International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: http://www.tandfonline.com/loi/tprs20

Effective bullwhip metrics for multi-echelon distribution systems under order batching policies with cyclic demand

Joaquim Jorge Vicente, Susana Relvas & Ana Paula Barbosa-Póvoa

To cite this article: Joaquim Jorge Vicente, Susana Relvas & Ana Paula Barbosa-Póvoa (2017): Effective bullwhip metrics for multi-echelon distribution systems under order batching policies with cyclic demand, International Journal of Production Research, DOI: 10.1080/00207543.2017.1367105

To link to this article: http://dx.doi.org/10.1080/00207543.2017.1367105

Published online: 29 Aug 2017.

International Journal of Production Research, 2017
https://doi.org/10.1080/00207543.2017.1367105

Effective bullwhip metrics for multi-echelon distribution systems under order batching policies with cyclic demand

Joaquim Jorge Vicente, Susana Relvas* and Ana Paula Barbosa-Póvoa

CEG-IST – Centre for Management Studies, Instituto Superior Técnico, Universidade de Lisboa, Lisboa, Portugal

(*Received 22 May 2015; accepted 7 August 2017*)

A large number of problems in a distribution supply chain require that decisions are made in the presence of the bullwhip effect phenomenon. The impact of the order batching policies on the bullwhip effect is analysed in this paper, when cycle demand on a multi-echelon supply chain operating is considered. While investigating which bullwhip effect metrics are more adequate to measure the bullwhip effect in these type of systems, the optimal reordering plan that minimises the operation costs of the overall system is calculated. A Mixed Integer Linear Programming (MILP) model is developed that takes into account an inventory and distribution system formed by multiple warehouses and retailers with lateral transshipments. The bullwhip effect is measured through four metrics: the echelon average inventory; the echelon inventory variance ratio; the echelon average order; and the echelon order rate variance ratio. As conclusion the inventory metrics suggest that (i) using batching policy reduces instability; (ii) batching may reduce in general order variance if using larger batches and (iii) cycle demand length has no major impact in the bullwhip effect. A motivational example and a real word case study are used and tested.

Keywords: supply chain management; order batching; bullwhip effect; inventory planning; wrap-around; mixed integer linear programming

1. Introduction

Products in distribution chains that are ordered and moved between echelons in batches are often characterised by using order batching policies. These policies appear in the literature as one of the main causes of the bullwhip effect as stated by Lee, Padmanabhan, and Whang (1997) and by Riddalls and Bennett (2001). Examples are the cases of retailers that order from wholesalers full truck loads or full containers that may represent a quantity discount while optimising transportation costs. Also for manufacturers, significant economies of scale can also be achieved by producing in large lot sizes. These may result in high inventories that increase holding costs or backorders, the latter due to the lack of stock (Hussain and Drake 2011). Additionally, long process set-up times are a major cause of large production batches with the corollary being that rapid changeovers are required to reduce batch sizes. These large batch sizes may cause a substantial oscillations in inventories since often a large batch is first produced, far in excess of the current demand, causing a rise in inventories that would eventually decrease, but at that point a large batch would enter the inventory pipeline again. However, finding an optimal batching decision is not easy since it directly relates to inventory holding and backlog costs.

Across supply chain management literature, the simulation-based method has been the most commonly used when analysing bullwhip effect (Cannella et al. 2013). In such studies, one of the assumptions most commonly used is the adoption of single-product conditions in linear supply chains structures. The main reason for this is that the methodological approach used for studying the bullwhip effect problem, i.e. continuous time and discrete time differential equation models are difficult to adapt to multiproduct and divergent systems. Therefore, choosing analytical methods as mixed integer linear programming models, where different complex distribution chains with multiple products can easily be modelled, can lead to an improvement in bullwhip effect research.

Distribution chains' demand can be characterised by a cyclic mode in certain cases, where orders tend to repeat across cycles (e.g. Monday of every week). The case of food retailers and some process supply chains are examples of cyclic operations (Moniz, Barbosa-Póvoa, and de Sousa 2014). Therefore, the study of such operation is also an interesting point to explore. Concluding, bullwhip effect studies in distribution supply chain considering order batching and a cycle demand appears as an important problem to be addressed.

*Corresponding author. Email: susana.relvas@tecnico.ulisboa.pt

In this context the aim of this paper is to contribute to the research in which adequate bullwhip effect metrics are chosen to measure the bullwhip effect in complex (divergent) multiproduct distribution supply chains operating in cyclic model. This work complements previous ones presented in conferences by the same authors (Vicente, Relvas, and Barbosa-Póvoa 2015a, 2015b). The additional scientific value of this paper comparing with previous ones is to (i) identify the network structures that combined with batching policies mitigate bullwhip effect throughout the distribution supply chain; (ii) study how the total costs are affected by the batching policy combined with supply chain structures; (iii) discussion and analysis of the bullwhip effect metrics performed, namely their strengths, weakness and suitability with related literature; (iv) and a complementary research, with three main family types of products competing for the capacity available. An optimisation approach is selected to be developed. This paper then explores this opportunity by introducing batching into a distribution supply chain mathematical programming model while analysing the bullwhip effect in terms of inventory instability and order amplification when a cyclic demand is considered. The impact of different order batch sizes and the impact of different cyclic demands on the bullwhip effect are studied. An inventory planning problem is considered running under several typical real world conditions (batch, transshipment, lead-time, multi-echelon, multi-warehouse, multi-retailer, multi-product, limited transportation, limited storage) so as to find the optimal solution that minimises total operation costs. Quantitative-phenomena are assessed, such as inventory instability and order amplification, by adopting specific performance measures that are tested in order to inform decision-makers on which best measures to use in such type of systems. An exact optimisation method is explored as it allows the modelling of complex systems and permits the optimisation of the distribution supply chain operation, aspects not yet enough explored in the bullwhip literature. Most contributions rely in simulation methods applied to single product and linear supply chains (Wang and Disney 2016). However, such structures do not really describe real world operations. Simulation methods while allowing the understanding of these systems' dynamics (Giard and Sali 2013) do not optimise their operation. Therefore, the use of analytic methods appears as promising and should be further explored as they have been widely used in supply chain literature under different contexts (Taticchi et al. 2015).

The structure of the rest of the paper is as follows. A literature review on metrics of the bullwhip effect and on batching is included in Section 2. In Section 3 a definition of the problem is presented. Section 4 presents the mathematical model and in Section 5 the bullwhip metrics are showed. In Section 6 experimental studies outcomes are shown. Section 7 provides the conclusions and future directions.

2. Literature review

In this literature review two main topics will be considered. Firstly, a review on the papers addressing the bullwhip effect related with batching will be performed. Secondly, in Section 2.2 relevant works characterising bullwhip effect metrics will be analysed.

2.1 *Order batching and bullwhip effect*

The relationship between order batching and bullwhip effect have been studied considering a range of ordering policies, including lot sizing (Pujawan 2004) or (R, S) systems (Potter and Disney 2006). The effect of batches, in the instability of inventory through multi echelon distribution supply chain, was recognised as a not yet solved issue and thus more studies should be performed when using batching (Hussain and Drake 2011). Additionally, many researchers emphasised the need to reduce the batch size as much as possible, but technical or economic problems may not allow the implementation of such policy.

The influence of order batching in a two-level distribution supply chain with various retailers and only one supplier was studied by Cachon (1999). This research proposes that reducing the bullwhip effect at the level of the suppliers may be possible by managing the retailer orders between longer order interval times and smaller batch sizes. Riddalls and Bennett (2001) studied the impact of batch production costs on the bullwhip effect. They proposed measuring the magnitude of the bullwhip effect in a two-tier distribution supply chain by observing the peak order rate of the upper level (the supplier). The optimisation model incorporates both a fixed batch production cost and a variable unit cost. The aim is to minimise the total cost. They found that the relationship between batch size and demand amplification is non-linear and depends on the remainder of the quotient of average demand and batch size.

Later on, Chen and Samroengraja (2004) researched a two-level production/distribution supply chain. The main goal of the model is to minimise the average total costs for the long-term. They concluded that reducing the bullwhip effect does not always increase the distribution's supply chain efficiency. Potter and Disney (2006) considered scenarios where orders, for stochastic and deterministic demand, are placed as a multiple of a fixed batch size. They concluded that through batching policies both minimisation of bullwhip, with a careful selection of the batch size, and savings could be accomplished. Potter et al. (2009) investigated the trade-off between bullwhip and inventory in a multi-product environment where some batching is inevitable. Conventional thinking suggests bullwhip should be minimised within the supply chain. While this may be straightforward when no constraints exist, in reality companies have to manage shared resources and are subject to capacity limits that affect their ability to do minimise bullwhip. Three case studies are presented where distinct clusters of product types exist, each with different observed levels of bullwhip.

Recently, Mikati (2010) studied the relationship between batching and lead times. This dependency is examined for different operational conditions using system dynamics simulation. It was shown that there is an optimal batch size that results in a minimum lead time and that the optimum inventory level matches the desired inventory.

Hussain and Drake (2011) studied the impact of lot sizing on bullwhip effect in a multi-echelon supply chain with information sharing. A systems dynamics approach was used. They found that large lot sizes when combined in integer multiples can result in an order rate that is close to the actual demand and leads to little demand amplification.

Giard and Sali (2013) reviewed and studied the effect of the bullwhip effect in supply chains. The authors stated that the existing research describes supply chains with only one product and they commonly consider it with a simple structure. As an answer to the complexity of real time systems, that are challenging to accomplish with analytical approaches, arose the simulation modelling. However, even within the simulation approaches, one of the most common simplifying assumptions in the literature is to assume that the supply chain adopts a sequential and linear structure (Cannella et al. 2013). Only few studies have considered complex (divergent) supply chain structures, namely multi-warehouse and multi-retailer systems, where transshipment and lead time are considered as stated by Wang and Disney (2016) in their recent literature survey.

Additionally, and from the literature analysed, the study of inventory problems under order batching policies with cyclic demand in divergent multi-echelon supply chain was not explored in literature. Cyclic demand, however, appears in research and in real world problems as it frequently describes both production (e.g. pharmaceutical production) and distribution systems (e.g. food retailer). A recent example was presented by Moniz, Barbosa-Póvoa, and de Sousa (2014) where simultaneous regular and non-regular production scheduling of multipurpose batch plants were studied in a real case.

2.2 *Metrics for measuring the bullwhip effect*

The bullwhip effect has been most commonly studied in regards of inventory instability and demand amplification that are described with the following quantitative metrics: inventory variance ratio (Disney and Towill 2002) and order rate variance ratio (Chen et al. 2000). The order rate variance ratio only informs whether the forces to amplify demand are stronger or weaker than the forces to attenuate demand (Cachon, Randall, and Schmidt 2007). Thus, it only represents one half of the bullwhip problems as the replenishment rule also influences the inventory dynamics (Disney and Towill 2002). The order rate variance ratio is defined as the ratio between the order variance at a generic node and the customer demand variance. According to Miragliotta (2006) modified versions of this metric were introduced by Warburton (2004), where the inventory variance ratio is defined as the ratio between the inventory variance at a generic node and the customer demand variance. Similarly, Coppini et al. (2010) state that more significant information on supply chain performance can be obtained from the bullwhip analysis (Coppini et al. 2010), as inventory variance ratio quantifies the fluctuations in inventory. An increased inventory variance results in higher holding and backlog costs, inflating the average inventory cost per period.

On the other hand, average inventory metric considers the echelon's inventory values over the time horizon. In distribution system analysis this metric is usually used to quantify the average holding cost over the time horizon and to evaluate concise information about inventory investments (Cannella and Ciancimino 2010). The inventory variance ratio is easily associated with the possible increase and the variation of the holding cost per unit. A complementary measure to this is the average inventory which measures, over the observation time, the average holding cost (Cannella et al. 2013).

Yungao et al. (2013) analysed the retailer's forecasting techniques based on the bullwhip effect on orders and inventory from the perspective of the entire supply chain. They used the sum of the two variance amplifications under different weightings as the performance measure to decide which forecasting technique the retailer should choose for different product characteristics. One year later, Dominguez, Framinan, and Cannella (2014) presented a comparative analysis of the bullwhip effect between a serial supply chain and a more complex divergent supply chain. They analyse the response of both supply chains under two different input demands: a stationary demand and an impulse demand.

Recently, Fu et al. (2015) developed a formulation for quantifying the bullwhip effect from control systems engineering approach. Each supply chain entity optimises for its own ordering policy so the variation of customer demand is able to be tracked. Excess inventory is minimised and the order variance amplification is effectively suppressed compared to traditional ordering policies.

The usage of bullwhip metrics is an important issue as it an essential performance measurement when planning supply chain distribution. This provides an informed analysis of the outcomes of the planning phase and as stated by Beamon (1998) it helps to quantify the efficiency and/or effectiveness of an existing system while allowing comparative analysis with alternative systems. The need to effectively develop and implement such measures so as to control bullwhip effect has been stressed through the seminal paper of Lee, Padmanabhan, and Whang (1997) and more recently in the bullwhip review made by Wang and Disney (2016). Some authors have proposed measurement systems regarding the measurement of bullwhip effect such as Sadeghi (2015) and Bandyopadhyay and Bhattacharya (2013). Other examples can be found in Chen and Lee (2012) and Cannella et al. (2013). The first authors developed a simple set of formulas that describe the traditional bullwhip measure as a combined outcome of several important drivers, such as finite capacity, batch-ordering, and seasonality. Building on the theoretical framework, they discuss the managerial implications of the bullwhip measurement. They show that the measurement can be completely non-informative about the underlying supply chain cost performance if it is not linked to the operational details (such as decision intervals and lead times). Specifically, they show that an aggregated measurement over relatively long time periods can mask the operational-level bullwhip. Cannella et al. (2013) presented a performance measurement system for the bullwhip effect analysis. The framework was designed to assess both individual (single/individual member supply chain) and systemic (whole/global supply chain) performances.

Having identified the need of modelling complex distribution supply chain systems while simultaneously considering adequate measures to quantify the system performance, the present paper aims to contribute to these goals. Thus, it focuses on modelling and analysing bullwhip effect metrics per supply chain echelon while understanding its practical and managerial implications in operating decisions. One important contribution of this work is that we aim to identify which bullwhip effect metric(s) is (are) more adequate to measure the bullwhip effect when operating cyclic demand distribution systems. Since cyclic demand patterns have not been addressed previously in bullwhip studies, our reasoning is to study the most common measures for testing in this setting and from that conclude their adequacy to monitoring bullwhip effect. Those metrics are: average inventory, inventory variance ratio, average order and order rate variance ratio. To this end, the metrics have to be adapted and incorporated in the optimisation-based methodology (see Section 5).

3. Problem definition

This section complements a previous one presented in a conference by the same authors (Vicente, Relvas, and Barbosa-Póvoa 2015a). In this study it is considered a generic distribution that accounts for multiple retailers and regional warehouses as shown in Figure 1, where the arrows represent the flow of multiple products that are distributed over a given time horizon.

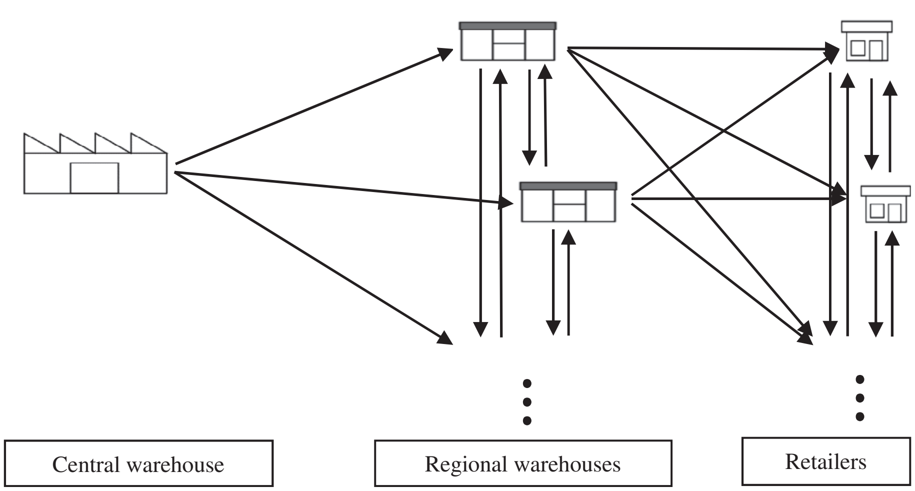


Figure 1. Supply chain structure with lateral transshipments.

In this supply chain structure the central warehouse restocks the regional warehouse inventories, while these restock the retailers' inventories. It accounts for lateral transhipment flow among retailers and regional warehouses. Transportation follows the placement of orders, and all transportation and storage have limits. A lost sale occurs when the demand of a given retailer in a given time period is not satisfied.

The overall operation costs are modelled and included in the model formulation. These include the ordering process; holding in storage; and holding in transit; transportation; transshipping and lost sales. Fixed ordering costs occur each time a regional warehouse or a retailer places an order and are related to the ordering activity, being although independent of the quantity ordered. Holding costs are defined for both storage and in transit inventory. The first ones are defined per unit stored and per time period on each regional warehouse or retailer. The second ones are defined per unit of product transported and are dependent of the lead times. Traditionally, holding cost is referred to the product being stored in warehouses. However, while being transported, inventory is still being held by the company. Since we assume a non-negligible transportation lead time, we are adding up the holding cost dependent on the duration of the transportation. Transportation costs are considered per unit of material transported between the different stages of the distribution supply chain. Related to these are the transshipping costs that represent the lateral transportations costs per unit that occurs within each stage between two identical entities. These can occur between warehouses or between retailers. Finally, lost sale costs are associated to the demand that cannot be satisfied and are defined per unit of product.

In order to optimise the overall operation costs, it is important to effectively represent and optimise the products' flows through the entire distribution supply chain. These aspects are considered within the problem formulation in study and the relevant decision that needs to be modelled is then to determine the shipping quantity to be sent from the regional warehouses to each retailer in each time period. A cyclic demand is assumed within the distribution chain and for that a single cycle is analysed. This aspect is later on characterised.

The definition of the studied problem is the following:

Given

- The planning time horizon and the defined discrete time scale;
- The number of regional warehouses and retailers;
- The number of products;
- The cyclic demand;
- The batch size;
- Customer demands for each product in all time periods;
- Storage capacities in each regional warehouse and retailer per time period;
- Transportation capacities between entities;
- Safety stock per product in each regional warehouse and retailer;
- Transportation lead times between entities;
- Ordering costs per order of each product at each regional warehouse and retailer (independent of order quantity);
- Unitary holding cost per time period and per product at each regional warehouse or retailer;
- Unitary holding transportation and transshipping cost per time period and per product in transit (dependent of lead time);
- Unitary transportation and transshipping cost per product;
- Unitary lost sale cost per product and per each time period.

Determine

- The inventory profile at the end of time periods for each product throughout the time horizon in each regional warehouse and retailer;
- The flows of products, considering order batching, across the distribution supply chain for each time period. These involve shipping quantities between entities on different distribution supply chain levels and transshipment quantities between entities on the same distribution supply chain level;
- The number of order batches per product, per time period and per each regional warehouse or retailer;
- The impact of different order batch sizes and the impact of different cyclic demands on bullwhip effect;
- The lost sale quantities of each product at each retailer at each time period.

So as to minimise an objective function that consists on the minimisation of the total operational costs for the time horizon considered.

As mentioned the demand pattern is cyclic, so when using the concept of cycle time T the modelling procedure is considered. The cycle time T is defined as the shortest time interval that it takes for a demand cycle to reoccur, and it will exist as decisions sequence, regarding the products inventory and shipping quantity, at every demand cycle.

Because of the assumption that all cycles are to be equal, the problem is formulated as a non-periodic operation within a single cycle where it is guaranteed that the inventory level is the same in the cycle beginning and at the end. It is allowed for the operation to overlap sequential cycles. Over the complete planning period extent a cycle is repeated so the modelling of the operation is done by wrapping around to the start of the same cycle. The wrap-around operator, in this cyclic operation, acts on inventories and flows among distribution supply chain entities. To do so, it used the wrap-around operator *WA(t)* defined by Shah, Pantelides, and Sargent (1993):

$$WA(t) = \begin{cases} t, & if \quad t \geq 1 \wedge t \leq T \\ WA(t+T), & if \quad t \leq 0 \end{cases}$$

This periodic mode of operation was also applied to optimally design and retrofit batch plants by Pinto, Barbosa-Póvoa, and Novais (2005) and other types of problems as stated in the review paper by Barbosa-Póvoa (2007). In the formulation of the model a time discretization is used where the cycle time is divided into T equal duration intervals. The cycle start and end time periods are $t = 1$ and $t = T$, respectively. The following cycle start coincides with the latter plus one. A planning horizon *(H)* is assumed which is divided into *N* equal duration cycles *(T)* which are also divided into a number of fixed duration elementary time periods *(δ)*, as shown in Figure 2, (Vieira, Pinto-Varela, and Barbosa-Póvoa 2015).

This problem is modelled through a mathematical programming model, which will be presented in the subsequent section.

4. Inventory planning model

The problem of inventory planning presented, which during the time horizon in study aims to minimise the total costs is modelled as a Mixed Integer Linear Programming (MILP) model, and it complements a previous one presented in a conference by the same authors (Vicente, Relvas, and Barbosa-Póvoa 2015a).

The notation, for the indices, constants, sets, parameters and variables (non-negative continuous, non-negative integer and binary) used in the model formulation, is the following:

Indices

i	product
j, k, l, m	entity node
t	time period

Constants

NP	number of products
NW	number of regional warehouses
NR	number of retailers
T	number of time periods in the cycle demand

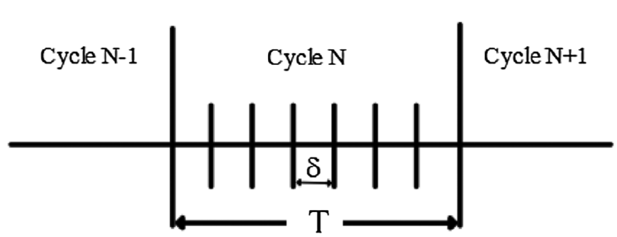


Figure 2. Time discretization for a single cycle.

Sets

$i \in P = \{1, 2, \ldots, NP\}$	products
$j, k, l, m \in I = \{0, 1, 2, \ldots, NW, NW + 1, NW + 2, \ldots, NW + NR\}$	supply chain nodes
$t \in D = \{1, 2, \ldots, T\}$	time periods
$W = \{1, 2, \ldots, NW\}, W \subset I$	regional warehouses
$R = \{1, 2, \ldots, NR\}, R \subset I$	retailers
$W_o = \{0\}, W_o \subset I$	central warehouse
$DN = \{1, 2, \ldots, NW, NW + 1, NW + 2, \ldots, NW + NR\}, DN \subset I$	demand nodes (regional warehouses and retailers)
$SN = \{0, 1, 2, \ldots, NW\}, SN \subset I$	supply nodes (central warehouse and regional warehouses)

Note: $W_o \cup W \cup R = I$

Parameters

BGM	a large positive number
BatSiz	batch size
CD_{ijt}	customer demand of the product i at entity j in time period t (note that customer demand occurs only at the retailers, but not at the warehouses)
H	planning horizon
HOC_{ij}	unitary holding cost of the product i at entity j per time period
HTC_{ijk}	unitary holding in transit cost of the product i from entity j to entity k (note that holding in transit is for shipping operations and for transshipping operations)
LSC_{ijt}	unitary lost sale cost of the product i at entity j in time period t
LTT_{jk}	transportation lead time from entity j to entity k
OC_{ij}	ordering cost of the product i at entity j (note that ordering cost is independent of quantity of product i)
SS_{ij}	safety stock of the product i at entity j
STC_{jt}	storage capacity at entity j in time period t
T	cycle time
$TRACMAX_{jk}$	maximum transportation capacity from entity j to entity k
$TRACMIN_{jk}$	minimum transportation capacity from entity j to entity k
TRC_{ijk}	unitary transportation cost of the product i from entity j to entity k (note that transportation is for shipping operations and for transshipping operations)

Non-negative continuous variables

FI_{ijt}	inventory of product i at entity j at the end of the time period t
LS_{ijt}	lost sales quantity of product i at entity j at the end of the time period t (note that lost sales only occur at the retailers)
SQ_{ijkt}	shipping quantity of product i from entity j to entity k during time period t

Non-negative integer variable

$NumBat_{ijkt}$	number of batches of product i between entity j and entity k at each time period t

Binary variable

$BV1_{ijt}$	equal to 1 if an order of product i is placed by entity j in time period t; 0 otherwise

Time operator

$WA(t)$	wrap-around time operator

Objective function

The objective function consists of the minimisation of the total cost is given as follows:

$$\text{Minimise total cost} = \left(\sum_{i \in P} \sum_{j \in I} \sum_{t \in T} \left(OC_{ij} \times BV1_{ijt} + HOC_{ij} \times FI_{ijt} + LSC_{ijt} \times LS_{ijt} \right) + \sum_{i \in P} \sum_{j \in I} \sum_{k \in I} \sum_{t \in T} \left(\left(HTC_{ijk} \times LTT_{jk} + TRC_{ijk} \right) \times SQ_{ijkt} \right) \right) \times H/T \tag{1}$$

The first term of the objective function (1) represents the ordering costs, holding costs and lost sale costs. The second term expresses the holding in transit costs and the transportation costs. Note that ordering costs are product quantity independent and holding in transit costs are lead time dependent. *H/T* multiplying factor represents the number of times that the cycle of operation mode repeats throughout the time horizon. Therefore, costs are accounted for per cycle and afterwards accounted for the entire time horizon.

Constraints

The model developed consists of different types of constraints. These are grouped into: inventory constraints, shipping constraints, storage capacities, transportation capacities, safety stock policy and non-negativity and binary conditions. Constraints will be presented grouped by type.

Inventory constraints

Inventory constraints have to be defined for both warehouses and retailers, taking into account all inputs and outputs at each time period.

Regional warehouses

The total incoming quantity at each regional warehouse j is equal to the shipping quantity from the central warehouse, plus the transshipping quantity from the others regional warehouses l, considering the transportation lead time through the introduction of a time lag. The total outgoing quantity at each regional warehouse j is equal to the shipping quantity to the retailers k plus the transshipping quantity to the other regional warehouses l, at time period t.

The inventory of product i at regional warehouse j at the end of the time period t is given by constraints (2).

$$FI_{ijt} = FI_{i,j,WA(t-1)} + SQ_{i,0,j,WA(t-LTT_{0j})}\big|_{LTT_{0j}<t} - \sum_{k \in R} SQ_{i,j,k,WA(t)} - \sum_{l \in W \wedge l \neq j} SQ_{i,j,l,WA(t)} + \sum_{l \in W \wedge l \neq j} SQ_{i,l,j,WA\left(t-LTT_{lj}\right)}\big|_{LTT_{lj}<t}, \quad i \in P, j \in W, t \in D \tag{2}$$

When the wrap-around operator *WA(t)* is applied, for instance to the variable $FI_{i,j,WA(t-1)}$, for $t-1 \le 0$ it leads to an inventory which will be equal to the one found at the end of time period $t-1+T$. Also for variable $SQ_{i,0,j,WA(t-LTT_{0j})}$, for $t - LTT_{0j} \le 0$ leads to an identical shipping quantity which will occur at time period $t - LTT_{0j} + T$.

Retailers

The total incoming quantity at each retailer k is equal to the shipping quantity from the regional warehouses j, plus the transshipping quantity from the other retailer m, at a previous time period t, considering the transportation lead time. The total outgoing quantity at each retailer k is equal to the customer demand minus the lost sale of that retailer k plus the transshipping quantity to the others retailers m, at time period t.

Constraints (3) are applicable for the inventory of product i at retailer k at the end of the time period t.

$$FI_{ikt} = FI_{i,k,WA(t-1)} + \sum_{j \in W} SQ_{i,j,k,WA\left(t-LTT_{jk}\right)}\big|_{LTT_{jk}<t} - \left(CD_{i,k,WA(t)} - LS_{i,k,WA(t)} \right) - \sum_{m \in R \wedge m \neq k} SQ_{i,k,m,WA(t)} + \sum_{m \in R \wedge m \neq k} SQ_{i,m,k,WA(t-LTT_{mk})|LTT_{mk}<t}, \quad i \in P,\ k \in R, t \in D \tag{3}$$

Shipping quantities among entities

Since transportation occurs after an order has been placed from a destination to its source, it is assumed that the fixed ordering cost is always incurred when the transportation occurs. Hence, if the transportation amount is not zero, then the binary variable

$BV1_{ijt}$ equals 1, as implied in constraints (4). The left-hand side of this constraint represents the quantity received by a warehouse, which can be supplied by the central warehouse (first term) or any other warehouse (second term) through a transshipment operation.

$$SQ_{i0jt} + \sum_{l \in W \wedge l \neq j} SQ_{iljt} \leq BGM \times BV1_{ijt}, \quad i \in P, j \in W, t \in D \tag{4}$$

Constraints equivalent to (4) are defined for retailers, as presented in constraints (5). The *BGM* parameter should attain a value such that it is a valid upper bound of any quantity that can be ordered by a regional warehouse or retailer.

$$\sum_{j \in W} SQ_{ijkt} + \sum_{m \in R \wedge m \neq k} SQ_{imkt} \leq BGM \times BV1_{ikt}, \quad i \in P, k \in R, t \in D \tag{5}$$

Batching size between entities

The number of batches of product i between node j and node k at each time period t is an integer number equal to the quotient between the shipping quantity and the batch size. This calculation is obtained through constraints (6). Please note that the complete order placed by one node may be constituted by more than one flow from different entities. Through constraint (6) it is consequently ensured that the complete order of one node is also a multiple of the batch size.

$$NumBat_{ijkt} = \frac{SQ_{ijkt}}{BatSiz}, \quad i \in P, j \in I, k \in I, j \neq k, t \in D \tag{6}$$

Storage capacities

The total inventory stored at any node, given by the sum of the inventory level of each product, must respect the storage capacity in each demand node j at any time period t which is enforced by constraints (7). Note that storage capacity is shared among products and the central warehouse has unlimited storage capacity.

$$\sum_{i \in P} FI_{ijt} \leq STC_{jt}, \quad j \in DN, t \in D \tag{7}$$

Transportation capacities

At any time period t, the sum of the shipping quantity of each product i must respect the lower and upper transportation limits between each two nodes j and k, as stated in constraints (8) and (9). Note that transportation capacity is shared among products and the transportation capacity from the central warehouse is unlimited.

$$\sum_{i \in P} SQ_{ijkt} \leq TRACMAX_{jk}, \quad j \in DN, k \in DN, j \neq k, t \in D \tag{8}$$

$$TRACMIN_{jk} \leq \sum_{i \in P} SQ_{ijkt}, \quad j \in DN, k \in DN, j \neq k, t \in D \tag{9}$$

Safety stock policy

Constraints (10) ensure that the inventory of each product i at each node j must be higher or equal than the required safety stock level for that product in that node in any time period t.

$$SS_{ij} \leq FI_{ijt}, \quad i \in P, j \in DN, t \in D \tag{10}$$

Non-negativity and binary conditions

As defined above, the model uses both non-negative integer (11) and continuous variables (12) and one binary variable (13).

$$NumBat_{ijkt} \in \mathbb{N}_0, \quad i \in P, j \in I, k \in I, t \in D \tag{11}$$

$$Q_{ijkt},\ II_{ijt},\ FI_{ijt},\ LS_{ijt} \geq 0, \quad i \in P, j \in I,\ k \in I, t \in D \tag{12}$$

$$BV1_{ijt} \in \{0,1\}, \quad i \in P, j \in I,\ t \in D \tag{13}$$

The above model formed by constraints (2) to (13) with objective function (1) describes the proposed inventory planning model. In order to compare the performance and adequacy of this model, the bullwhip effect is analysed in terms of inventory instability and order amplification per echelon under order batching policy and cyclic time period demand. Such metrics are characterised in the following section.

5. Echelon metrics for measuring the bullwhip effect

Echelon average inventory and echelon inventory variance ratio have been adopted for measuring the bullwhip effect in terms of inventory instability. Additionally, for measuring the bullwhip effect in terms of order amplification, the echelon average order and echelon order rate variance ratio are considered (Chen et al. 2000).

Disney and Towill (2002) introduced the inventory variance ratio (*InvVarR*) as a measure for multi-echelon system instability. This is obtained by quantifying the inventory variance magnitude as the inventory variance at a generic node and customer demand variance ratio as shown in Equation (14).

$$InvVarR = \frac{\sigma^2_{inv}/\mu_{inv}}{\sigma^2_d/\mu_d} \tag{14}$$

where σ^2_{inv} is the inventory variance and μ_{inv} is the average inventory, while customer demand variance is given by σ^2_d and μ_d is the average customer demand.

It is considered and adopted this measure per echelon and per product (*InvVarRat*) as shown in Equation (15).

$$InvVarRat = \frac{InvVar/InvAve}{CdeVar/CdeAve} \tag{15}$$

where *InvVar* is the inventory variance, determined per product and per echelon. Under the same perspective of representation per product and per echelon, *InvAve* is the average inventory. Customer demand variance is given by *CdeVar* and *CdeAve* is the average customer demand.

Thus for the warehouses' echelon, inventory variance ratio per product i ($InvVarRatW_i$) is given by Equation (16), whereas the same metric applied to the retailers' echelon ($InvVarRatR_i$) is given in Equation (17).

$$InvVarRatW_i = \frac{InvVarW_i/InvAveW_i}{CdeVarR_i/CdeAveR_i}, \quad i \in P \tag{16}$$

$$InvVarRatR_i = \frac{InvVarR_i/InvAveR_i}{CdeVarR_i/CdeAveR_i}, \quad i \in P \tag{17}$$

In order to calculate these metrics, the reasoning behind these calculations will be presented. The notation and information are that the one of the proposed MILP model.

The inventory quantity at the warehouses' echelon by product i at the end of time period t ($InvW_{it}$) is given by Equation (18).

$$InvW_{it} = \sum_{j \in W} FI_{ijt}, \quad i \in P, t \in D \tag{18}$$

while the average inventory quantity at the warehouses' echelon by product i ($InvAveW_i$) is given by Equation (19).

$$InvAveW_i = \frac{\sum_{t \in D} InvW_{it}}{T}, \quad i \in P \tag{19}$$

Thus, the inventory quantity variance at warehouses' echelon by product i ($InvVarW_i$) is given by Equation (20).

$$InvVarW_i = \frac{\sum_{t \in D} (InvW_{it} - InvAveW_i)^2}{T}, \quad i \in P \tag{20}$$

For the retailers' echelon, it is used a similar notation and thus the average inventory quantity at the retailers' echelon by product i ($InvAveR_i$) is given by Equation (21).

$$InvAveR_i = \frac{\sum_{t \in D} InvR_{it}}{T}, \quad i \in P \tag{21}$$

Customer demand quantity at the retailers' echelon by product i at time period t ($CdeR_{it}$) is given by Equation (22).

$$CdeR_{it} = \sum_{k \in R} CD_{ikt}, \quad i \in P, t \in D \tag{22}$$

while the average customer demand quantity at the retailers' echelon by product i ($CdeAveR_i$) is given by Equation (23).

$$CdeAveR_i = \frac{\sum_{t \in D} CdeR_{it}}{T}, \quad i \in P \tag{23}$$

Customer demand variance at the retailers' echelon of product i ($CdeVarR_i$) is given by Equation (24).

$$CdeVarR_i = \frac{\sum_{t \in D} (CdeR_{it} - CdeAveR_i)^2}{T}, \quad i \in P \tag{24}$$

Therefore, echelon average inventory metrics are given by Equations (19) and (21) and echelon inventory variance ratio metrics are given by Equations (16) and (17).

The measure of multi-echelon system demand amplification is obtained by quantifying the ratio of the order variance at a generic node and the variance of the customer demand: the order rate variance ratio (*OrdRVarR*) as given in Equation (25).

$$OrdRVarR = \frac{\sigma^2_{ord} / \mu_{ord}}{\sigma^2_d / \mu_d} \tag{25}$$

where σ^2_{ord} is the order variance and μ_{ord} is the average order. Customer demand variance is given by σ^2_d and μ_d is the average customer demand.

Such measure, as before, is adapted per echelon and per product (*OrdRVarRat*) as shown in Equation (26).

$$OrdRVarRat = \frac{OrdVar / OrdAve}{CdeVar / CdeAve} \tag{26}$$

where *OrdVar* is the order variance. Under the same perspective of representation per product and per echelon, *OrdAve* is the average order. Customer demand variance is given by *CdeVar* and *CdeAve* is the average customer demand.

So, order rate variance ratio at the warehouses' echelon expressed by product i ($OrdRVarRatW_i$) is given by Equation (27), whereas the same metric applied to the retailers' echelon ($OrdRVarRatR_i$) is given in Equation (28).

$$OrdRVarRatW_i = \frac{OrdVarW_i / OrdAveW_i}{CdeVarR_i / CdeAveR_i}, \quad i \in P \tag{27}$$

$$OrdRVarRat R_i = \frac{OrdVarR_i / OrdAveR_i}{CdeVarR_i / CdeAveR_i}, \quad i \in P \tag{28}$$

The involved terms are given in the following Equations (29)–(31).

$$OrdW_{it} = \sum_{j \in W} SQ_{i0jt} + \sum_{l \in W \wedge l \neq j} \sum_{j \in W} SQ_{iljt}, \quad i \in P, t \in D \tag{29}$$

$$OrdAveW_i = \frac{\sum_{t \in D} OrdW_{it}}{T}, \quad i \in P \tag{30}$$

$$OrdVarW_i = \frac{\sum_{t \in D} (OrdW_{it} - OrdAveW_i)^2}{T}, \quad i \in P \tag{31}$$

For the retailers' echelon, similar notation is used. The average order quantity by product i ($OrdAveR_i$) is given by Equation (32).

$$OrdAveR_i = \frac{\sum_{t \in D} OrdR_{it}}{T}, \quad i \in P \tag{32}$$

Summarising echelon average order metrics are given by Equations (30) and (32) and echelon order rate variance ratio metrics are given by Equations (27) and (28).

The usage of these metrics is recognised in the literature (Chen et al. 2000 and Disney and Towill 2002) as an adequate indicator for the presence of bullwhip effect in distribution supply chains. High metric values correspond to higher probability of bullwhip in the operation of such systems. The usage of these metrics will be exemplified through an experimental example in the next section.

6. Experimental studies

6.1 *Motivational example*

In this section a motivational example is solved to test the effect of different order batching dimensions (under the same cyclic time period demand) on the bullwhip effect in terms of inventory instability and order amplification for different distribution supply chain structures. The aim is to identify the network structures that combined with batching policies mitigate instability and amplification throughout the distribution supply chain. This will be measured using the metrics defined in the previous section.

The model was implemented in GAMS 23.5 modelling language and solved using CPLEX 12.2 solver in an Intel Core i7 CPU 2.20 GHz and 8 GB RAM. The stopping criterion was the determination of the optimal solution.

Three different distribution supply chain structures without lateral transshipments are considered. These are respectively: structure A (Figure 3) – one central warehouse, one regional warehouse and one retailer; structure B (Figure 4) – one central warehouse, one regional warehouse and two retailers; structure C (Figure 5) – one central warehouse, two regional warehouses and two retailers. The proposed structures represent different configurations that may occur in distribution networks.

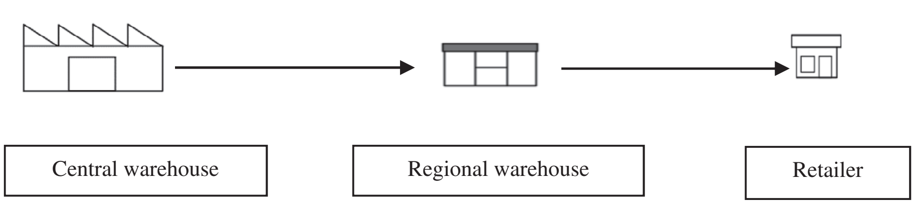


Figure 3. Supply chain structure A.

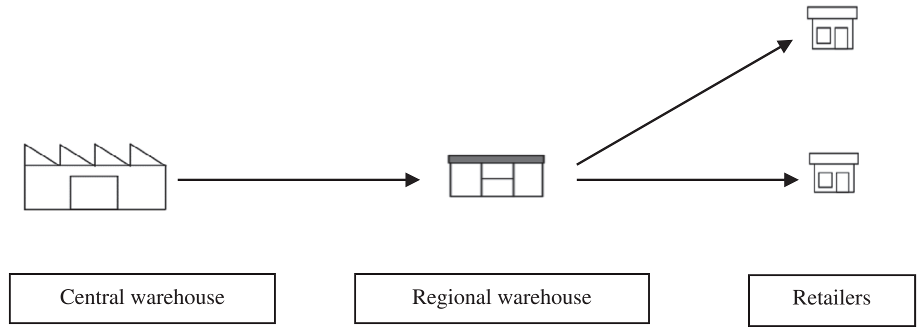


Figure 4. Supply chain structure B.

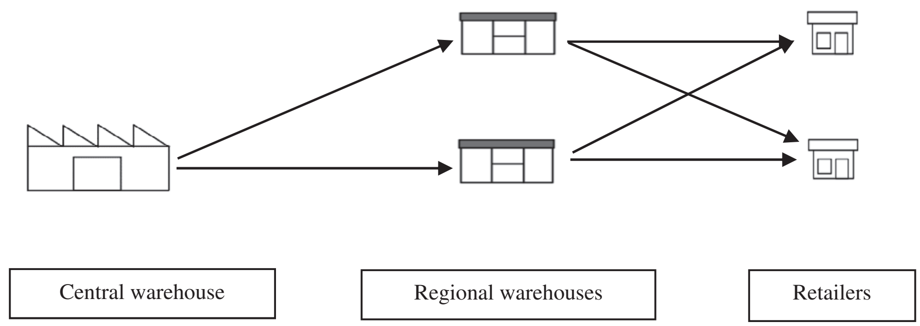


Figure 5. Supply chain structure C.

Storage and transportation capacities are unlimited. The safety stock policy is assumed to hold no stock for all entities. One main family type of products is considered. A 56-time period planning horizon with a 7 period length cycle was assumed to test the model (modelled in Section 4).

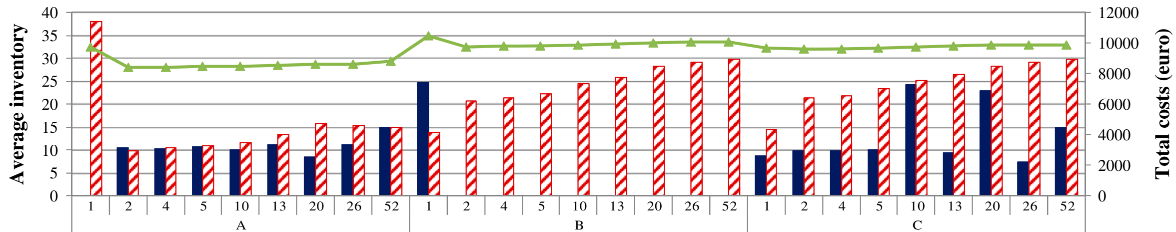
In Appendix 1, Tables from A1 to A4 present the parameters' values considered for the experimental example, including general model parameters, transportation costs, customer demand and lead time. Customer demands are shown in Table A3 (Appendix 1), aggregated in the retailers' echelon. Thus, customer demand is equal to the sum of customer demand per time period.

Different order batching policies (1, 2, 4, 5, 10, 13, 20, 26 and 52 units) were considered under the same cyclic time period demand. These values were considered randomly and aim to describe small and large batch sizes situations.

The results are structured along four main sections related to the measures in use: average inventory; inventory variance ratio; average order; order rate variance ratio.

6.1.1 *Echelon average inventory*

Figure 6 shows the echelon average inventory by batch size policy and by supply chain structure. It can be seen that when batching is introduced (batches starting with a size of 2 units) the main trend is to have higher inventory being held at retailers. However, the distribution supply chain structure has a strong influence in this trend. The linear structure A balances inventories at both echelons, while in structure B regional warehouses act as throughput nodes with no inventory being held. In structure C the regional warehouses hold stock so as to take advantage of a pull system. On the other hand, the experiments with no batching (batch size equal to 1, allowing any combination of flows) return different results since only with structure B it is observed that inventory is higher at warehouses. These results suggest that for different network structures this metric can be used in order to access bullwhip effect when different batch policies are used.



	A									B									C								
	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52
Warehouse´s echelon	0	10,57	10,29	10,71	10	11,14	8,57	11,14	14,86	24,71	0	0	0	0	0	0	0	0	8,71	9,71	9,71	10	24,29	9,29	22,86	7,43	14,86
Retailer's echelon	38	9,86	10,43	10,86	11,57	13,29	15,86	15,43	15	13,71	20,71	21,29	22,29	24,57	25,71	28,29	29,14	29,86	14,57	21,43	21,71	23,29	25,14	26,57	28,29	29,14	29,86
Total costs	9728	8421	8437	8456	8472	8542	8600	8614	8802	10453	9771	9790	9824	9901	9939	10026	10054	10078	9694	9632	9642	9698	9760	9795	9898	9861	9906

Batch size policy by supply chain structure

Figure 6. Echelon average inventory and total costs by batch size policy (1, 2, 4, 5, 10, 13, 20, 26 and 52 units) and by supply chain structure (A, B and C).

6.1.2 *Echelon inventory variance ratio*

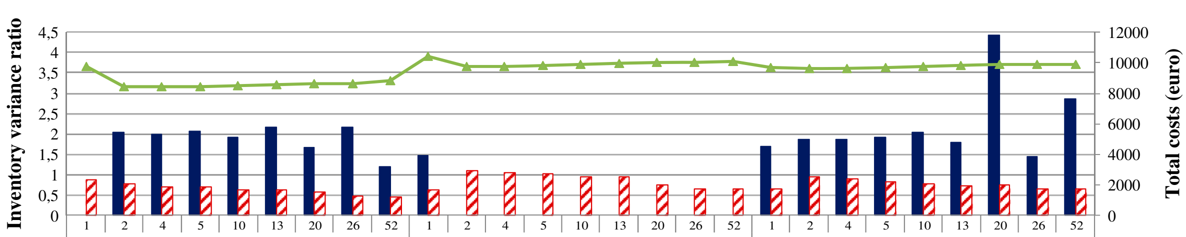
From Figure 7 it can be seen that inventory variance ratio is more stable at the retailers' echelon than at the warehouses' echelon and there is no clear relation between batching and the behaviour of the metric. The results suggest that the batch size has no major influence in the inventory variance ratio. However, for structure C, it can be observed that in two of the larger batch sizes (20 and 52) some influence exists. Therefore, these results suggest that for network structures that predominantly have more than one entity per echelon, this metric allows the analysis of the behaviour of the bullwhip effect.

6.1.3 *Echelon average order*

On the echelon average order by batch size policy and by distribution supply chain structure (Figure 8) it is observed the same value for all situations. This is because the warehouse's echelon order and the retailer's echelon order aggregated on cycle time period (7 time periods) is equal to the customer demand aggregated on retailer's echelon and on cycle time period. Therefore, as conclusion, this metric does not reflect alterations along the chain and thus is not suited for measuring the bullwhip effect in cyclic operations.

6.1.4 *Echelon order rate variance ratio*

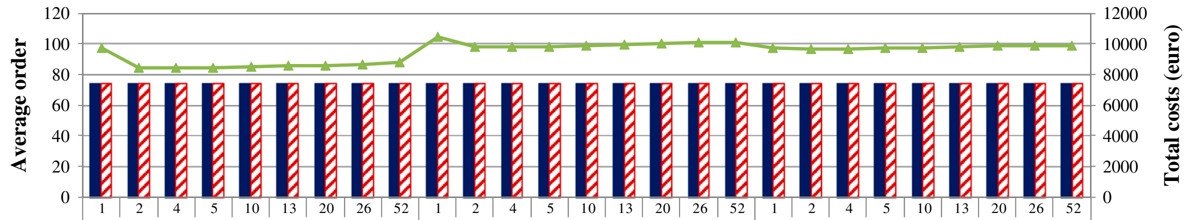
Finally, and from Figure 9, it can be seen that for the different echelons the order rate variance ratio is somewhat balanced between both echelons, suggesting that reduced amplification is verified. This amplification is more visible in structure C when compared to structures A and B. In this sense, linear structures control the forces that lead to the amplification of demand at warehouses, using the reasoning of Cachon, Randall, and Schmidt (2007). Also, and on average, the order rate variance ratio of the distribution supply chain increases with the increase of nodes in the network, as it can be seen in Table 1. This suggests that more complex networks are more affected with order amplification. The relation between batch



	A									B									C								
	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52
Warehouse´s echelon	0	2,04	1,99	2,07	1,93	2,16	1,66	2,16	1,2	1,46	0	0	0	0	0	0	0	0	1,69	1,88	1,88	1,93	2,04	1,8	4,42	1,44	2,87
Retailer's echelon	0,88	0,77	0,7	0,7	0,62	0,62	0,58	0,47	0,44	0,63	1,1	1,04	1,02	0,96	0,95	0,74	0,66	0,64	0,66	0,94	0,91	0,82	0,78	0,73	0,74	0,66	0,64
Total costs	9728	8421	8437	8456	8472	8542	8600	8614	8802	10453	9771	9790	9824	9901	9939	10026	10054	10078	9694	9632	9642	9698	9760	9795	9898	9861	9906

Batch size policy by supply chain structure

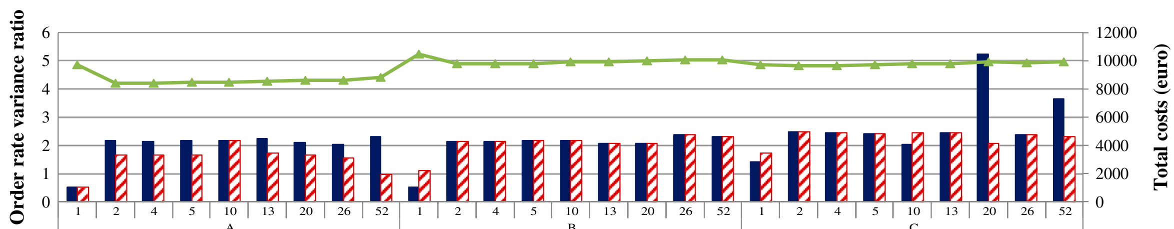
Figure 7. Echelon inventory variance ratio and total costs by batch size policy (1, 2, 4, 5, 10, 13, 20, 26 and 52 units) and by supply chain structure (A, B and C).



	A									B									C								
	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52
Warehouse´s echelon	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3
Retailer's echelon	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3	74,3
Total costs	9728	8421	8437	8456	8472	8542	8600	8614	8802	1045	9771	9790	9824	9901	9939	1002	1005	1007	9694	9632	9642	9698	9760	9795	9898	9861	9906

Batch size policy by supply chain structure

Figure 8. Echelon average order and total costs by batch size policy (1, 2, 4, 5, 10, 13, 20, 26 and 52 units) and by supply chain structure (A, B and C).



	A									B									C								
	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52	1	2	4	5	10	13	20	26	52
Warehouse´s echelon	0,53	2,16	2,15	2,18	2,18	2,24	2,12	2,05	2,3	0,52	2,15	2,13	2,16	2,18	2,07	2,07	2,38	2,3	1,4	2,49	2,46	2,42	2,04	2,45	5,24	2,38	3,64
Retailer's echelon	0,53	1,66	1,65	1,67	2,18	1,73	1,67	1,54	0,96	1,11	2,15	2,13	2,16	2,18	2,07	2,07	2,38	2,3	1,71	2,47	2,46	2,42	2,45	2,45	2,07	2,38	2,3
Total costs	9728	8421	8437	8456	8472	8542	8600	8614	8802	10453	9771	9790	9824	9901	9939	10026	10054	10078	9694	9632	9642	9698	9760	9795	9898	9861	9906

Batch size policy by supply chain structure

Figure 9. Echelon order rate variance ratio and total costs by batch size policy (1, 2, 4, 5, 10, 13, 20, 26 and 52 units) and by supply chain structure (A, B and C).

Table 1. Mean of order rate variance ratio by supply chain structure (A, B and C) and by echelon.

Supply chain structure	Warehouse's echelon	Retailer's echelon
A	2.00	1.52
B	2.01	2.07
C	2.72	2.32

size and order rate variance ratio is less visible, except for two batch sizes in structure C (20 and 52). So, the results suggest that for network structures that predominantly have more than one entity per echelon, this metric is adequate to assess bullwhip effect.

From the above analysis and as main insights it can be concluded that the echelon average inventory, the echelon inventory variance ratio and the echelon order rate variance ratio metrics appear as suitable metrics for measuring the bullwhip effect on multi-echelon systems under order batching policies with cyclic demand. Additionally, and on the total costs, it can be seen that these slightly increase with the batch size increase for all distribution supply chain structures. However, these costs can be controlled if economies of scale are explored ordering larger batches.

6.2 *Real world case study*

In this section a real world case study based on a Portuguese Retail Company is studied. The cycle time periodic demand, on the bullwhip effect in order amplification and inventory instability, and different order batching impact are tested. The data provided has been changed, due to confidentiality reasons, but it still describes the real operation.

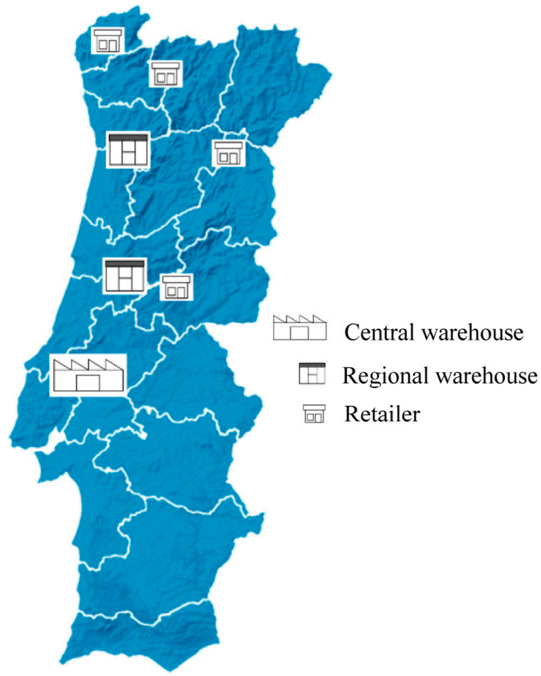


Figure 10. Supply chain structure of the Portuguese Retail Company.

The model was implemented in GAMS 23.5 modelling language and solved using CPLEX 12.2 solver in an Intel Core i7 CPU 2.20 GHz and 8 GB RAM. The stopping criteria were either a computational time limit of 3600 s or the optimal solution determination. A central warehouse, two regional warehouses and four retailers, are used in this distribution supply chain as shown in Figure 10.

In this company it is allowed to do lateral transhipments. It considers one main product family type and the storage capacity limit is 50 for warehouses, 25 units for retailers and between 5 and 40 for shipping. Since it is being used a just-in-time policy, all entities have a zero safety stock policy. To test the model, that uses orders placed at a variable time intervals with a variable order quantity, it was assumed a 56-time period planning horizon.

In Appendix 2, Tables from B1 to B4 present the parameters' values considered for this case study, including general model parameters, transportation costs, customer demand and lead time. Additionally, Table B5 (Appendix 2) shows the computational statistics.

It is measured, under different cyclic demands, the bullwhip effect with the following three order batching policies: batching of 1 unit (s1), 5 units (s2) and 10 units (s3). Please note that, in Table B1 (Appendix 2), it is considered that ordering different batch sizes implies different costs per ordering – scale economies. These costs were estimated for this company and were related with the administrative effort taken to put an order. Therefore, when ordering without batching, the ordering cost is higher than within a batching policy.

The goal of this section is to provide the company with some more insights on how to develop their inventory planning, namely in selecting batching and cycle policies that enhance costs without implying a negative impact on bullwhip effect.

6.2.1 *Results: costs and number of orders placed*

Two important results are analysed. These are related to the costs involved and to the number of orders placed.

Table 2 provides the total costs and costs' breakdown for the three scenarios of order batching. Also, three different cycle lengths are considered: 7, 14 and 28, that will span over the complete planning horizon (56 time periods).

On the costs side, the major difference among the three order batching scenarios is on the ordering costs: 20 euro, 10 euro and 5 euro/order, respectively, for cases s1, s2 and s3. From the results it can be seen that case s3 presents the lowest total costs, a result that was expected given the economies of scale present in the system.

Table 2. Costs per nature for the three case of batching for 7/14/28 cyclic time periodic demand of 56 time period planning horizon (euro).

	1 unit batching (s1)	5 units batching (s2)	10 units batching (s3)
Ordering	3360.00/3360.00/3400.00	1760.00/1800.00/1700.00	1160.00/1200.00/1230.00
Holding	936.00/944.00/920.00	736.00/701.60/882.00	577.60/586.40/617.60
Holding transportation in transit	4228.80/4239.60/4219.80	4416.00/4470.00/4398.00	4128.00/4164.00/4164.00
Holding transshipment in transit	0/0/0	0/0/0	0/0/0
Transportation	2206.88/2192.84/2202.20	2054.00/2015.00/2054.00	2163.20/2147.60/2147.60
Transshipment	0/0/0	0/0/0	0/0/0
Lost sales	0/0/0	0/0/0	0/0/0
Total	10,731.68/10,736.44/10,742.00	8966.00/8986.60/9034.00	8028.80/8098.00/8159.20

Table 3. Number of orders for the three case of batching for 7/14/28 cyclic time periodic demand of 56 time period planning horizon.

	1 unit batching (s1)	5 units batching (s2)	10 units batching (s3)
Orders	168/168/170	176/180/170	232/240/246

When analysing the results obtained while varying the order batching and the cycle time periods considered it can be seen that for instance for 1 unit batching, the total costs for a cycle of 7 days are of 10,731 €, for 14 days they are of 10,736 €, while for 28 days are of 10,742 €. No costs related to holding transshipment in transit, transshipment and lost sales are observed in the system for any of the situations studied. Despite the differences being quite small, costs increase with the size of the cycle. So if demand patterns are present in weekly cycles, optimisation may improve system costs.

The results related to the number of orders are shown in Table 3. It can be seen that the total number of orders during the complete time horizon increases when the batch size is increased. This is due to the fact that ordering different batch sizes implies different costs per ordering. Cycle length has no major impact in the number of orders.

6.2.2 *Results: bullwhip analysis in terms of instability*

The metrics' results are shown in Figures 11 and 12. Figure 11 illustrates the echelon average inventory by batch size policy and by cycle time extent. A decrease of the value of warehouses' and retailers' echelon average inventory can be seen with the increase of order batching. On the other hand, changing the batching policy by cycle time period does not present significant changes on the warehouses' and retailers' echelons average inventory. Thus, one can conclude that within a cyclic operation, batching may mitigate high inventory levels, whereas cycle extent has reduced impact on bullwhip effect mitigation. Therefore, the best strategy for the company is to use a 10-unit batching policy with economies of scale.

Figure 12 presents the echelon inventory variance ratio by batch size policy and by cycle time period. It is observed that the retailers' echelon inventory variance ratio decreases with the increase of the batch size. On the other hand, the warehouses' echelon inventory variance ratio has a different behaviour. The warehouses' echelon faces more instability, namely with a batch size of 5. Additionally, it can be seen that changing the batch size policy by cycle time period does not present significant changes on the warehouses' and retailers' echelon inventory variance ratio, excepted for the batch size of 5. Concluding, as the inventory variance ratio quantifies the fluctuations in inventory, an increased inventory variance results in higher holding costs, inflating the average inventory cost per period. Thus, this metric confirms that the best strategy for the company is to use a 10-unit batching policy with economies of scale. Related to the cycle length the option choice is again indifferent.

The results obtained in this section provide new findings and managerial implications on distribution supply chains with cyclic demand. Thus, the analysis of the inventory related metrics suggest that using batching policy reduces instability. On the other hand, the length of the time period has no major impact at retailers and warehouses echelons. In this way, the variance becomes more balanced between echelons and the average inventory can be seen as a complementary measure of the inventory variance ratio. The latter is associated with the variation and the increase of the holding costs, whereas the former quantifies the average holding cost over the observation time.

6.2.3 *Results: bullwhip analysis in terms of amplification*

The results for echelon average order metric are not showed as this metric was considered as not suited for measuring bullwhip in cyclic operations, as it was seen in Section 6.1.

Figure 13 shows the echelon order rate variance ratio by batch size policy and by cycle time period. It can be seen that the warehouses' and retailers' echelon order rate variance ratio decreases with the increase of the batch size. Order metrics suggest that batching may reduce in general order variance if using larger batches. Cycle length has no major impact in order rate variance ratio. The analyses from Figures 11–13 are aligned with the previous analyses namely, regarding holding and ordering costs in Table 2.

As a conclusion and through the measurement of the bullwhip amplification it can be said that batching has no effect on the amplification when looking upstream in the distribution supply chain. On the other hand, batching may reduce in general order variance if using larger batches, as stated by Hussain and Drake (2011). Additionally, cycle length has no major impact in order rate variance ratio.

Inventory and order metrics suggest that batching may reduce in general variance if using larger batches while cycle length has no major impact in variance. Nevertheless, these conclusions assume that larger orders (using larger batch sizes) will have a lower ordering cost due to economies of scale. Summing up, conclusions are maintained between the both studies.

6.2.4 *Final recommendation for the company*

As final recommendation, and within this case, the company should use a 10-units batching policy. In this way the total cost of the operation is lower and the bullwhip effect control is more effective. Additionally, a complementary research, with three main family types of products is presented in Appendix 3 in order to analyse whether the conclusions are valid when more than one product is present. It is verified that even when two or more products compete for the capacity the conclusions remain the same.

6.2.5 *Analysis of results*

This section sums up the discussion and analysis of the bullwhip effect metrics performed above, namely it identifies their relative strengths, weaknesses and suitability. Additionally, the reasons for suitability classification and the analysis with related literature are also presented. Table 4 summarises these results.

The final column presents the comparison of the results obtained against the main literature in the area and revised previously in Section 2. Please note that the studies published address different distribution systems configurations when compared to the ones analysed in the present paper, as discussed above. In this last column we indicate whether our results are aligned (+) or not (–) with other papers in the scientific literature in terms of strengths (S), weaknesses (W)

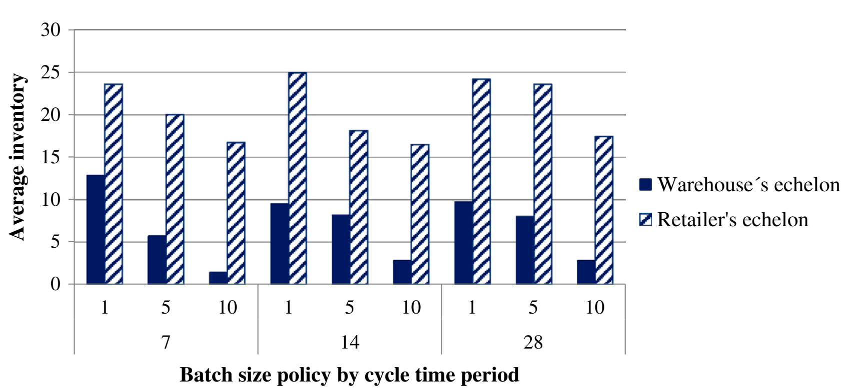


Figure 11. Echelon average inventory by batch size policy (1, 5 and 10 units) and by cyclic demand (7, 14 and 28 time periods).

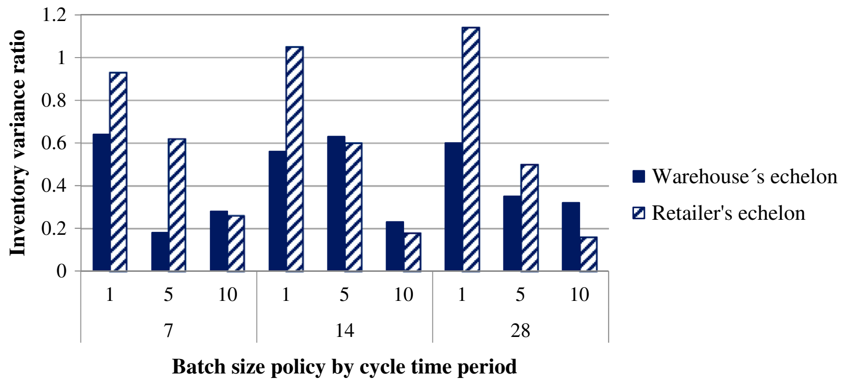


Figure 12. Echelon inventory variance ratio by batch size policy (1, 5 and 10 units) and by cyclic demand (7, 14 and 28 time periods).

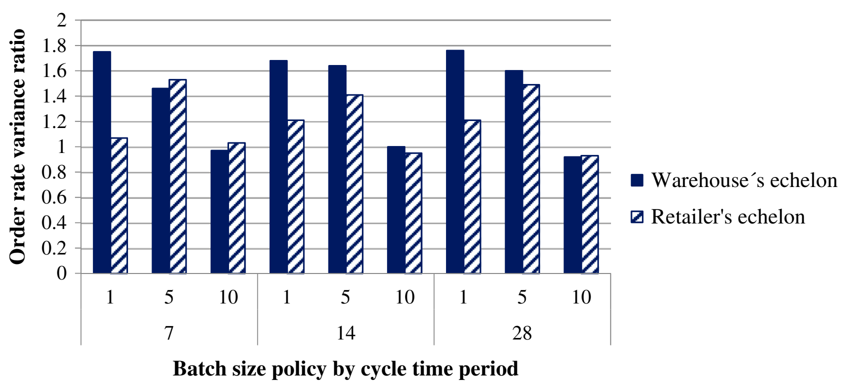


Figure 13. Echelon order rate variance ratio by batch size policy (1, 5 and 10 units) and by cyclic demand (7, 14 and 28 time periods).

Table 4. Bullwhip effect metrics analysis within the context of the paper and with literature.

	Analysis within the context of the paper				Results comparison with the literature
Metric	Strengths of Metric	Weaknesses of Metric	Suitability of Metric	Reasons for Suitability Classification	Suitability in the paper vs. Suitability in literature
Average inventory	Concise information on inventory investment	Complementary measure of the inventory variance ratio	Yes	Quantifies the average holding cost over the observation time	Cannella and Ciancimino (2010); (+)(S,W,U)
Inventory variance ratio	Quantifies the fluctuations in inventory	Needs information of the average inventory metric	Yes	Gives the variation and the potential increment of the holding cost per unit	Coppini et al. (2010); (+)(S,W,U)
Average order	Gives information on customer demand	Bullwhip in cyclic demand systems	No	Constant behaviour in cyclic demand systems	Not yet studied
Order rate variance ratio	Ability to monitor the magnitude of the bullwhip effect	Only informs whether the forces to amplify demand are stronger or weaker than the forces to attenuate demand	Yes	Provides information on potential unnecessary costs for suppliers	Miragliotta (2006); (+)(S,W,U), Cachon, Randall, and Schmidt (2007); (+)(S,W,U)

and/or suitability (U) of metric. From the breakdown of this column it can be concluded that our results in terms of average inventory; inventory variance ratio; and order rate variance ratio; are aligned with the main literature published in the area – classification (+)(S,W,U); while the average order has not yet been studied in the literature published.

7. Conclusions

The aim of this paper is to minimise operational costs for different common real world conditions while measuring the bullwhip effect which is analysed through the measurement of the order amplification and inventory instability. It addresses an inventory planning problem in multi-period/multi-warehouse/multi-retailer distribution supply where different operational aspects like batch, transshipment, lead time, limited storage and limited transportation, are considered.

Over the time planning horizon a cycle operation is considered and to allow the modelling of a single cycle a wrapping around operator is used to model such operation. Through a mixed linear programming model the system is modelled, leading to a minimisation of the total costs under cyclic demand. The use of such analytic method permits the modelling of complex systems and, for these, obtaining information that can help managers supporting their supply chain operation decisions.

One of the main insights from the developed work is that, in multi-echelon systems under order batching policies with cyclic demand, when trying to measure the bullwhip effect the following metrics should be considered: echelon average inventory, echelon inventory variance ratio and echelon order rate variance ratio. The echelon average order was found to be inadequate. Additionally, the total costs slightly increased with the increase of the batch size for all the structures of the distribution supply chain. However, these costs can be controlled if when ordering large batches based on economies of scale are considered.

Also as the main finding it was observed that the increase of batch size didn't impact the bullwhip effect if optimisation procedures are considered. On the other hand, the study of bullwhip metrics associated with the inventory leads to conclude that a batching policy reduces instability and when using larger batches it may, in general, decrease order variance. Finally, the variance among echelons becomes more balanced, since the cycle length doesn't have a major impact at retailers and warehouses echelons.

A complementary research, with three main family types of products is presented in Appendix 3 in order to analyse whether the conclusions are valid when more than one product is present. It is verified that even when two or more products compete for the capacity the conclusions remain the same.

As future research directions, is suggested the study of real distribution supply chains with more complex structures and also subject to a diverse range of cycle time extents and with different costs and order batching policies, in order to reinforce the achieved conclusions. Another extension of this research could be adding more products that compete for transportation and storage capacities or the relationship between lateral transshipments among both warehouses and retailers with the bullwhip effect. Lastly, uncertainty on products demand could be another option for a follow up to this study.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Bandyopadhyay, S., and R. Bhattacharya. 2013. "A Generalized Measure of Bullwhip Effect in Supply Chain with ARMA Demand Process under Various Replenishment Policies." *The International Journal of Advanced Manufacturing Technology* 68: 963–979.

Barbosa-Póvoa, A. P. 2007. "A Critical Review on the Design and Retrofit of Batch Plants." *Computers and Chemical Engineering* 31: 833–855.

Beamon, B. M. 1998. "Supply Chain Design and Analysis: Models and Methods." *International Journal of Production Economics* 55 (3): 281–294.

Cachon, G. P. 1999. "Managing Supply Chain Demand Variability with Scheduled Ordering Policies." *Management Science* 45: 843–856.

Cachon, G. P., T. Randall, and G. M. Schmidt. 2007. "In Search of the Bullwhip Effect." *Manufacturing and Service Operations Management* 9: 457–479.

Cannella, S., and E. Ciancimino. 2010. "On the Bullwhip Avoidance Phase: Supply Chain Collaboration and Order Smoothing." *International Journal of Production Research* 48: 6739–6776.

Cannella, S., A. P. Barbosa-Póvoa, J. M. Framinan, and S. Relvas. 2013. "Metrics for Bullwhip Effect Analysis." *Journal of the Operational Research Society* 64: 1–16.

Chen, L., and H. L. Lee. 2012. "Bullwhip Effect Measurement and Its Implications." *Operations Research* 60: 771–784.

Chen, F., and R. Samroengraja. 2004. "Order Volatility and Supply Chain Costs." *Operations Research* 52: 707–722.

Chen, F., Z. Drezner, J. K. Ryan, and D. Simchi-Levi. 2000. "Quantifying the Bullwhip Effect in a Simple Supply Chain: The Impact of Forecasting, Lead times, and Information." *Management Science* 46: 436–443.

Coppini, M., C. Rossignoli, T. Rossi, and F. Strozzi. 2010. "Bullwhip Effect and Inventory Oscillations Analysis Using the Beer Game Model." *International Journal of Production Research* 48: 3943–3956.

Disney, S. M., and D. R. Towill. 2002. "A Discrete Transfer Function Model to Determine the Dynamic Stability of a Vendor Managed Inventory Supply Chain." *International Journal of Production Research* 40: 179–204.

Dominguez, R., J. M. Framinan, and S. Cannella. 2014. "Serial Vs. Divergent Supply Chain Networks: A Comparative Analysis of the Bullwhip Effect." *International Journal of Production Research* 52: 2194–2210.

Fu, D., C. Ionescu, E. Aghezzaf, and R. De Keyser. 2015. "Quantifying and Mitigating the Bullwhip Effect in a Benchmark Supply Chain System by an Extended Prediction Self-adaptive Control Ordering Policy." *Computers and Industrial Engineering* 81: 46–57.

Giard, V., and M. Sali. 2013. "The Bullwhip Effect in Supply Chains: A Study of Contingent and Incomplete Literature." *International Journal of Production Research* 51: 3880–3893.

Hussain, M., and P. R. Drake. 2011. "Analysis of the Bullwhip Effect with Order Batching in Multi-Echelon Supply Chains." *International Journal of Physical Distribution and Logistics Management* 41: 797–814.

Lee, H. L., V. Padmanabhan, and S. Whang. 1997. "Information Distortion in a Supply Chain: The Bullwhip Effect." *Management Science* 43: 546–558.

Mikati, N. 2010. "Dependence of Lead Time on Batch Size Studied by a System Dynamics Model." *International Journal of Production Research* 48: 5523–5532.

Miragliotta, G. 2006. "Layers and Mechanisms: A New Taxonomy for the Bullwhip Effect." *International Journal of Production Economics* 104: 365–381.

Moniz, S., A. P. Barbosa-Póvoa, and J. P. de Sousa. 2014. "Simultaneous Regular and Non-regular Production Scheduling of Multipurpose Batch Plants: A Real Chemical-pharmaceutical Case Study." *Computers and Chemical Engineering* 67: 83–102.

Pinto, T., A. P. Barbosa-Póvoa, and A. Q. Novais. 2005. "Optimal Design and Retrofit of Batch Plants with a Periodic Mode of Operation." *Computers and Chemical Engineering* 29: 1293–1303.

Potter, A., and S. M. Disney. 2006. "Bullwhip and Batching: An Exploration." *International Journal of Production Economics* 104: 408–418.

Potter, A., D. Towill, T. Böhme, and S. M. Disney. 2009. "The Influence of Multi-product Production Strategy on Factory Induced Bullwhip." *International Journal of Production Research* 47: 5739–5759.

Pujawan, I. N. 2004. "The Effect of Lot Sizing Rules on Order Variability." *European Journal of Operational Research* 159: 617–635.

Riddalls, C. E., and S. Bennett. 2001. "The Optimal Control of Batched Production and Its Effect on Demand Amplification." *International Journal of Production Economics* 72: 159–168.

Sadeghi, A. 2015. "Providing a Measure for Bullwhip Effect in a Two-product Supply Chain with Exponential Smoothing Forecasts." *International Journal of Production Economics* 169: 44–54.

Shah, N., C. C. Pantelides, and R. W. H. Sargent. 1993. "Optimal Periodic Scheduling of Multipurpose Batch Plants." *Annals of Operations Research* 42: 193–228.

Taticchi, P., P. Garengo, S. Nudurupati, F. Tonelli, and R. Pasqualino. 2015. "A Review of Decision-support Tools and Performance Measurement and Sustainable Supply Chain Management." *International Journal of Production Research* 53 (21): 6473–6494.

Vicente, J. J., S. Relvas, and A. P. Barbosa-Póvoa. 2015a. "Multi-period and Multi-product Inventory Management Model with Lateral Transshipments." 16th National Conference of the APDIO, Bragança, Portugal, 425–444, June 3–5, 2013.

Vicente, J. J., S. Relvas, and A. P. Barbosa-Póvoa. 2015b. "Bullwhip Effect Metrics for Multi-Echelon Systems under Order Batching Policies with Cyclic Demand." International Conference on Industrial Engineering and Systems Management, Seville, Spain, 771–780, October 21–23, 2015.

Vieira, M., T. Pinto-Varela, and A. P. Barbosa-Póvoa. 2015. "Periodic versus Non-periodic Multipurpose Batch Plant Scheduling: A Paint Industry Case Study." *Operational Research.* 445–465.

Wang, X., and S. Disney. 2016. "The Bullwhip Effect: Progress, Trends and Directions." *European Journal of Operational Research* 250: 691–701.

Warburton, R. D. H. 2004. "An Analytical Investigation of the Bullwhip Effect." *Production and Operations Management* 13: 150–160.

Yungao, M., W. Nengmin, C. Ada, H. Yufei, and X. Jinpeng. 2013. "The Bullwhip Effect on Product Orders and Inventory: A Perspective of Demand Forecasting Techniques." *International Journal of Production Research* 51: 281–302.

Appendix 1. Data for the motivational example

Tables A1 to A4 list data for the motivational example, in Section 6.1.

Table A1. General experimental example parameters (euro).

Parameters	Value
ordering cost $(\mathrm{OC}_{ij}, j \in \mathrm{DN}, \forall_i)$	20
holding cost $(\mathrm{HOC}_{ij}, j \in W, \forall_i)$	0.2
holding cost $(\mathrm{HOC}_{ik}, k \in R, \forall_i)$	0.6
holding in transit cost $(\mathrm{HTC}_{i0j}, j \in W, \forall_i)$	0.3
holding in transit cost $(\mathrm{HTC}_{ijk}, j \in W, k \in R, \forall_i)$	0.9
lost sales cost $(\mathrm{LSC}_{ikt}, k \in R, \forall_{it})$	25

Table A2. Unitary product transportation costs for experimental example (euro).

		Warehouse 1	Warehouse 2	Retailer 1	Retailer 2
TRC	Warehouse 0	0.15	0.15	–	–
	Warehouse 1	–	–	0.22	0.52
	Warehouse 2	–	–	0.52	0.22

Table A3. Customer Demand (CD) at the retailers in each time period for cyclic time period demand of 7 time periods for experimental example (unit).

		Retailer 1	Retailer 2
CD	Period 1	13	13
	Period 2	16	17
	Period 3	21	19
	Period 4	24	29
	Period 5	36	39
	Period 6	73	69
	Period 7	77	74

Table A4. Lead Time (LTT) of product for experimental example (time period).

		Warehouse 1	Warehouse 2	Retailer 1	Retailer 2
LTT	Warehouse 0	1	1	–	–
	Warehouse 1	–	–	1	1
	Warehouse 2	–	–	1	1

Appendix 2. Data and computational statistics for the real world case study

Tables B1 to B4 list data for the real world case study, in Section 6.2.

Table B1. General case study parameters (euro).

Parameters	Value
ordering cost ($OC_{ij}, j \in DN, \forall_i$)for s1/s2/s3	20/10/5
holding cost ($HOC_{ij}, j \in W, \forall_i$)	0.2
holding cost ($HOC_{ik}, k \in R, \forall_i$)	0.6
holding in transit cost ($HTC_{i0j}, j \in W, \forall_i$)	0.3
holding in transit cost ($HTC_{ijk}, j \in W, k \in R, \forall_i$)	0.9
holding in transit cost ($HTC_{ijl}, j \in W, l \in W, j \neq l, \forall_i$)	0.3
holding in transit cost ($HTC_{ikm}, k \in R, m \in R, k \neq m, \forall_i$)	0.9
lost sales cost ($LSC_{ikt}, k \in R, \forall_{it}$)	25

Table B2. Unitary product transportation costs for case study (Euro).

		Warehouse 1	Warehouse 2	Retailer 1	Retailer 2	Retailer 3	Retailer 4
TRC	Warehouse 0	0.15	0.52	–	–	–	–
	Warehouse 1	–	0.35	0.22	0.2	0.32	0.38
	Warehouse 2	0.35	–	0.68	0.52	0.34	0.4
	Retailer 1	–	–	–	0.1	0.4	0.65
	Retailer 2	–	–	0.1	–	0.15	0.5
	Retailer 3	–	–	0.4	0.15	–	0.18
	Retailer 4	–	–	0.65	0.5	0.18	–

Table B3. Customer Demand (CD) at the retailers in each time period for cyclic time periodic demand of 7, 14 and 28 time periods for the case study (unit).

		Retailer 1	Retailer 2	Retailer 3	Retailer 4
CD	Period 1	7	6	6	7
	Period 2	8	9	8	8
	Period 3	11	10	10	9
	Period 4	10	16	14	13
	Period 5	20	18	16	21
	Period 6	36	35	37	34
	Period 7	38	36	39	38
	Period 8	6	7	5	8
	Period 9	7	8	6	9
	Period 10	10	11	9	10
	Period 11	12	14	16	13
	Period 12	20	16	18	19
	Period 13	36	36	37	33
	Period 14	39	38	39	38
	Period 15	7	6	6	5
	Period 16	8	9	8	7
	Period 17	11	12	9	8
	Period 18	13	13	17	12
	Period 19	19	20	19	24
	Period 20	35	34	35	36
	Period 21	37	36	36	38
	Period 22	5	8	6	8
	Period 23	9	9	7	9
	Period 24	10	12	9	11
	Period 25	11	15	16	12
	Period 26	22	15	19	21
	Period 27	34	34	36	34
	Period 28	39	37	37	35

Table B4. Lead Time (LTT) of product for case study (time period).

		Warehouse 1	Warehouse 2	Retailer 1	Retailer 2	Retailer 3	Retailer 4
LTT	Warehouse 0	1	1	–	–	–	–
	Warehouse 1	–	1	1	1	1	1
	Warehouse 2	1	–	2	2	1	0
	Retailer 1	–	–	–	1	1	1
	Retailer 2	–	–	1	–	1	1
	Retailer 3	–	–	1	1	–	1
	Retailer 4	–	–	1	1	1	–

Table B5 shows the computational statistics for the model (Section 6.2) with different order batching policies for this case study. The optimal solution was obtained for most of the situations. Nevertheless, it may be seen that the relative gaps, when different than 0, are below 2%. The computational effort increases with the increase of the duration of the cycle applied to the cyclic demand. The dimension of the model is independent of the order batching policy.

Table B5. Computational statistics for the three scenarios of batching for 7/14/28 cyclic time periodic demand of 56 time period planning horizon.

	1 unit batching	5 units batching	10 units batching
MIP solution	10,731.68/10,736.44/10,742.00	8966.00/8986.60/9034.00	8028.80/8098.00/8159.20
Best possible	10,731.68/10,736.44/10,706.51	8966.00/8986.60/8881.58	8028.80/8057.32/8011.53
Relative gap	0%/0%/0.33%	0%/0%/1.69%	0%/0.50%/1.81%
Single equations	943/1881/3757	943/1881/3757	943/1881/3757
Single variables	922/1839/3673	922/1839/3673	922/1839/3673
Discrete variables	497/994/1988	497/994/1988	497/994/1988
Computational time used (s)	4.58/35.45/3600	7.11/800.30/3600	4.94/3600/3600

Appendix 3. Complementary research of real world case study

In this appendix it is presented a complementary research, with three main family types of products, for the case study.

The model was implemented in GAMS 23.5 modelling language and solved using CPLEX 12.2 solver in an Intel Core i7 CPU 2.20 GHz and 8 GB RAM. The stopping criteria were either a computational time limit of 3600 s or the determination of the optimal solution.

The supply chain involves one central warehouse, two regional warehouses and four retailers. Lateral transshipments are allowed in the operation of this company. The maximum storage capacity for warehouses is of 500 and for retailers is of 50 units. The shipping quantity limit is considered between 5 and 80 units. Note that the storage and transportation capacity are limited and shared among products. The safety stock policy is equal to zero for all entities, since a just-in-time policy is being used.

In Appendix 2, Tables B1, B2 and B4 present the parameters' values considered for this complementary research, including general model parameters, transportation costs and lead time. Customer demands for the three products are given by Table C1.

In this complementary research the bullwhip effect, with order batching policies per product, is measured only under the same cycle demand of 7 time periods of 56 time periods planning horizon.

Three different order lot size policies were studied, leading to the following cases: 1 unit batching or without batching (s1), batching of 5 units (s2) and batching of 10 units (s3). Please note that, in Table B1 (Appendix 2), it is considered that ordering different batch sizes implies different costs per ordering. This cost was estimated for this company and is related with the administrative effort taken to put an order. Therefore, when ordering without batching, the ordering cost is higher than with a batching policy.

Table C1. Customer Demand (CD) at the retailers by product (p1, p2 and p3) in each time period for a 7 cyclic time periodic demand of 56 time periods of planning horizon (unit).

		Retailer 1			Retailer 2			Retailer 3			Retailer 4		
	Product	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3
CD	Period 1	7	4	3	6	6	8	6	7	5	7	4	7
	Period 2	8	8	10	9	7	9	8	8	6	8	5	9
	Period 3	11	11	10	10	9	11	10	11	10	9	14	11
	Period 4	10	12	17	16	17	12	14	16	14	13	17	16
	Period 5	20	26	20	18	19	16	16	16	20	21	17	20
	Period 6	36	32	34	35	34	35	37	35	36	34	36	32
	Period 7	38	37	36	36	38	39	39	37	39	38	37	35

Results: costs and number of orders placed

Table C2 provides the total costs and costs per nature for the three scenarios of order batching per product and for a 7 cyclic time period demand over the complete planning horizon (56 time periods). The major difference cost factor among the three order batching scenarios is on the ordering costs. The cost per order is of 20, 10 and 5 euro, respectively, for cases s1, s2 and s3. From the results it can be seen that case s3 presents the lowest total costs, as expected.

Table C2 also presents results per product (p1, p2 and p3) under each order batching policy. So, taking as example the 1 unit batching, the total costs for product1 (p1) are of 11,077 €, for product2 (p2) they are of 10,840 €, while for product3 (p3) are of 10,771 €. No lost sales occur in the system for any of the situations.

One important result to also analyse is the number of orders observed. From Table C3, it can be seen that the total number of orders during the complete time horizon increases when the batch size is increased. This is due to the fact that ordering different batch sizes implies different costs per ordering, this is, when ordering without batching leads to an ordering cost higher than when a batching policy is followed.

Table C2. Costs per nature for the three case of batching per product (p1/p2/p3) for 7 cyclic time periodic demand of 56 time period planning horizon (euro).

	1 unit batching (s1)	5 units batching (s2)	10 units batching (s3)
Ordering	3200.00/3040.00/2880.00	1760.00/1920.00/1680.00	1080.00/1200.00/1200.00
Holding	881.60/963.20/1382.40	688.00/910.40/1022.40	808.00/612.80/856.00
Holding transportation in transit	4344.00/4675.20/4430.40	4416.00/4128.00/4524.00	4272.00/4344.00/4128.00
Holding transshipment in transit	165.60/36.00/0	36.00/216.00/0	144.00/0/0
Transportation	2366.88/2120.00/2078.96	2088.00/2294.80/2066.00	2128.00/2069.60/2350.40
Transshipment	119.60/6.00/0	26.00/60.00/0	92.00/0/0
Lost sales	0/0/0	0/0/0	0/0/0
Total	11,077.68/10,840.40/10,771.76	9014.00/9529.20/9292.40	8524.00/8226.40/8534.40

Table C3. Number of orders for the three case of batching per product (p1/p2/p3) for 7 cyclic time periodic demand of 56 time period planning horizon (euro).

	1 unit batching (s1)	5 units batching (s2)	10 units batching (s3)
Orders	160/152/144	176/192/168	216/240/240

Results: bullwhip analysis in terms of instability

The metrics results used to analyse the bullwhip in terms of instability are shown in Figures C1 and C2. The echelon average inventory by batch size policy and by product is illustrated in Figure C1. In a generalised way, a decrease of the value of warehouses' and retailers' echelon average inventory with the increase of order batching is observed for all products, excepted on 10-unit batching policy for product 1 (p1) in retailers' echelon and for product 3 (p3) in warehouses' echelon.

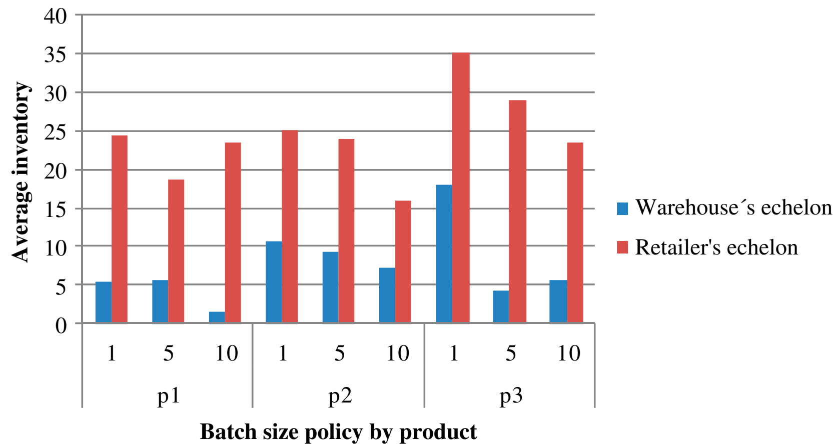


Figure C1. Echelon average inventory by batch size policy (1, 5 and 10 units) and by product (p1, p2 and p3).

Figure C2 presents the echelon inventory variance ratio by batch size policy and by product. The retailer’s echelon inventory variance ratio decreases with the increase of the batch size for all products. The warehouse’s echelon inventory variance ratio has a different behaviour only for product 1 (p1).

Thus, the analysis of the inventory related metrics suggest that using batching policy reduces instability. Note that the products (p1, p2 and p3) compete for the limited capacity of the storage and the transportation.

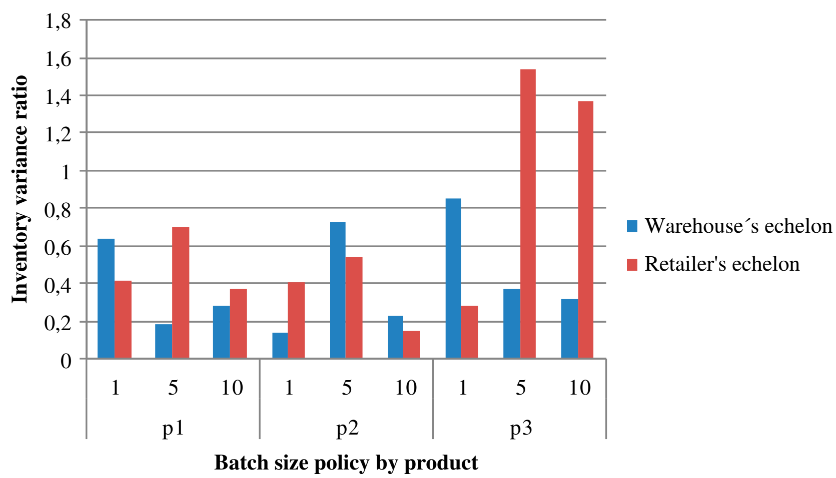


Figure C2. Echelon inventory variance ratio by batch size policy (1, 5 and 10 units) and by product (p1, p2 and p3).

Results: bullwhip analysis in terms of amplification

The results for echelon average order metric are not shown because this metric is not suited for measuring bullwhip in cyclic operations, as it was seen in Section 6.

Figure C3 shows the echelon order rate variance ratio by batch size policy and by product. The warehouses’ and retailers’ echelon order rate variance ratio decreases in general for product 2 (p2) and product 3 (p3) or not increases significantly with the increase of the batch size for product 1 (p1).

Thus, by the analysis of the order related metric it can be said that batching may reduce in general order variance if using larger batches.

Table C4 shows the computational statistics for this complementary research with different order batching policies for 7 time periods cycle demand of 56 time periods of planning horizon. The optimal solution was not obtained for any of the situations. Even so the relative gaps are below 2%.

For analyses with 14 and 28 time periods of cyclic demand a further research is needed to improve model formulation because relative gaps are observed to be higher than 5% after one hour of computation.

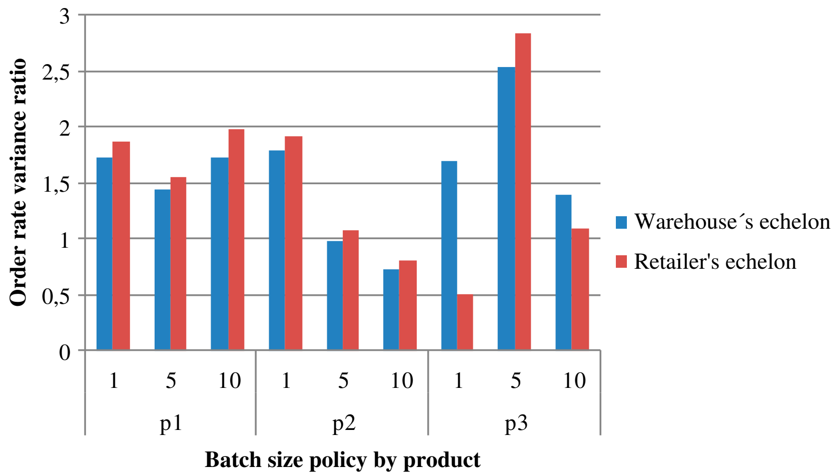


Figure C3. Echelon order rate variance ratio by batch size policy (1, 5 and 10 units) by product (p1, p2 and p3).

Table C4. Computational statistics for the three scenarios of batching for 7 cyclic time periodic demand of 56 time period of planning horizon.

	1 unit batching	5 units batching	10 units batching
MIP solution	32,689.84	27,835.60	25,284.80
Best possible	32,254.68	27,288.13	24,985.48
Relative gap	1.33%	1.97%	1.18%
Single equations	1903	1903	1903
Single variables	2456	2456	2456
Discrete variables	1183	1183	1183
Computational time used (s)	3600	3600	3600