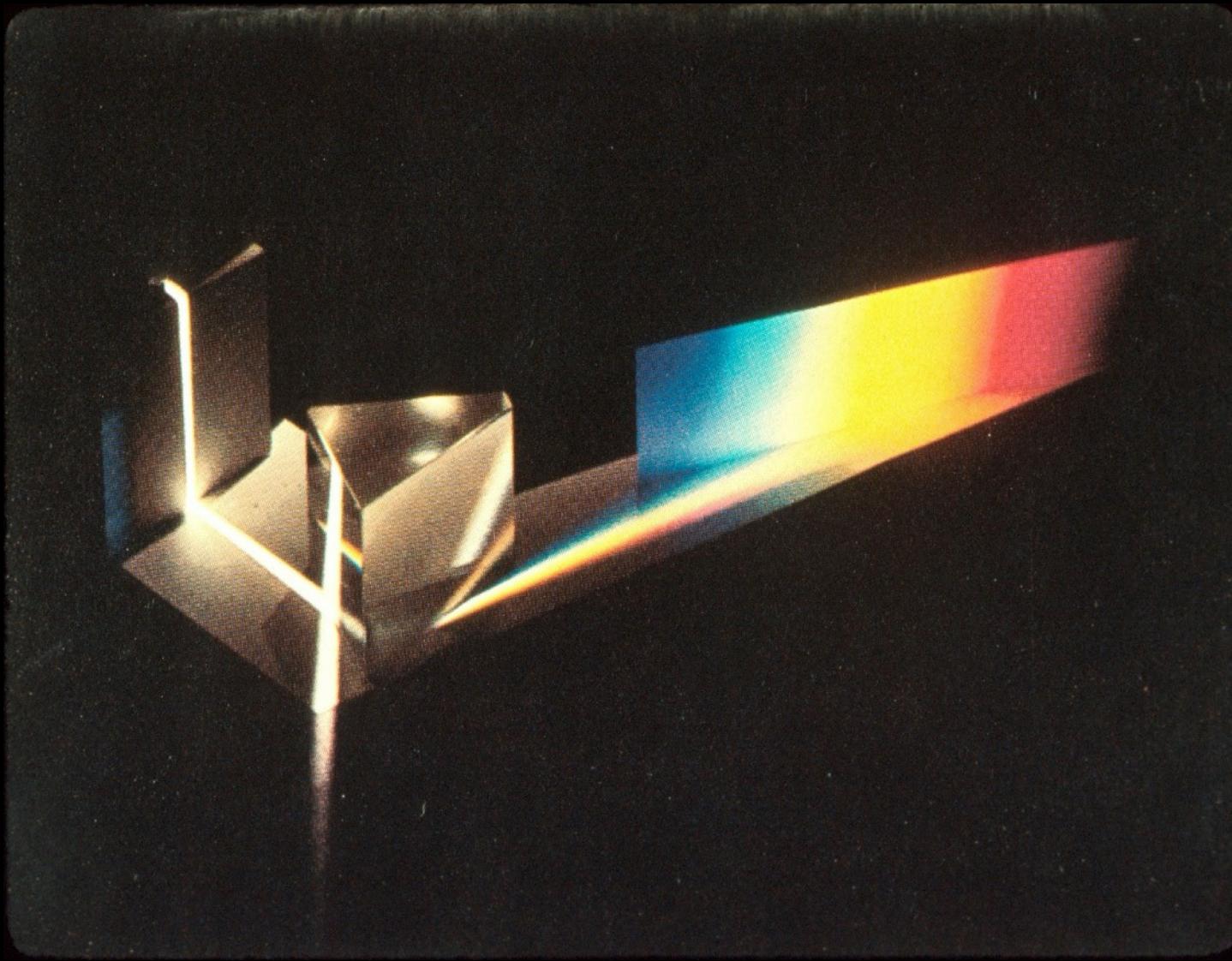


Color Science

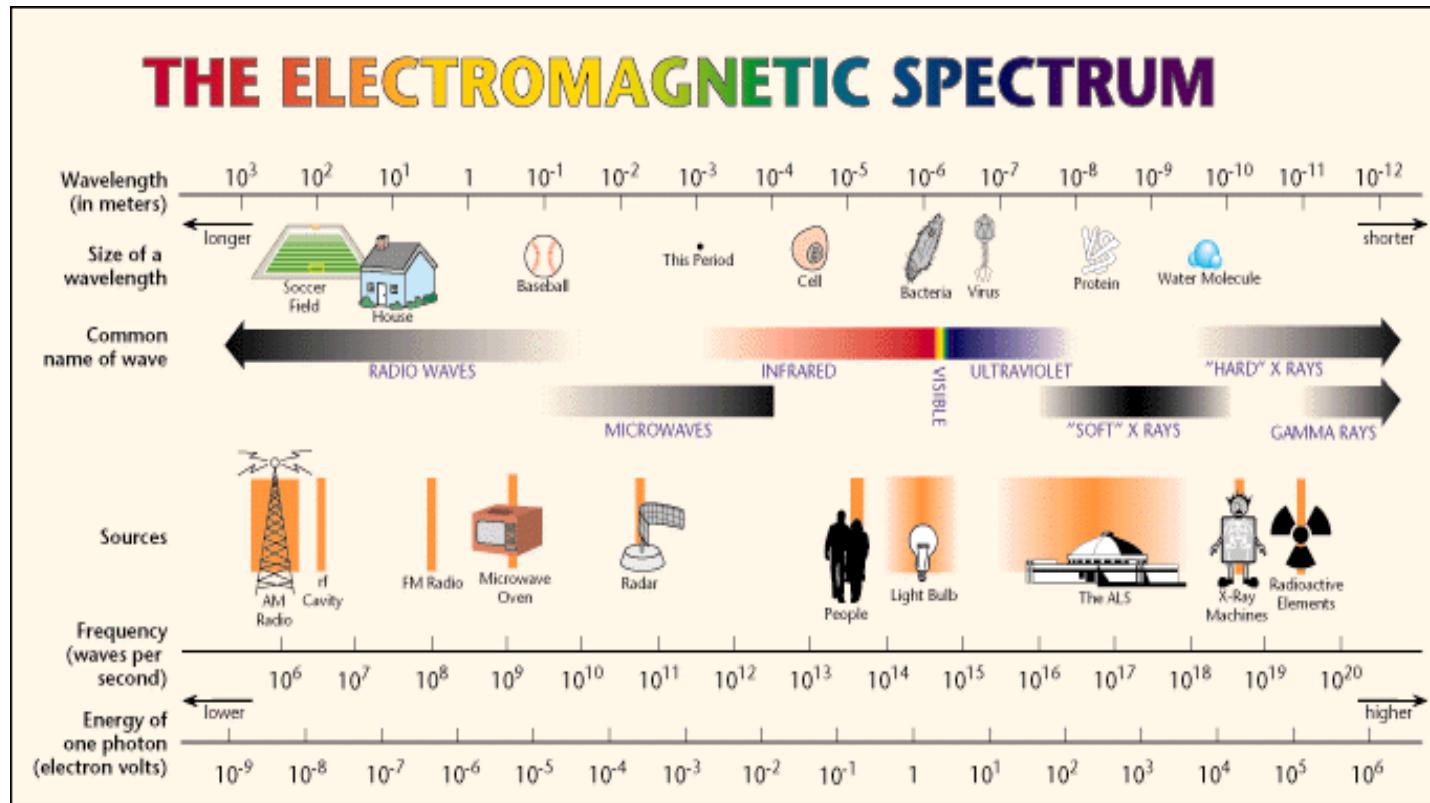
CS 4620 Lecture 23



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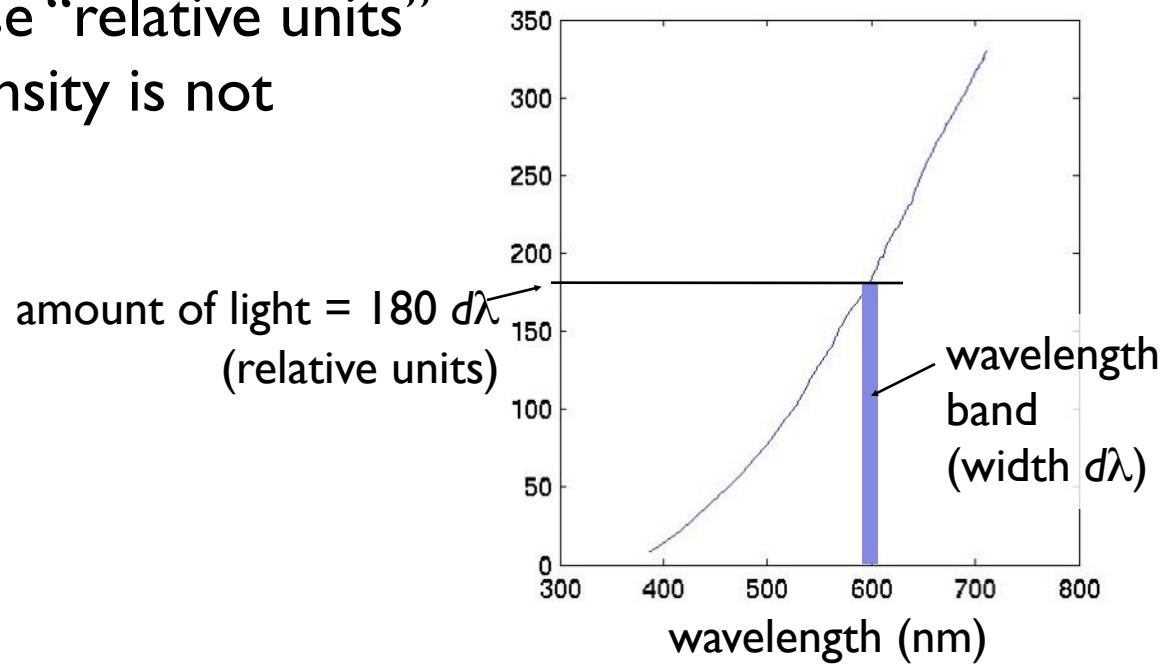
What light is

- Light is electromagnetic radiation
 - exists as oscillations of different frequency (or, wavelength)



Measuring light

- Salient property is the *spectral power distribution* (SPD)
 - the amount of light present at each wavelength
 - units: Watts per nanometer (tells you how much power you'll find in a narrow range of wavelengths)
 - for color, often use “relative units” when overall intensity is not important



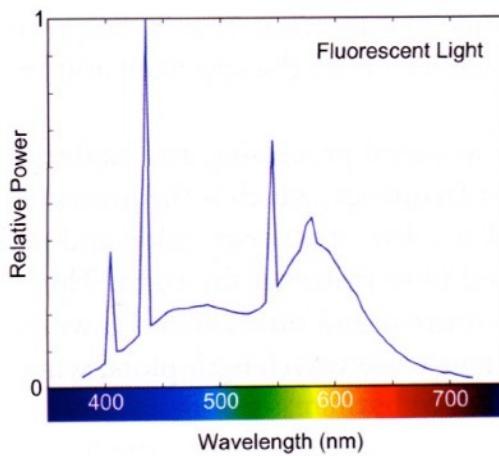
What color is

- Colors are the sensations that arise from light energy of different wavelengths
 - we are sensitive from about 380 to 760 nm—one “octave”
- Color is a phenomenon of human perception; it is **not** a universal property of light
- Roughly speaking, things appear “colored” when they depend on wavelength and “gray” when they do not.

The problem of color science

- Build a model for human color perception
- That is, map a *Physical light description* to a *Perceptual color sensation*

[Stone 2003]



Physical



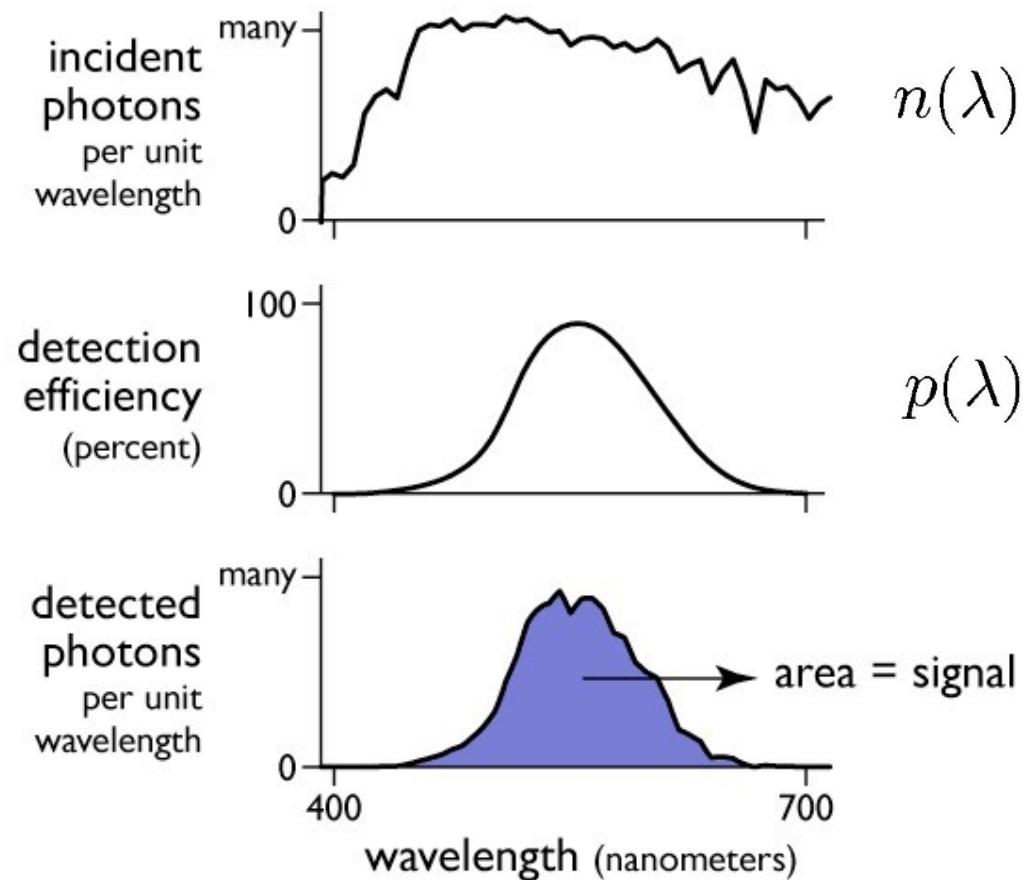
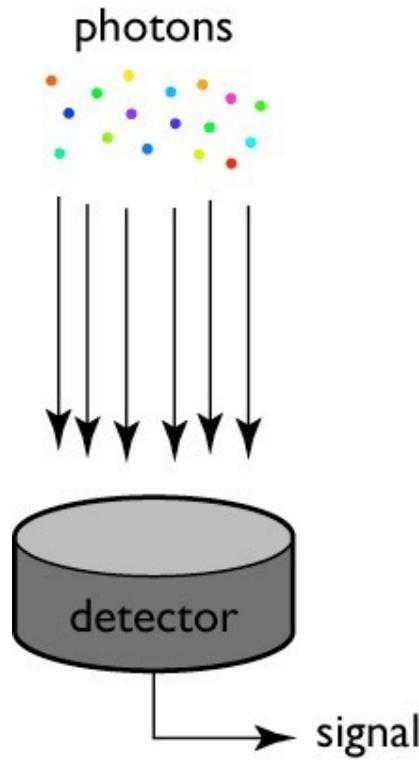
?

Perceptual

A simple light detector

- Produces a scalar value (a number) when photons land on it
 - this value depends strictly on the number of photons detected
 - each photon has a probability of being detected that depends on the wavelength
 - there is no way to tell the difference between signals caused by light of different wavelengths: there is just a number
- This model works for many detectors:
 - based on semiconductors (such as in a digital camera)
 - based on visual photopigments (such as in human eyes)

A simple light detector



$$X = \int n(\lambda)p(\lambda) d\lambda$$

Light detection math

- Same math carries over to power distributions
 - spectrum entering the detector has its spectral power distribution (SPD), $s(\lambda)$
 - detector has its *spectral sensitivity* or *spectral response*, $r(\lambda)$

$$X = \int s(\lambda)r(\lambda) d\lambda$$

The diagram shows the mathematical expression $X = \int s(\lambda)r(\lambda) d\lambda$. Three vertical lines extend downwards from the terms $s(\lambda)$, $r(\lambda)$, and $d\lambda$ respectively. Labels are placed below each line: 'measured signal' is under the first line, 'detector's sensitivity' is under the second line, and 'input spectrum' is under the third line.

Light detection math

- If we think of s and r as vectors, this operation is a dot product (aka inner product) $X = s \cdot r$
 - in fact, the computation is done exactly this way, using sampled representations of the spectra.
 - let λ_i be regularly spaced sample points $\Delta\lambda$ apart; then:

$$X = \int s(\lambda)r(\lambda) d\lambda$$

$$\tilde{s}[i] = s(\lambda_i); \tilde{r}[i] = r(\lambda_i)$$

$$\int s(\lambda)r(\lambda) d\lambda \approx \sum_i \tilde{s}[i]\tilde{r}[i] \Delta\lambda$$

- this sum is very clearly a dot product

Human observation

- Human eye observes electro-magnetic wavelengths
 - Humans ‘see’ different spectra as different colors
 - Color is a phenomenon of human perception; it is not a universal property of light
- Other animals observe other wavelengths
 - Bees: 340 – 540 nm
 - (they see no red, but can see ultra-violet)

Insects and color

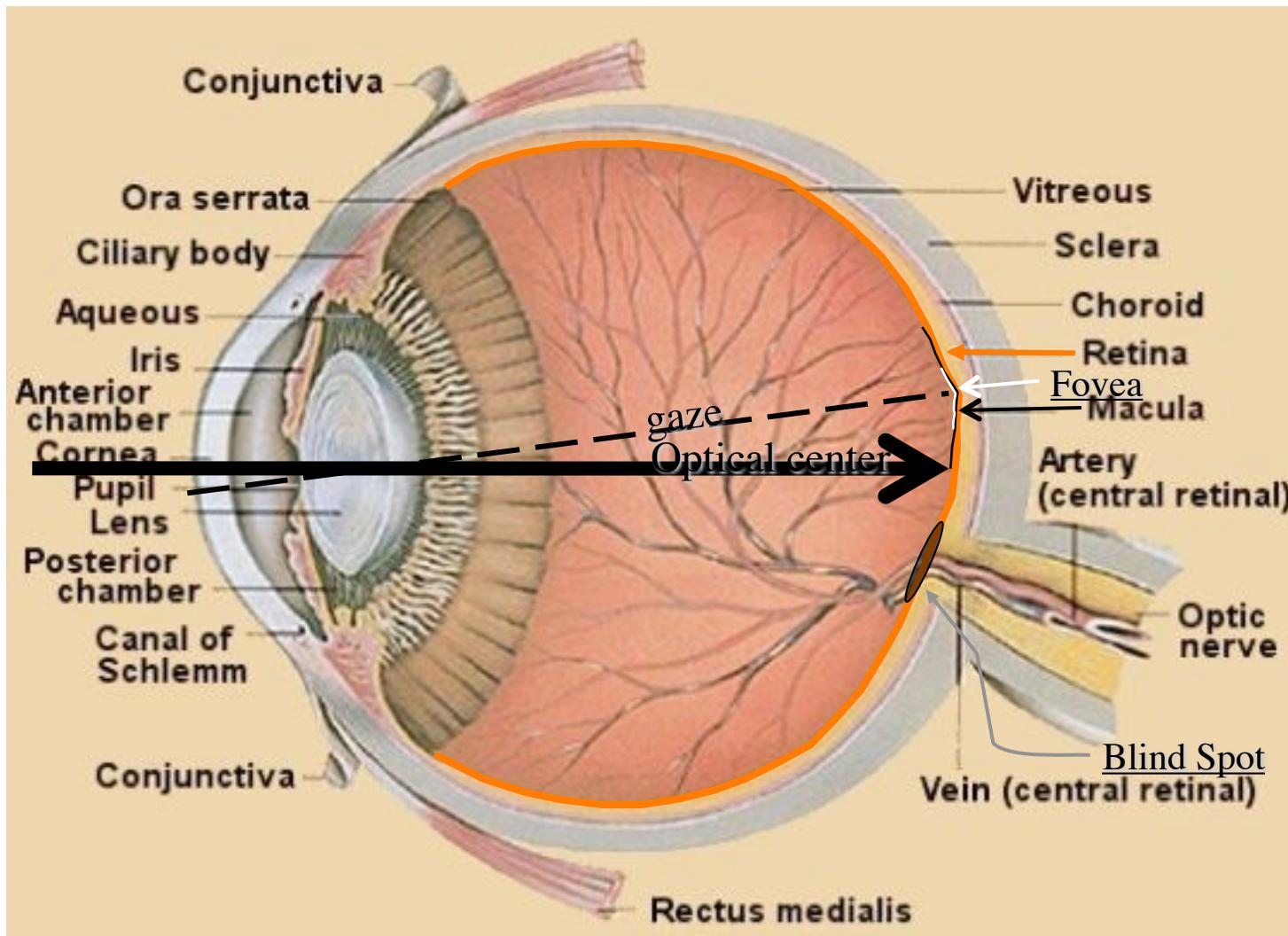


‘Human’



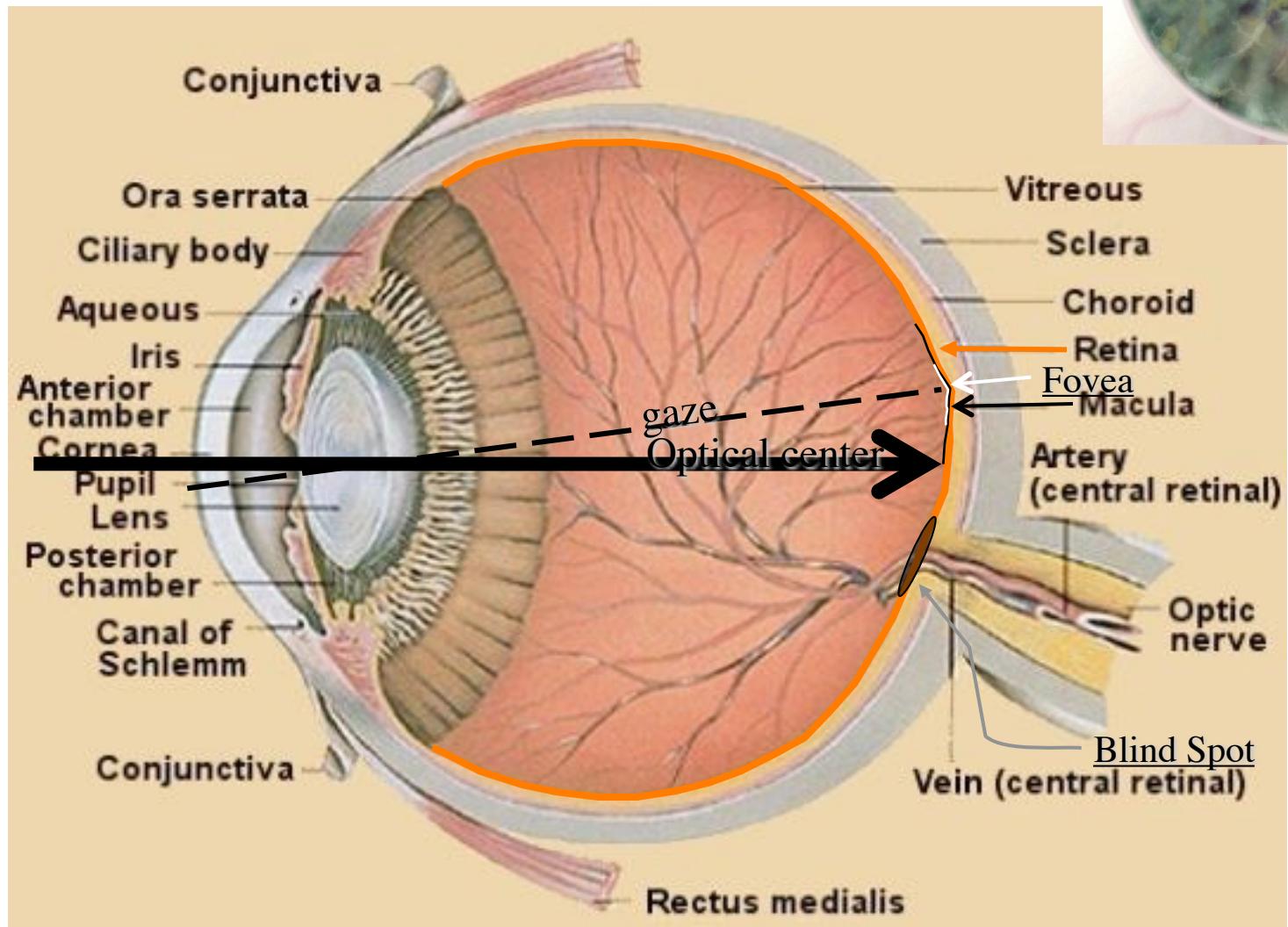
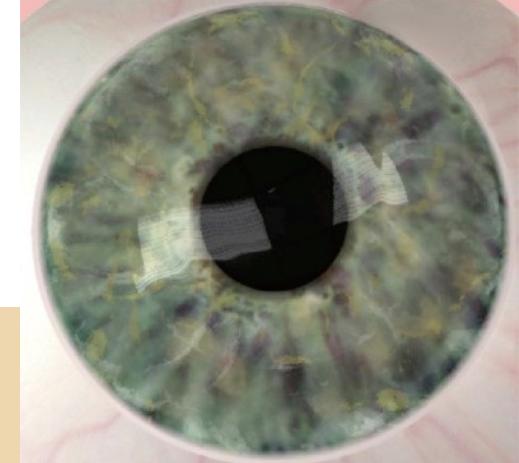
‘Honey Bee’

Human eye



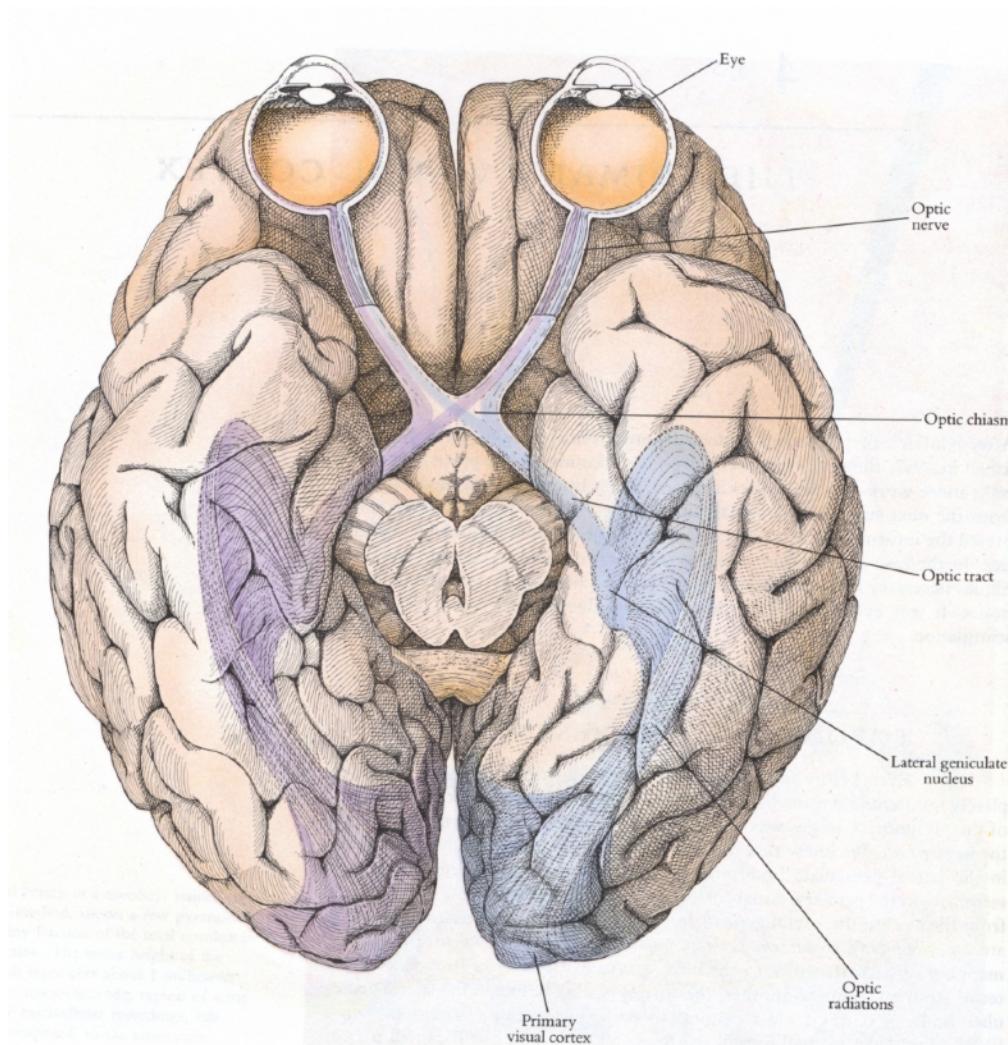
62

Human eye



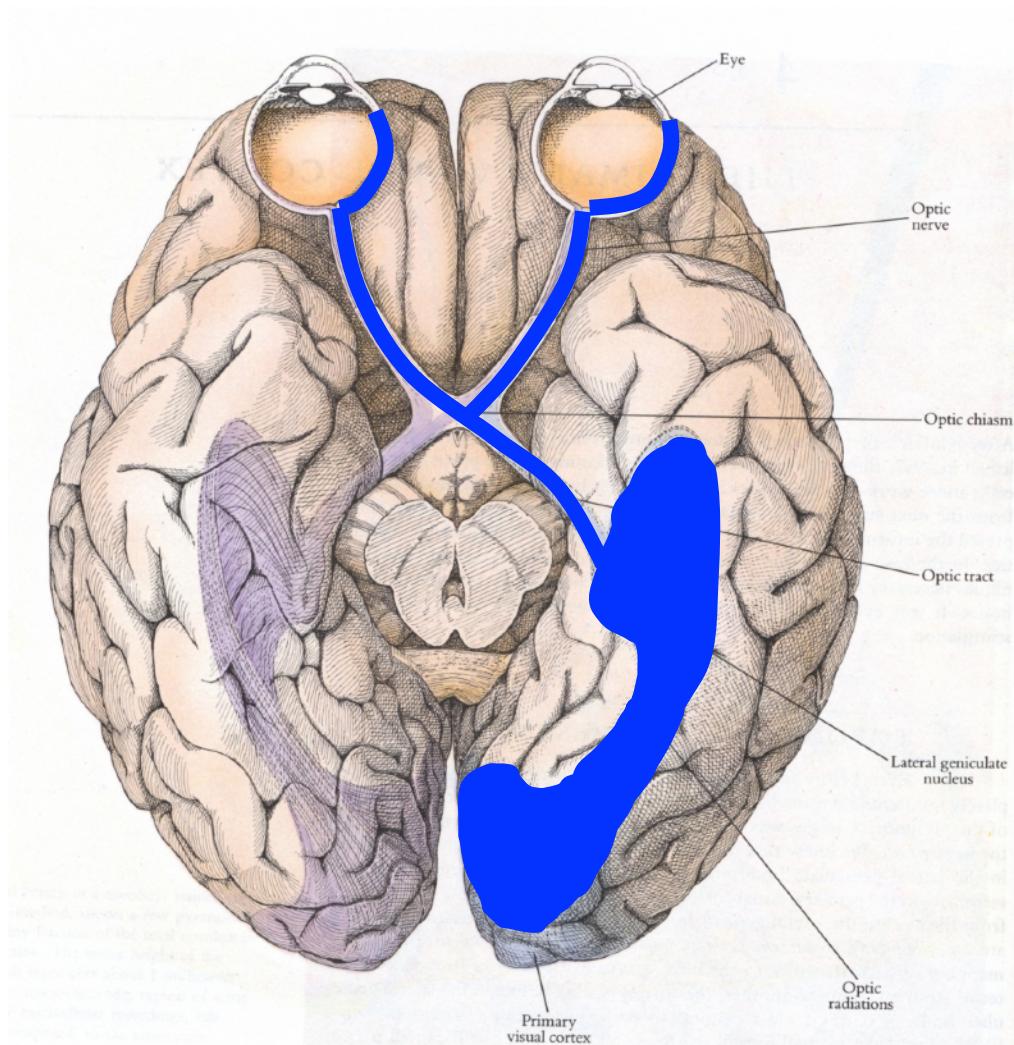
62

Human Brain



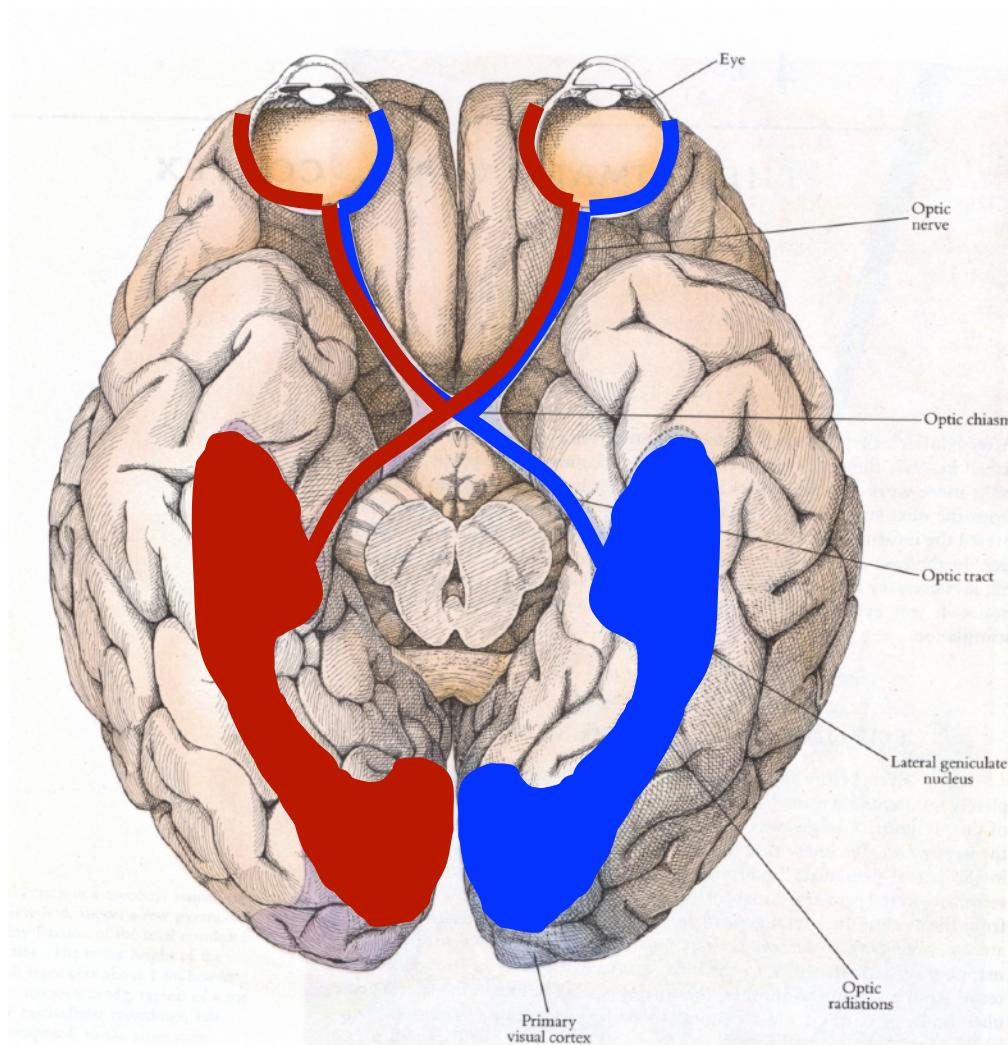
63

Human Brain



63

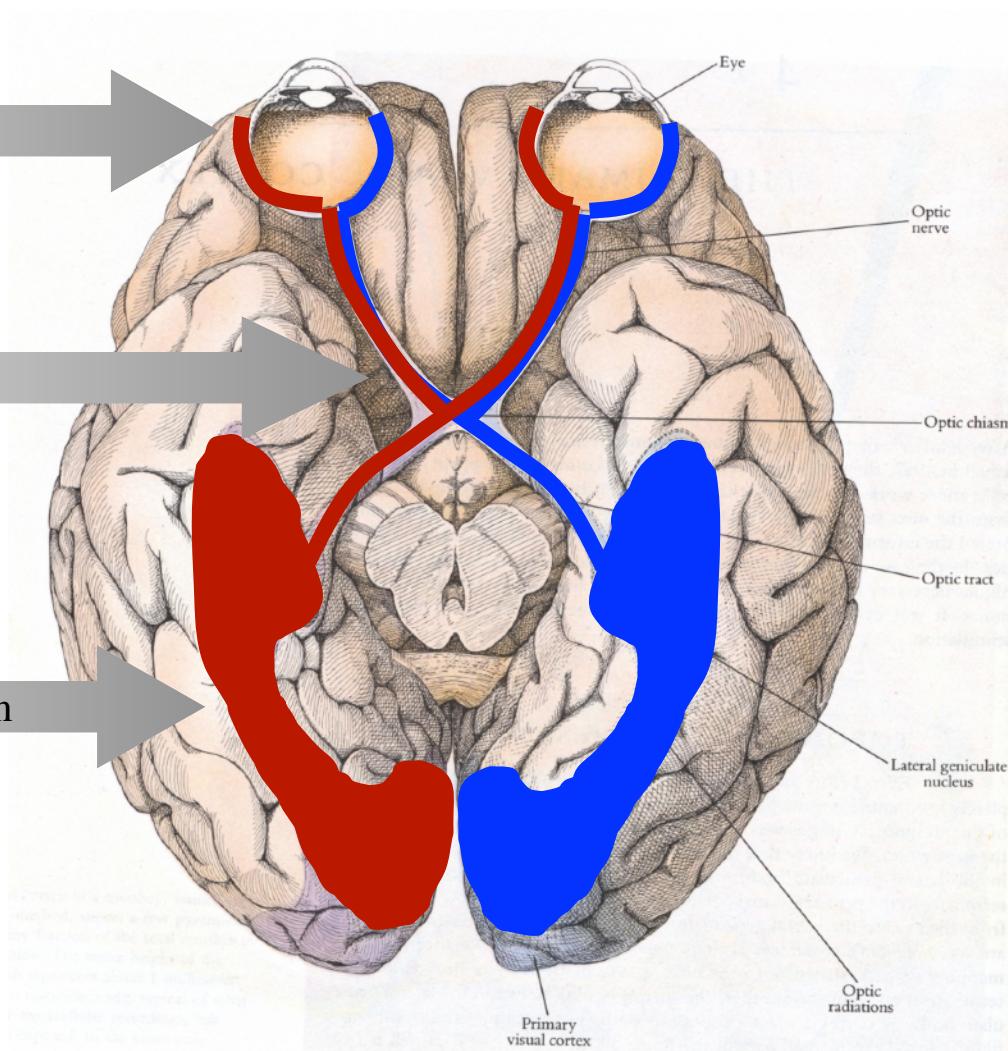
Human Brain



63

Human Brain

imperfect measuring instrument

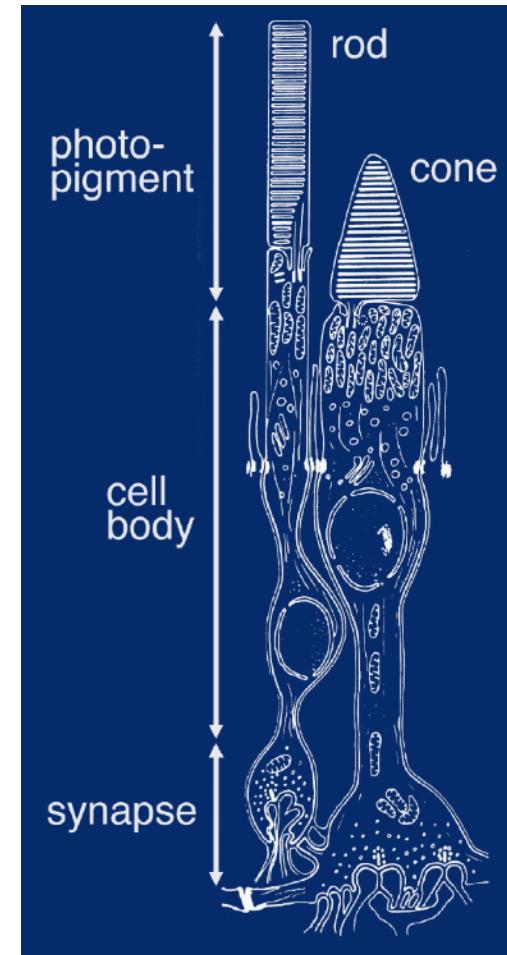
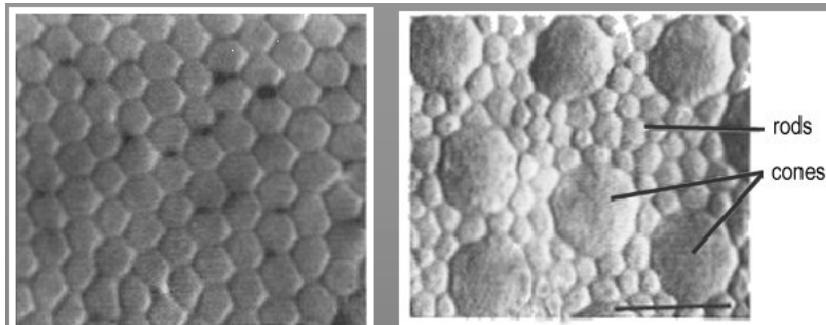
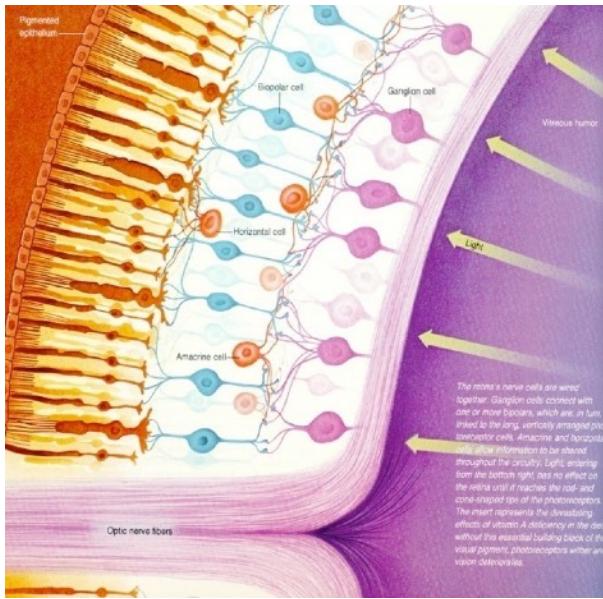


signals

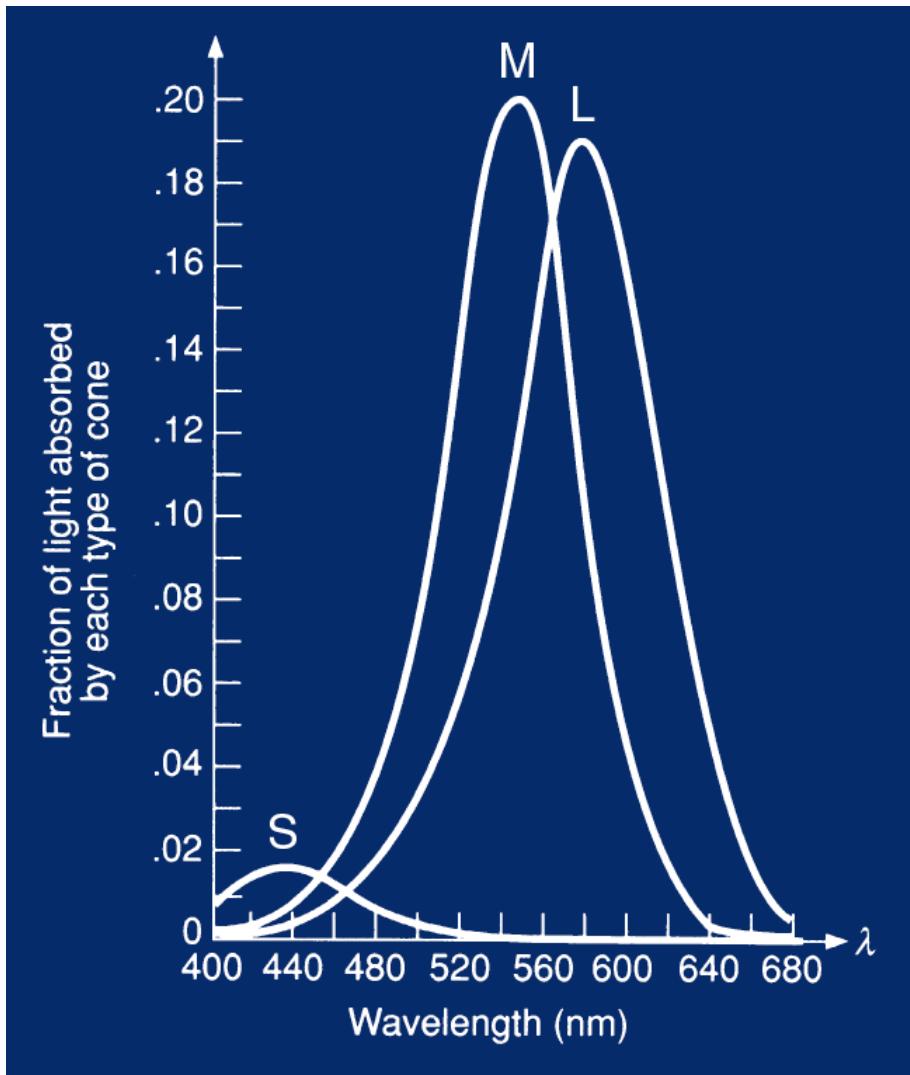
cognitive & visual interpretation

Human eye: retina

Light passes through blood vessels & retinal layers before reaching the light-sensitive cells (“rods” & “cones”)



Cone Responses



- S,M,L cones have broadband spectral sensitivity
- S,M,L neural response is integrated w.r.t. λ
 - we'll call the response functions r_S, r_M, r_L
- Results in a trichromatic visual system
- S, M, and L are *tristimulus values*

[source unknown]

Cone responses to a spectrum s

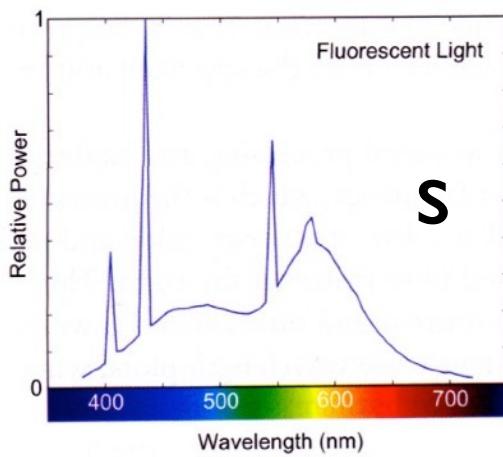
$$S = \int r_S(\lambda) s(\lambda) d\lambda = r_S \cdot s$$

$$M = \int r_M(\lambda) s(\lambda) d\lambda = r_M \cdot s$$

$$L = \int r_L(\lambda) s(\lambda) d\lambda = r_L \cdot s$$

Colorimetry: an answer to the problem

- Wanted to map a *Physical light description* to a *Perceptual color sensation*
- Basic solution was known and standardized by 1930
 - Though not quite in this form—more on that in a bit



Physical



$$S = r_S \cdot s$$

$$M = r_M \cdot s$$

$$L = r_L \cdot s$$

Perceptual

Basic fact of colorimetry

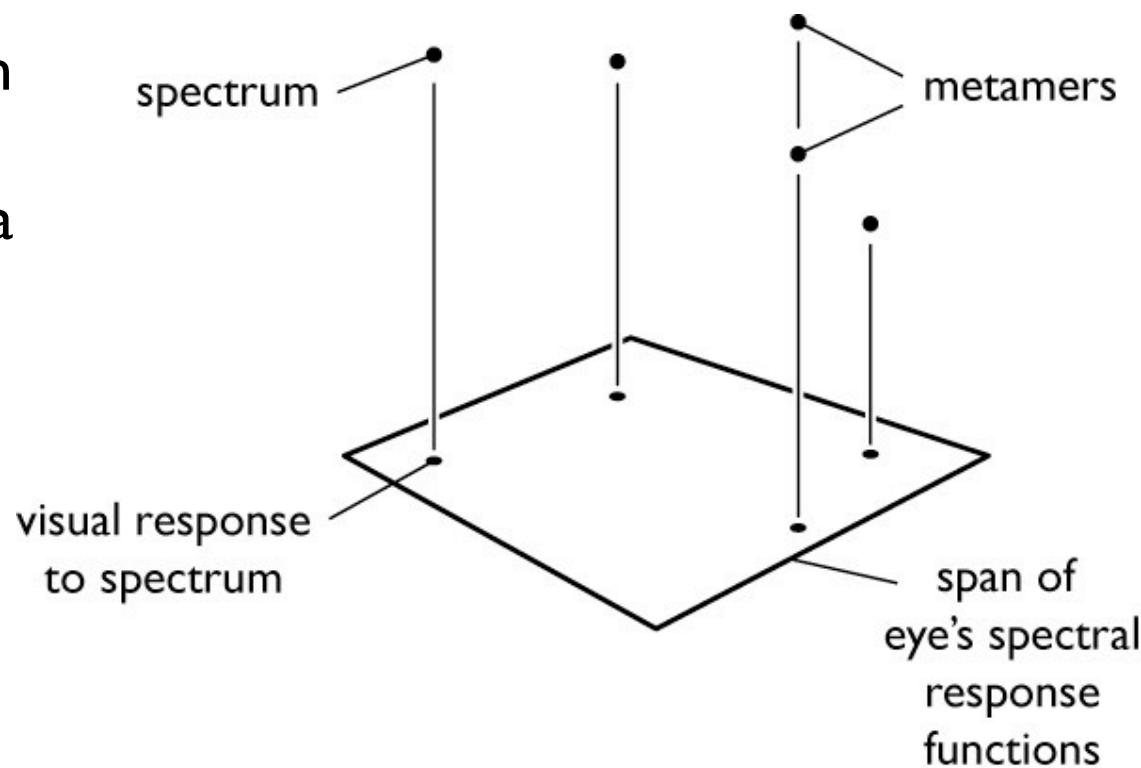
- Take a spectrum (which is a function)
- Eye produces three numbers
- This throws away a lot of information!
 - Quite possible to have two different spectra that have the same S, M, L tristimulus values
 - Two such spectra are *metamers*

Pseudo-geometric interpretation

- A dot product is a projection
- We are projecting a high dimensional vector (a spectrum) onto three vectors
 - differences that are perpendicular to all 3 vectors are not detectable
- For intuition, we can imagine a 3D analog
 - 3D stands in for high-D vectors
 - 2D stands in for 3D
 - Then vision is just projection onto a plane

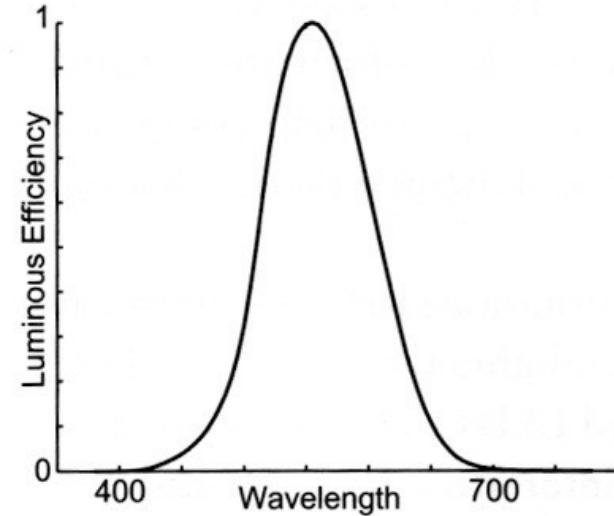
Pseudo-geometric interpretation

- The information available to the visual system about a spectrum is three values
 - this amounts to a loss of information analogous to projection on a plane
- Two spectra that produce the same response are metamers



Basic colorimetric concepts

- Luminance
 - the overall magnitude of the visual response to a spectrum (independent of its color)
 - corresponds to the everyday concept “brightness”
 - determined by product of SPD with the *luminous efficiency function* V_λ that describes the eye’s overall ability to detect light at each wavelength
 - e.g. lamps are optimized to improve their luminous efficiency (tungsten vs. fluorescent vs. sodium vapor)



[Stone 2003]

Luminance, mathematically

- Y just has another response curve (like S , M , and L)

$$Y = r_Y \cdot s$$

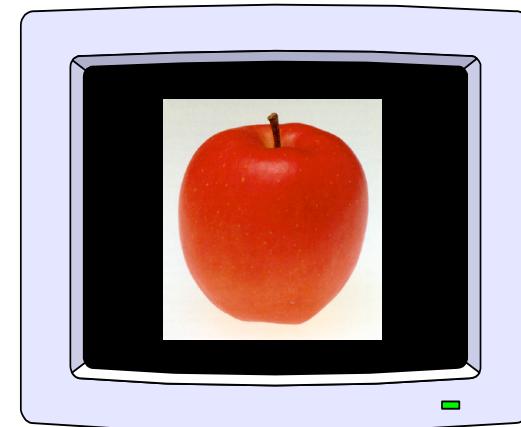
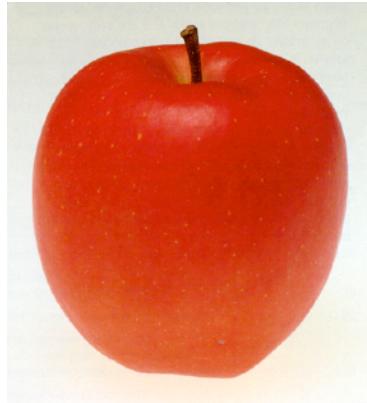
- r_Y is really called “ V_λ ”
- V_λ is a linear combination of S , M , and L
 - Has to be, since it's derived from cone outputs

More basic colorimetric concepts

- Chromaticity
 - what's left after luminance is factored out (the color without regard for overall brightness)
 - scaling a spectrum up or down leaves chromaticity alone
- Dominant wavelength
 - many colors can be matched by white plus a spectral color
 - correlates to everyday concept “hue”
- Purity
 - ratio of pure color to white in matching mixture
 - correlates to everyday concept “colorfulness” or “saturation”

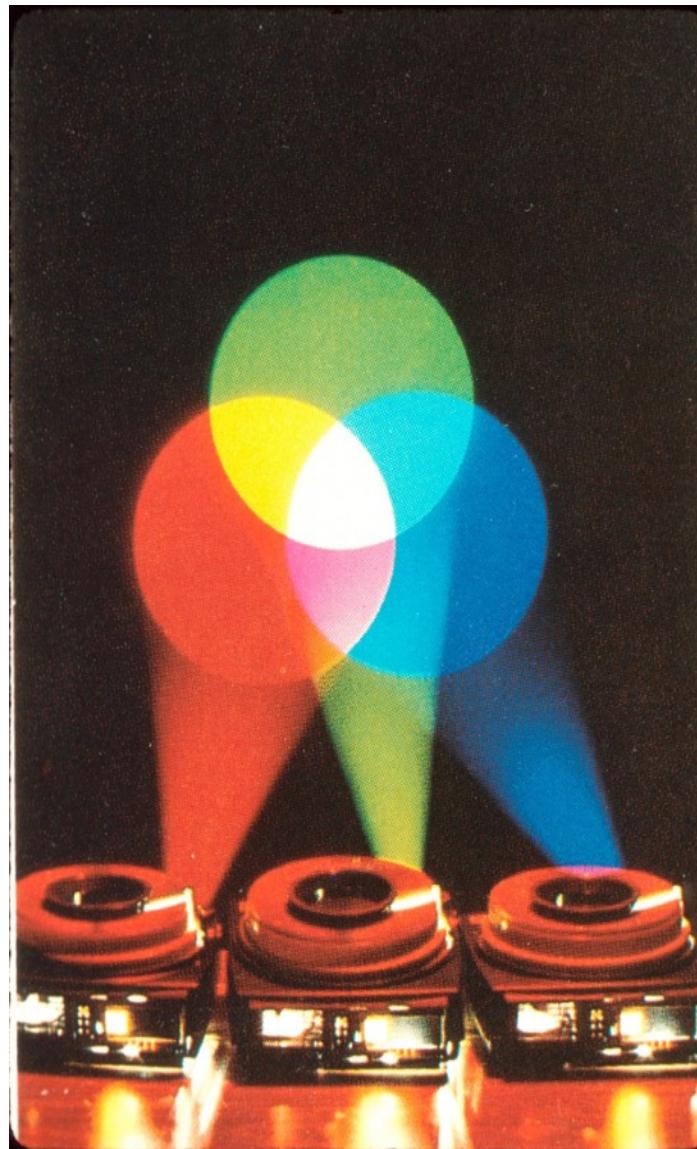
Color reproduction

- Have a spectrum s ; want to match on RGB monitor
 - “match” means it looks the same
 - any spectrum that projects to the same point in the visual color space is a good reproduction
- Must find a spectrum that the monitor *can* produce that is a metamer of s



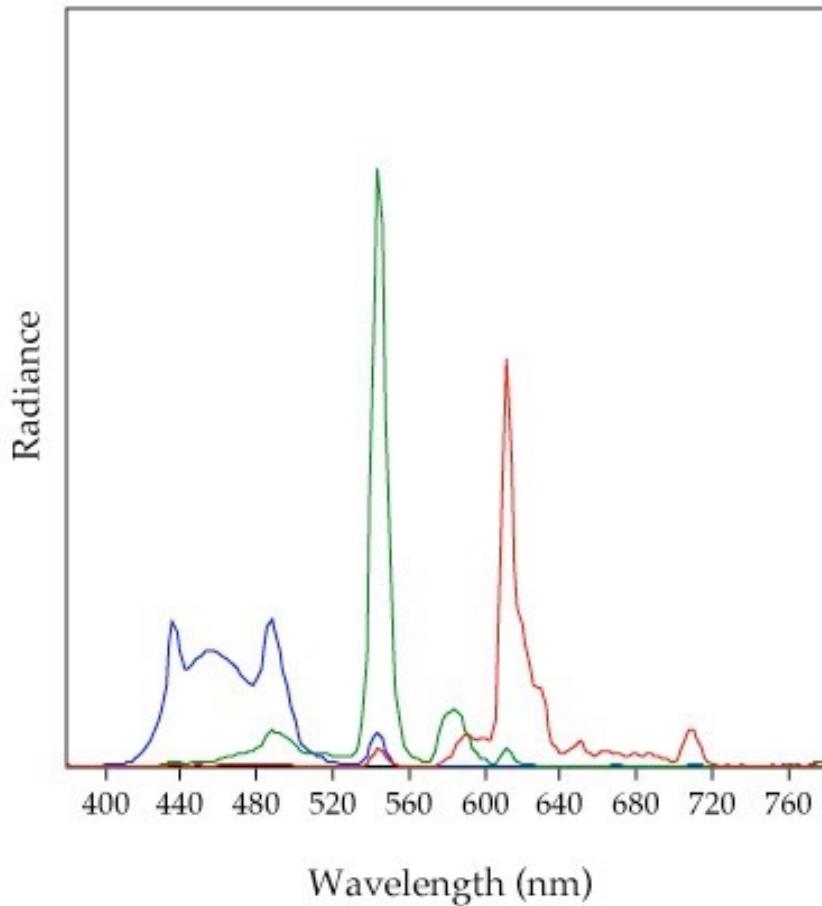
R, G, B?

Additive Color



[source unknown]

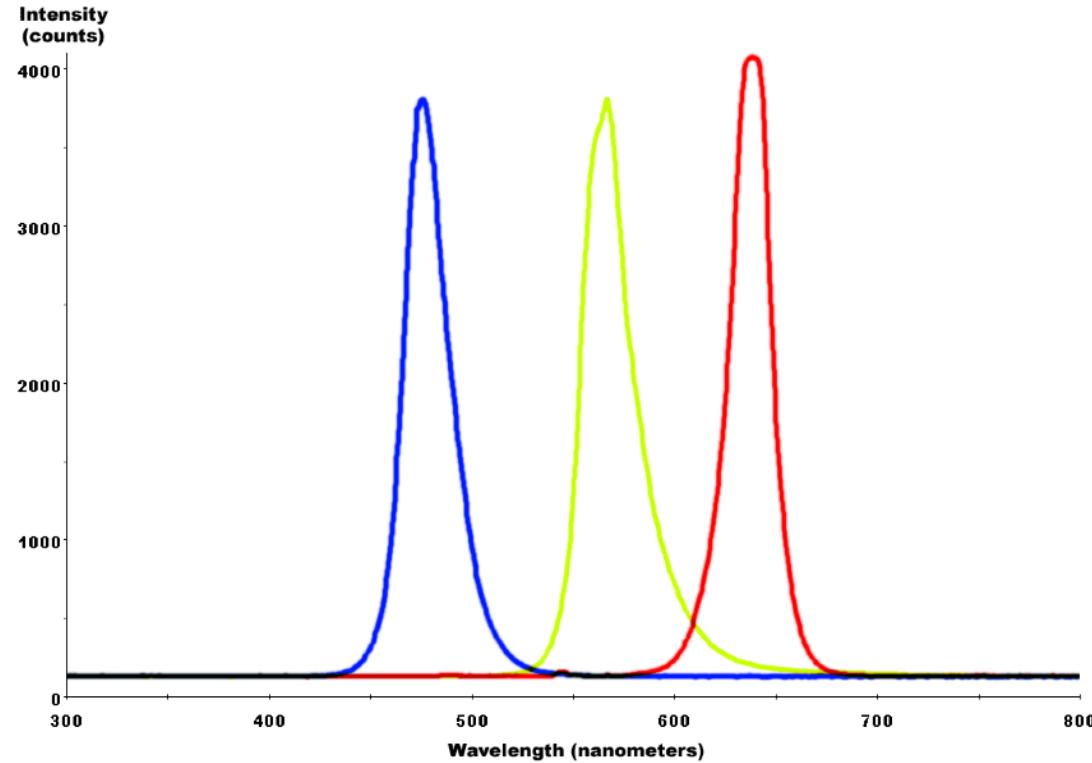
LCD display primaries



[Fairchild 97]

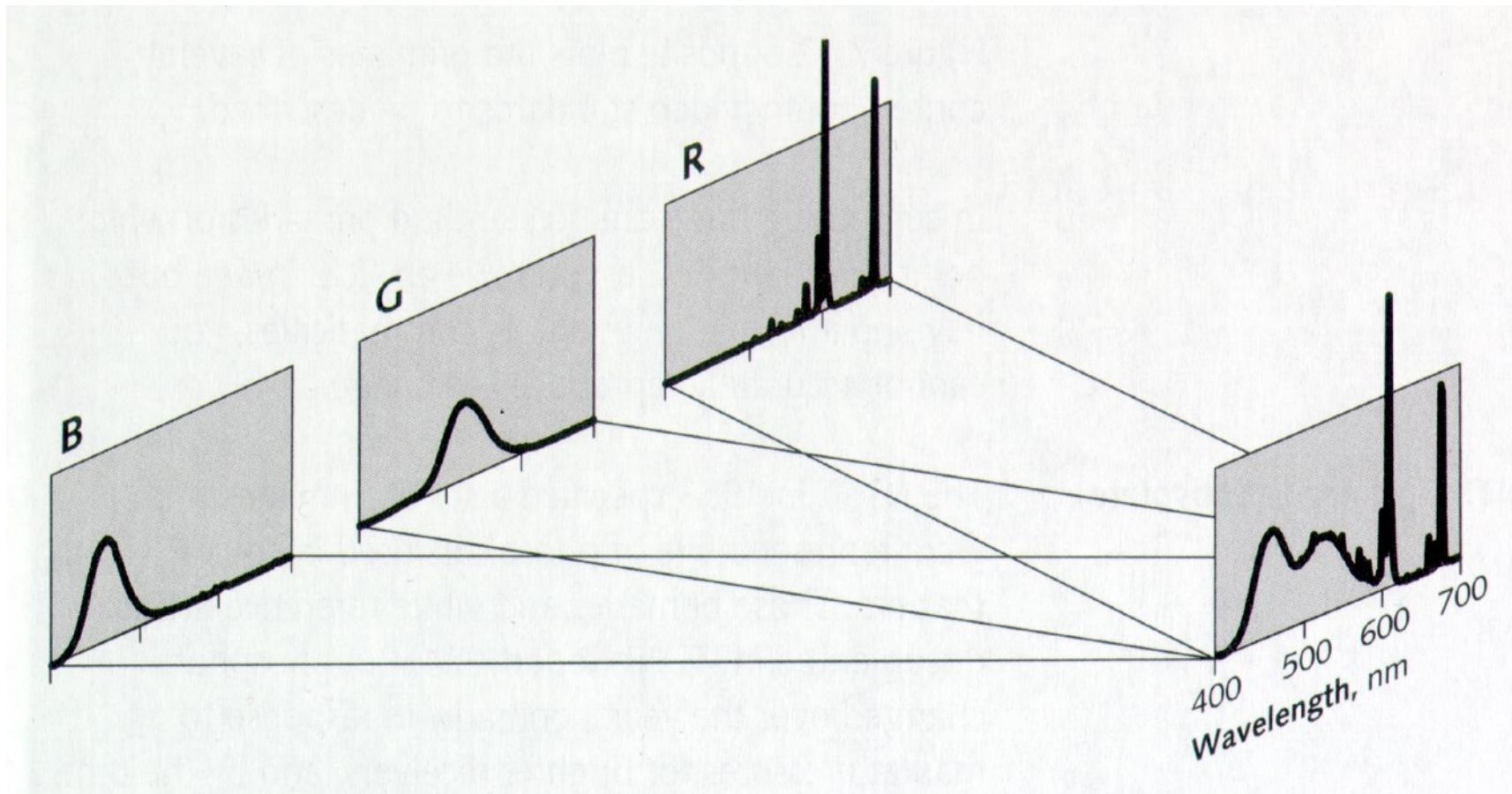
Curves determined by (fluorescent or LED) backlight and filters

LED display primaries



- Native emission curves of 3 LED types

Combining Monitor Phosphors with Spatial Integration



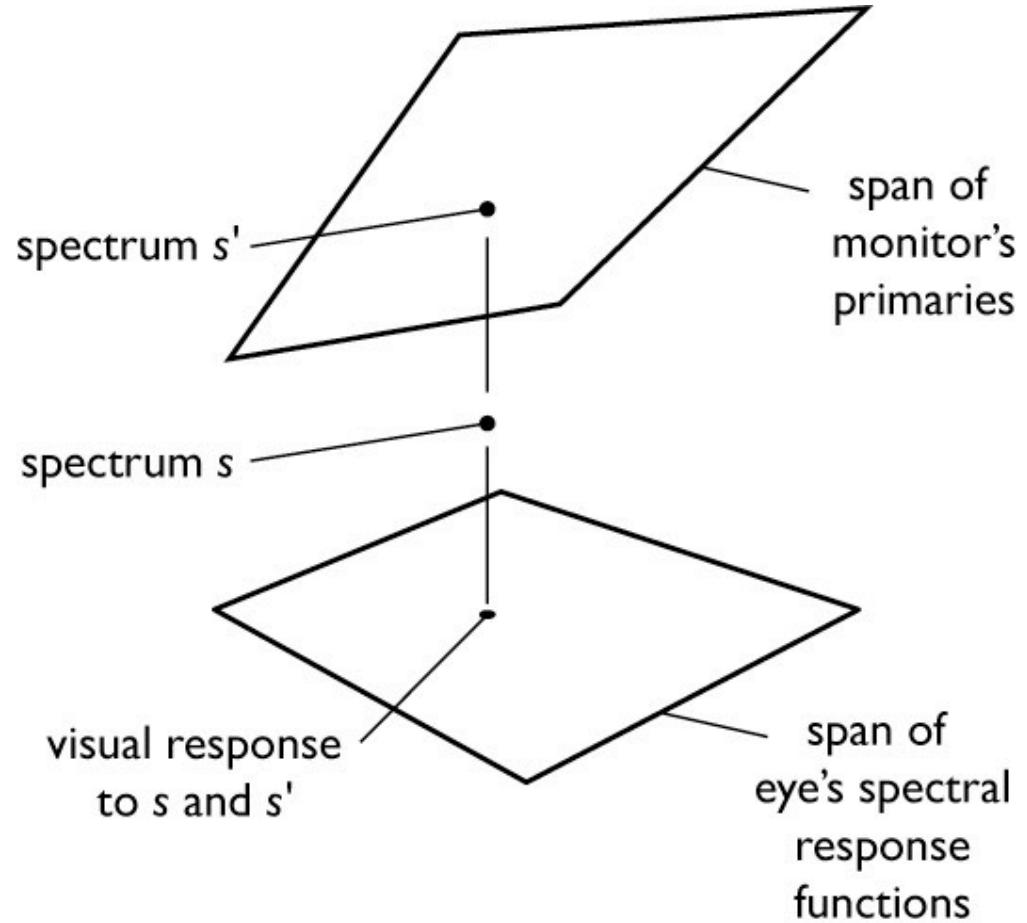
[source unknown]

Color reproduction

- Say we have a spectrum s we want to match on an RGB monitor
 - “match” means it looks the same
 - any spectrum that projects to the same point in the visual color space is a good reproduction
- So, we want to find a spectrum that the monitor can produce that matches s
 - that is, we want to display a metamer of s on the screen

Color reproduction

- We want to compute the combination of r, g, b that will project to the same visual response as s .



Color reproduction as linear algebra

- The projection onto the three response functions can be written in matrix form:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} -r_S- \\ -r_M- \\ -r_L- \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

or,

$$V = M_{SML} s.$$

Color reproduction as linear algebra

- The spectrum that is produced by the monitor for the color signals R, G, and B is:

$$s_a(\lambda) = R s_r(\lambda) + G s_g(\lambda) + B s_b(\lambda).$$

- Again the discrete form can be written as a matrix:

$$\begin{bmatrix} | \\ s_a \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} =$$

or,

$$s_a = M_{RGB} C.$$

Color reproduction as linear algebra

- What color do we see when we look at the display?
 - Feed C to display
 - Display produces s_a
 - Eye looks at s_a and produces V

$$V = M_{SML}M_{RGB}C$$

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color reproduction as linear algebra

- Goal of reproduction: visual response to s and s_a is the same:

$$M_{SML} \tilde{s} = M_{SML} \tilde{s}_a.$$

- Substituting in the expression for s_a ,

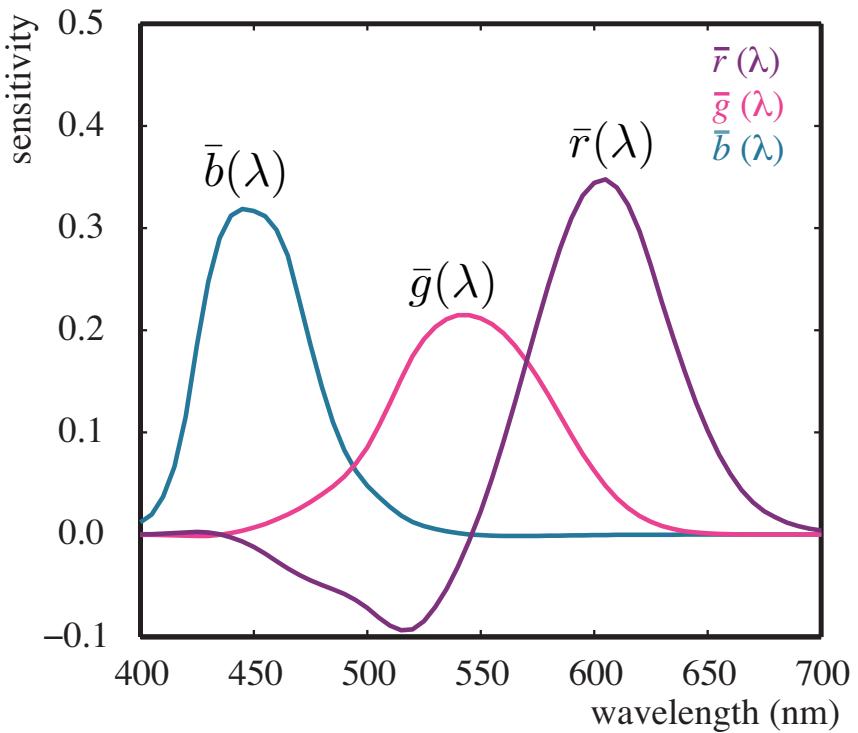
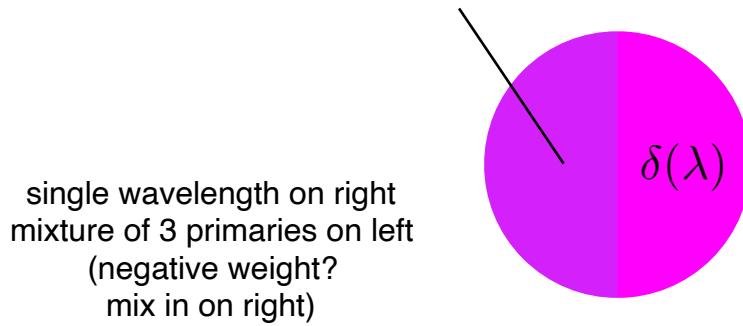
$$M_{SML} \tilde{s} = M_{SML} M_{RGB} C$$

$$C = \underbrace{(M_{SML} M_{RGB})^{-1}}_{\text{color matching matrix for RGB}} M_{SML} \tilde{s}$$

Color matching functions

- For given primaries, how much are needed to match each spectral color?
 - can be determined experimentally without knowing S, M, L
 - experiment:

$$Rs_r(\lambda) + Gs_g(\lambda) + Bs_b(\lambda)$$

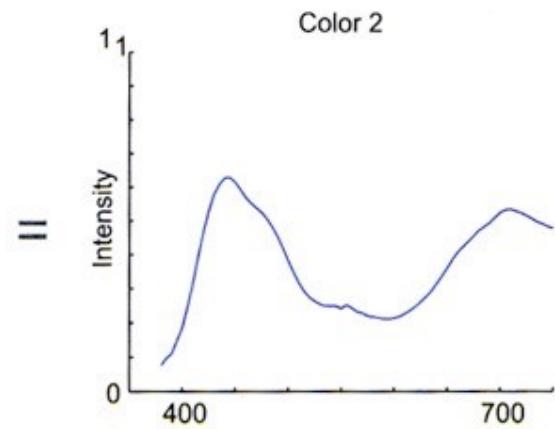
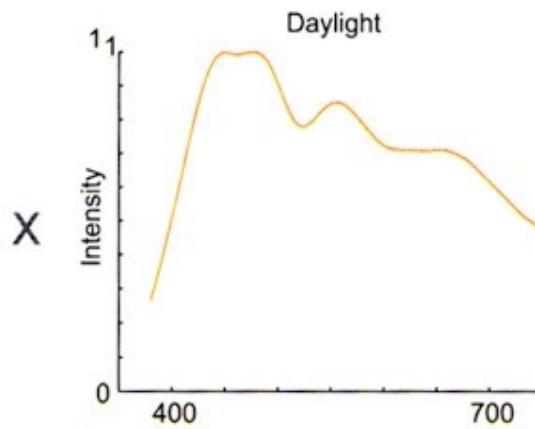
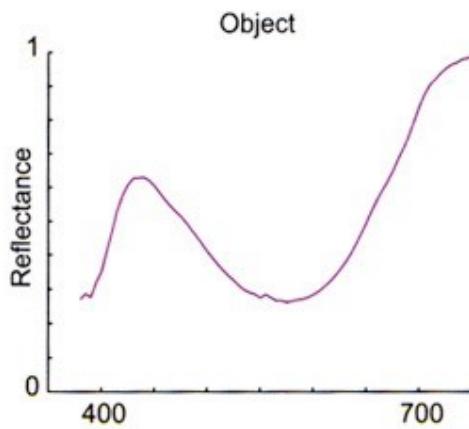
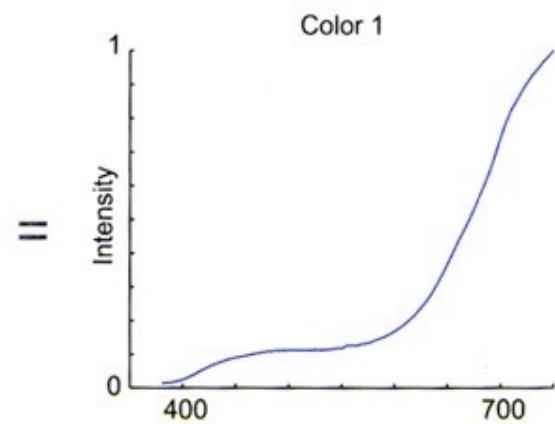
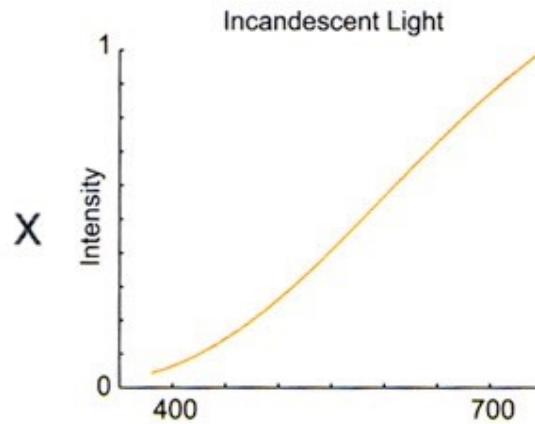
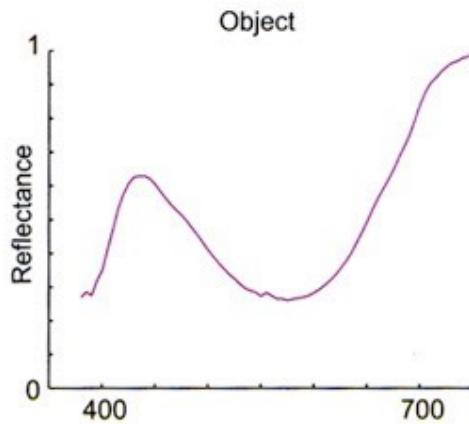


Subtractive Color



[source unknown]

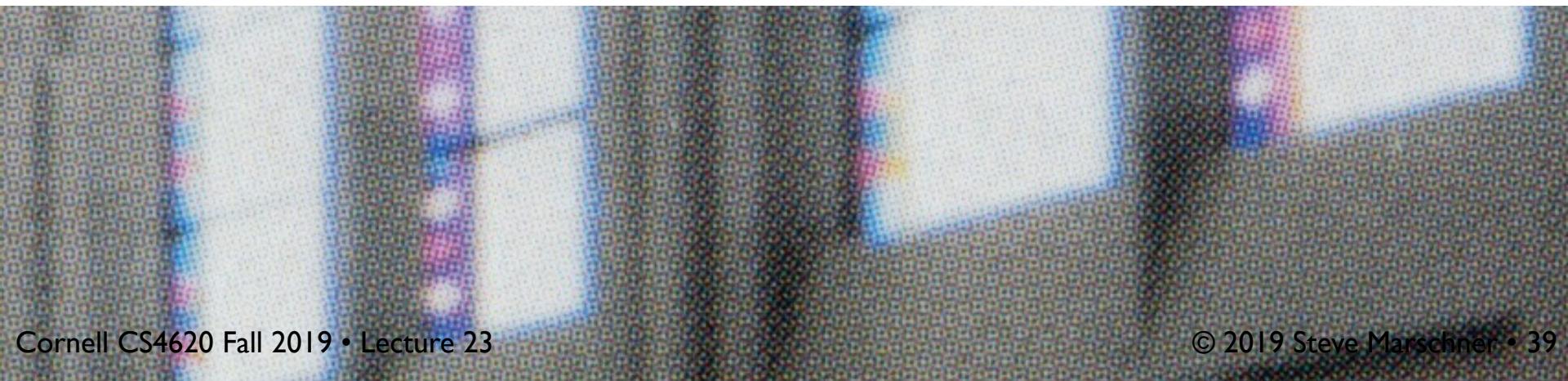
Reflection from colored surface



[Stone 2003]

Subtractive color

- Produce desired spectrum by *subtracting* from white light (usually via absorption by pigments)
- Photographic media (slides, prints) work this way
- Leads to C, M, Y as primaries
- Approximately, $I - R, I - G, I - B$
- Real subtractive processes are more complex
 - usually modeled empirically using calibration tables



Color spaces

- Need three numbers to specify a color
 - but what three numbers?
 - a color space is an answer to this question
- Common example: monitor RGB
 - define colors by what R, G, B signals will produce them on your monitor
 - (in math, $s = R\mathbf{R} + G\mathbf{G} + B\mathbf{B}$ for some spectra $\mathbf{R}, \mathbf{G}, \mathbf{B}$)
 - device dependent (depends on gamma, phosphors, gains, ...)
 - therefore if I choose RGB by looking at my monitor and send it to you, you may not see the same color
 - also leaves out some colors (limited gamut), e.g. vivid yellow

Standard color spaces

- Standardized RGB (sRGB)
 - makes a particular monitor RGB standard
 - other color devices simulate that monitor by calibration
 - sRGB is usable as an interchange space; widely adopted today
 - gamut is still limited

A universal color space: XYZ

- Standardized by CIE (*Commission Internationale de l'Eclairage*, the standards organization for color science)
- Based on three “imaginary” primaries X , Y , and Z
 - (in math, $s = XX + YY + ZZ$)
 - imaginary = only realizable by spectra that are negative at some wavelengths
 - key properties
 - any stimulus can be matched with positive X , Y , and Z
 - separates out luminance: X , Z have zero luminance, so Y tells you the luminance by itself

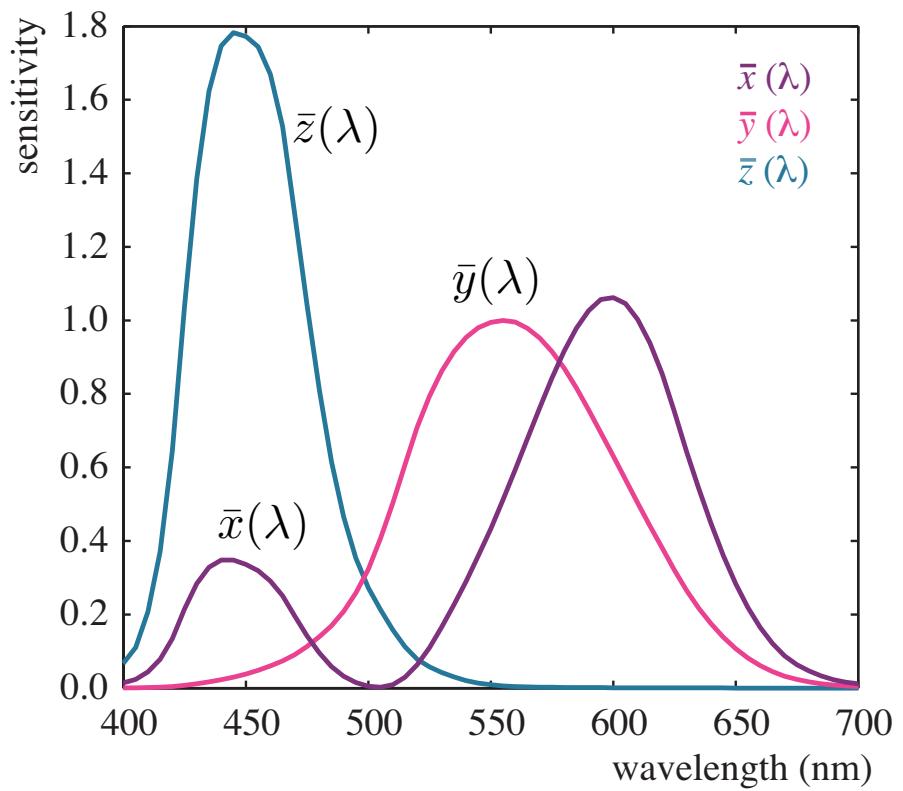
XYZ color matching functions

- XYZ primaries are not specified because they are not needed; instead the color matching functions are in the standard.

$$X = k \int \bar{x}(\lambda) s(\lambda) d\lambda$$

$$Y = k \int \bar{y}(\lambda) s(\lambda) d\lambda$$

$$Z = k \int \bar{z}(\lambda) s(\lambda) d\lambda$$



XYZ color matching functions

- They are linear combinations of S, M, L
 - as are all sets of color matching functions for humans
- They are what is standardized
- All other color spaces standardized in terms of XYZ
 - if you have a spectrum to convert to a color, the first step is to integrate it against these functions

Separating luminance, chromaticity

- Luminance: Y
- Chromaticity: x, y, z , defined as

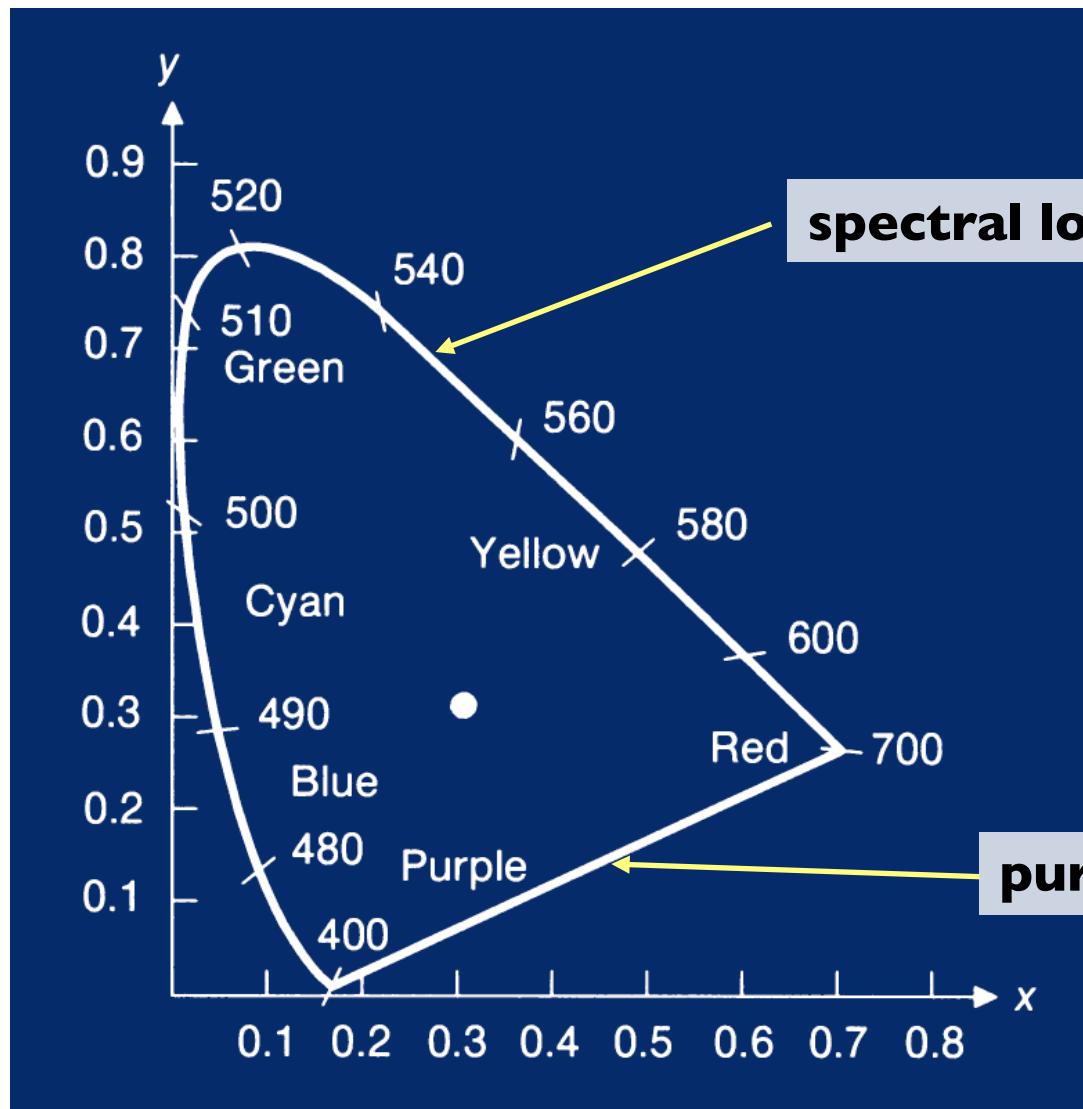
$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

- since $x + y + z = 1$, we only need to record two of the three
 - usually choose x and y , leading to (x, y, Y) coords

Chromaticity Diagram



$$X = k \int \bar{x}(\lambda) \delta(\lambda) d\lambda = k x(\lambda)$$

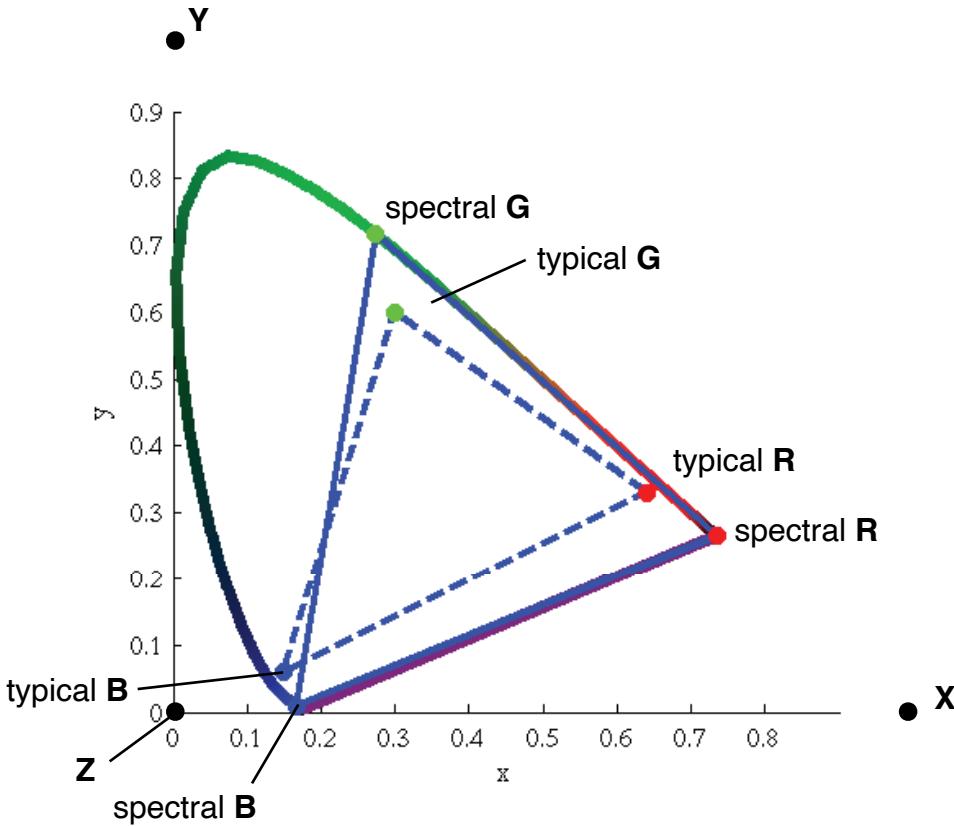
$$x(\lambda) = \frac{x(\lambda)}{x(\lambda) + y(\lambda) + z(\lambda)}$$

...and similarly for $y(\lambda)$

[source unknown]

Color Gamuts

[Marschner & Shirley ch. 19 by Reinhard & Johnson]

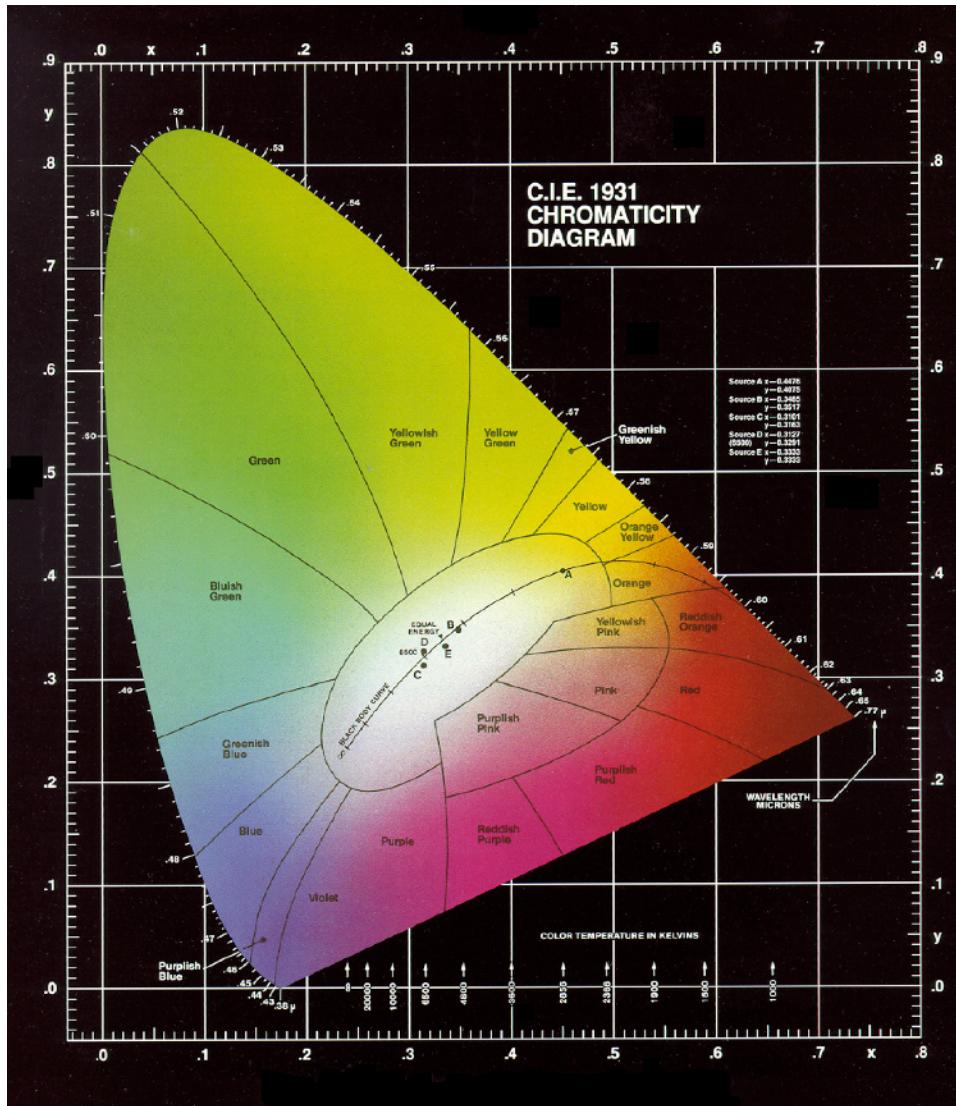


Monitors/printers can't produce all visible colors

Reproduction is limited to a particular domain

For additive color (e.g. monitor) gamut is the triangle defined by the chromaticities of the three primaries.

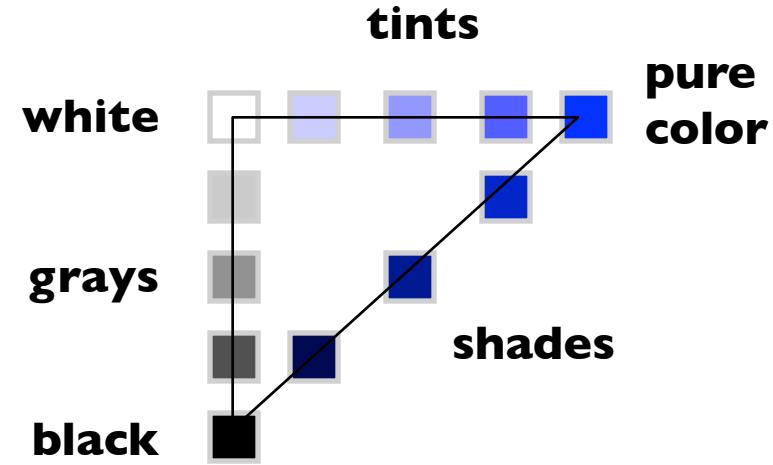
Chromaticity Diagram



[source unknown]

Perceptually organized color spaces

- Artists often refer to colors as *tints*, *shades*, and *tones* of pure pigments
 - tint: mixture with white
 - shade: mixture with black
 - tones: mixture with black and white
 - gray: no color at all (aka. neutral)
- This seems intuitive
 - tints and shades are inherently related to the pure color
 - “same” color but lighter, darker, paler, etc.



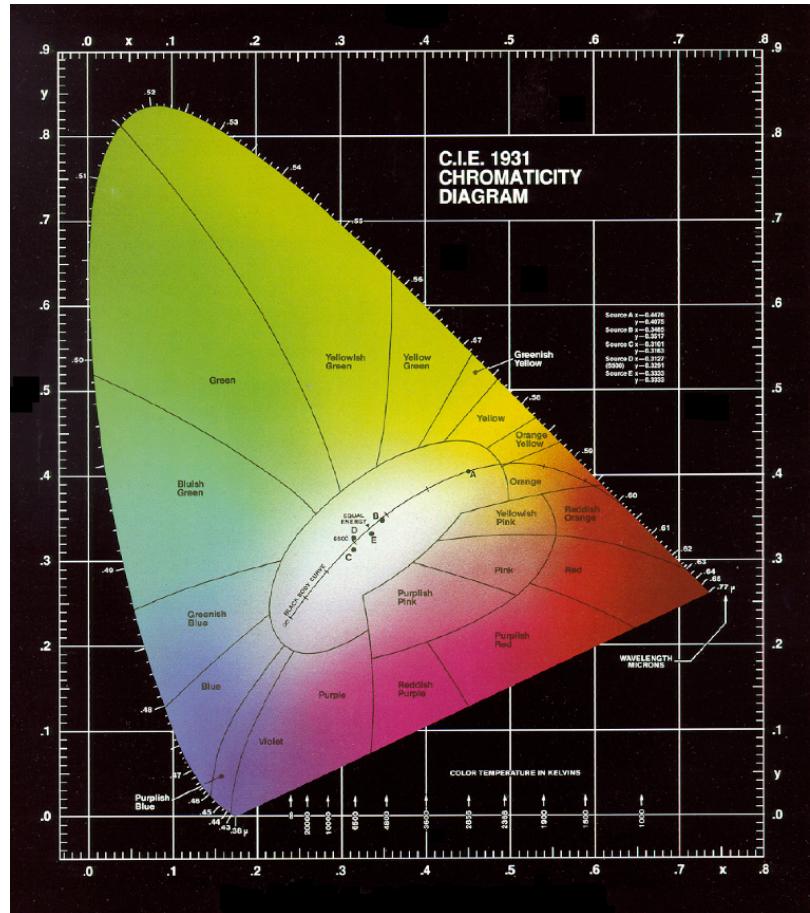
[after FvDFH]

Perceptual dimensions of color

- Hue
 - the “kind” of color, regardless of attributes
 - colorimetric correlate: dominant wavelength
 - artist’s correlate: the chosen pigment color
- Saturation
 - the “colorfulness”
 - colorimetric correlate: purity
 - artist’s correlate: fraction of paint from the colored tube
- Lightness (or value)
 - the overall amount of light
 - colorimetric correlate: luminance
 - artist’s correlate: tints are lighter, shades are darker

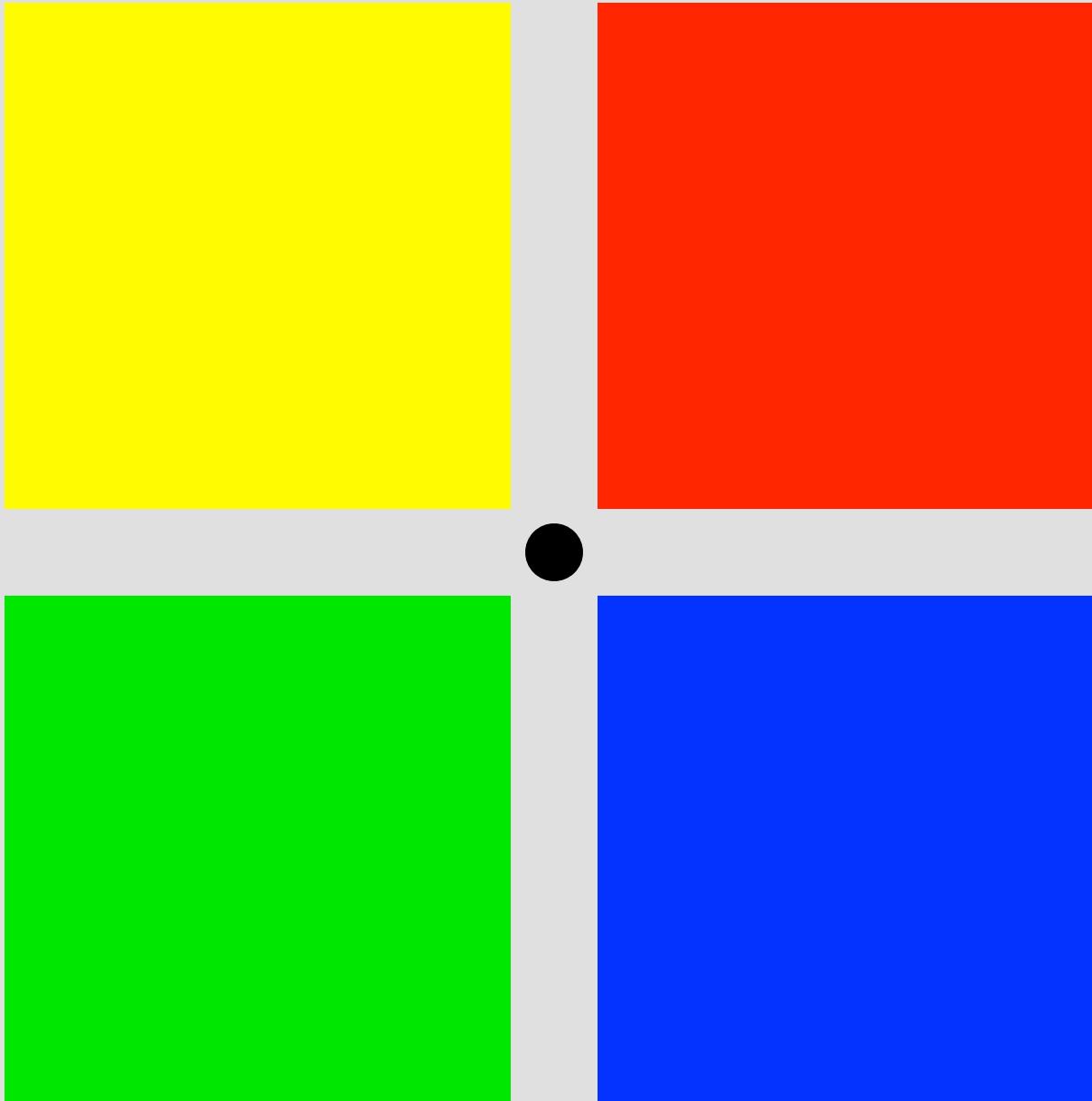
Perceptual dimensions: chromaticity

- In x, y, Y (or another luminance/chromaticity space), Y corresponds to lightness
- hue and saturation are then like polar coordinates for chromaticity (starting at white, which way did you go and how far?)



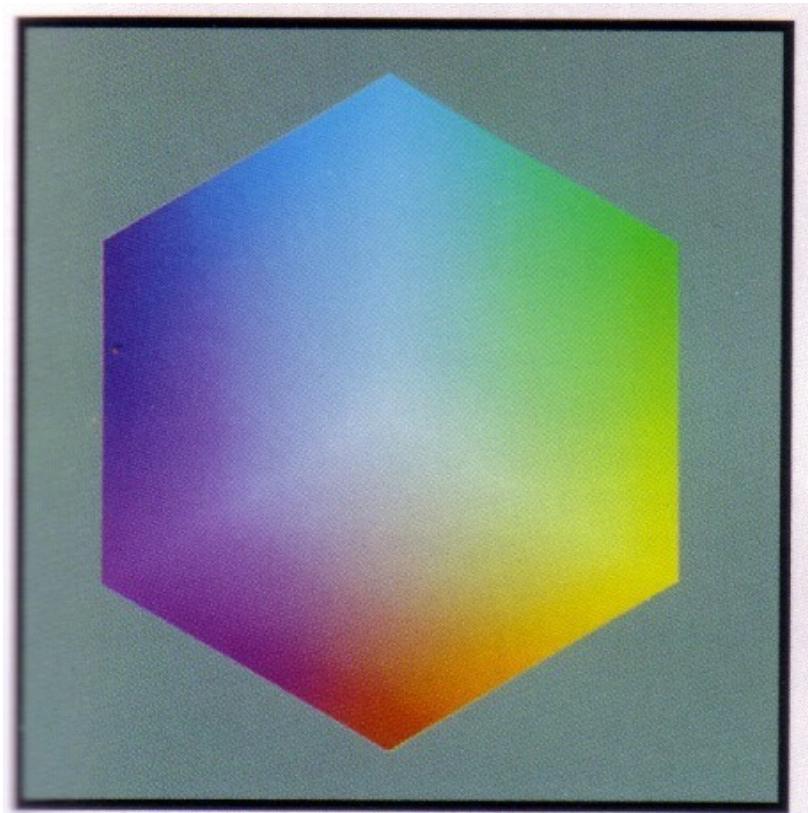
Perceptual dimensions of color

- There's good evidence ("opponent color theory") for a neurological basis for these dimensions
 - the brain seems to encode color early on using three axes:
 - white — black, red — green, yellow — blue
 - the white—black axis is lightness; the others determine hue and saturation
 - one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can't have a reddish green (just doesn't make sense)
 - thus red is the *opponent* to green
 - another piece of evidence: afterimages (next slide)



RGB as a 3D space

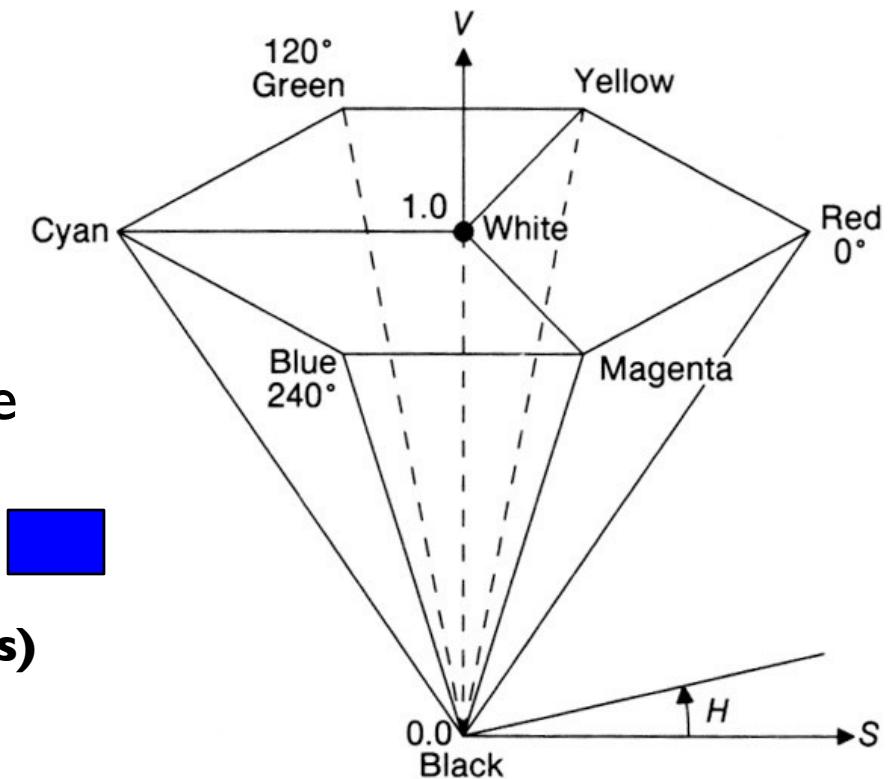
- A cube:



Perceptual organization for RGB: HSV

- Uses hue (an angle, 0 to 360), saturation (0 to 1), and value (0 to 1) as the three coordinates for a color
 - the brightest available RGB colors are those with one of R,G,B equal to 1 (top surface)
 - each horizontal slice is the surface of a sub-cube of the RGB cube

(demo of HSV color pickers)



[FvDFH]

Perceptually uniform spaces

- Two major spaces standardized by CIE
 - designed so that equal differences in coordinates produce equally visible differences in color
 - LUV: earlier, simpler space; L^* , u^* , v^*
 - LAB: more complex but more uniform: L^* , a^* , b^*
 - both separate luminance from chromaticity
 - including a gamma-like nonlinear component is important