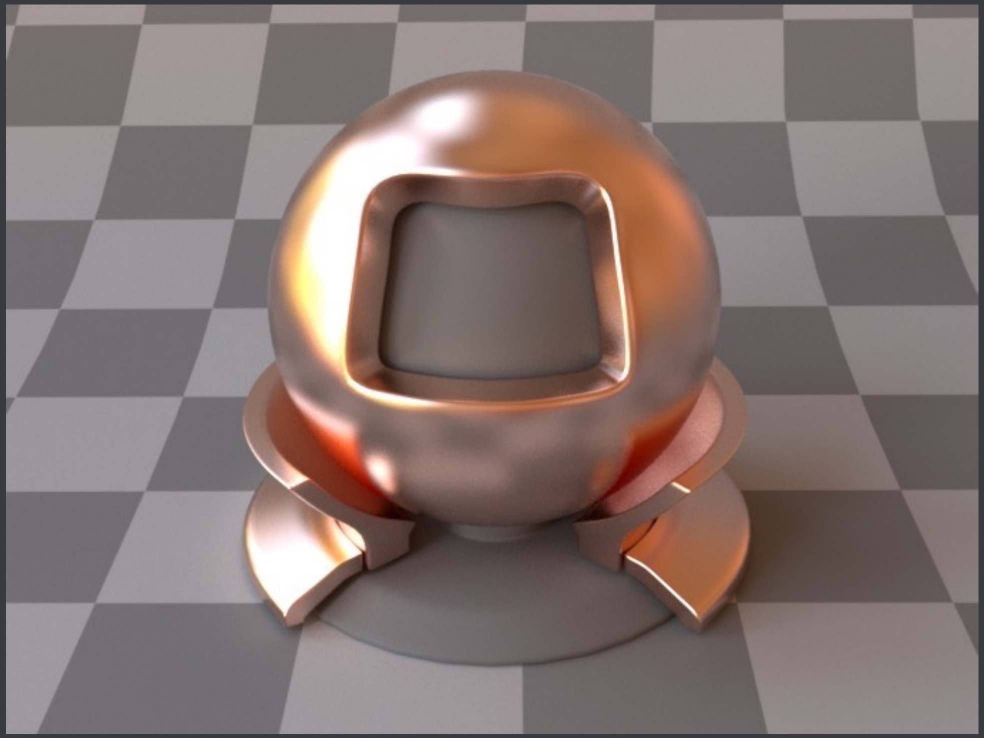
Rough interfaces

- metal interfaces: blurred reflection
- dielectric interfaces: blurred transmission

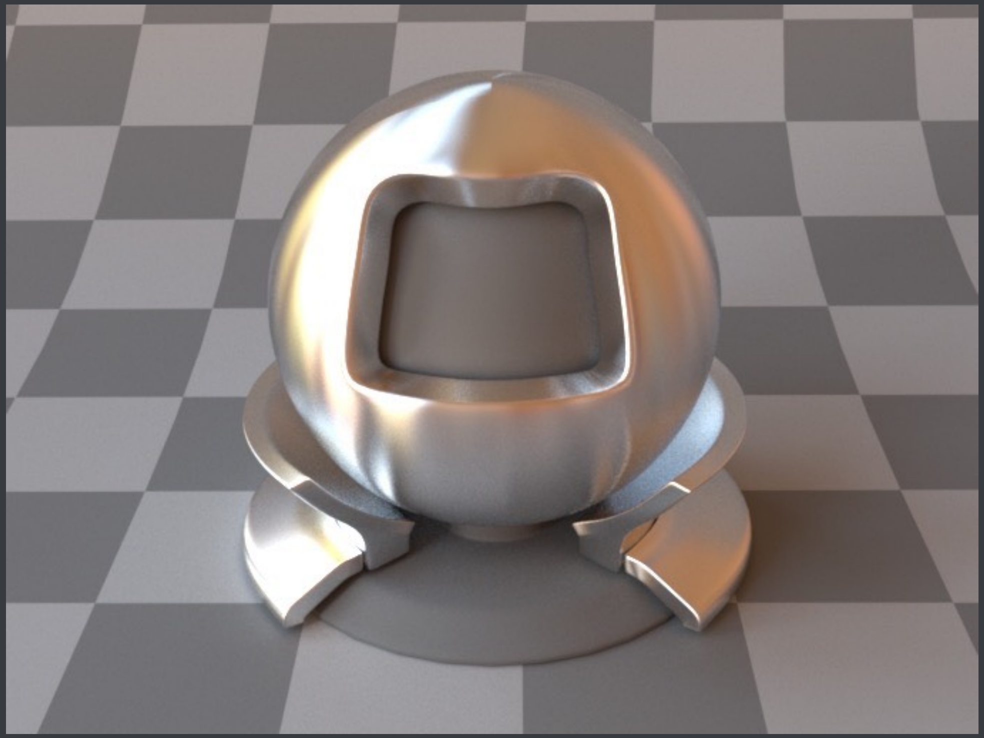
Subsurface scattering

- liquids—milk, juice, beer, …
- coatings—paint, glaze, varnish, …
- natural materials—wood, marble, …
- biological materials—skin, plants, …
- low optical density leads to *translucency*

Reflection from rough metal interfaces



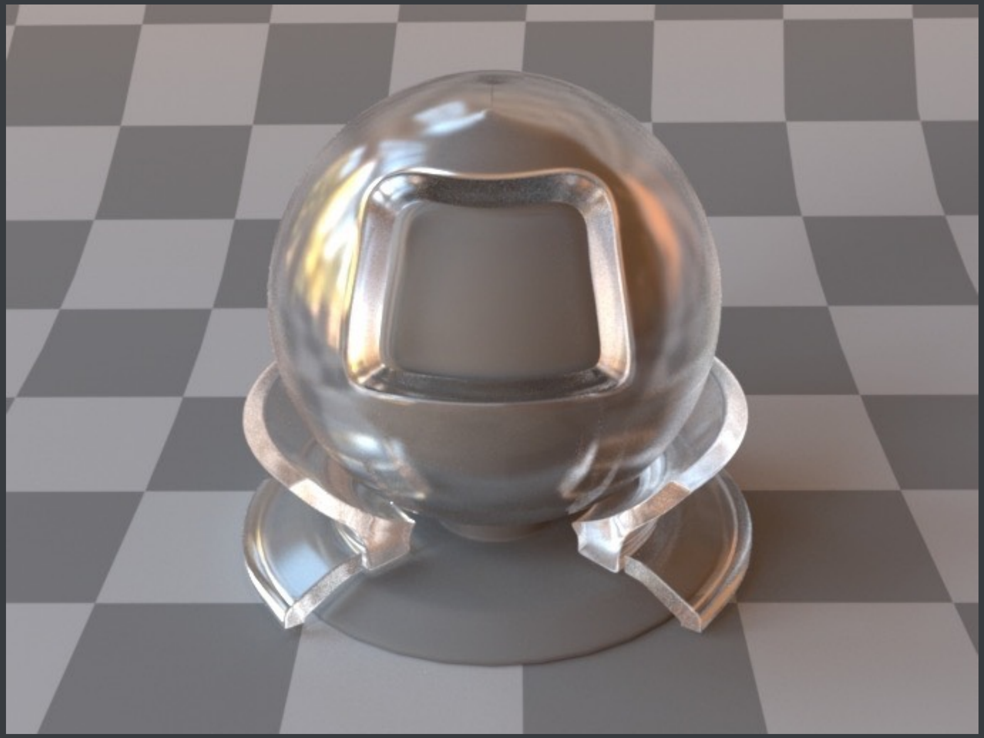
Cu (α = 0.1)



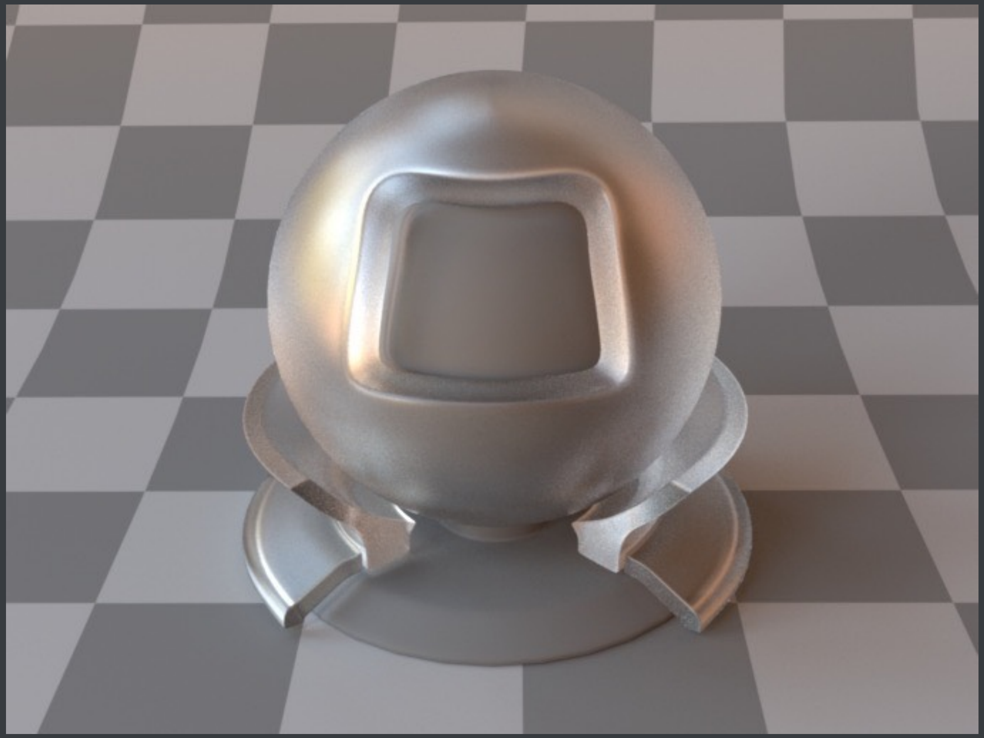
Al (anisotropic)

Wenzel Jakob / Mistuba

Reflection and refraction at rough dielectric interfaces



Anti-glare glass (α = 0.02)



Etched glass (α = 0.1)

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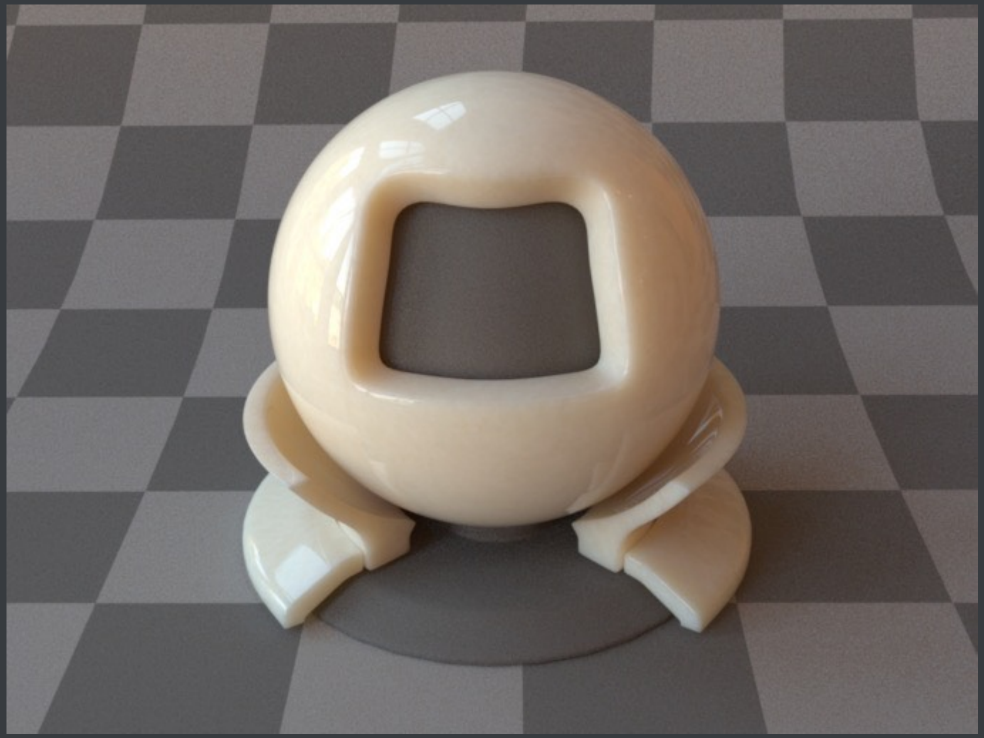
Translucent materials



Wenzel Jakob / Mistuba

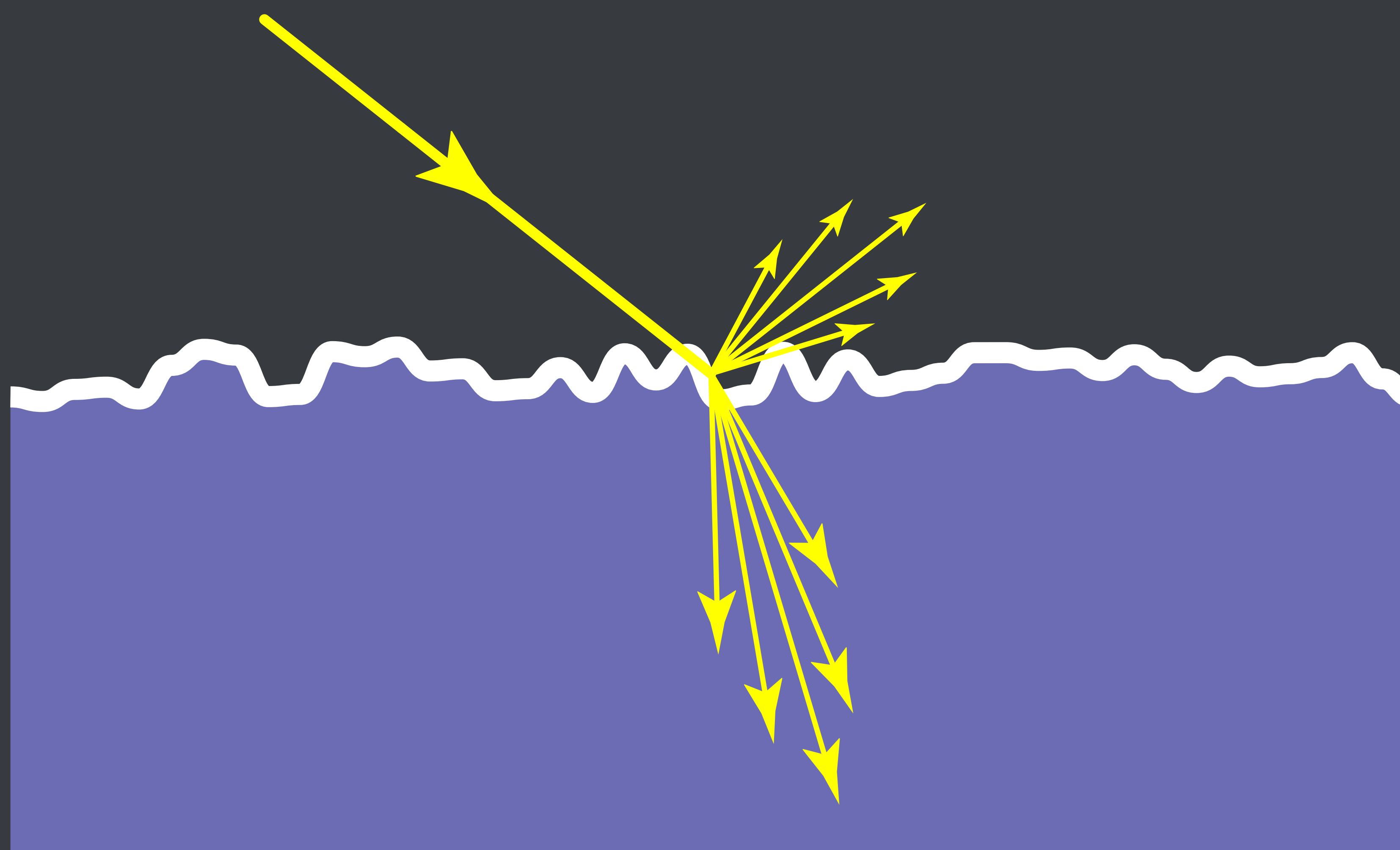
“skim milk”

low optical density

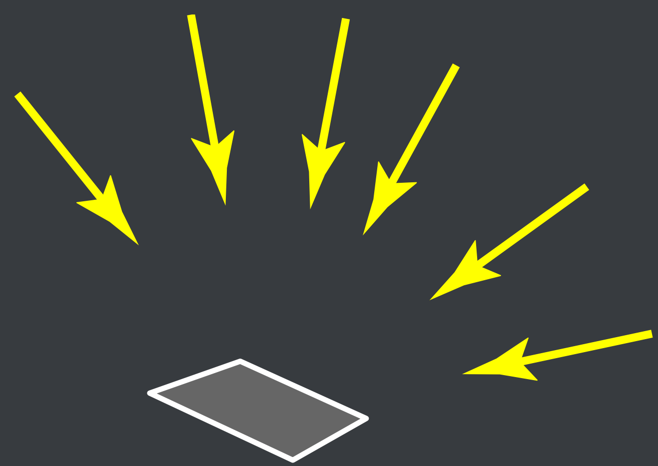
Wenzel Jakob / Mistuba

high optical density

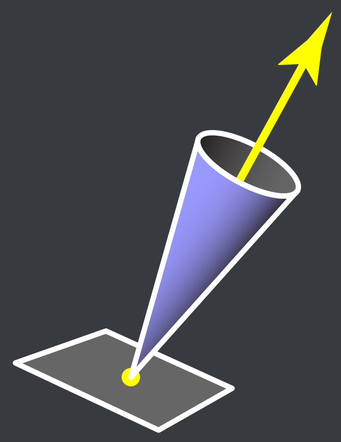
Rough surface scattering



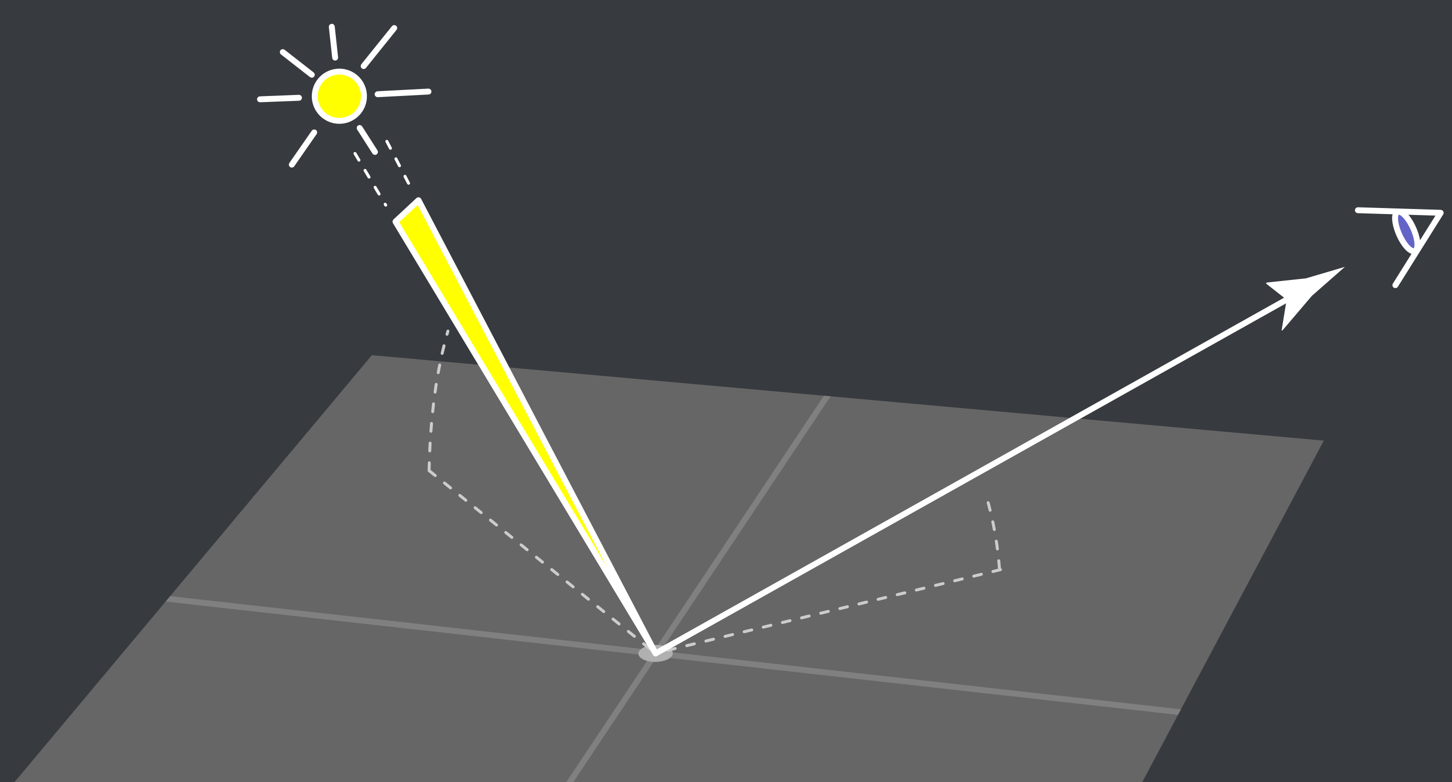
Units for reflection



irradiance [W/m²]
power per unit area
$E(\mathbf{x})$

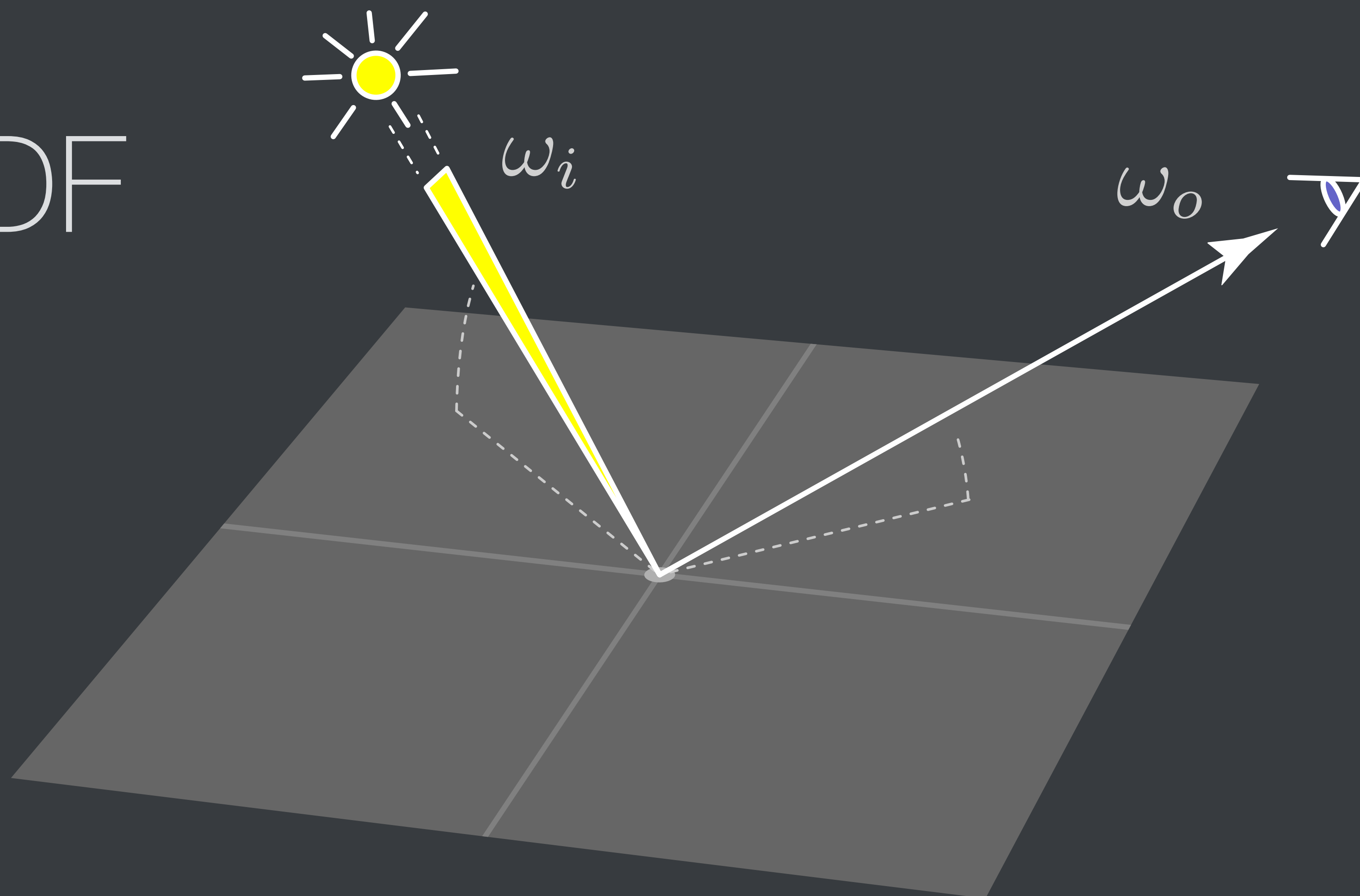


radiance [W/(m² sr)]
power per unit area
per unit solid angle
$L(\mathbf{x}, \omega)$



BRDF [1/sr]
reflected radiance
per unit incident irradiance
$f_r(\mathbf{x}, \omega_i, \omega_o)$

BRDF/BSDF

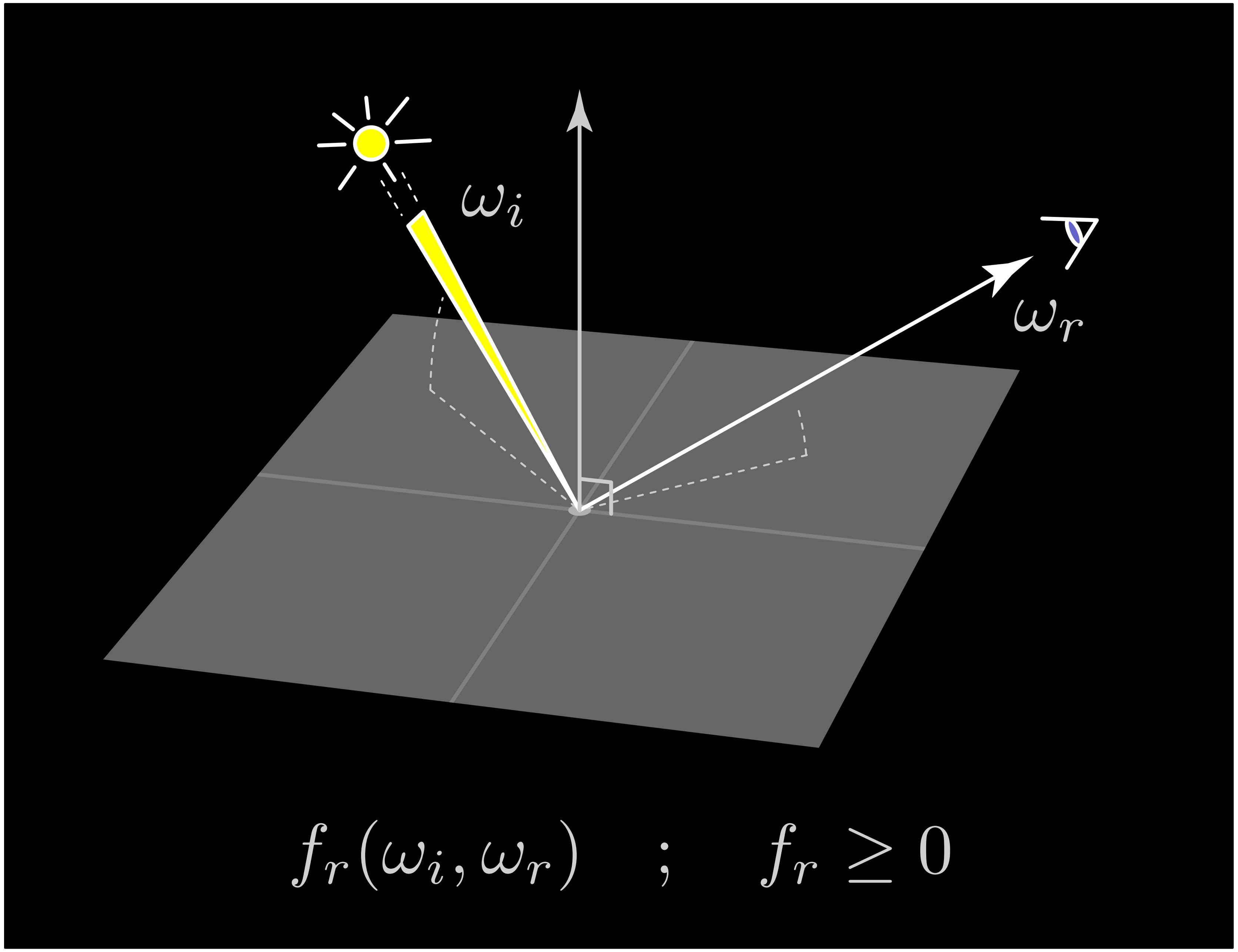


$$f_r(\mathbf{x}, \omega_i, \omega_o) \quad ; \quad f_r \geq 0$$

Bidirectional Reflectance Distribution Function
(both dirs on same side)

Bidirectional Scattering Distribution Function
(dirs can be on either side—includes transmission)

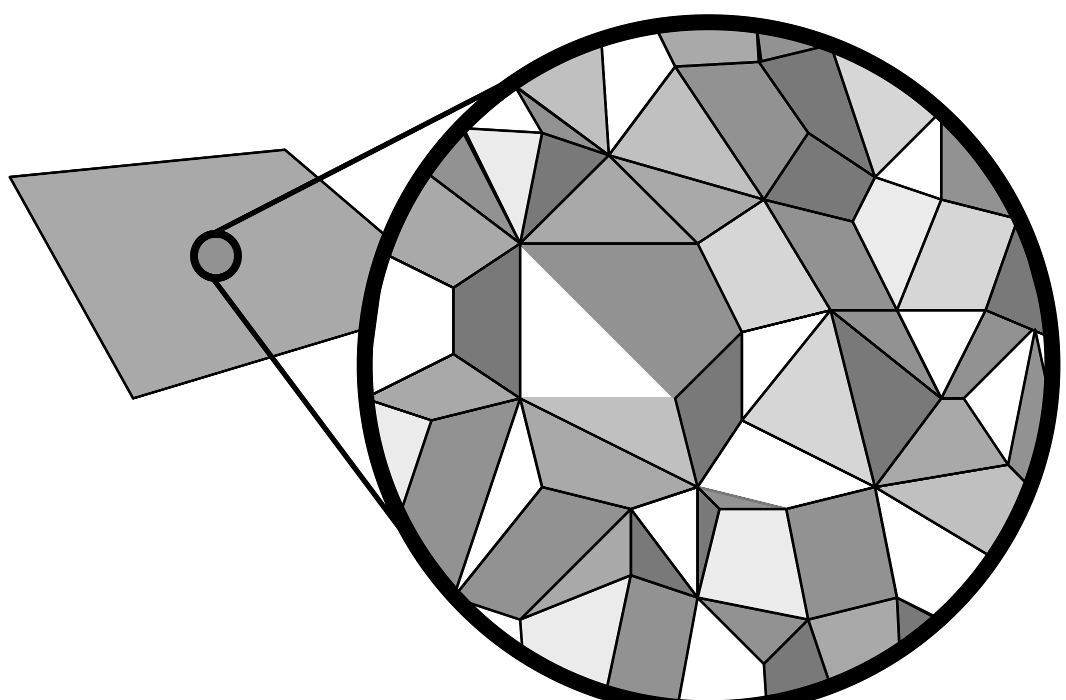
BRDF



Bidirectional Reflectance Distribution Function

Microfacet BRDF Model

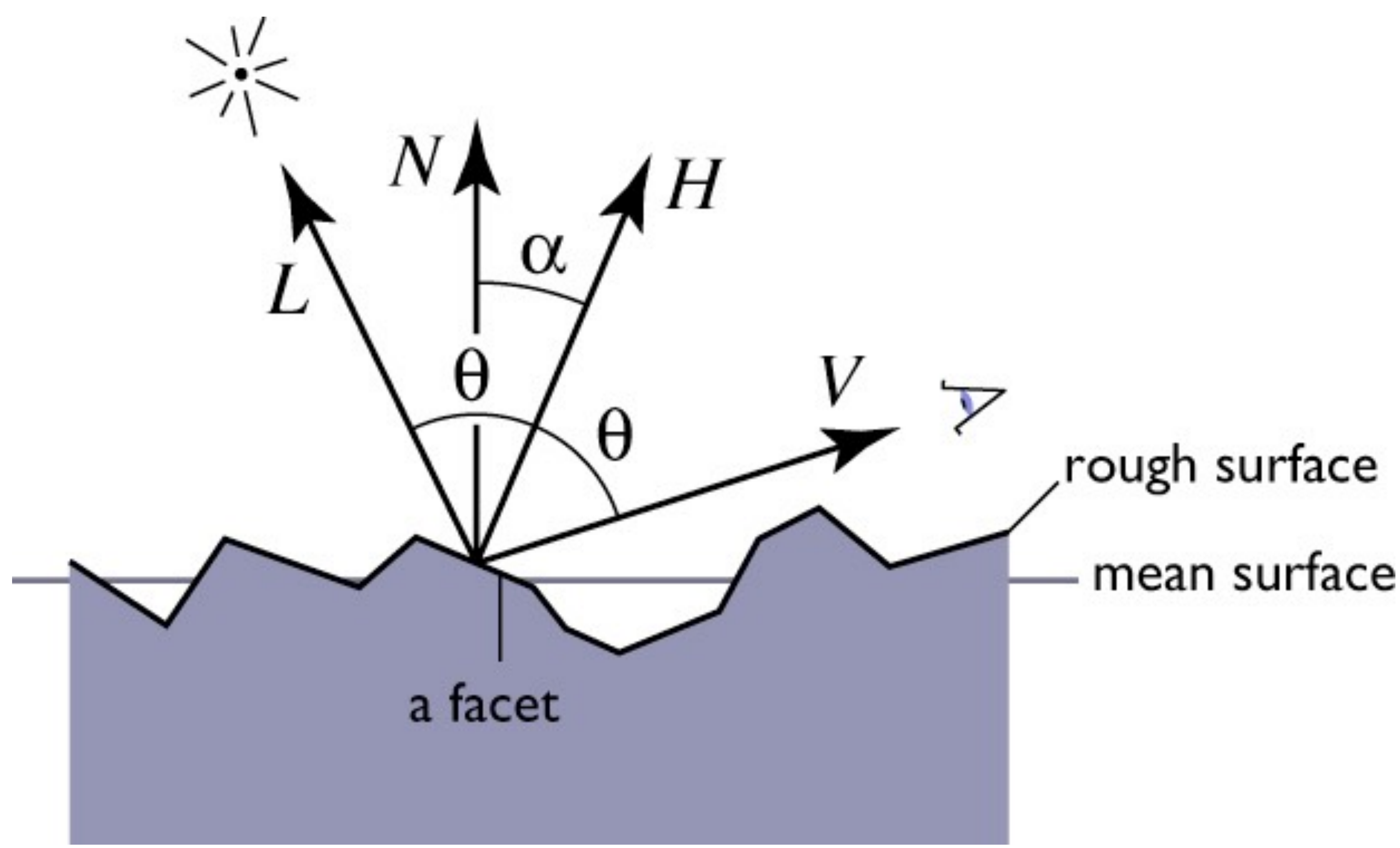
- **The *microfacet* idea**
  - surface modeled as random collection of planar facets
  - an incoming ray hits exactly one facet, at random
- **Key input: probability distribution of facet angle**



[Stephen Westin]

Facet Reflection

- ***H* vector used to define facets that contribute**
  - L and V determine H; only facets with that normal matter
  - reflected light is proportional to number of facets



Microfacet BRDF Model

$$f_r(\mathbf{n}, \mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h}) D(\mathbf{h}) G(\mathbf{l}, \mathbf{v}, \mathbf{h})}{4 |\mathbf{n} \cdot \mathbf{l}| |\mathbf{n} \cdot \mathbf{v}|}$$

Cook-Torrance BRDF Model

Fresnel Reflectance

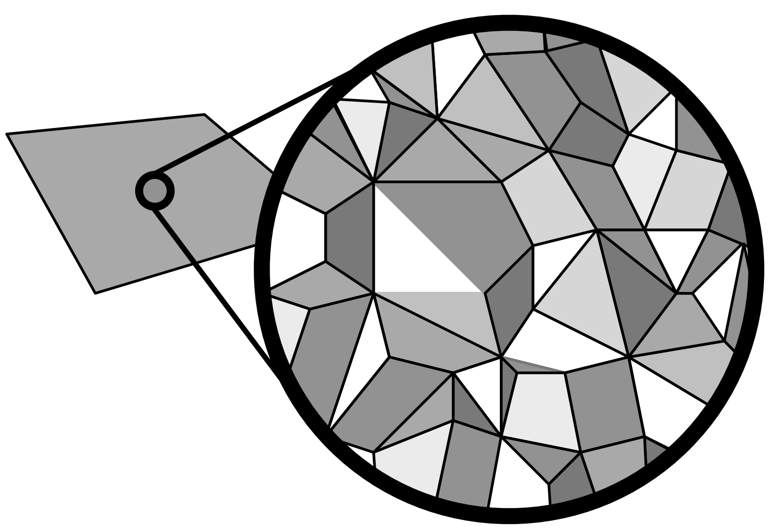
$$f_r(\mathbf{n}, \mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h}) D(\mathbf{h}) G(\mathbf{l}, \mathbf{v}, \mathbf{h})}{4 |\mathbf{n} \cdot \mathbf{l}| |\mathbf{n} \cdot \mathbf{v}|}$$

- **Fresnel reflectance for smooth facet**
  - more light reflected at grazing angles

Microfacet BRDF Model

Facet distribution

$$f_r(\mathbf{n}, \mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h})D(\mathbf{h})G(\mathbf{l}, \mathbf{v}, \mathbf{h})}{4|\mathbf{n} \cdot \mathbf{l}||\mathbf{n} \cdot \mathbf{v}|}$$



[Stephen Westin]

Facet Distribution

- **D function describes distribution of h**
- **Many choices, depending on surface characteristics**
- **A classic choice is due to Beckmann**
  - derivation based on Gaussian random processes

$$D(\mathbf{h}) = \frac{e^{-\frac{\tan^2(\mathbf{h},\mathbf{n})}{m^2}}}{\pi m^2 \cos^4(\mathbf{h},\mathbf{n})}$$

Cook-Torrance BRDF Model

Masking/shadowing

$$f_r(\mathbf{n}, \mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h}) D(\mathbf{h}) G(\mathbf{l}, \mathbf{v}, \mathbf{h})}{4|\mathbf{n} \cdot \mathbf{l}||\mathbf{n} \cdot \mathbf{v}|}$$

Masking and Shadowing

- **Many options; Smith shadowing-masking follows from *D***
  - long story, it is an integral related to *D* that often doesn't have a closed form solution
  - generally it is assumed that masking and shadowing are statistically independent

$$G(\omega_i, \omega_r, \mathbf{h}) = G_1(\omega_i, \mathbf{h}) G_1(\omega_r, \mathbf{h})$$

  - for Beckmann, recommend using this rational approximation due to Bruce Walter:

$$G_1(\mathbf{v}, \mathbf{h}) \;=\; \chi^+\left(\frac{\mathbf{v}\cdot\mathbf{h}}{\mathbf{v}\cdot\mathbf{n}}\right) \begin{cases} \frac{3.535a + 2.181a^2}{1 + 2.276a + 2.577a^2}, & \text{if } a < 1.6 \\ 1 & \text{otherwise} \end{cases}$$

Microfacet BRDF Model

$$f_r(\mathbf{n}, \mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h}) D(\mathbf{h}) G(\mathbf{l}, \mathbf{v}, \mathbf{h})}{4 |\mathbf{n} \cdot \mathbf{l}| |\mathbf{n} \cdot \mathbf{v}|}$$

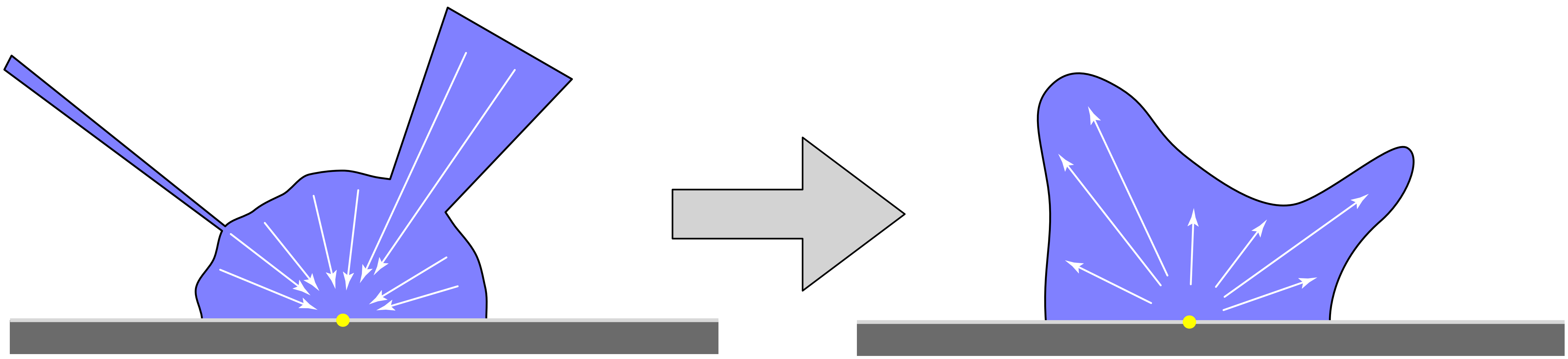
– reasons for cosine terms in denominator
– if one is there they clearly both have to be there (by reciprocity)

Behaviors of surface reflections

- **diffuse reflections**
  - depict general shape well
  - independent of (or weakly dependent on) view direction
  - very often colored
  - generally produce pixel values that end up below 1.0
- **ideal specular reflections (Fresnel equations)**
  - reflect environment with sharp details
  - on dielectrics, brighter reflections at increasing angles
- **rough-surface specular reflections (microfacet model)**
  - reflect light sources and environment with a blur (blur increases with roughness)
  - on dielectrics, brighter reflections at increasing angles

Light reflection: full picture

- **when writing a shader, think like a bug standing on the surface**
  - bug sees an incident distribution of light arriving at the surface
  - physics question: what is the outgoing distribution of light?



incident distribution
(function of direction)

reflected distribution
(function of direction)

Surface illumination integral (as sum)

- **BRDF tells you how light from a single direction is reflected**
- **Light coming from a small source behaves similarly**
- **What about light coming from everywhere?**
  - approximate incoming light with many small sources on a sphere (the little bug can't tell the difference…)
  - reflected light is sum of reflected light due to each source (each source has its size Ω_k, brightness L_k, and direction $\boldsymbol{\omega}_k$)

$$\underbrace{L_r(\omega_r)}_{\text{reflected light in direction } \boldsymbol{\omega}_r} = \sum_k \underbrace{\Omega_k L_k}_{\text{“intensity” of light source } k} \underbrace{f_r(\omega_k, \omega_r)}_{\text{BRDF}} \underbrace{|\omega_k \cdot \mathbf{n}|}_{\text{cosine factor}}$$

Surface illumination integral

- **Take the limit as the little area sources get smaller**
  - collection of separate brightnesses L_k becomes a function $L_i(\omega_i)$
  - size of sources turns into an integration measure $d\sigma$

$$L_r(\omega_r) = \int_{S^2_+} L_i(\omega_i) f_r(\omega_i, \omega_r) |\omega_i \cdot \mathbf{n}| d\sigma(\omega_i)$$

"The light reflected to direction ω_r is the integral, over the positive unit hemisphere, of the incoming light times the BRDF times the incoming cosine factor, with respect to surface area."

