14 Real-time Particle Physics

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Overview

Particles and Springs

- Matrix notation and the Mass Matrix
- Equations of motion
- Forces as derivatives of energy and the Stiffness matrix

Time Integration Algorithms

- Forward, Backward, and Symplectic Euler

Constraints and Solvers

- Iterative Methods
- Manifold Projection

Examples of Particle Systems

Particle Dreams [Karl Sims, 1988]

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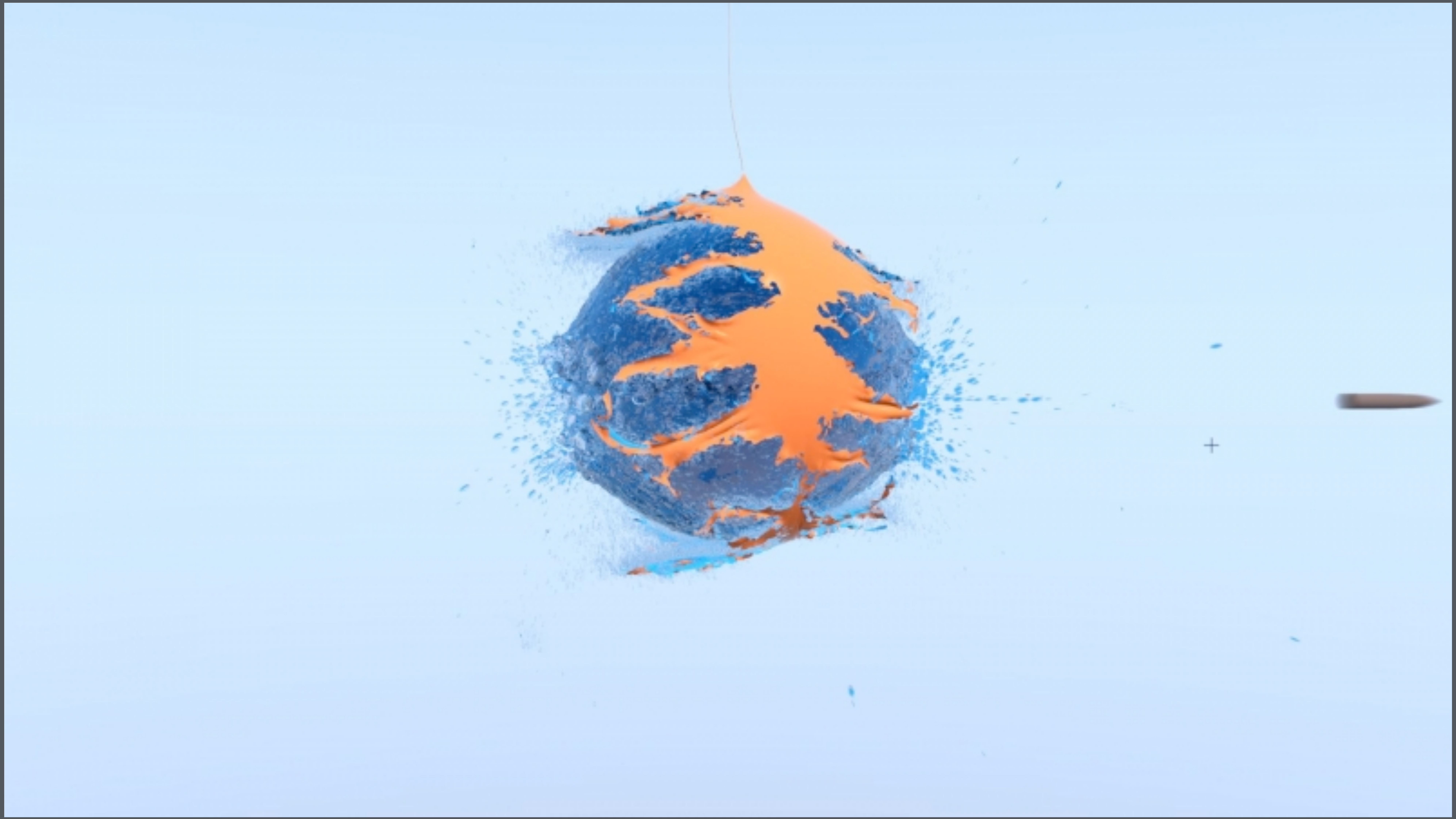
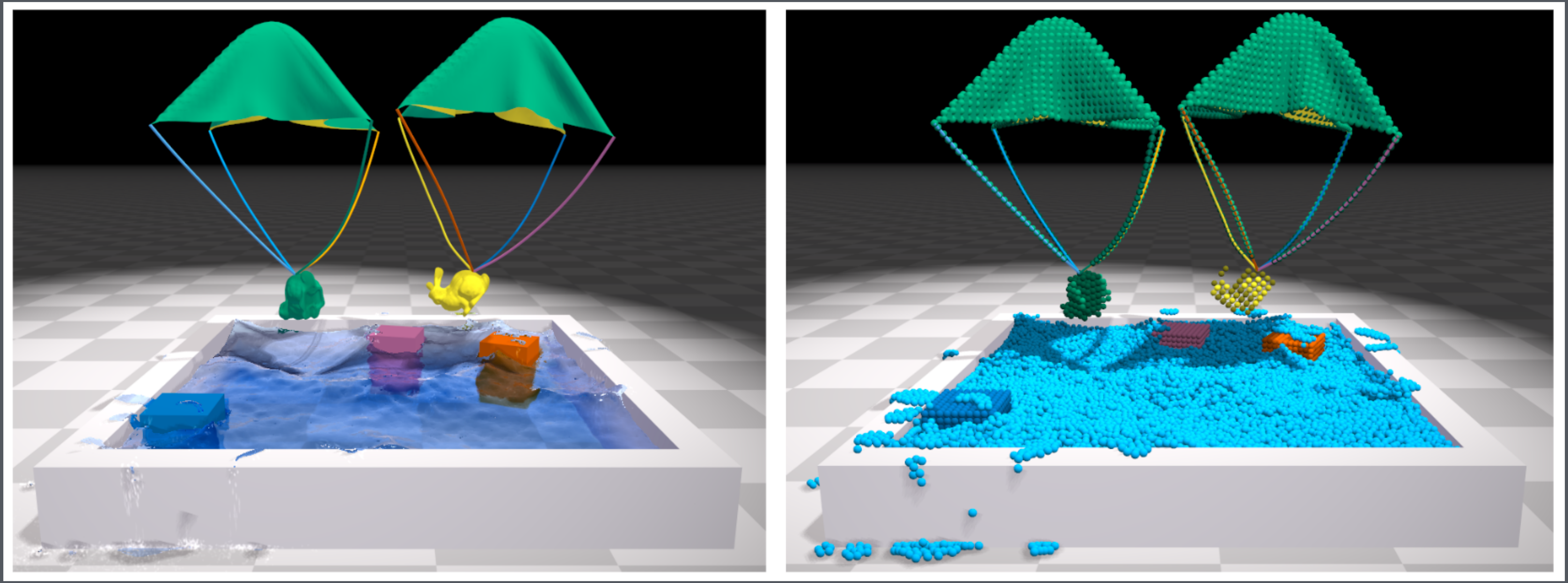
Examples of Particle Systems



Balloon Burst [Macklin et. al, SIGGRAPH 2015]

Examples of Particle Systems



Unified Particle Physics for Real-Time Applications [Macklin et. al, SIGGRAPH 2014]

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Particle System Review

Each particle has a mass

Each particle’s movement is determined by a sum of forces

- Forces depend on particles’ positions and velocities, and maybe time

F = ma determines how the particle moves

- Forces determine acceleration at any given time given the position and velocity
- Differential equation determines the entire motion given initial position and velocity

Basic Algorithm

1) **Clear forces from previous calculations**

2) **Calculate/accumulate forces for each particle**

3) **Solve for particle's state (position, velocity) for the next time step**

Unary Forces

Constant

- Gravity

Position/Time-Dependent

- Force fields, e.g. wind

Velocity-Dependent

- Drag

Matrix Notation

If we have multiple particles, it is nice to group variables together

- Example: 1D System
- Example: 3D System

Mass matrix

- An n x n matrix that represents the mass distribution of n particles
- For simple systems, this is block-diagonal, where the ith block represents the mass of the ith particle
- Each block is a scaled identity matrix $m\mathbf{I}$ where m is the particle's mass and the size of $\mathbf{I}$ is the number of dimensions of the domain

Integration Algorithm 1

Calculating Particle State from Forces: First attempt

- Use forces to update velocity
- Use old velocity to update position

Issues

- Unstable in certain cases!
- Reducing time step can help, but this becomes computationally expensive

This technique is called Forward (Explicit) Euler Integration

Binary, *n*-ary Forces

Much more interesting behaviors to be had from particles that interact

Simplest: binary forces, e.g. springs

$$\mathbf{f}_i(\mathbf{x}_i, \mathbf{x}_j) = -k_s(|\mathbf{x}_i - \mathbf{x}_j| - r_0)\frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|}$$

Nice example project with mass-spring systems:

- https://vimeo.com/73188339

More sophisticated models for deformable things use forces relating 3 or more particles

Forces as Derivatives of Energy

If energies depend on particle position, forces can be determined by taking the derivative with respect to each particle's position

E.g. Hooke's Law

- $E = 0.5*k|\mathbf{x} - \mathbf{x}_0|^2$
- $F = -\nabla E = k(\mathbf{x}_0 - \mathbf{x})$

See Kass course notes and Baraff & Witkin 98 for detailed explanation

Stiffness Matrix

Relates particle displacement to spring-like forces between particles

- For a system with *n* particles, this is an *n* x *n* matrix.
- For a 1D spring connecting to a single particle, this is just a single scalar equal to the spring's stiffness

If forces depend linearly on positions (e.g. Hookean forces), then Stiffness matrix is constant up to rotation of the system

Integration Algorithms

Another attempt

- Update velocity with forces at next time step determined by solving a (non-)linear system
- Use new velocity to update position

Benefits

- Unconditionally stable if the system is linear!

Issues

- Solving a system at each step can become expensive
- Can introduce artificial viscous damping

This technique is called Backward (Implicit) Euler Integration

Integration Algorithms

Next attempt: A compromise

- Update velocity using current forces
- Use this updated velocity to update the position

Benefits

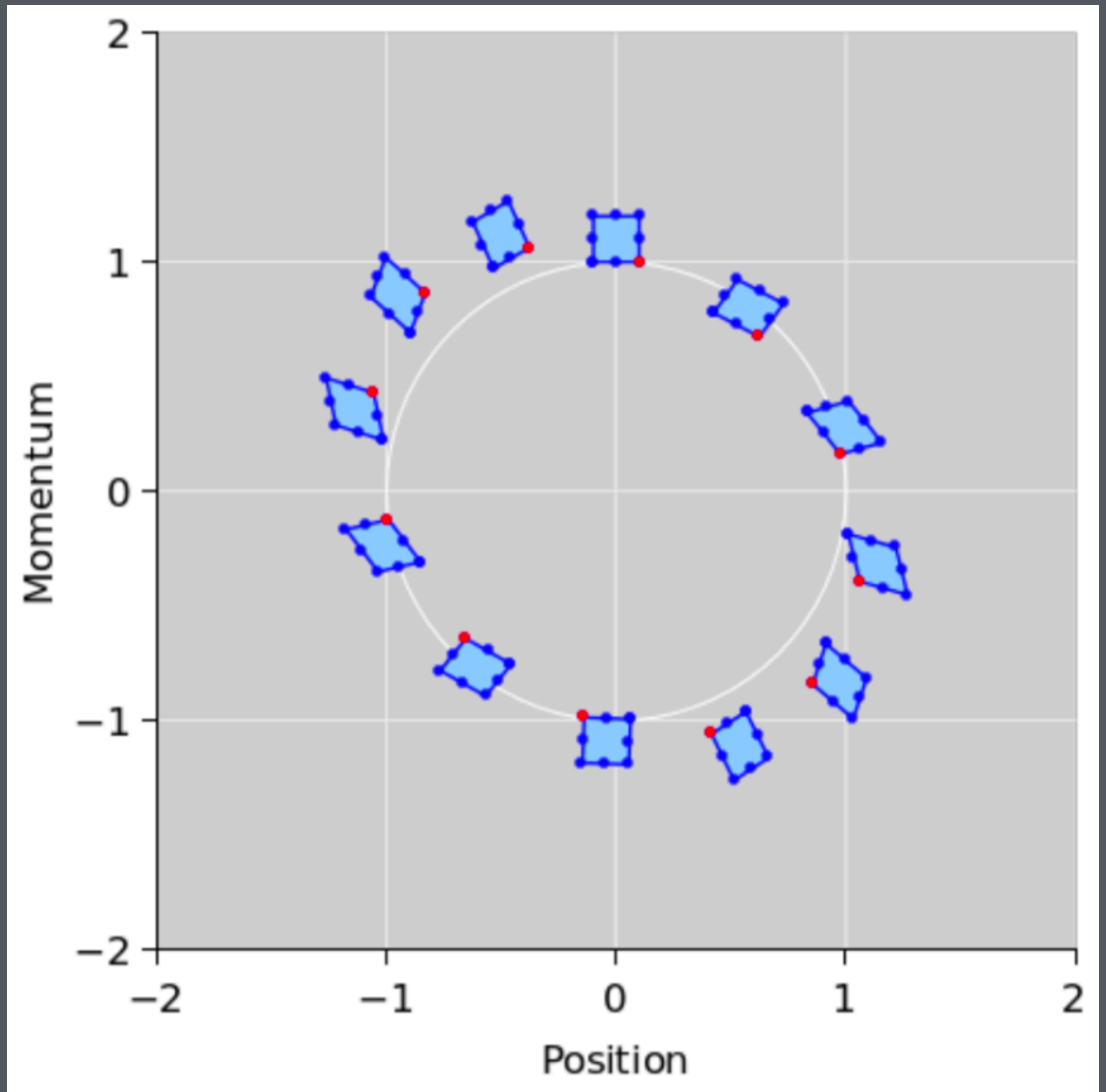
- All the speed benefits of Forward Euler, but much more stable!
- You should basically always choose this algorithm over Forward Euler

Issues

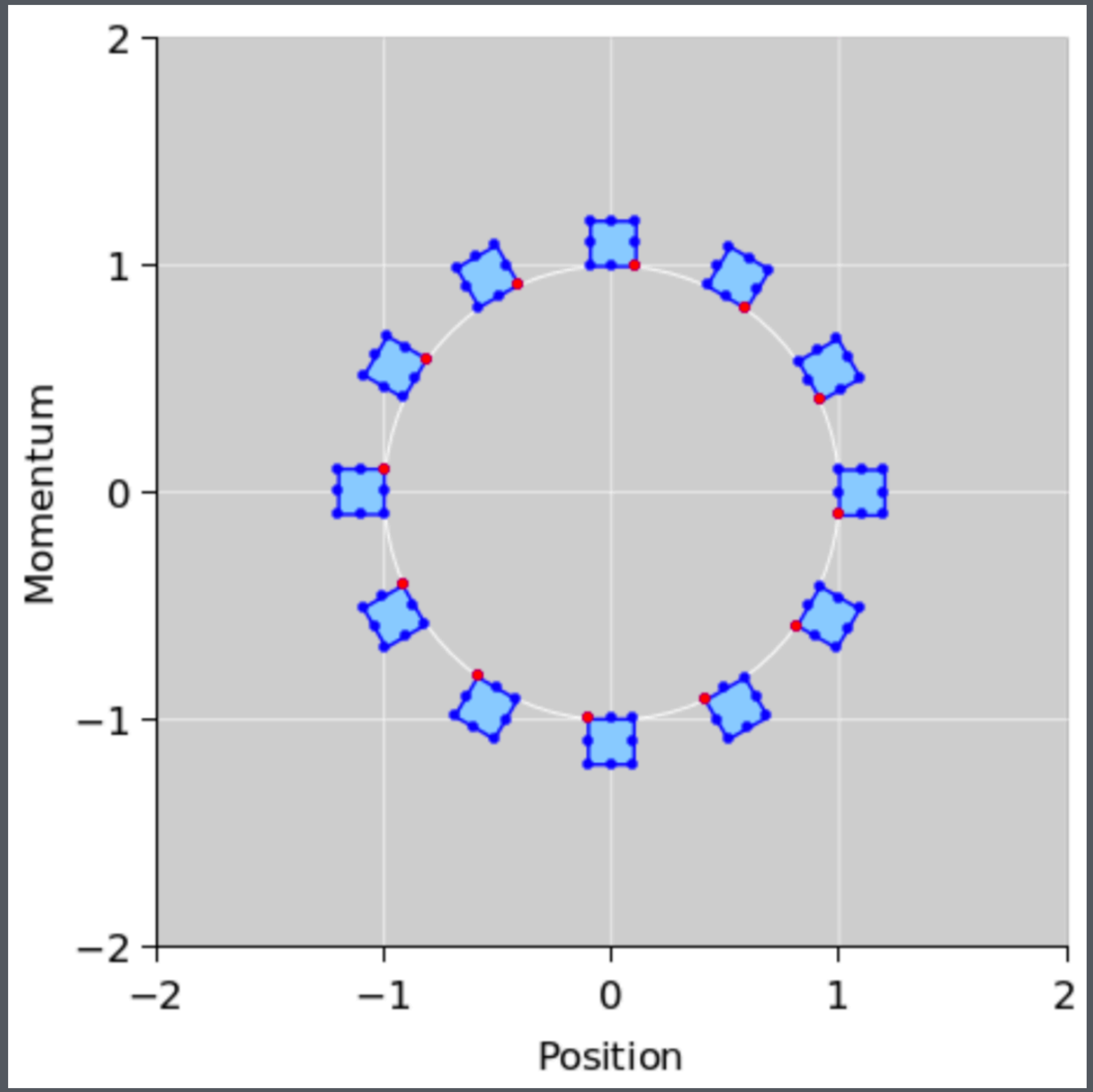
- Still not unconditionally stable, though

This technique is called Symplectic (Semi-implicit) Euler Integration

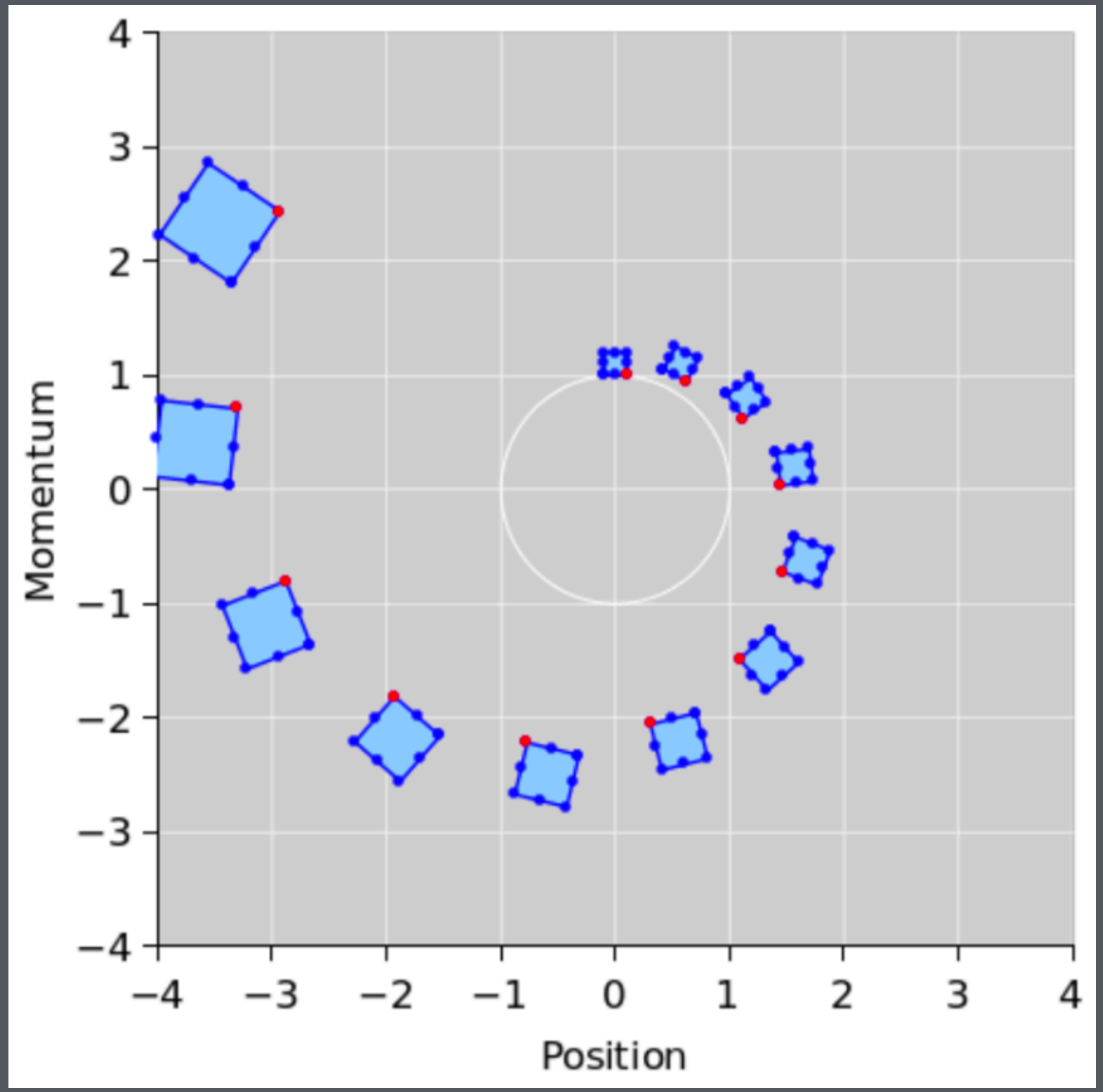
Euler variants viewed in phase space



Symplectic Euler



Exact solution



Forward Euler

Stelian Coros, CMU

Other Integration techniques

Midpoint

Newmark-β

Verlet

RK-4

Many more (complicated) schemes

- RK family
- Exponential Integrators

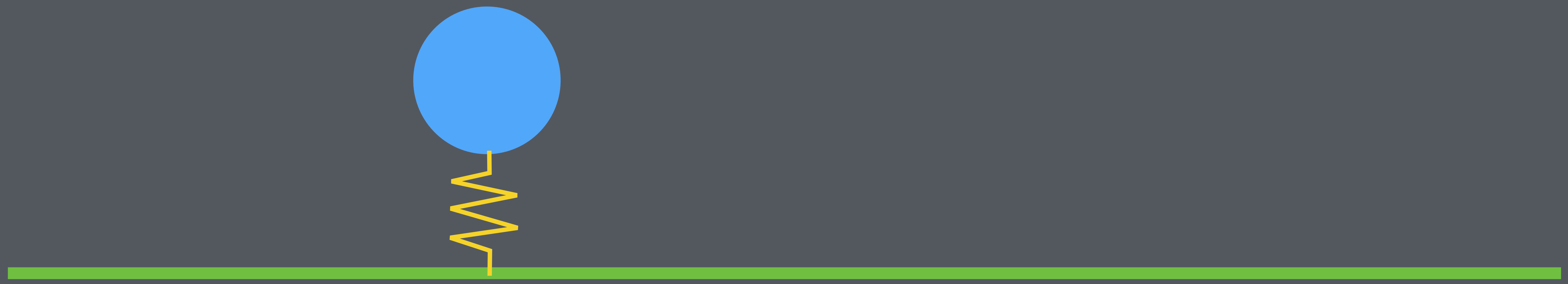
Computational Stiffness

E.g. Bead on Wire

- Can use a spring force to bind bead (particle) to a wire
- If the spring is weak, the particle may drift too far away
- If the spring is strong, we need very small time steps to ensure stability

Known as a “stiff” problem

- One stiff spring makes the whole system stiff!



Constraints

- **At the end of each step (i.e. after integration), enforce certain properties of the system**
    - e.g., the bead should not leave the wire
- **Idea: push unconstrained system towards acceptable configuration by modifying particle momentum as little as possible**

Constraint Equations

- **Usually of the form $C(x) = 0$ or $C(x) \geq 0$**
- **When finding a solution, we are usually interested in the derivatives of these equations with respect to position (x)**
- **These are similar to forces, but are non-physical**

Constraint Jacobian Matrix

Collection of derivatives of constraints into a single matrix

Not necessarily square: relates *n* particle positions to *m* constraints

Similar to a stiffness matrix

Enforcing Constraints

First attempt: Apply constraint equation derivatives iteratively

Benefits

- Fast, parallelizable over particles

Issues

- Constraint application order matters!
- Convergence not guaranteed!
- Successive Over-Relaxation can help (i.e., apply a scaled version of the constraint derivative)
    - But this is finicky, finding the right scaling value can be difficult (or it might not exist)

Enforcing Constraints

Another attempt: Lagrange Multipliers

- Solve a global linear system over all constraints
- Add an extra row/column for each 1D constraint

Benefits

- Order of constraints doesn't matter
- Solves simultaneous constraints exactly in one pass

Issues

- Non-parallelizable, global linear solve (but this can be done quickly using, e.g., conjugate gradient)
- Doesn't work for nonlinear constraints, which are fairly common in practice

Enforcing Constraints

Another attempt: Fast Manifold Projection

- Solve a linear equation over constraints to project particles to “nearest” valid position
- Iterate until convergence

Benefits

- Typically very few iterations needed; system size depends on number of constraints, not number of particles

Issues

- Again, requires a global, non-parallelizable system solve

The New Algorithm

1) Clear forces from previous calculations

2) Calculate/accumulate forces for each particle

3) Use time integration algorithm of choice to update particle to unconstrained position

4) Enforce constraints with algorithm of choice