Perspective

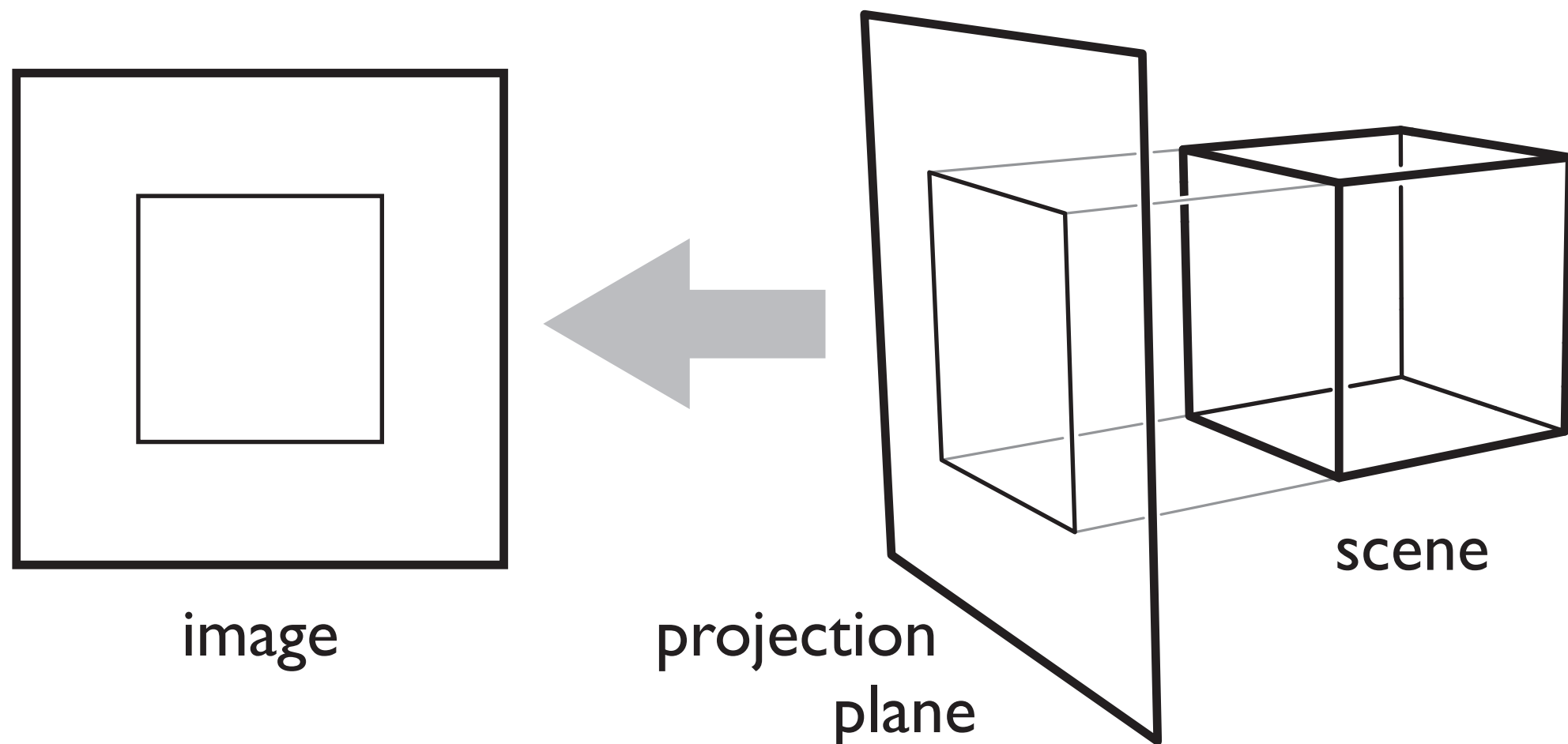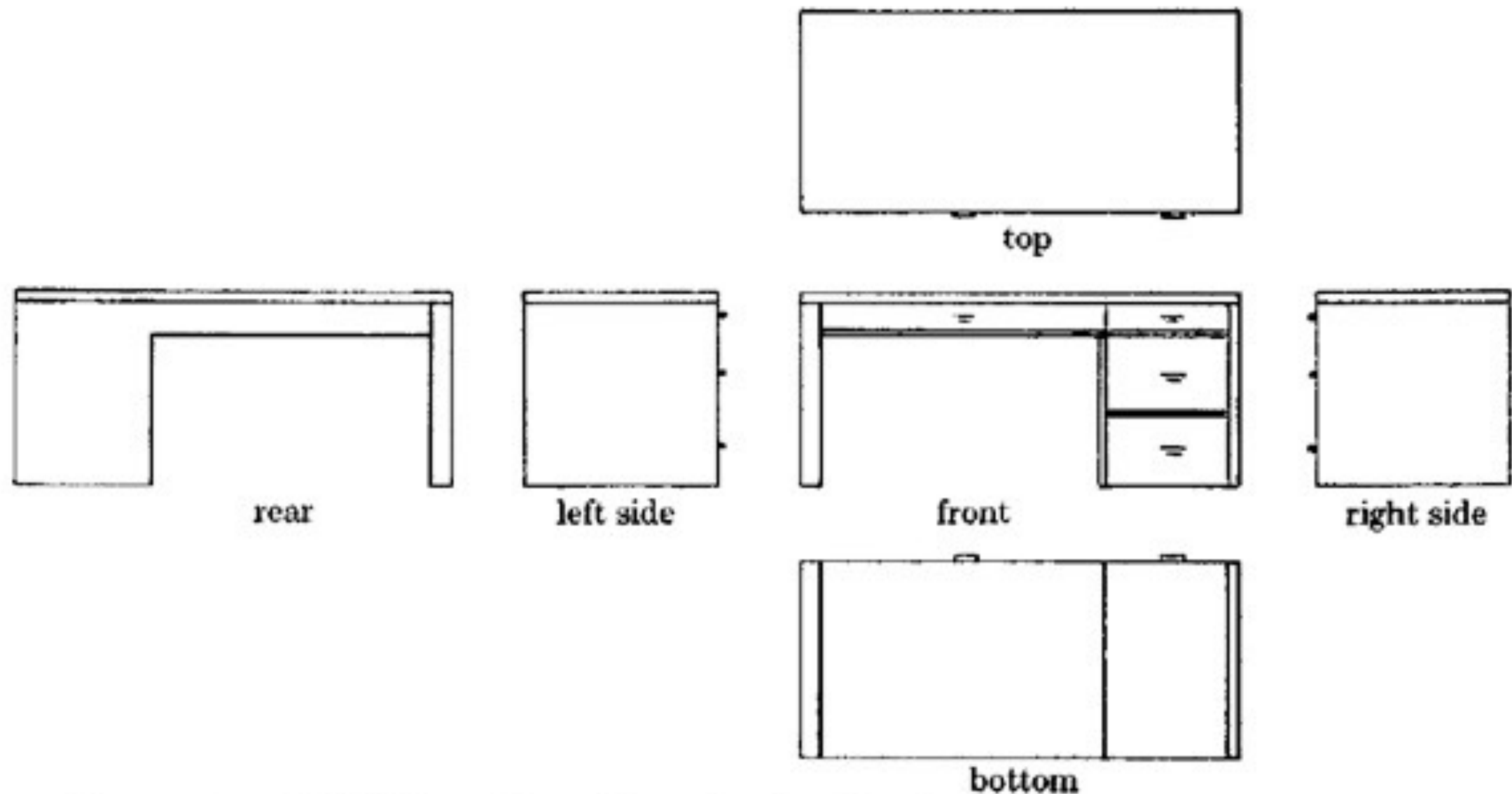**CS 4620 Lecture 6**

Parallel projection

- **To render an image of a 3D scene, we *project* it onto a plane**
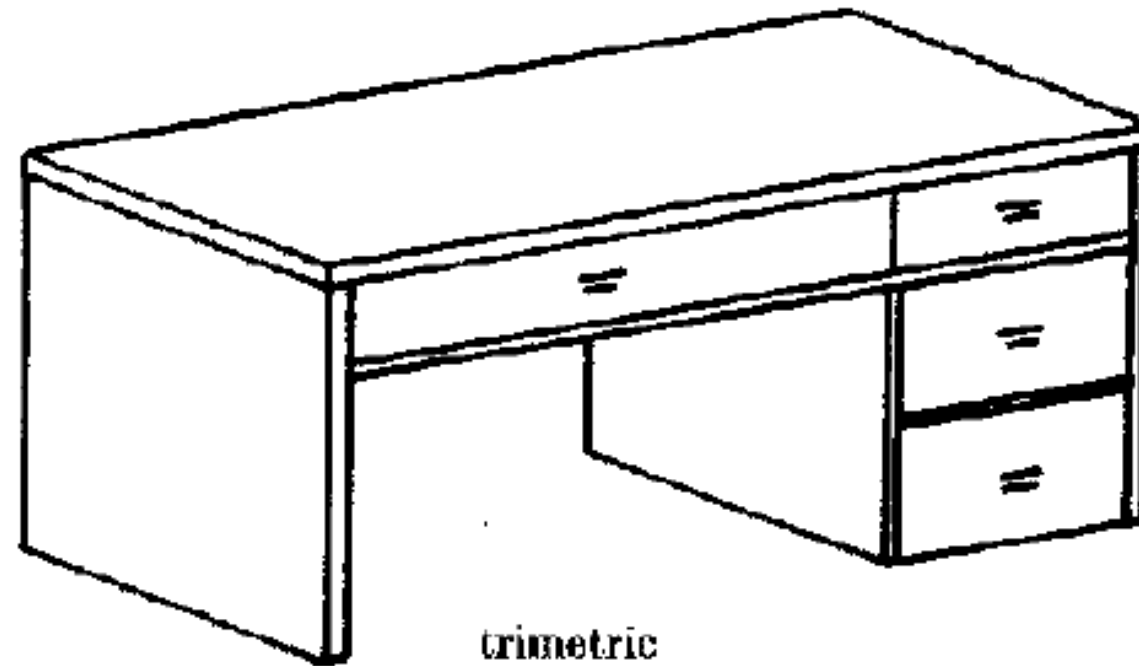- **Simplest kind of projection is *parallel projection***

image   projection plane   scene

Orthographic in traditional drawing

top

rear

left side

front

right side

bottom

– projection plane parallel to a coordinate plane
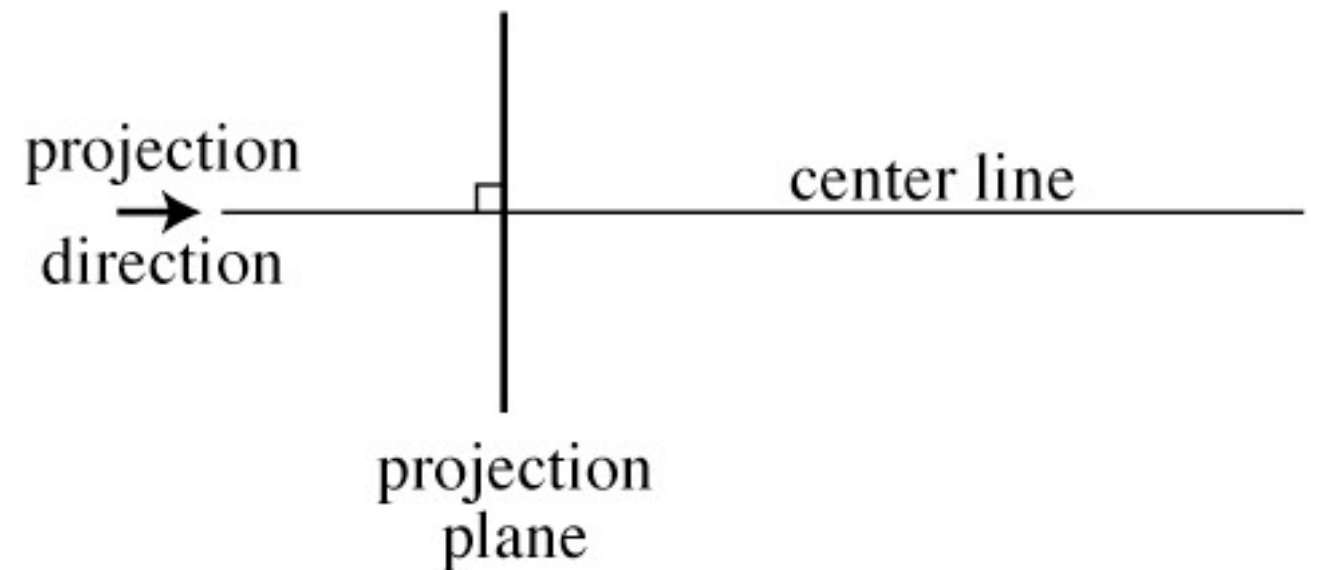– projection direction perpendicular to projection plane

Other parallel in traditional drawing

trimetric

axonometric: projection plane perpendicular to projection direction but not parallel to coordinate planes

"Orthographic" projection

- **In graphics usually we lump axonometric with orthographic**
  - projection plane perpendicular to projection direction
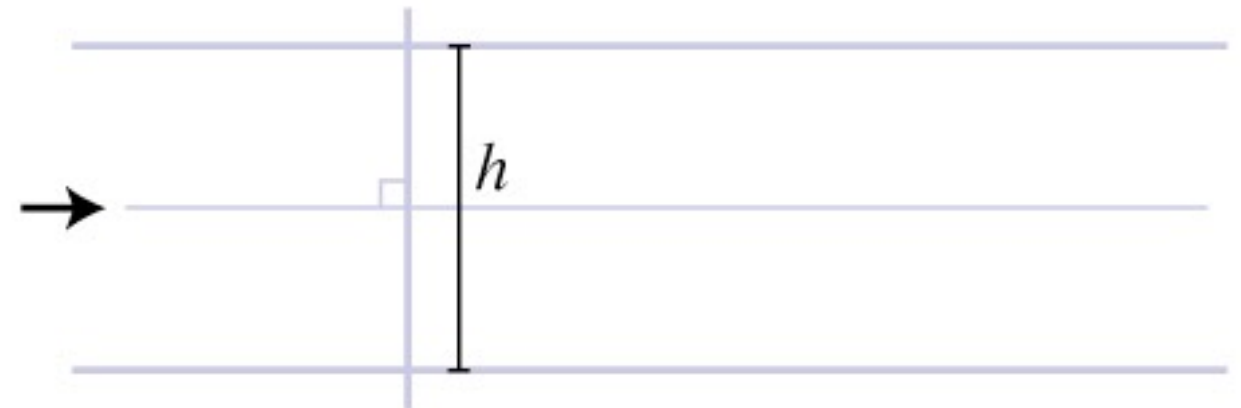 - image height determines size of objects in image

"Orthographic" projection

- **In graphics usually we lump axonometric with orthographic**
  - projection plane perpendicular to projection direction
  - image height determines size of objects in image

"Orthographic" projection

- **In graphics usually we lump axonometric with orthographic**
  - projection plane perpendicular to projection direction
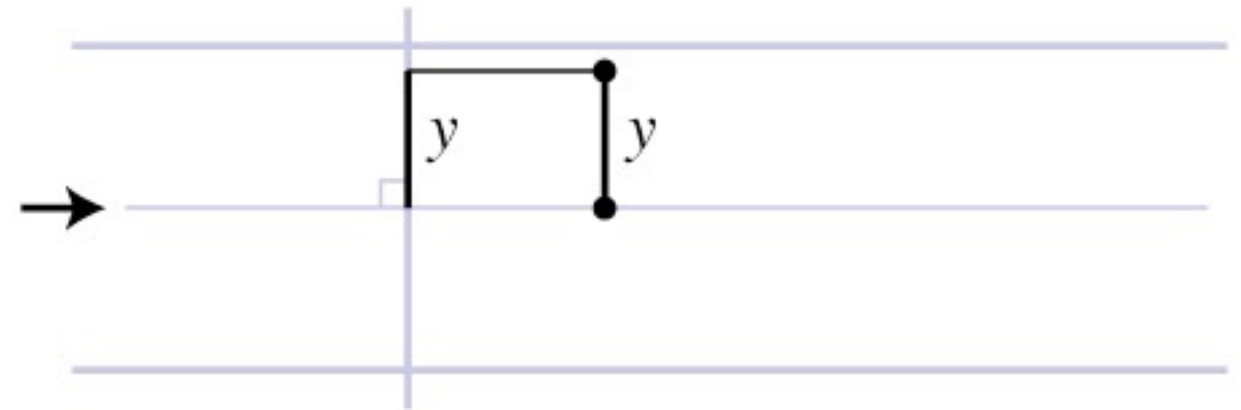 - image height determines size of objects in image

"Orthographic" projection

- **In graphics usually we lump axonometric with orthographic**
  - projection plane perpendicular to projection direction
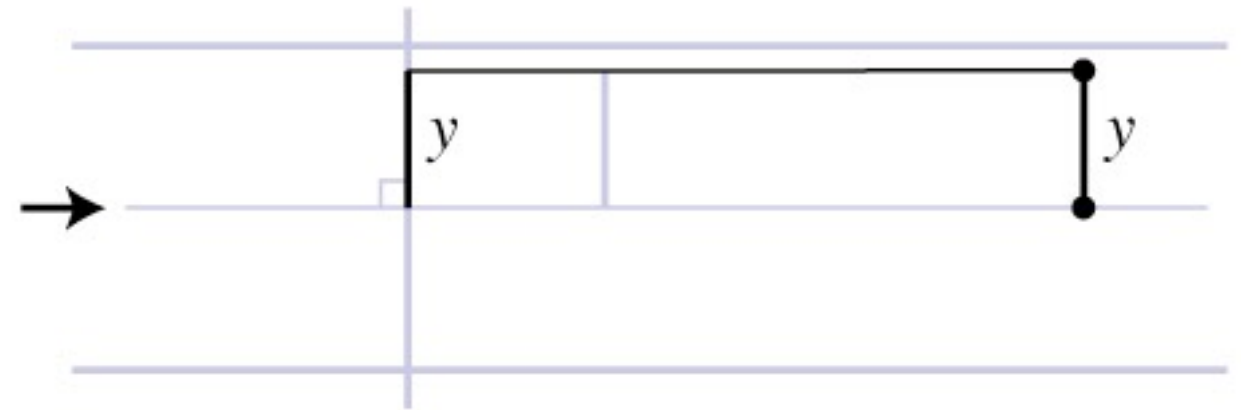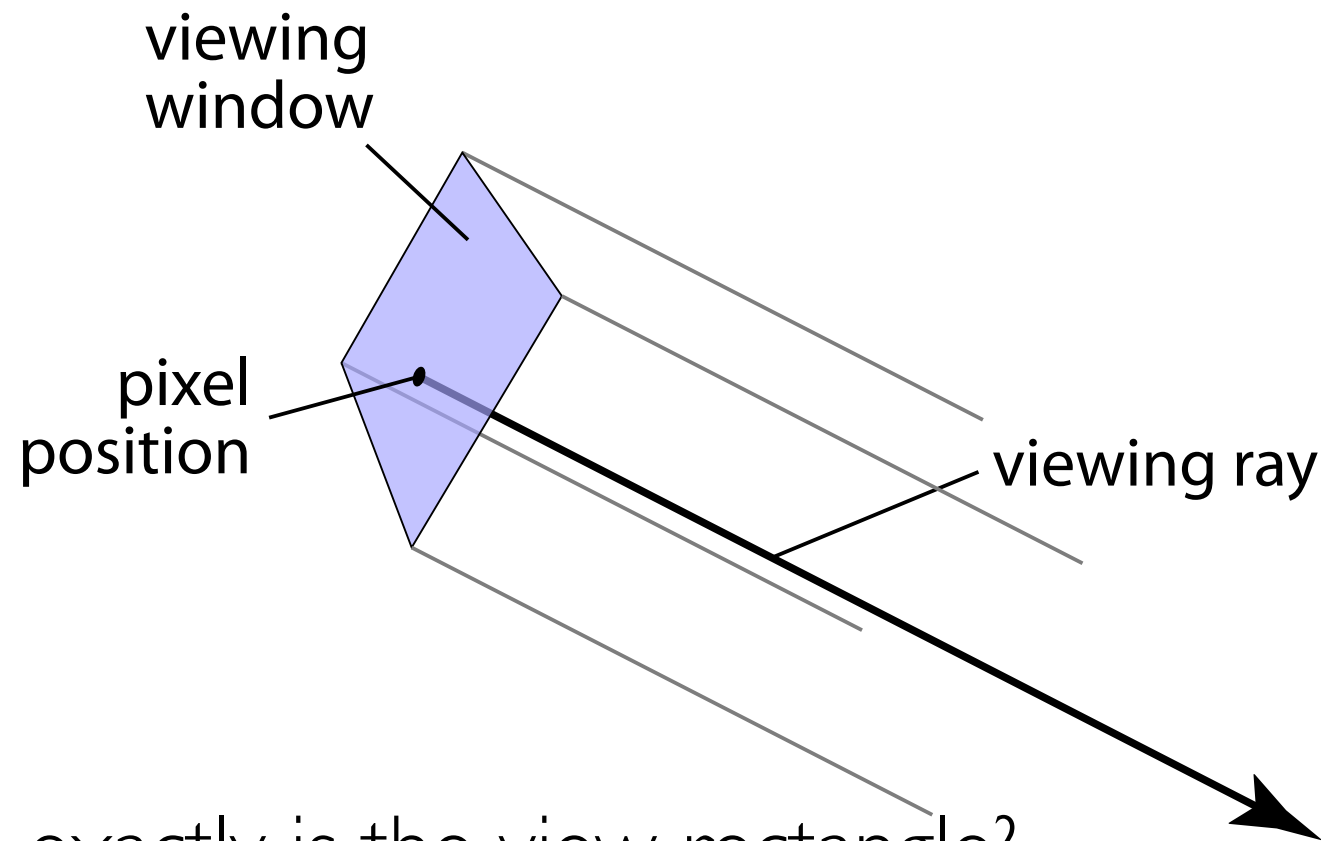 - image height determines size of objects in image

Generating eye rays—orthographic

- **Ray origin (varying): pixel position on viewing window**
- **Ray direction (constant): view direction**

viewing window

pixel position

viewing ray

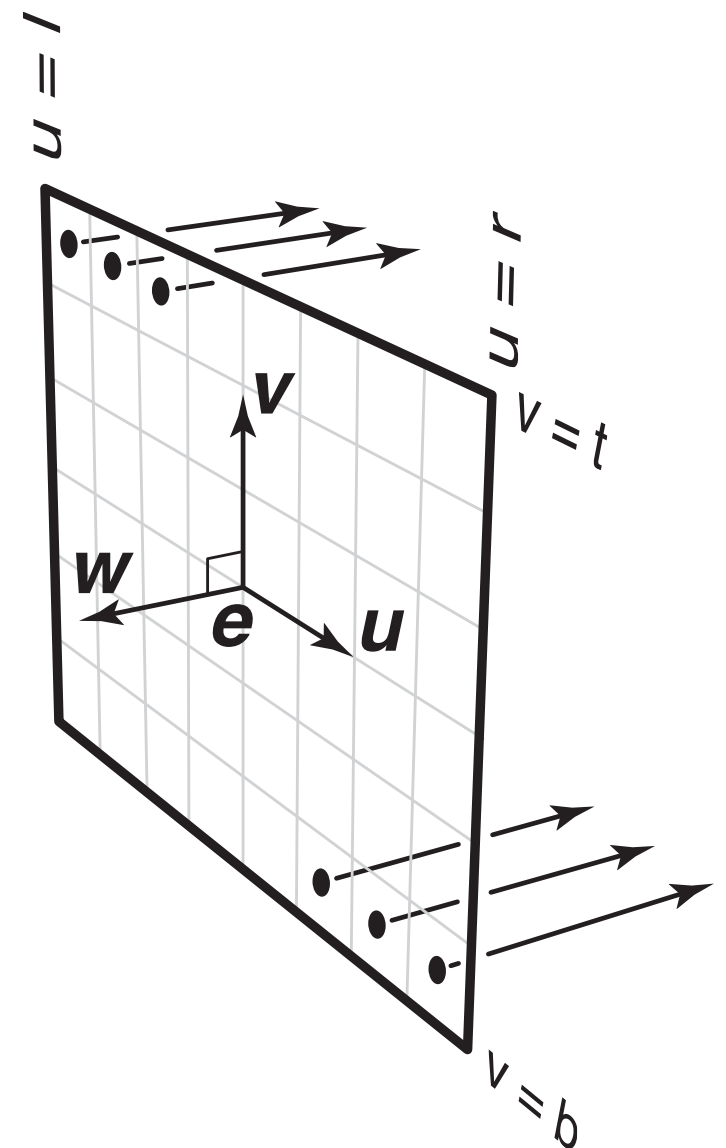– but where exactly is the view rectangle?

Generating eye rays—orthographic

- **Positioning the view rectangle**
  - establish three vectors to be *camera basis:* **u**, **v**, **w**
  - view rectangle is in **u**–**v** plane, specified by l, r, t, b (often $l = -r$ and $b = -t$)
- **Generating rays**
  - for (u, v) in $[l, r] \times [b, t]$
  - ray.origin = **e** $+ u$ **u** $+ v$ **v**
  - ray.direction = $-$**w**

Establishing the camera basis

- **Could require user to provide e, u, v, and w**
  - but this is error prone and unintuitive
- **Instead, calculate basis from things the user cares about**
  - viewpoint: where the camera is → **e**
  - view direction: which way the camera is looking → **d**
  - view plane normal (normally same as view direction)
  - up vector: how the camera is oriented
- **This is enough to calculate u, v, and w**
  - set **w** parallel to v.p. normal, facing away from **d**
  - set **u** perpendicular to **w** and perpendicular to up-vector
  - set **v** perpendicular to **w** and **u** to form a right-handed ONB

Camera basis
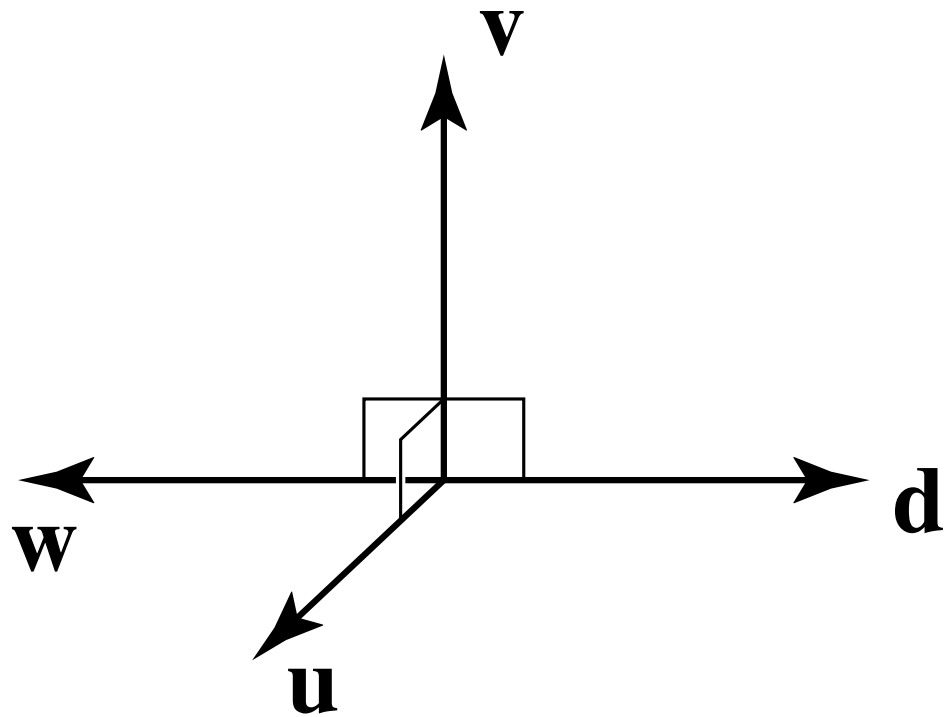
forming the basis
with **d** and **v** given
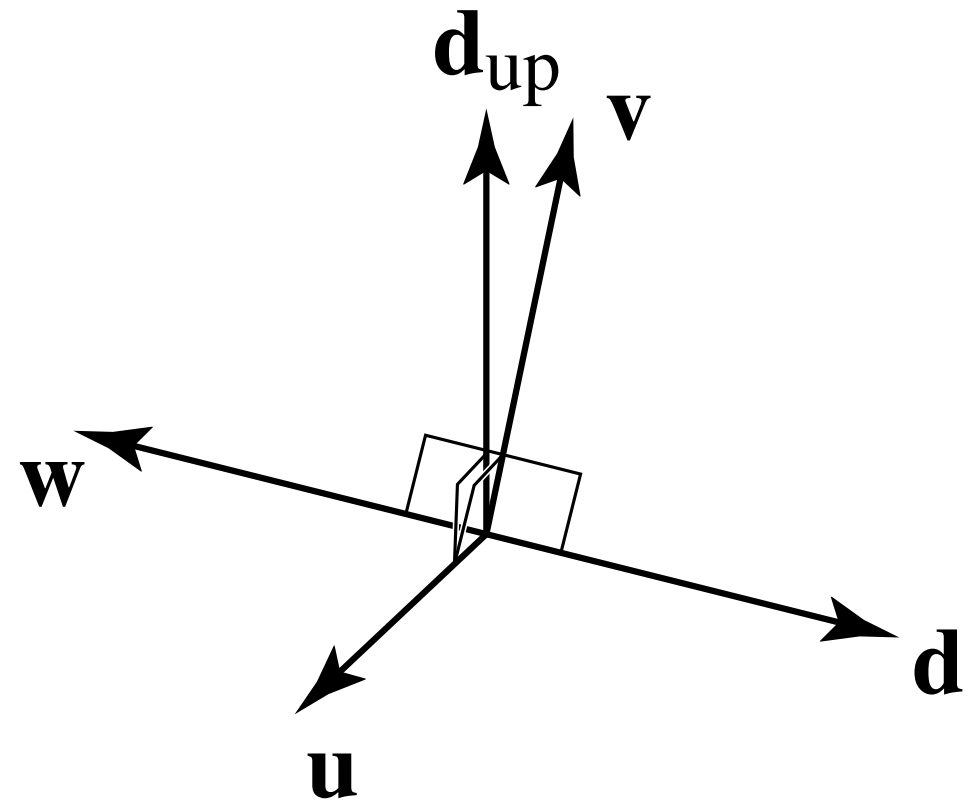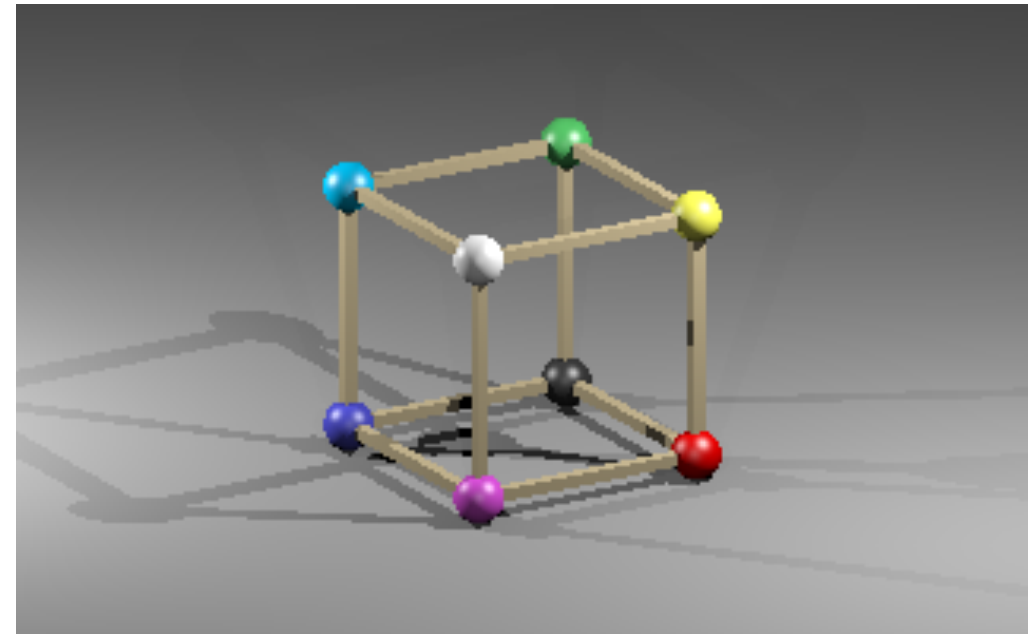
forming the basis
with **d** and up vector given

Orthographic views in Ray 1
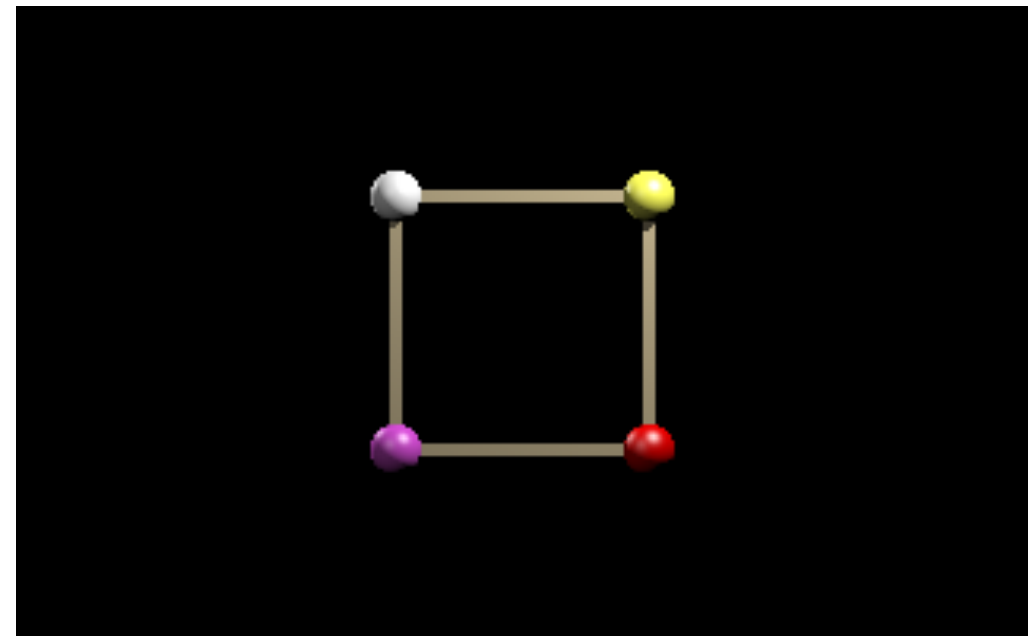
```
<camera type="OrthographicCamera">
  <viewPoint>10 4.2 6</viewPoint>
  <viewDir>-5 -2.1 -3</viewDir>
  <viewUp>0 1 0</viewUp>
  <viewWidth>8</viewWidth>
  <viewHeight>5</viewHeight>
</camera>
```



```
<camera type="OrthographicCamera">
  <viewPoint>10 0 0</viewPoint>
  <viewDir>-1 0 0</viewDir>
  <viewUp>0 1 0</viewUp>
  <viewWidth>8</viewWidth>
  <viewHeight>5</viewHeight>
</camera>
```
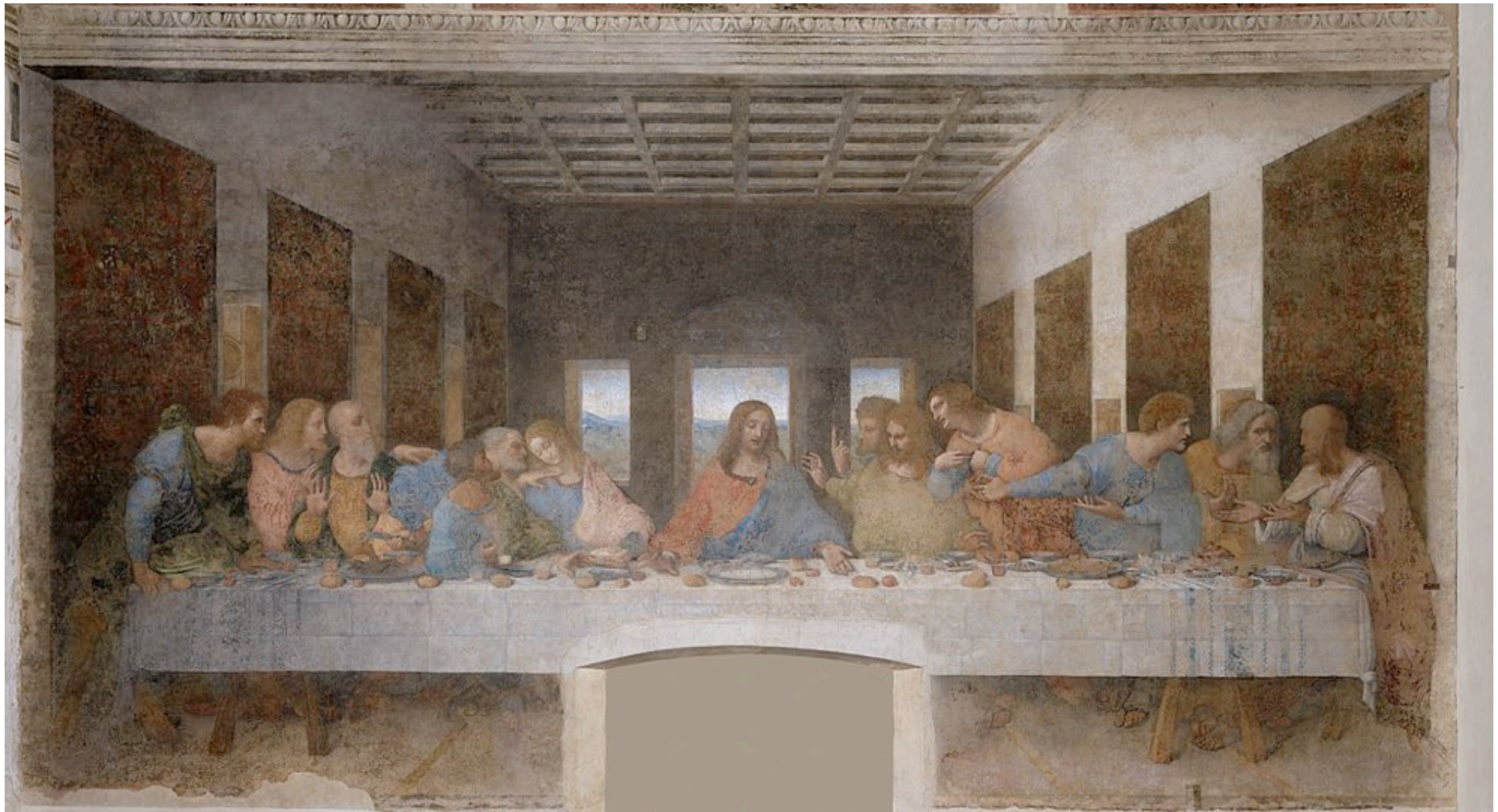
History of projection

- **Ancient times: Greeks wrote about laws of perspective**

- **Renaissance: perspective is adopted by artists**
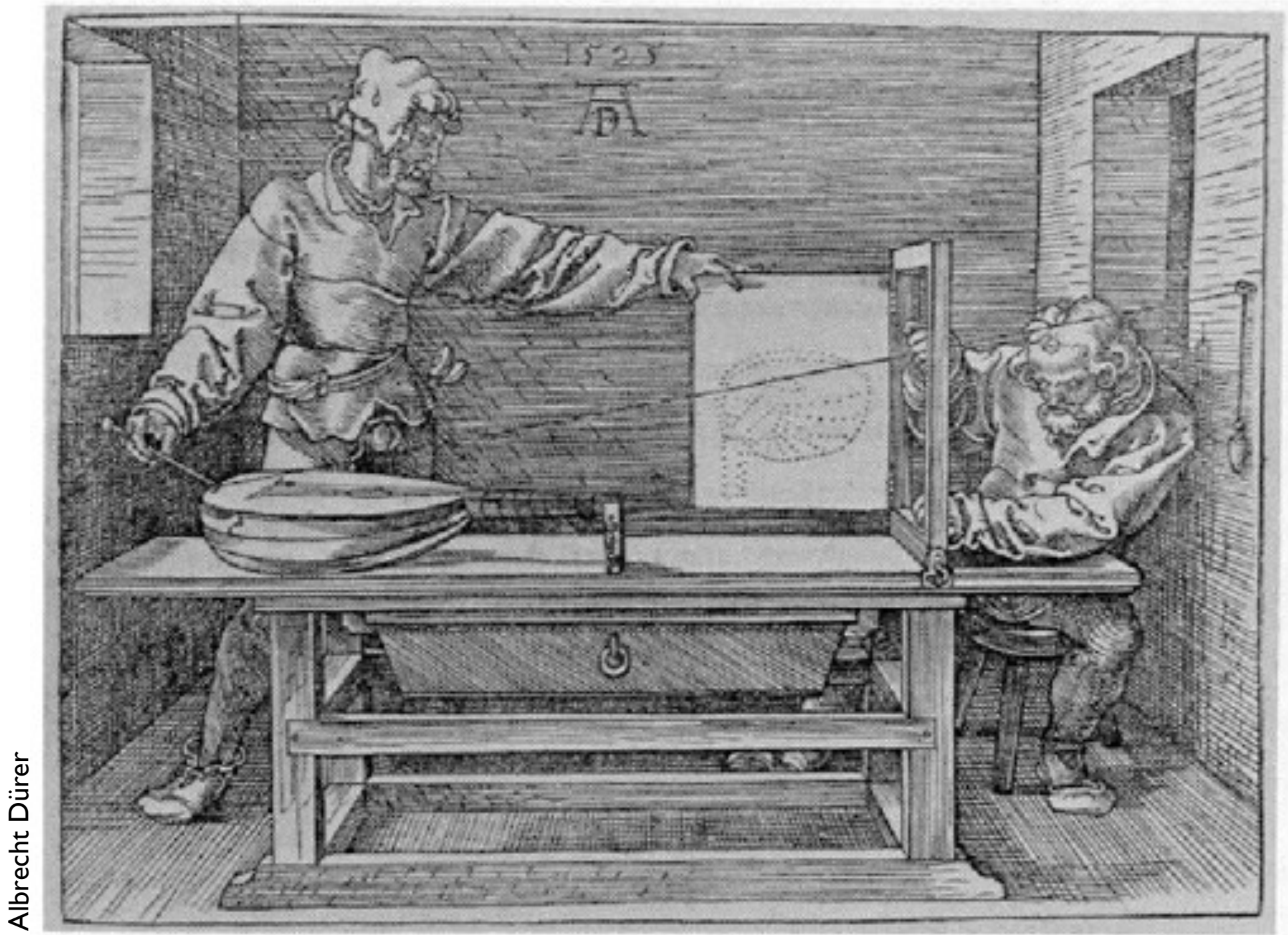

Duccio c. 1308

History of projection
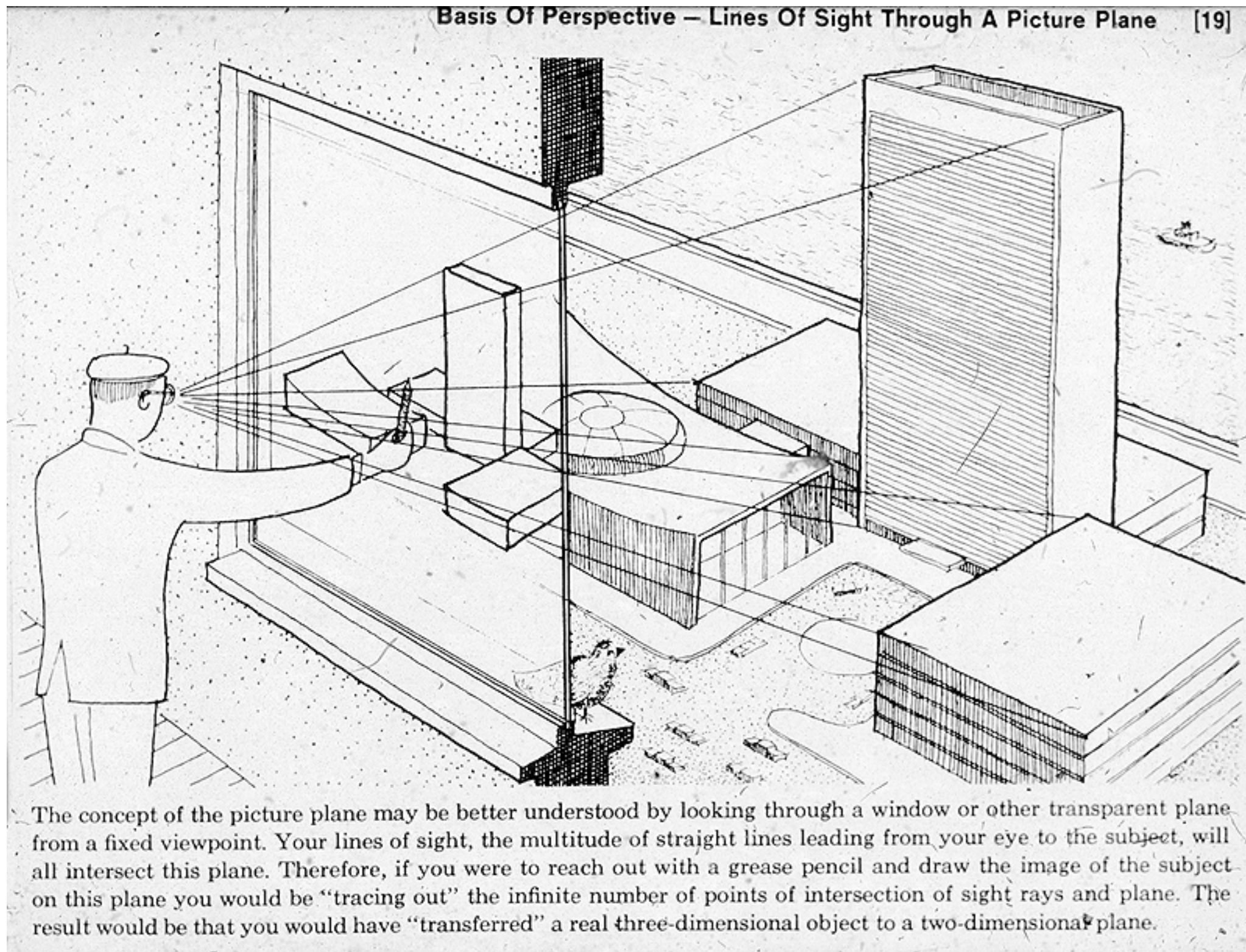
- **Later Renaissance: perspective formalized precisely**

da Vinci c. 1498

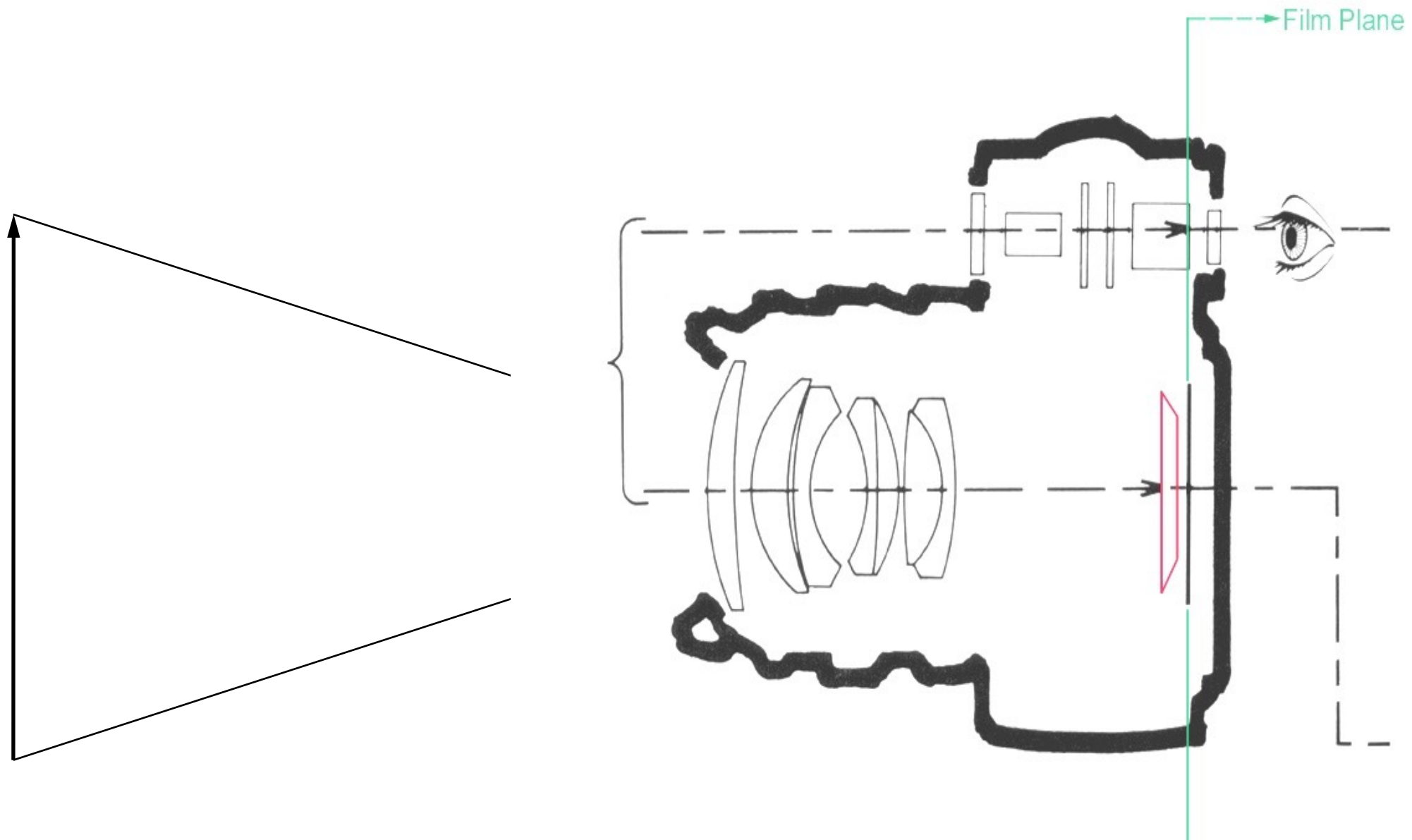# Plane projection in drawing

Albrecht Dürer

Plane projection in drawing

Basis Of Perspective — Lines Of Sight Through A Picture Plane [19]

The concept of the picture plane may be better understood by looking through a window or other transparent plane from a fixed viewpoint. Your lines of sight, the multitude of straight lines leading from your eye to the subject, will all intersect this plane. Therefore, if you were to reach out with a grease pencil and draw the image of the subject on this plane you would be "tracing out" the infinite number of points of intersection of sight rays and plane. The result would be that you would have "transferred" a real three-dimensional object to a two-dimensional plane.

Plane projection in photography

- **This is another model for what we are doing**
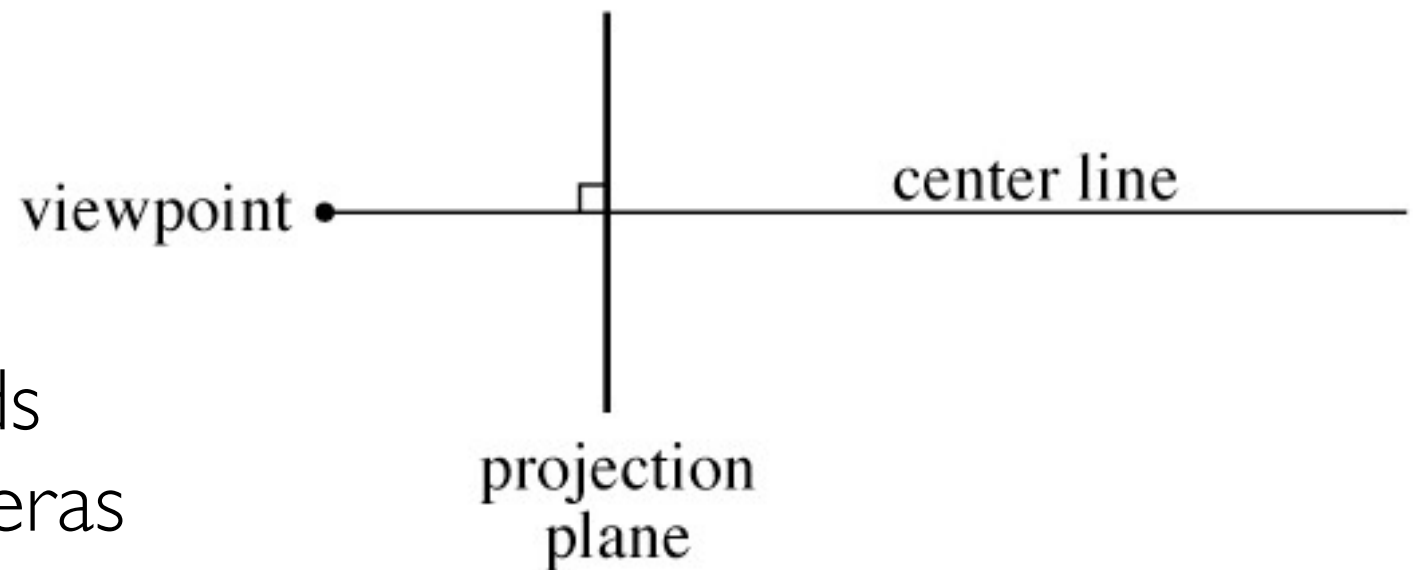 - applies more directly in realistic rendering

[Source unknown]

Plane projection in photography

[Richard Zakia]

Perspective projection (normal)

- **Perspective is projection by lines through a point; "normal" = plane perpendicular to view direction**
  - magnification determined by:
    - image height
    - object depth
    - image plane distance
  - f.o.v. $\alpha = 2\ \text{atan}(h/(2d))$
  - $y' = d\,y\,/\,z$
  - "normal" case corresponds to common types of cameras

viewpoint •

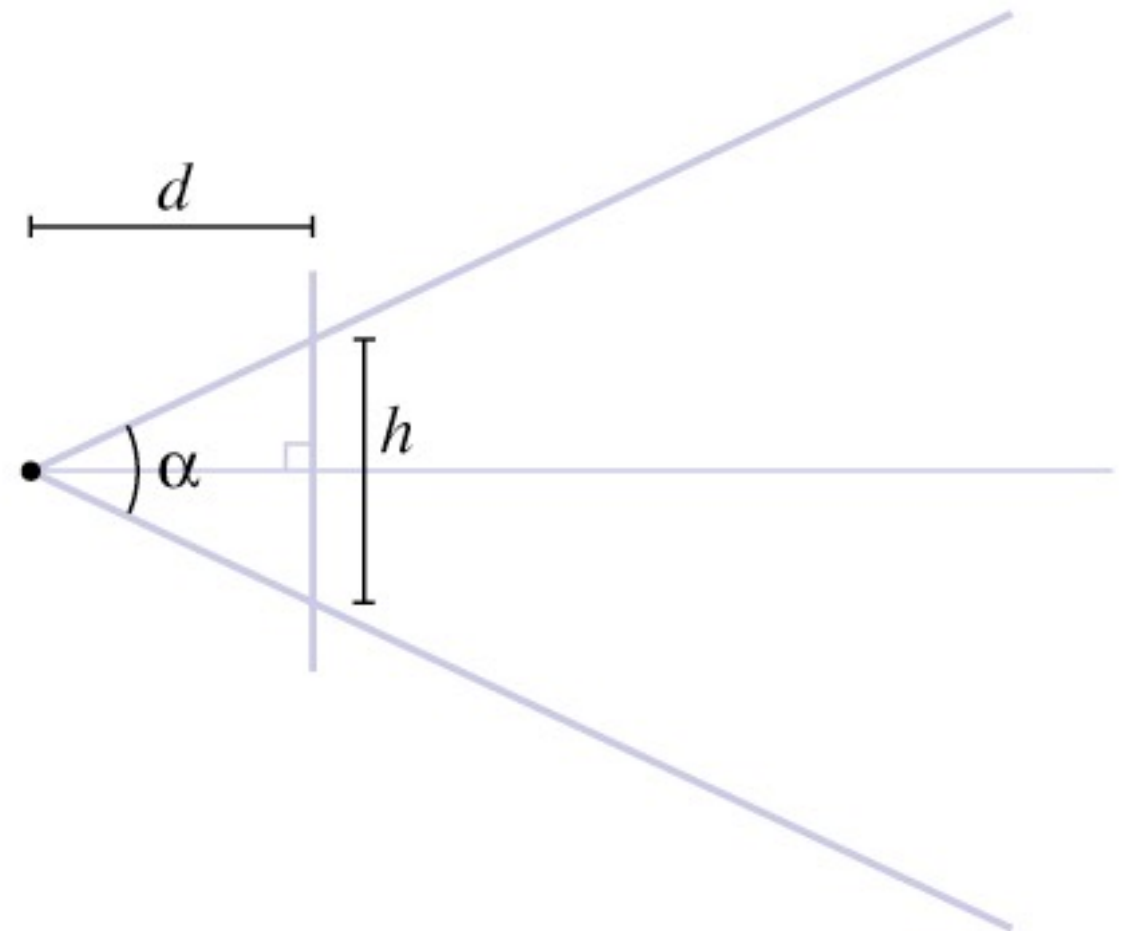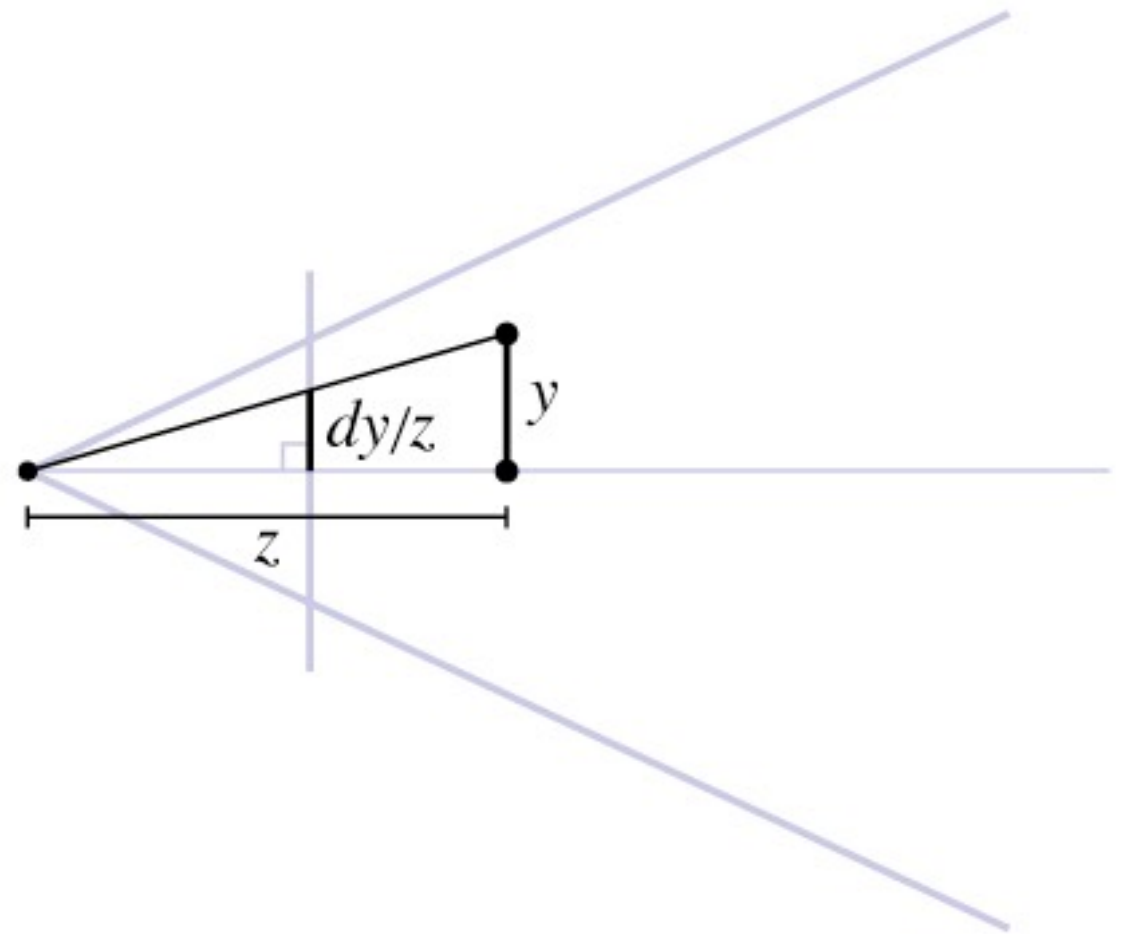center line

projection plane

Perspective projection (normal)

- **Perspective is projection by lines through a point; "normal" = plane perpendicular to view direction**
  - magnification determined by:
    - image height
    - object depth
    - image plane distance
  - f.o.v. α = 2 atan($h/(2d)$)
  - **$y' = d\, y\, /\, z$**
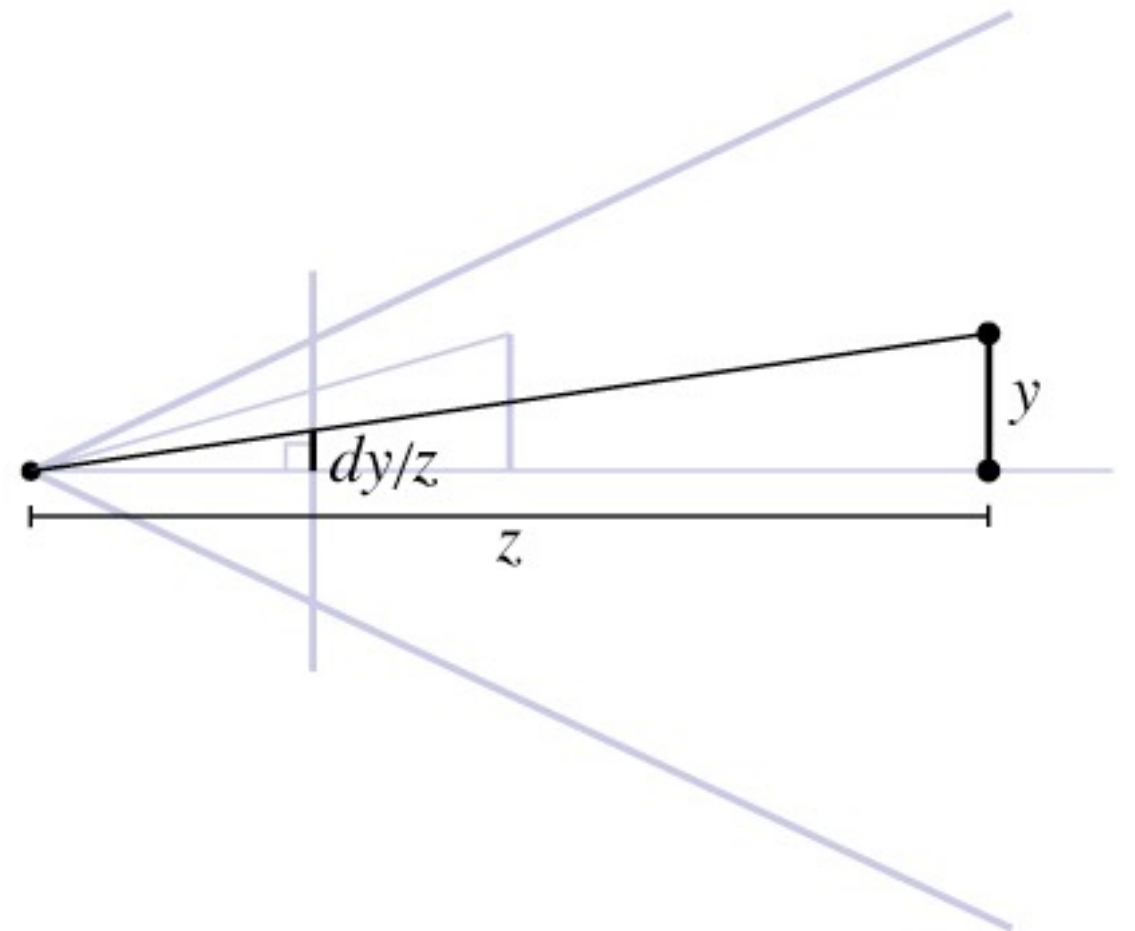 - "normal" case corresponds to common types of cameras

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  - **$y' = d\,y\,/\,z$**
  - "normal" case corresponds to common types of cameras

Generating eye rays—perspective

- **Use window analogy directly**
- **Ray origin (constant): viewpoint**
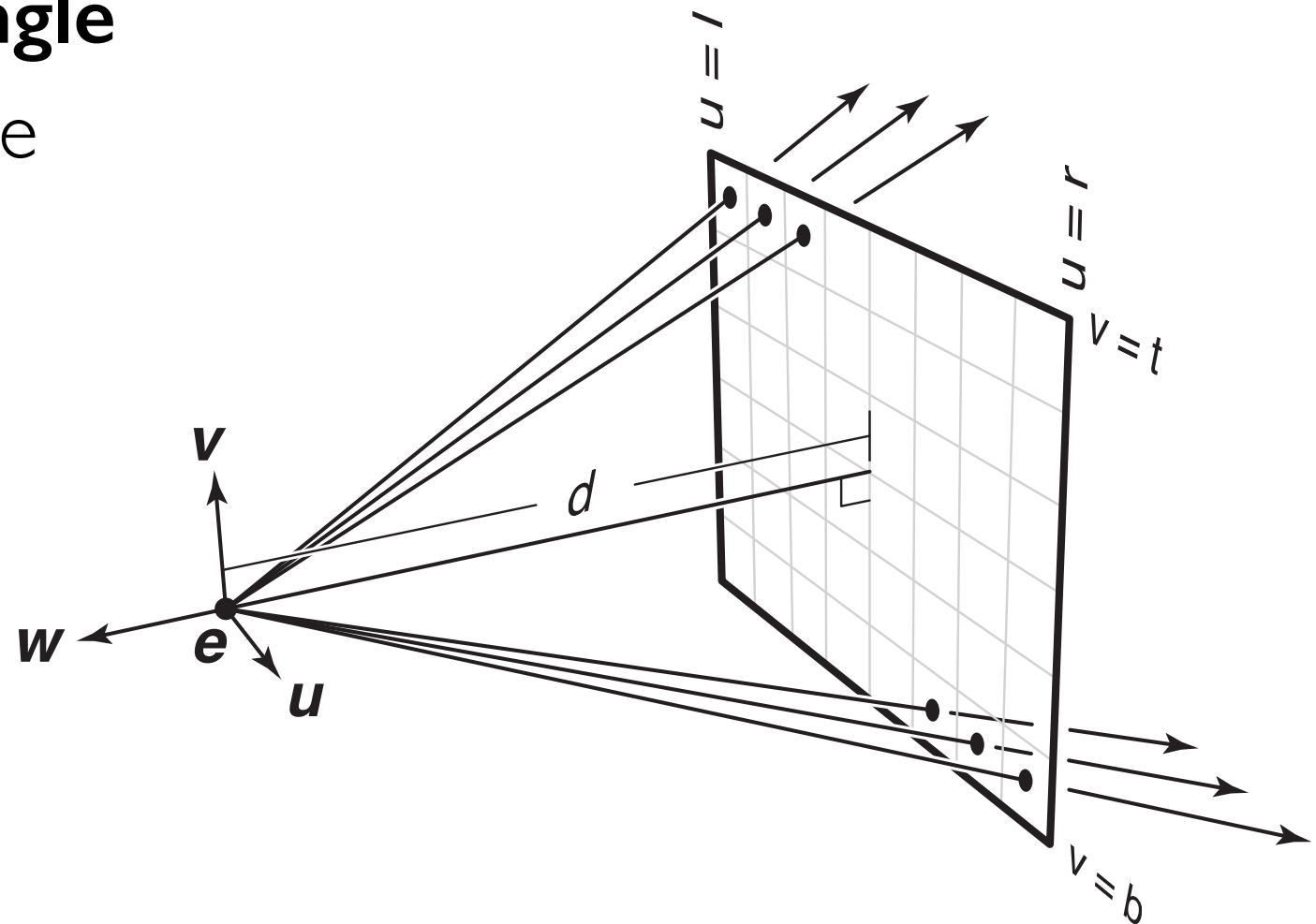- **Ray direction (varying): toward pixel position on viewing window**
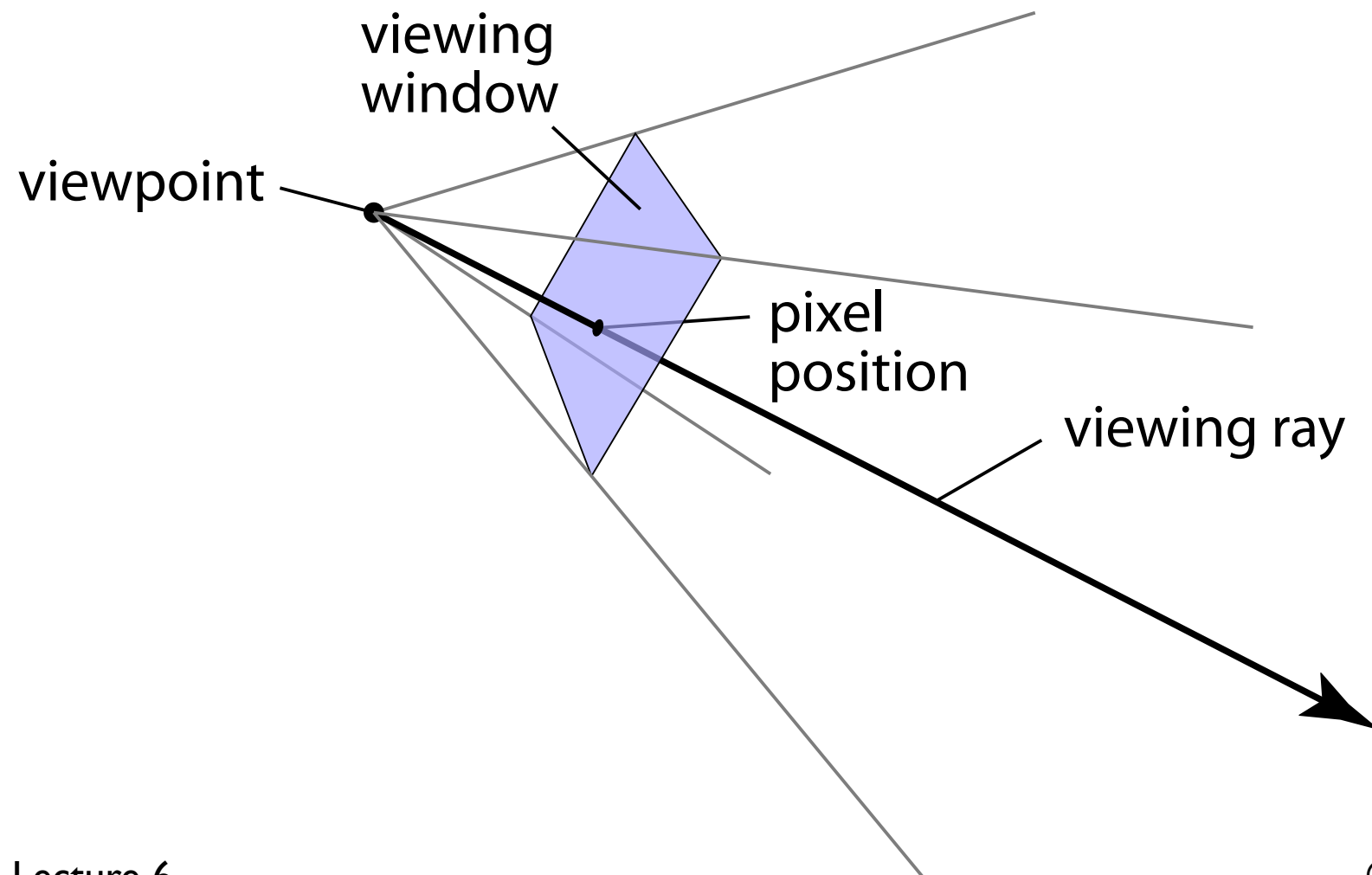
Generating eye rays—perspective

- **Positioning the view rectangle**
  - establish three vectors to be *camera basis:* **u**, **v**, **w**
  - view rectangle is parallel to **u**–**v** plane, at $w = -d$, specified by l, r, t, b
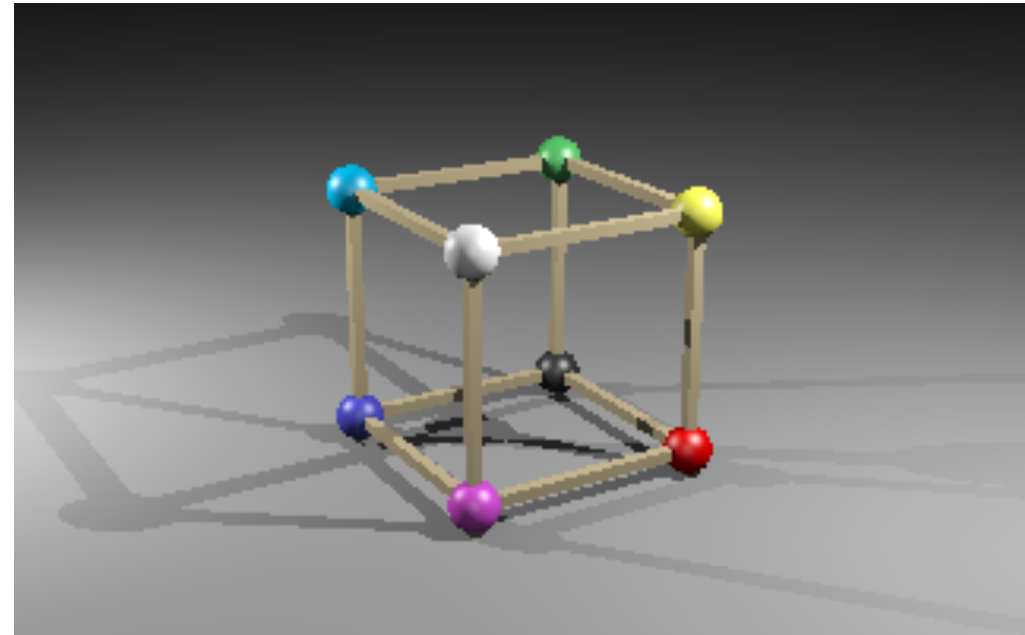- **Generating rays**
  - for (u, v) in $[l, r] \times [b, t]$
  - ray.origin = **e**
  - ray.direction = $-d$ **w** + u **u** + v **v**

Perspective views in Ray 1

```
<camera type="PerspectiveCamera">
   <viewPoint>10 4.2 6</viewPoint>
   <viewDir>-5 -2.1 -3</viewDir>
   <viewUp>0 1 0</viewUp>
   <projDistance>12</projDistance>
   <viewWidth>8</viewWidth>
   <viewHeight>5</viewHeight>
</camera>
```
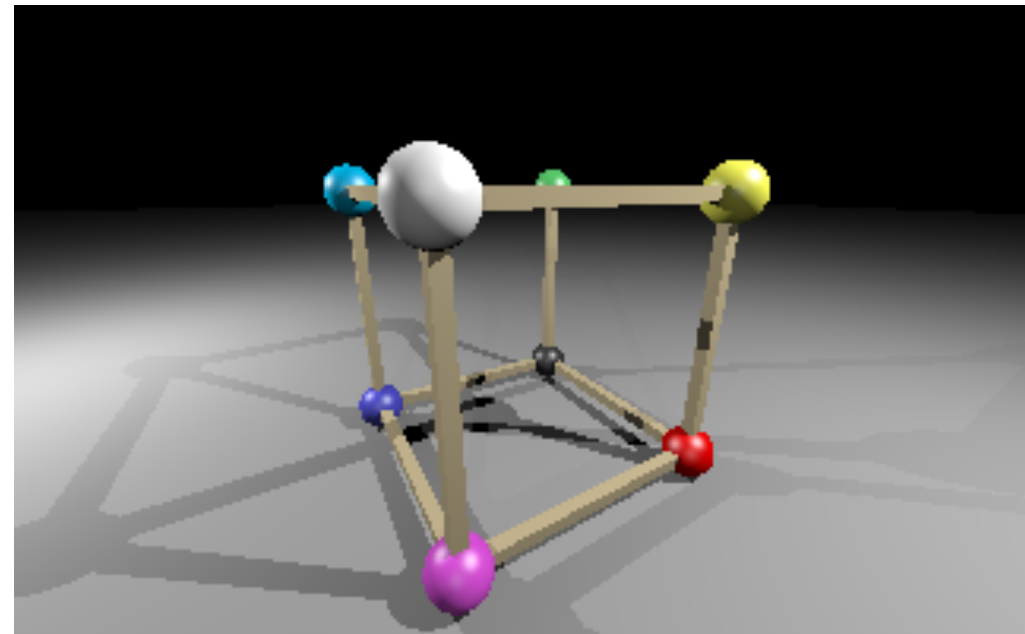


```
<camera type="PerspectiveCamera">
   <viewPoint>2.5 1.05 1.5</viewPoint>
   <viewDir>-5 -2.1 -3</viewDir>
   <viewUp>0 1 0</viewUp>
   <projDistance>3</projDistance>
   <viewWidth>8</viewWidth>
   <viewHeight>5</viewHeight>
</camera>
```

Field of view (or f.o.v.)

- **The angle between the rays corresponding to opposite edges of a perspective image**
  - simpler to compute for "normal" perspective
  - have to decide to measure vert., horiz., or diag.
- **In cameras, determined by focal length**
  - confusing because of many image sizes
  - for 35mm format (36mm by 24mm image)
    - 18mm = 67° v.f.o.v. — super-wide angle
    - 28mm = 46° v.f.o.v. — wide angle
    - 50mm = 27° v.f.o.v. — "normal"
    - 100mm = 14° v.f.o.v. — narrow angle ("telephoto")

Field of view

- **Determines "strength" of perspective effects**

close viewpoint
wide angle
large scale differences

far viewpoint
narrow angle
small scale differences

[Ansel Adams]
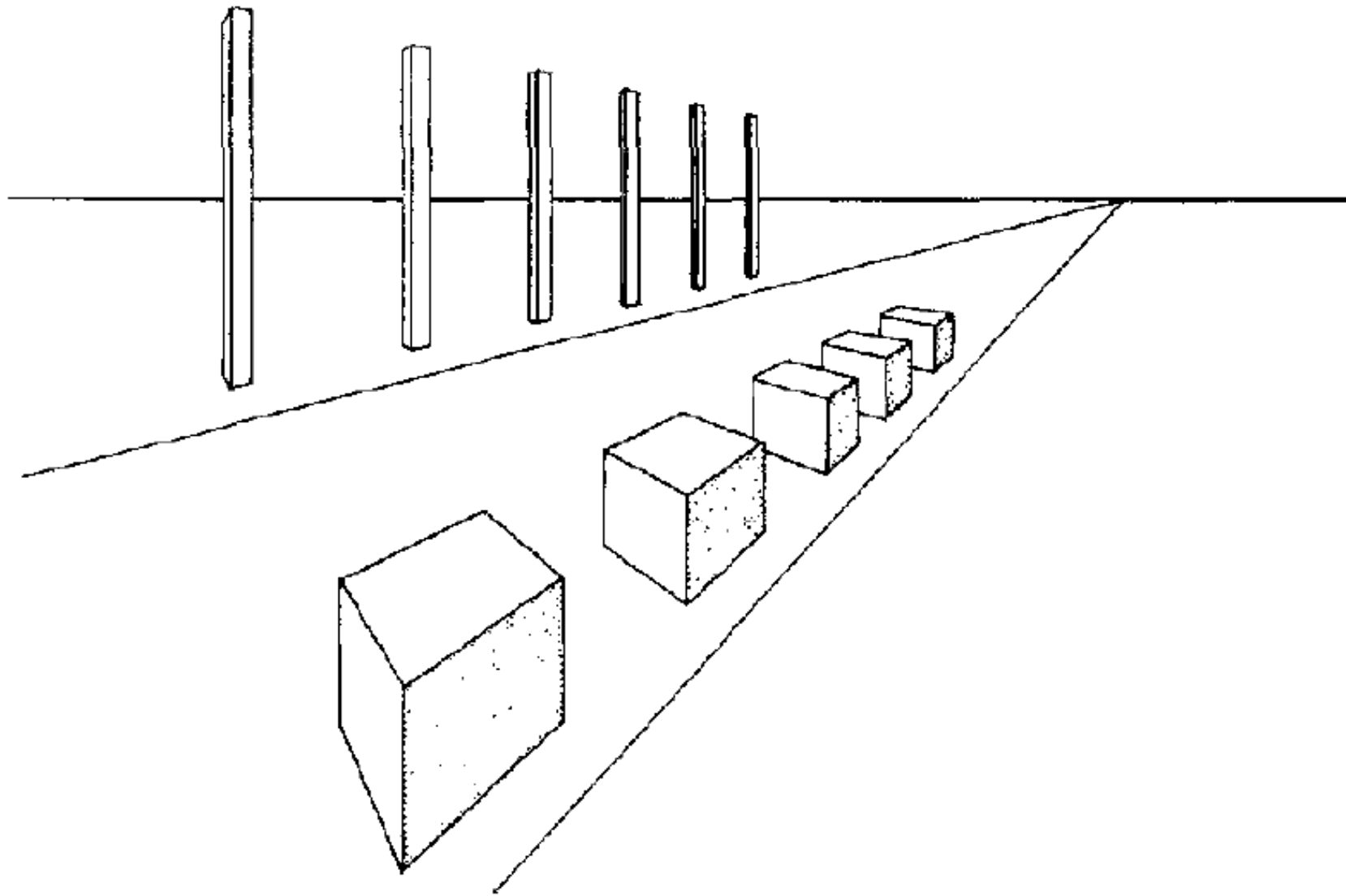
Choice of field of view

- **In photography, wide angle lenses are specialty tools**
  - "hard to work with"
  - easy to create weird-looking perspective effects
- **In graphics, you can type in whatever f.o.v. you want**
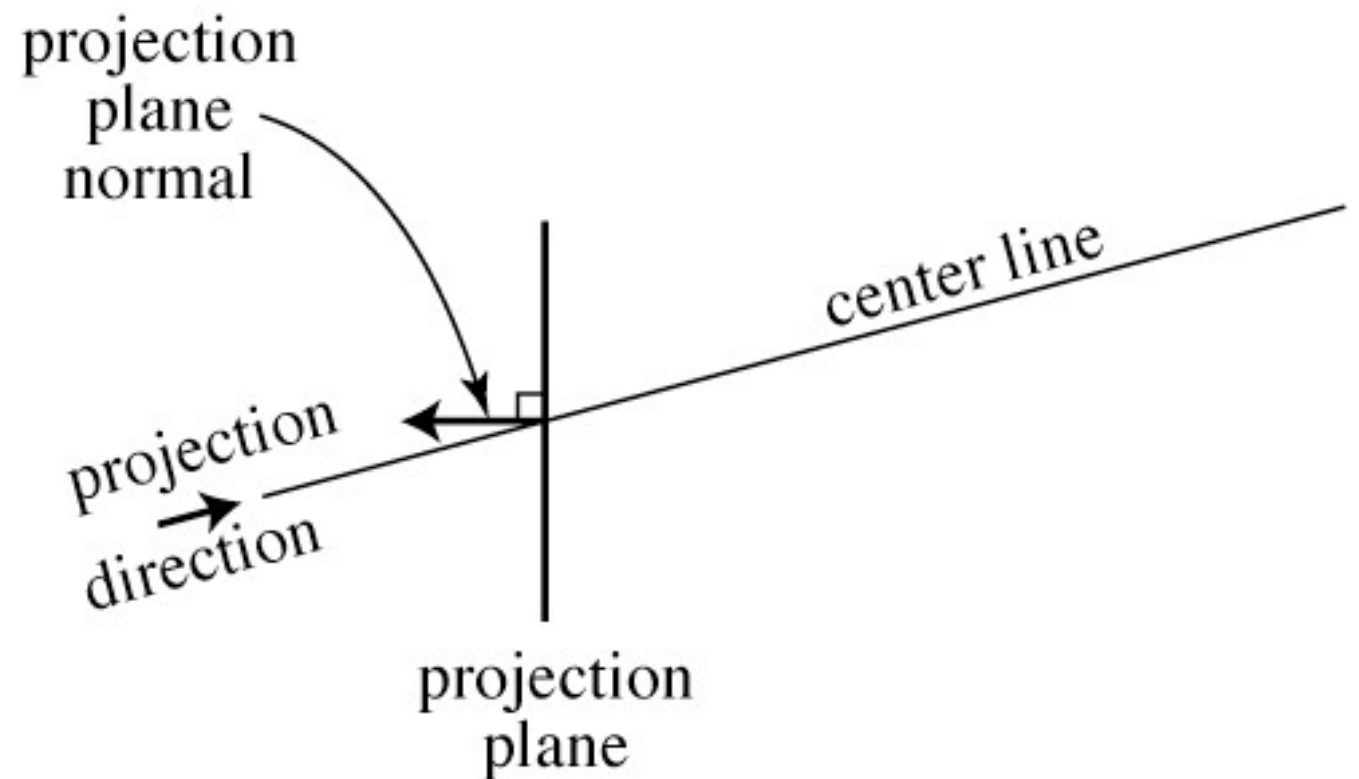 - and people often type in big numbers!

[Ken Perlin]

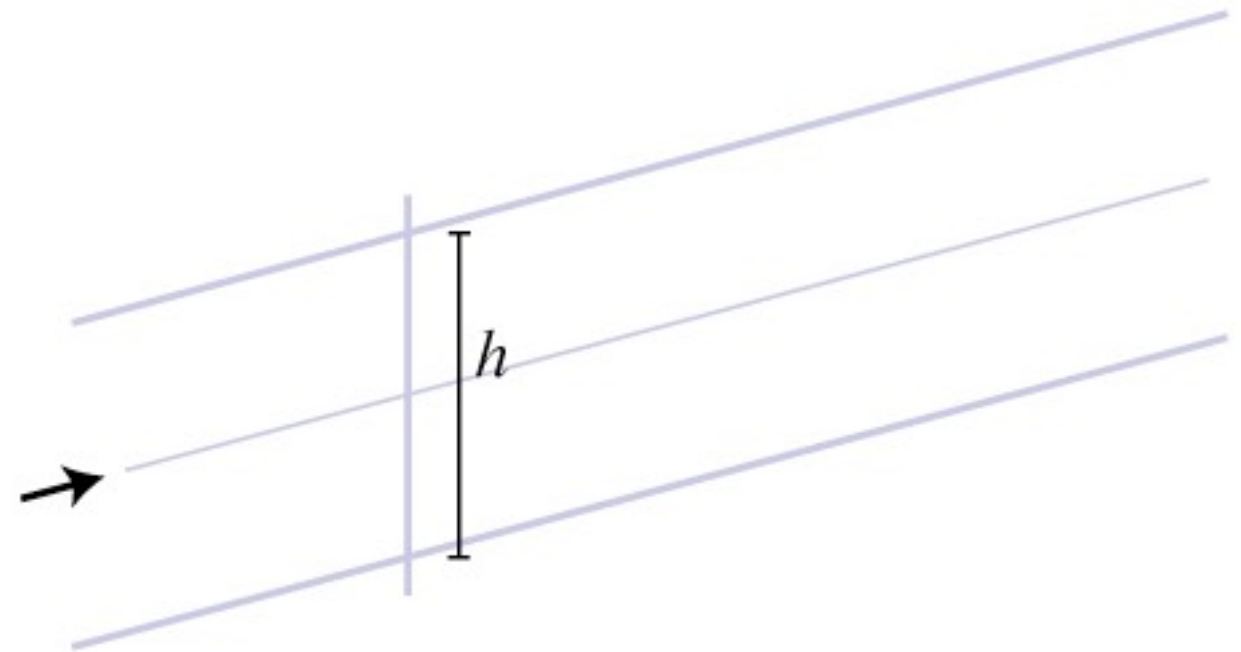# Perspective "distortions"

- **Lengths, length ratios**

Oblique projection

- **View direction no longer coincides with projection plane normal (one more parameter)**
  - objects at different distances still same size
  - objects are shifted in the image depending on their depth
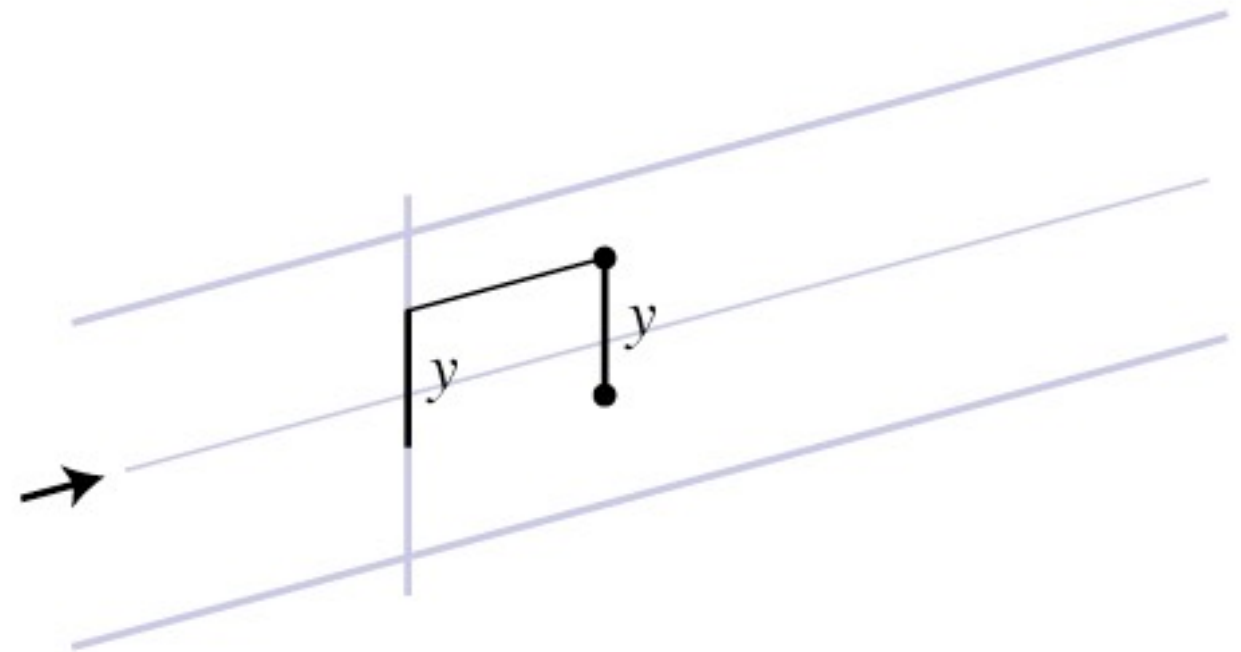
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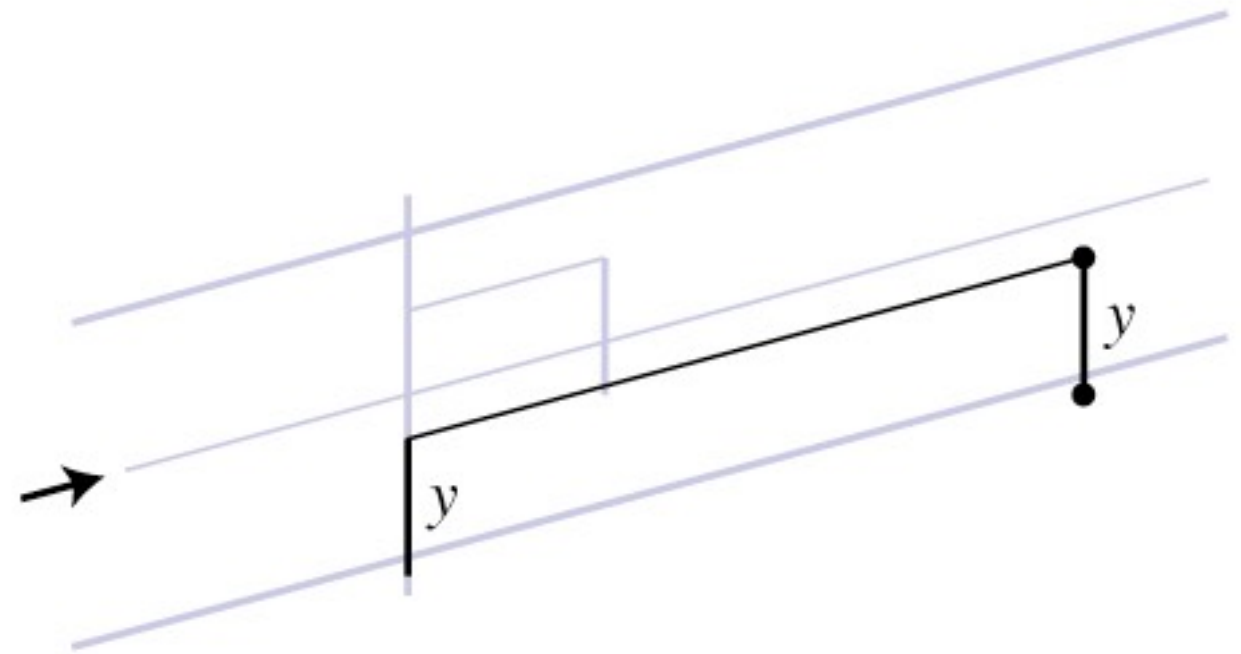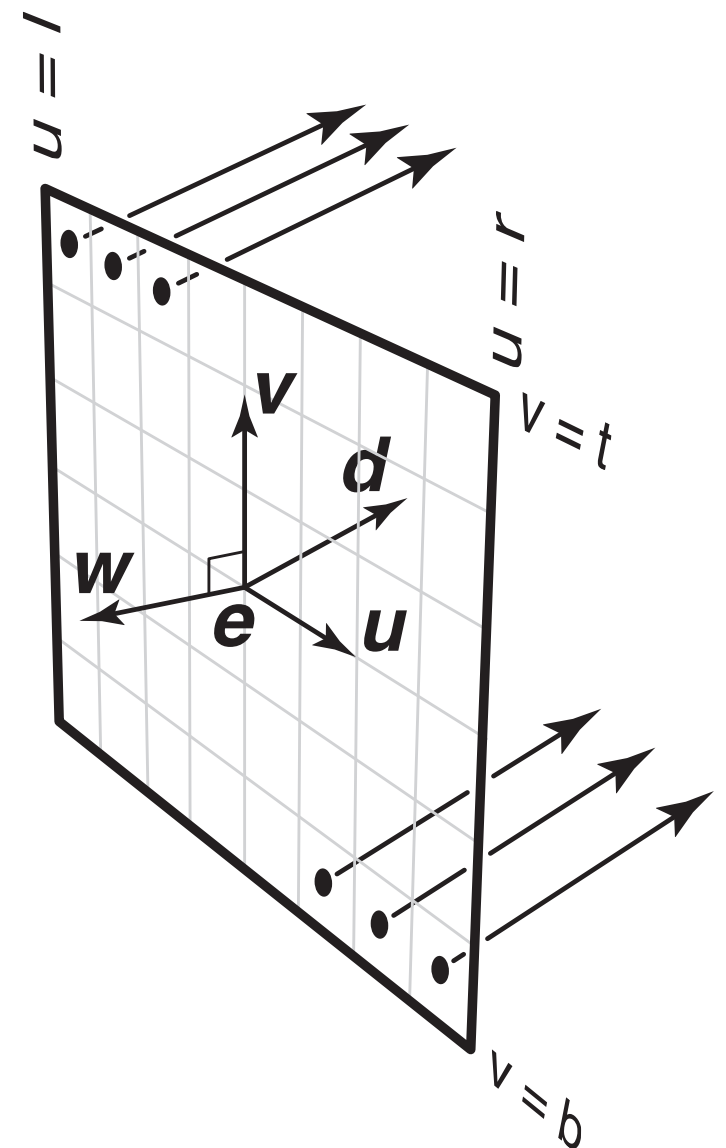
Oblique projection

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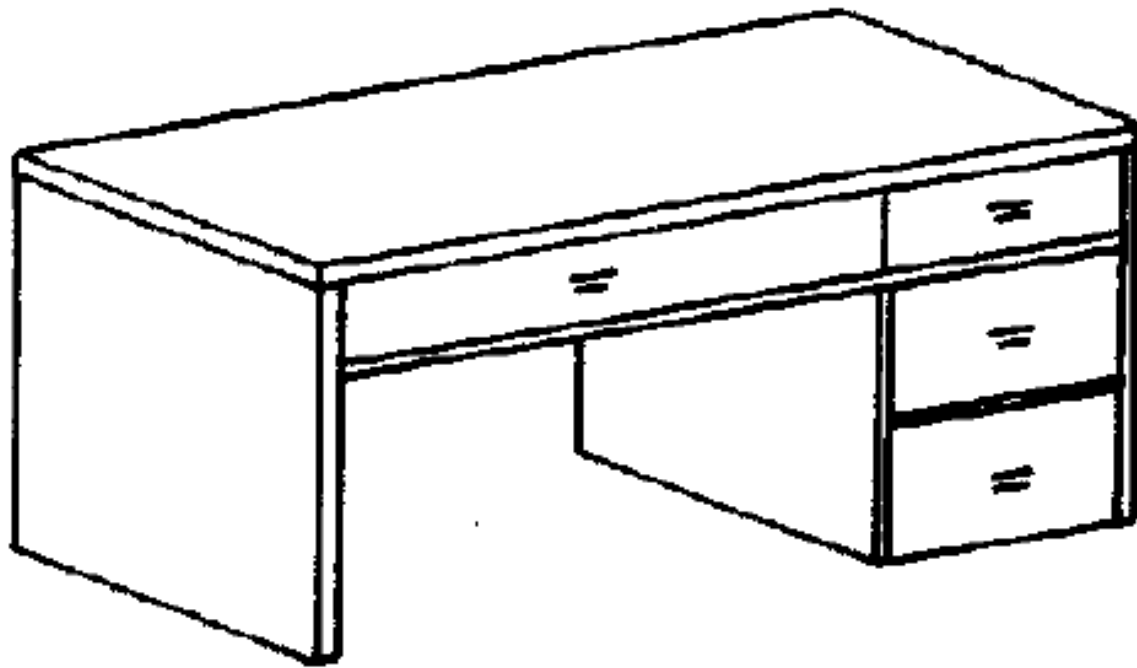 - objects are shifted in the image depending on their depth
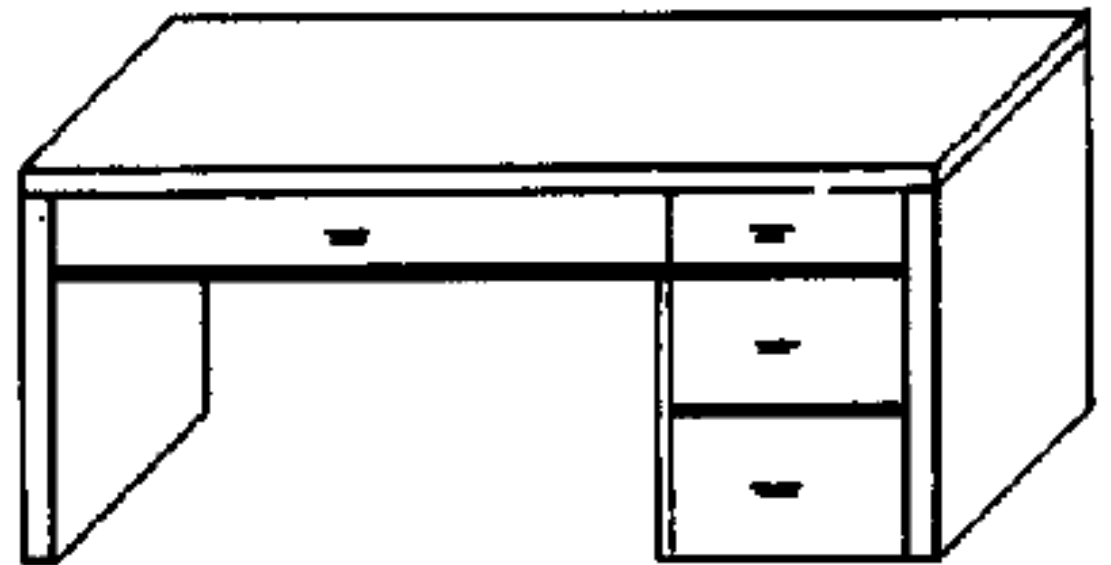
Oblique parallel views

- **View rectangle is the same**
  - ray origins identical to orthographic
  - view direction **d** differs from −**w**
- **Generating rays**
  - for (u, v) in $[l, r] \times [b, t]$
  - ray.origin = **e** + u **u** + v **v**
  - ray.direction = **d**

Off-axis parallel

[Carlbom & Paciorek 78]

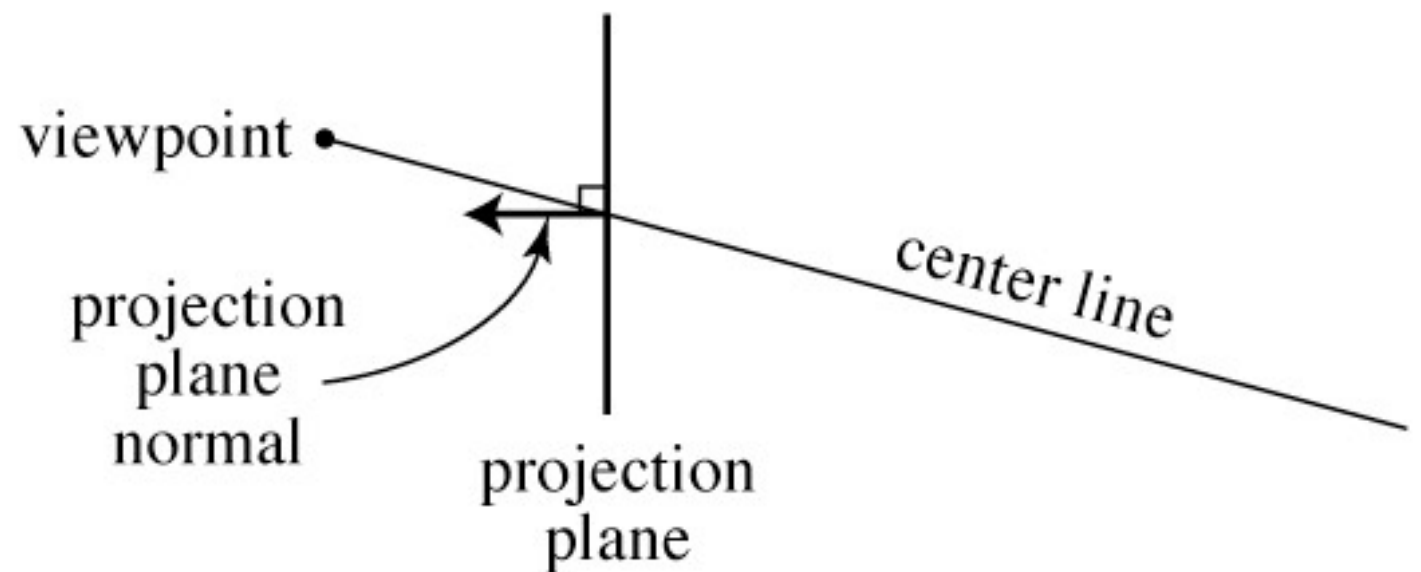**axonometric**: projection plane perpendicular to projection direction but not parallel to coordinate planes

oblique: projection plane parallel to a coordinate plane but not perpendicular to projection direction.

Shifted perspective projection

- **Perspective but with projection plane not perpendicular to view direction**
  - additional parameter: projection plane normal
  - exactly equivalent to cropping out an off-center rectangle from a larger "normal" perspective
  - corresponds to *view camera* in photography

viewpoint

projection plane normal

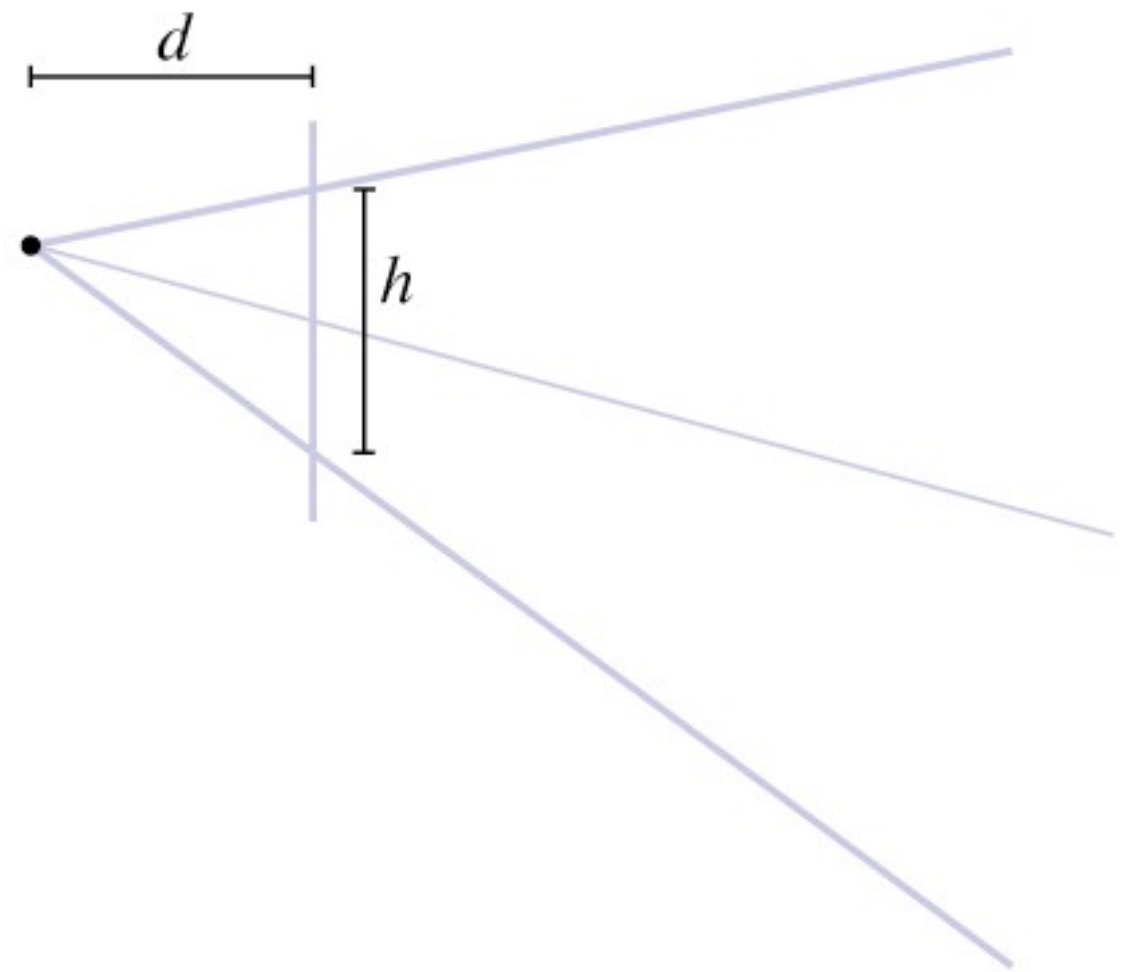projection plane

center line

Shifted perspective projection

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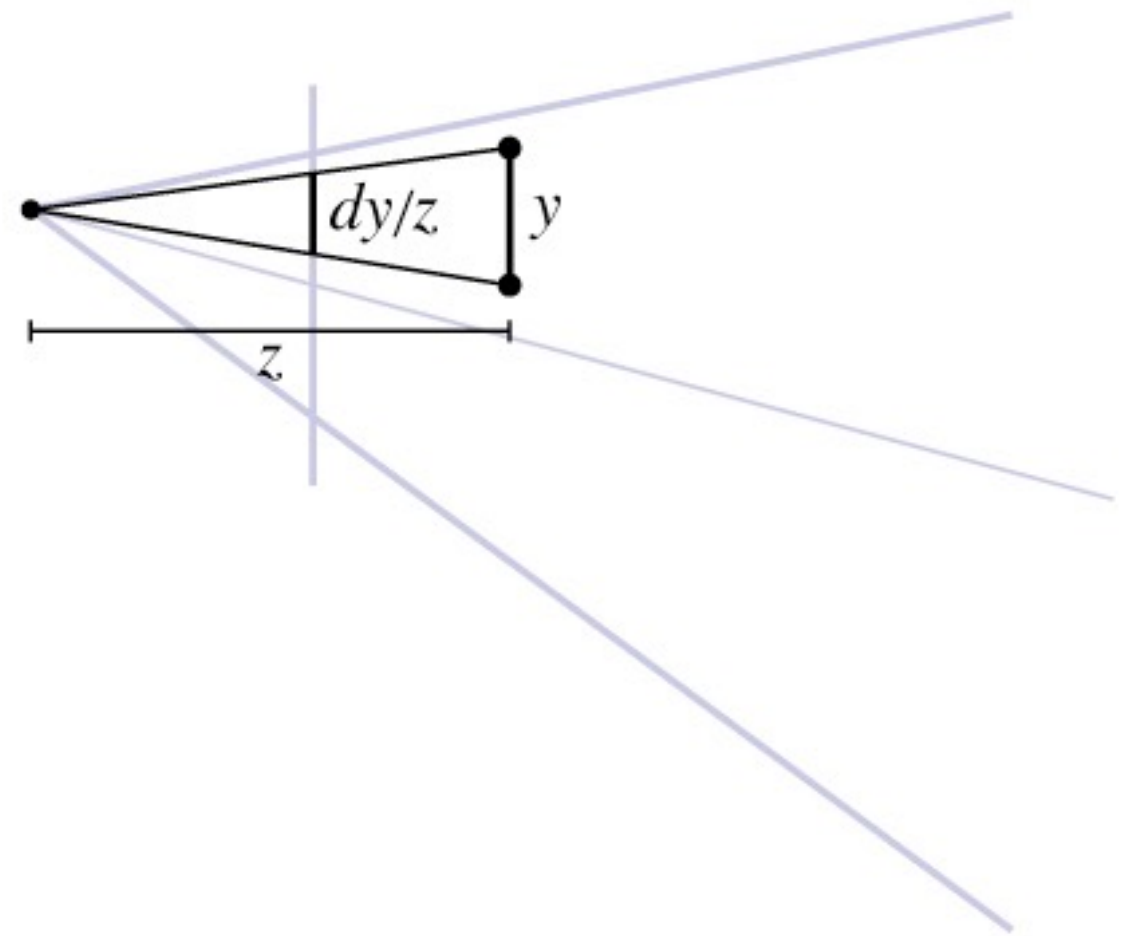 - corresponds to *view camera* in photography

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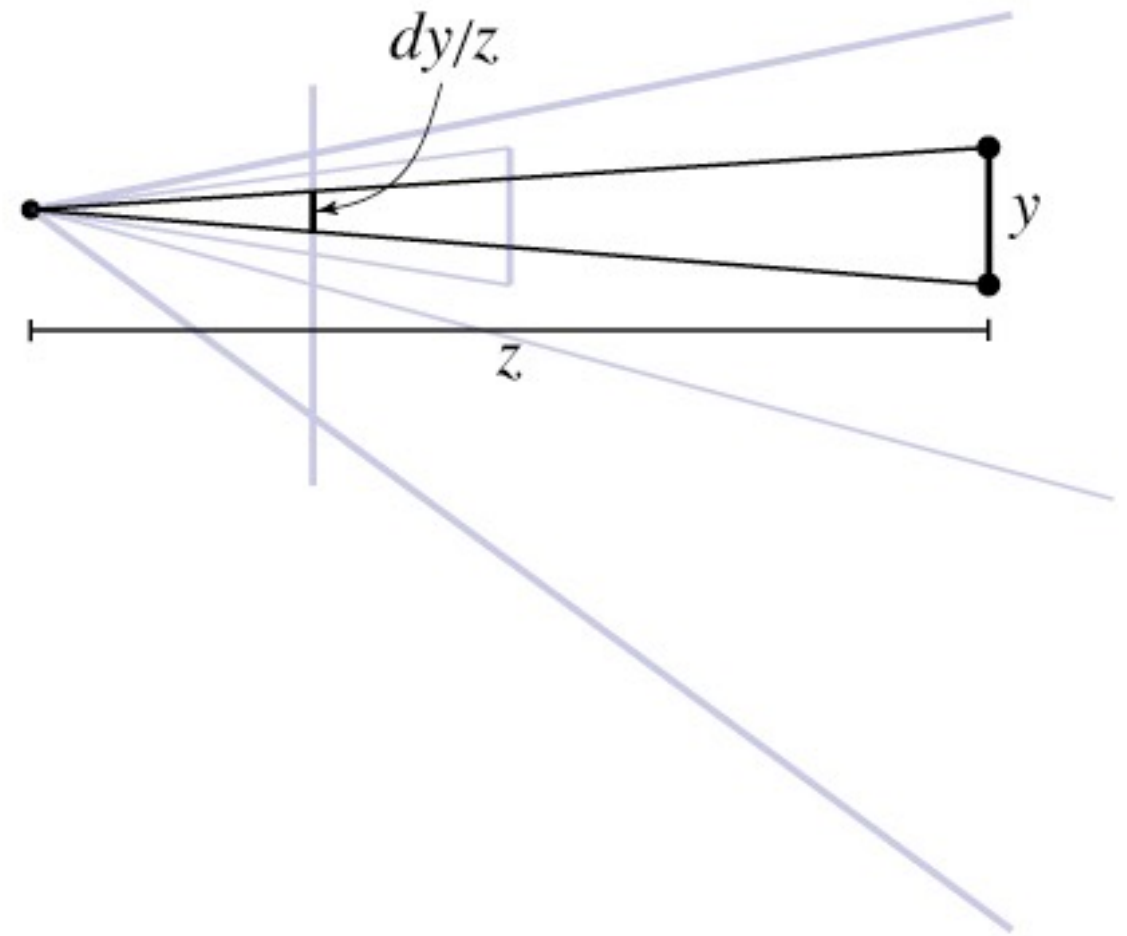
dy/z y

z

Shifted perspective projection

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  - additional parameter: projection plane normal
  - exactly equivalent to cropping out an off-center rectangle from a larger "normal" perspective
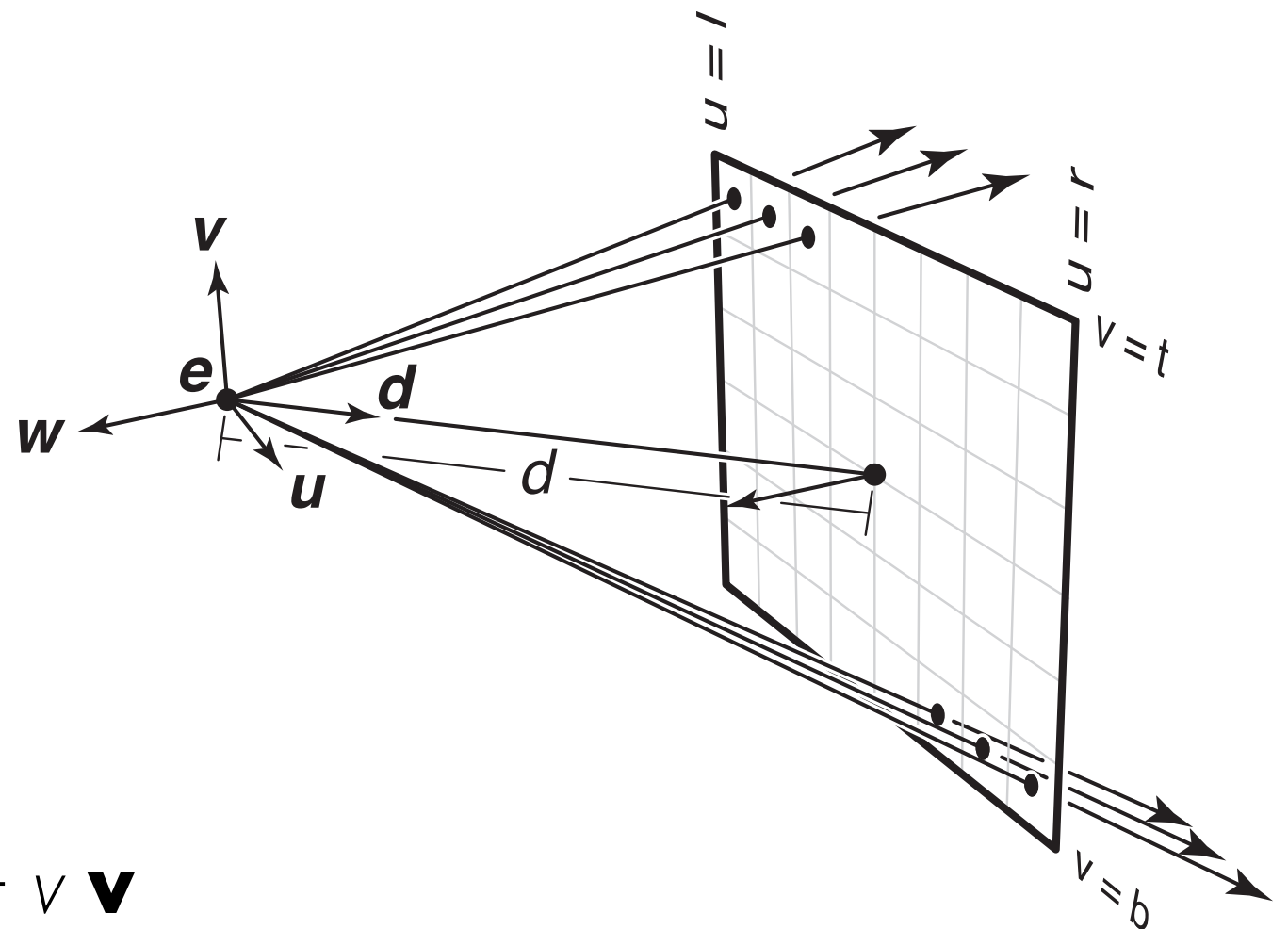 - corresponds to *view camera* in photography

Oblique perspective views

- **Positioning the view rectangle**
  - establish three vectors to be *camera basis:* **u**, **v**, **w**
  - view rectangle is the same, but shifted so that the center is in the direction **d** from **e**
- **Generating rays**
  - for (u, v) in $[l, r] \times [b, t]$
  - ray.origin = **e**
  - ray.direction = $d\, \mathbf{d} + u\, \mathbf{u} + v\, \mathbf{v}$

Why shifted perspective?

- **Control convergence of parallel lines**
- **Standard example: architecture**
  - buildings are taller than you, so you look up
  - top of building is farther away, so it looks smaller
- **Solution: make projection plane parallel to facade**
  - top of building is the same distance *from the projection plane*
- **Same perspective effects can be achieved using post-processing**
  - (though not the focus effects)
  - choice of *which* rays vs. arrangement of rays in image

camera tilted up: converging vertical lines

lens shifted up: parallel vertical lines