

# 09 Real time area lights

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# Area lighting

**when lights are not point-like, we have an integration problem**

$$L_r(\mathbf{v}) = \int_{\Omega} f_r(\mathbf{v}, \mathbf{l}) L_i(\mathbf{l}) (\mathbf{l} \cdot \mathbf{n}) d\mathbf{l}$$

- for real time we need fast analytic solutions to (approximations of) this definite integral
- the form of the solution depends on the form of  $L_i$  and  $f_r$ .

## diffuse

- polygonal lights: analytical solution due to Lambert (1760)
- environment lights: irradiance environment maps

This is all with no shadows! That is a (large-ish) topic for later in the course.

## specular

- polygonal lights: linearly transformed cosines
- environment lights: prefiltered environment maps

# Diffuse, polygonal lights

**In this case the illumination integral simplifies to**

$$L_r = \frac{R}{\pi} \int_{\Omega(S)} (\mathbf{n} \cdot \mathbf{l}) d\mathbf{l}$$

**Lambert worked out a solution to this in 1760**

$$L_r = \frac{R}{2\pi} \sum_{i=1}^n \cos^{-1}(\mathbf{l}_i \cdot \mathbf{l}_{i+1}) \frac{\mathbf{l}_i \times \mathbf{l}_{i+1}}{\|\mathbf{l}_i \times \mathbf{l}_{i+1}\|}$$

- where the index is understood to wrap around for the last term
- (derivation on board; refer to Eric Heitz's notes linked on the schedule)

# Specular shading, polygonal lights

**Now the integral (for unit source radiance) reads**

$$L_r(\mathbf{v}) = \int_{\Omega(S)} f_r(\mathbf{v}, \mathbf{l}) (\mathbf{n} \cdot \mathbf{l}) d\mathbf{l}$$

**and analytic solutions are only available for special forms of the BRDF**

- Phong BRDF:  $f_r(\mathbf{v}, \mathbf{l}) \propto (\mathbf{r} \cdot \mathbf{l})^n$   
solution due to Arvo via a generalization of Lambert's formula
- Other BRDFs: approximate the function  $f_r(\mathbf{v}, \mathbf{l})$  by some convenient function  $g(\mathbf{l})$
- Clever idea due to Heitz: linearly transformed cosine function:

$$g(\mathbf{l}) \propto (\mathbf{M}^{-1}\mathbf{l}) \cdot \mathbf{n}$$

# Linearly transformed cosines

**See Heitz's slides, video, and WebGL demo**

- <https://eheitzresearch.wordpress.com/415-2/>

# Real-time environment illumination

**Now incident radiance field is stored in a texture, not defined by a polygon**

- this makes it easy to do mirror reflections: single cubemap lookup

**Two special kinds of BRDFs are convenient**

- diffuse BRDFs: irradiance depends only on surface normal (not  $\mathbf{v}$ )  
...leads to **irradiance environment maps**
- rotationally symmetric lobes (e.g. Phong): it's a convolution of the environment map  
...leads to **prefiltered environment maps**

# Standard reflection map

**For mirror-specular surface, the illumination integral reduces to**

$$L_r(\mathbf{v}) = L_i(\mathbf{r}(\mathbf{v}, \mathbf{n}))$$

**This is a function only of  $\mathbf{r}$**

- so store it in a cubemap and look up using  $\mathbf{r}(\mathbf{v}, \mathbf{n})$ .

# Irradiance environment map

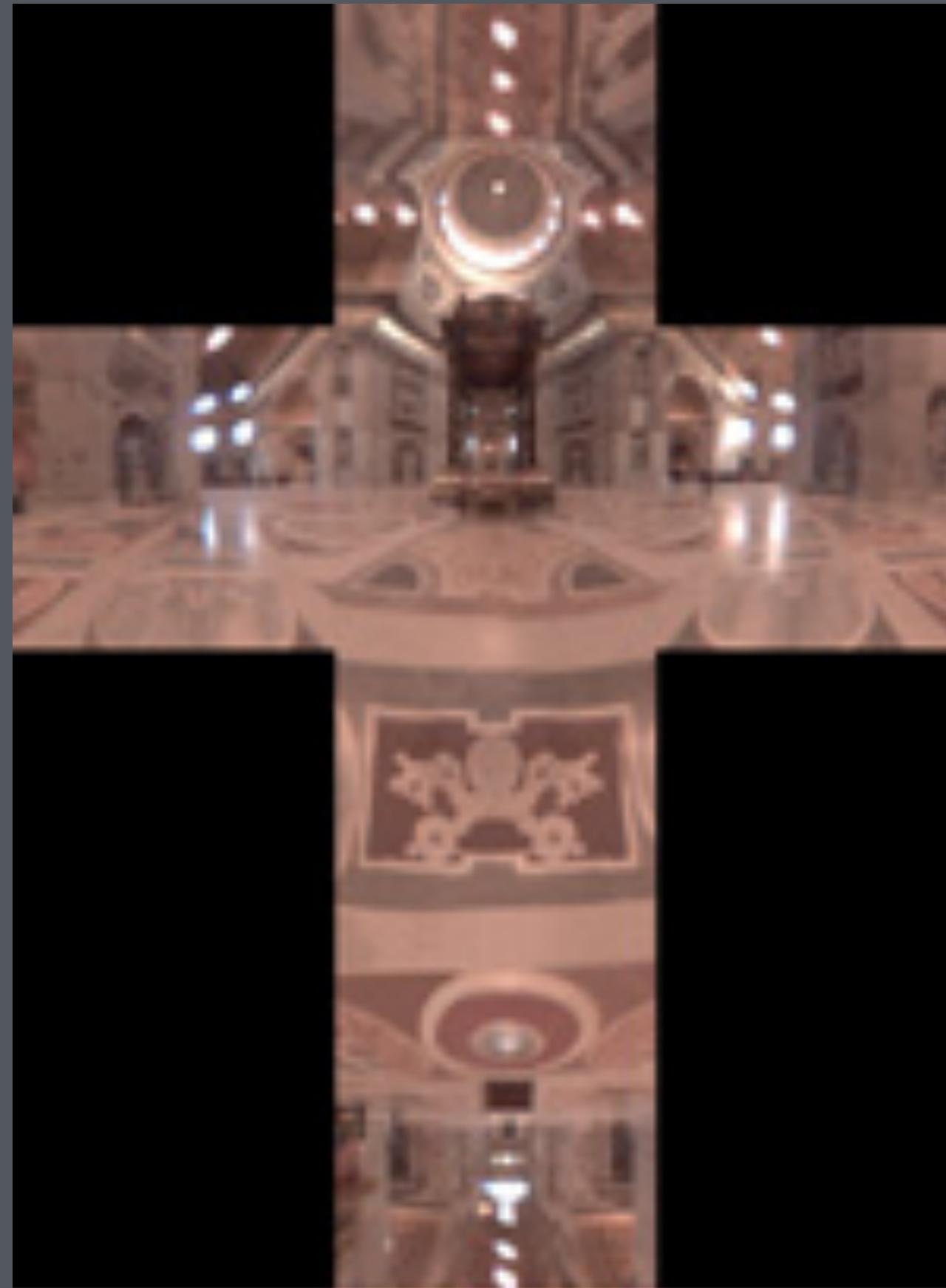
**For diffuse surface, illumination integral reduces to**

$$L_r = \frac{R}{\pi} \int_{\Omega} L_i(\mathbf{l}) (\mathbf{n} \cdot \mathbf{l}) d\mathbf{l}$$

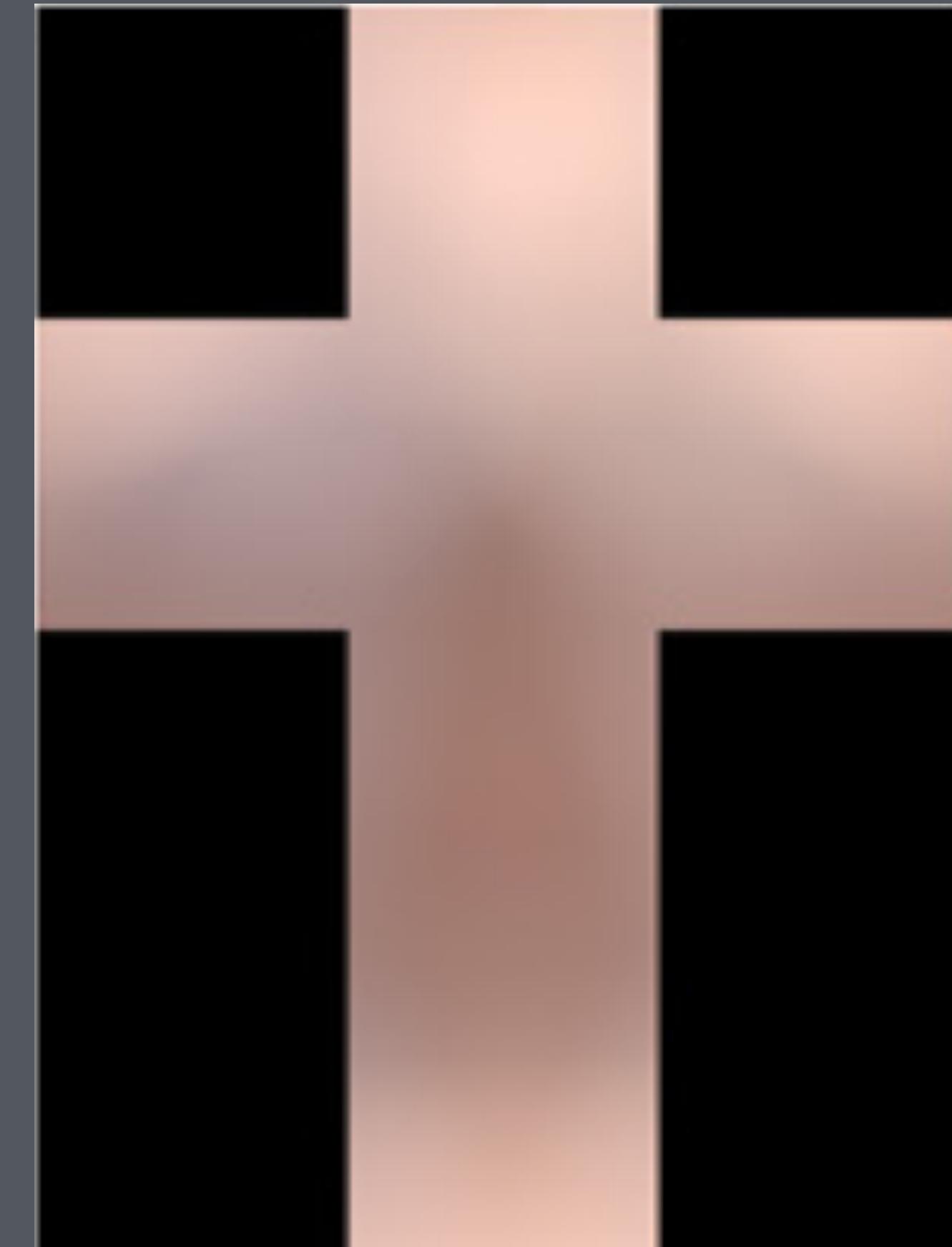
**This is a function only of n**

- so store it in a cubemap and look up using  $\mathbf{n}$ .

# Irradiance environment map

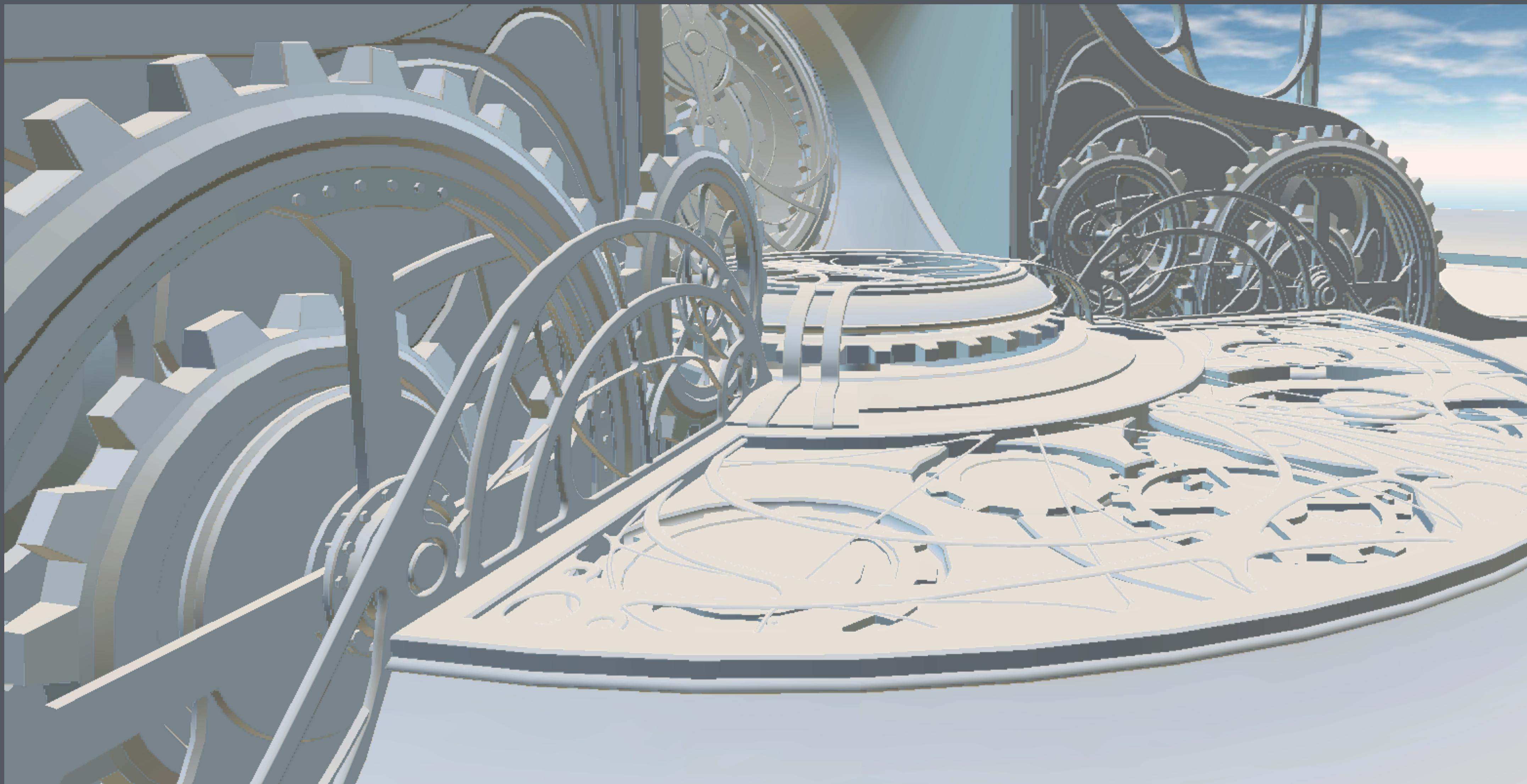


environment map



irradiance map

# Irradiance map illumination



# Irradiance map illumination



McGuire et al. HPG '11 [10.1145/2018323.2018327](https://doi.org/10.1145/2018323.2018327)

# Prefiltered environment map

**For a general specular surface, approximate the BRDF by a function**

$$f_r(\mathbf{v}, \mathbf{l}) (\mathbf{n} \cdot \mathbf{l}) \approx g_\sigma(\mathbf{l} \cdot \mathbf{r})$$

- where  $g$  defines a lobe of width  $\sigma$  that is symmetric around  $\mathbf{r}$

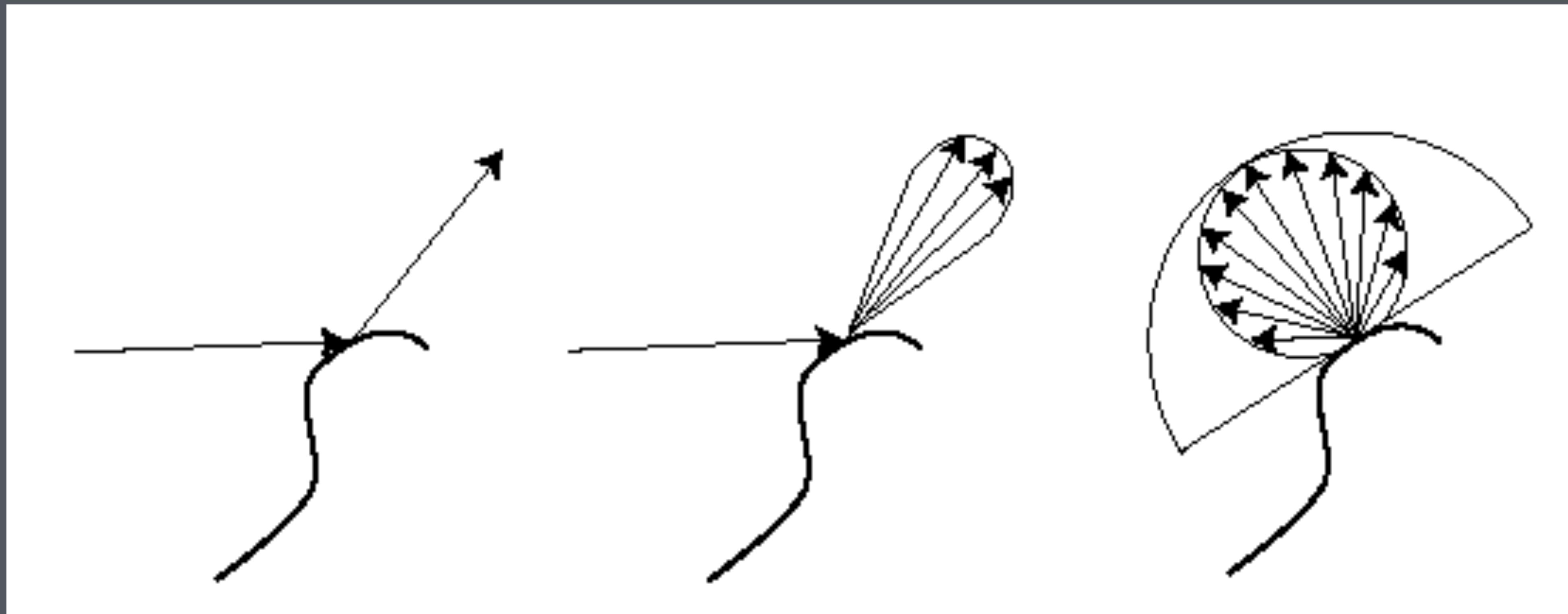
**The illumination integral is**

$$L_r(\mathbf{v}) \approx \int_{\Omega} g_\sigma(\mathbf{l} \cdot \mathbf{r}) L_i(\mathbf{l}) d\mathbf{l}$$

**This depends only on  $\mathbf{r}$  and the scalar  $\sigma$**

- so store it in an array of cubemaps indexed by  $\sigma$ , and look up using  $\mathbf{r}$

# Irradiance environment mapping

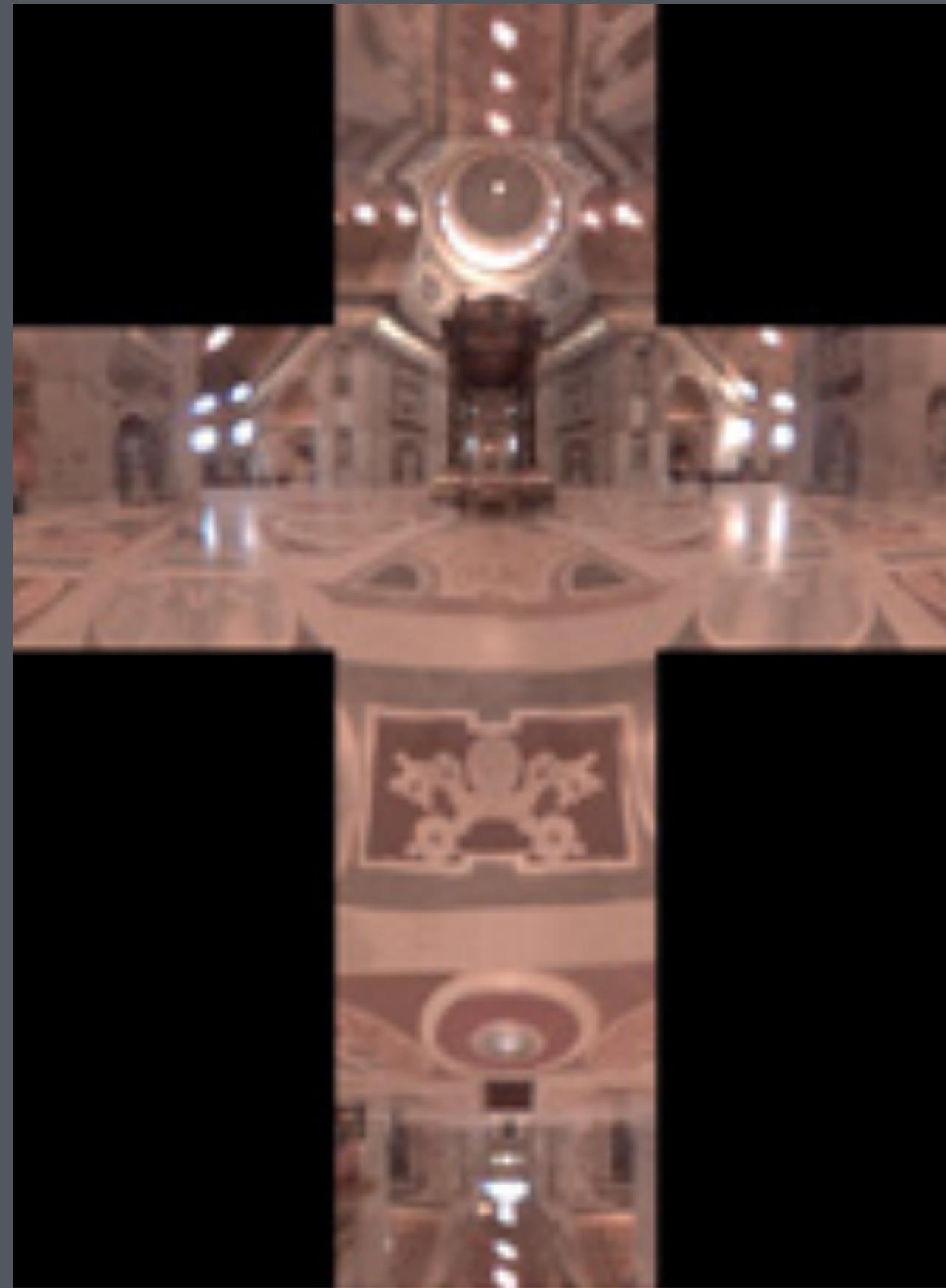


environment map  
for specular surface

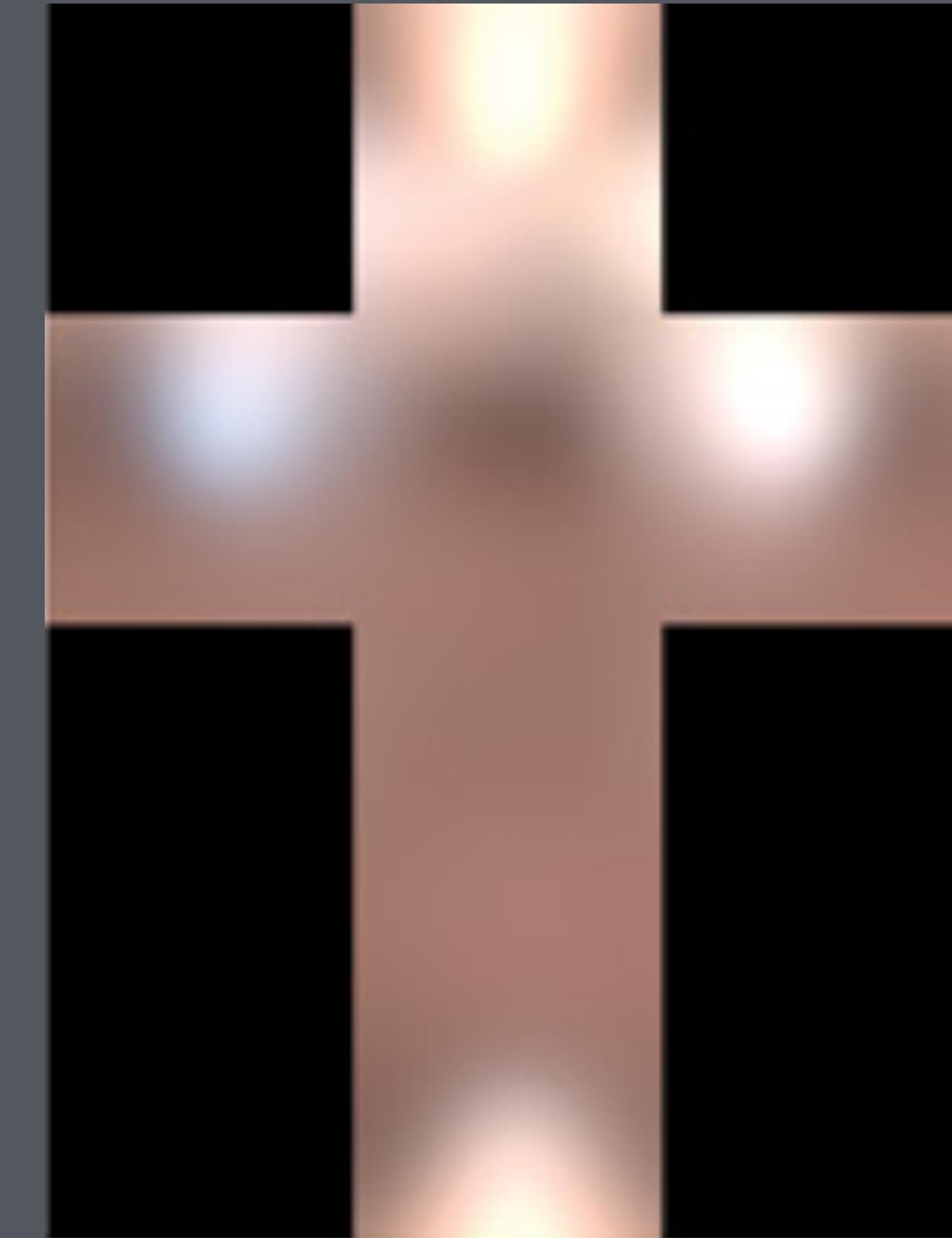
prefiltered map  
for glossy surface

prefiltered map  
for diffuse surface

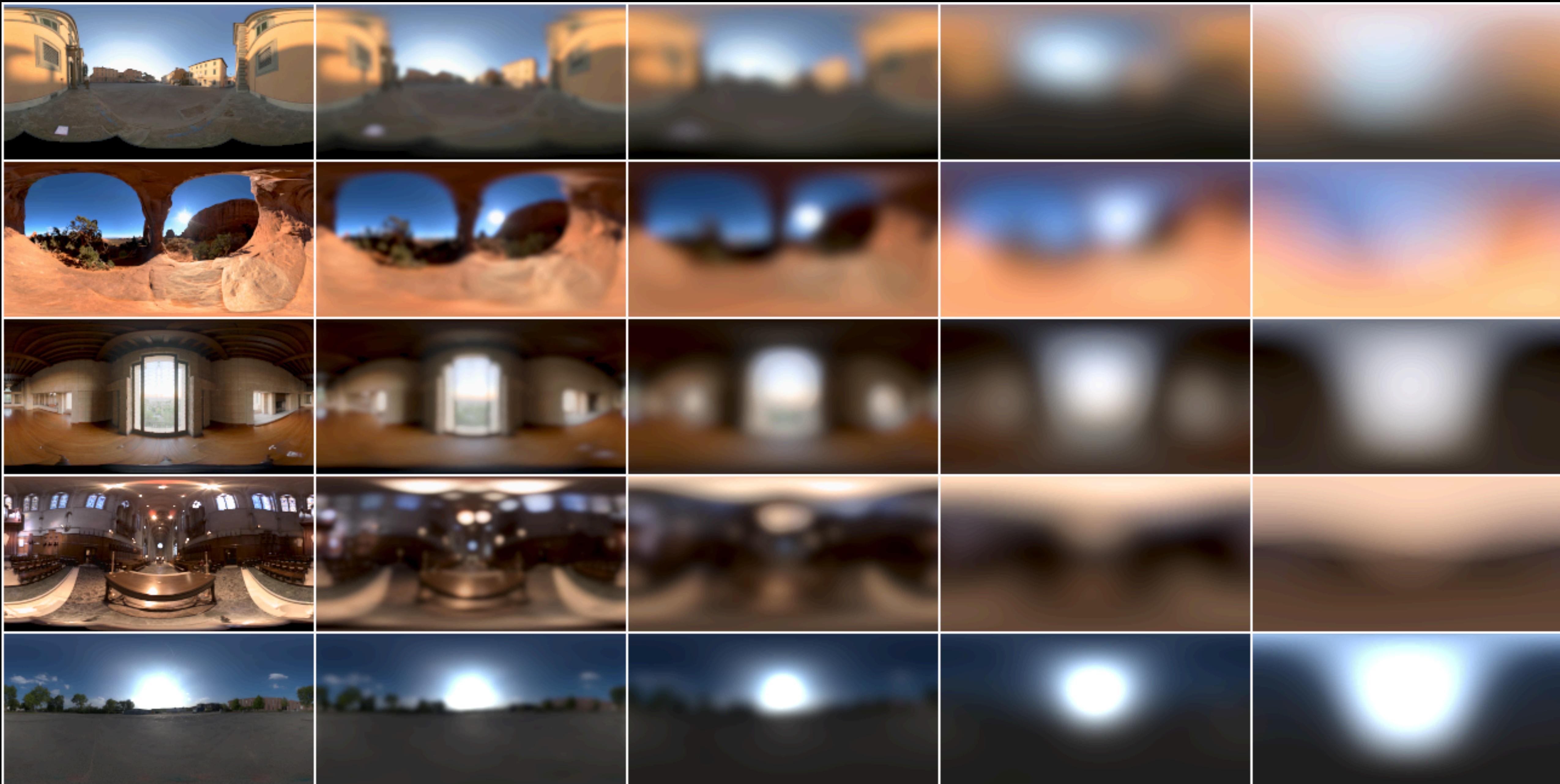
# Prefiltered environment map



environment map



prefiltered for Phong



# Prefiltered map variants

## **Many approaches to approximating the BRDF with symmetric lobes**

- classic is to use a single lobe, loses stretching at grazing angles
- many techniques use multiple samples, similar to anisotropic texture filtering
- modern thought process is to treat the whole precomputation as approximating more accurate BRDF lobes using multiple samples from arbitrary maps stored in mipmaps

Reference Our method



Manson & Sloan 2016