Learning Goals and Difficulty Levels in the Function Dungeon game

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Mathematical content and educational goals

Understanding the subject functions involves e.g. knowledge about its meaning as a mathematical object, recognizing different types of functions and its main properties, interpreting and applying functions in different contexts and within different representations, relating different representations of the same function and using functions as a process. In the Function Dungeon game we focus on linear functions.

The purpose of the Function Dungeon game is to support the understanding of the concept linear functions and practicing specific skills such as relating different function representations and analyzing functions. When using games for learning, the game should be set up targeting specific learning objectives (see below).

Levels of learning and game levels

Levels in the game are meant to enact feelings of progression and accomplishing. They are based on didactical considerations and the taxonomy of Bloom.

The progression is related with the didactical considerations. Informed by theory and our experience as teachers and mathematical educators about the learning of functions we establish a possible learning journey for the student-player:

- playing with the fundamental characteristics of a linear function first individually and then in combination: constant rate of change; y-intercept (start value); sign and magnitude of the slope
- applying the fundamental characteristics to transform one representation (verbal, graph, formula, table) into another in a playful way
- solving quests, tasks and problems involving multiple representations of linear functions (verbal, graph, formula, table)
- Applying properties of linear functions to solve equations in a playful way
    - It has one root (x-intercept, also called the “zero” of the linear function) and one y-intercept
    - ○ The y-intercept of the graph of a linear function is the value of the y-coordinate of the point where it crosses the y-axis. To find the y-intercept, compute f(0); the result is the value of b.
    - ○ The x-intercept of the graph of a linear function is the value of the x-coordinate of the point where it crosses the x-axis. To find the x-intercept, set f(x) = 0, giving equation 0 = mx + b and solve for x. We get x = -b/m (assuming m is not zero).
- Applying the appropriate function representation to solve a question or problem in the game

The taxonomy of Bloom informs the progression in the sense that in each level there are tasks requiring lower (recall, understand, apply) and higher cognitive activities (analyse, evaluate, create).

Goal 1

Understand the fundamental characteristics of a linear function, which are: constant rate of change; y-intercept (start value); sign and magnitude of the slope

Subgoal 1 (S1) *recognizing* constant rate of change	*Task example:* Given a dot-pattern. What is the next figure? How many dots will have the next figure? What is the missing value in the table?
S2 *identifying* the y-intercept	Given three linear graphs: which lines goes through point (0,…) Given three tables with x-values different from zero. Which table contain point (0,…) Given three formulas. Which one contains the point (0,…)
S3 sign of the slope	Which of the following formulas matches with an increasing linear function?
S4 magnitude of the slope	See the road from A to H. What is the slope of the line segment CD? Given the two tables representing different functions. How do their slopes compare to each other?

Goal 2

Apply the properties of linear functions to transform one representation ((verbal, graph, formula, table) into another one.

In G2 the different functions are given but all the function are described by the same representation. A question is asked about a property (y-intercept, sign of slope, magnitude of slope).

Structure for questions belonging to G2 is:

Below you see the [representation] of [x] linear functions (Can we use the phrase linear function?).

S1 connect numerical and graphical representations	

S2 connect verbal and graphical representations	Line ... … with slope [a] … with y-intercept [b] … through (c,d) … slants upwards/downwards
S3 connect numerical and analytical/algebraic representations	numerical (table)
S4 connect graphical and analytical/algebraic representations	
S5 connect verbal and numerical representations	Increasing/decreasing linear function ... … with slope [a] … with y-intercept [b] … through (c,d) …
S6 connect verbal and analytical/algebraic representations	Given the formulas y=ax and y=ax+5 for a certain number a. How do their slopes compare to each other? (Q016) Given the formulas y=ax and y=ax+3 for a certain number a. What do you know about the rate of change (Q017)

Goal 3

Analyse or evaluate multiple representations of linear functions (verbal, graph, formula, table)

S1 interpret, compare and reflect on the y-intercept in 3 or more different representations	Q028 and 4 more variations

S2 interpret, compare and reflect on the slope diraction in 3 or more different representations	Q026 and 4 more variations
S3 interpret, compare and reflect on theslope magnitude in 3 or more different representations	Q027 and 4 more variations
S4 combine the properties	Q024, 25

Goal 4
Apply properties of linear functions to solve equations.

S1 reasoning about slope (parallel and perpendicular)	Q021 variation: Q022,Q043 Q023 variation Q044
S2 reasoning with algebraic representations, eg ax+by=c and y-y1=m(x-x1)	Q020 Q048 variation Q049 Q045
S3 setting up the equation	Q012 variation Q015 Q050
S4 solving equations graphically	Q046 variation Q047

Goal 5
apply the appropriate representation to solve a question or problem

S1 solving word problems	Q008 Q011

S2
finding the pattern