PREPRINTED PROCEEDINGS

21st CONGRESS

International Association for Hydraulic Research

MELBOURNE
19-23 AUGUST 1985

VOLUME 6

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21e CONGRES

Association Internationale de Recherches Hydrauliques

MELBOURNE
19 AU 23 AOÛT 1985

VOLUME 6

Conférences Principales
Rapports Généraux
Rapports des Séminaires d'expérts Invites.

THE INSTITUTION OF ENGINEERS, AUSTRALIA
NATIONAL CONFERENCE PUBLICATION NO. 85/13

21st IAHR Congress,
Melbourne, Australia,
19-23 August 1985

SEMINAR A.

Head — Discharge Relations for Culverts

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SUMMARY Head-discharge relations for circular pipe and box culverts are derived and compared with published design procedures. The relations are all of similar form. The main classifications of culvert flow are inlet control with inlet not submerged, inlet control with inlet submerged and outlet control. Of these, inlet control governs in the great majority of cases. Generalised equations are presented for the head-discharge relation under inlet control.

1 INTRODUCTION

Determination of the culvert size required to convey a design flowrate without exceeding a specified upstream water depth has two contrasting aspects. The cost of each individual culvert is relatively small, however large numbers of culverts are used nationally and the cost of waterway crossings (including culverts) has been estimated at 10% to 40% of road costs (Pilgrim and Cordery, 1974) giving a large total expenditure.

Since the culvert is a relatively minor drainage structure, its hydraulic design needs to be simple and time efficient. However, culvert hydraulics are quite complex and require consideration of uniform flow, varied flow and critical depth as well as full and part full pipe flow. Complete analysis using these techniques can take a considerable time. Fortunately many experimental investigations have been carried out and design procedures are available which simplify the design while maintaining the accuracy of complete analysis techniques.

This paper reviews the hydraulics of culverts, with particular attention on the head-discharge relation. Results obtained using several well known design procedures are compared, and generalised equations are presented for the most common form of culvert flow.

Throughout the text SI units of metres and seconds are used.

2 TYPES OF CULVERT FLOW

Figure 1 shows the range of flow types commonly encounted in culverts. The distinguishing features are inlet submergence by the headwater depth HW and outlet submergence by the tailwater depth TW. The tailwater depth is determined by conditions in the downstream channel, including channel control sections, obstructions, uniform flow and water levels at a downstream confluence. Most natural channels are wide relative to the culvert and the tailwater depth is less than critical depth at the outlet, thus making the tailwater depth ineffective in the head-discharge relation.

The headwater depth HW is set by the level of the upstream energy line required to convey the flowrate through the culvert. Because the flow contracts on entering the culvert the inlet will be submerged only when HW exceeds the culvert height D. This value of HW is generally in the range 1.2D to 1.5D, depending on the particular inlet geometry.

The most important factor in determining the head-discharge relation for a culvert is whether the flow is subject to inlet control or outlet control. The energy line in the headwater pond must provide sufficient energy to convey the flowrate into the culvert inlet, and also sufficient head difference above the tailwater to meet the entrance loss, friction loss in the barrel and exit loss. For a given flowrate, flow control is determined by the larger of the headwater depths required for inlet or outlet control.

Inlet control can occur with inlet submerged and outlet not submerged (Figure 1a). The flow contracts to a supercritical jet immediately downstream of the inlet. If the culvert is laid at a steep

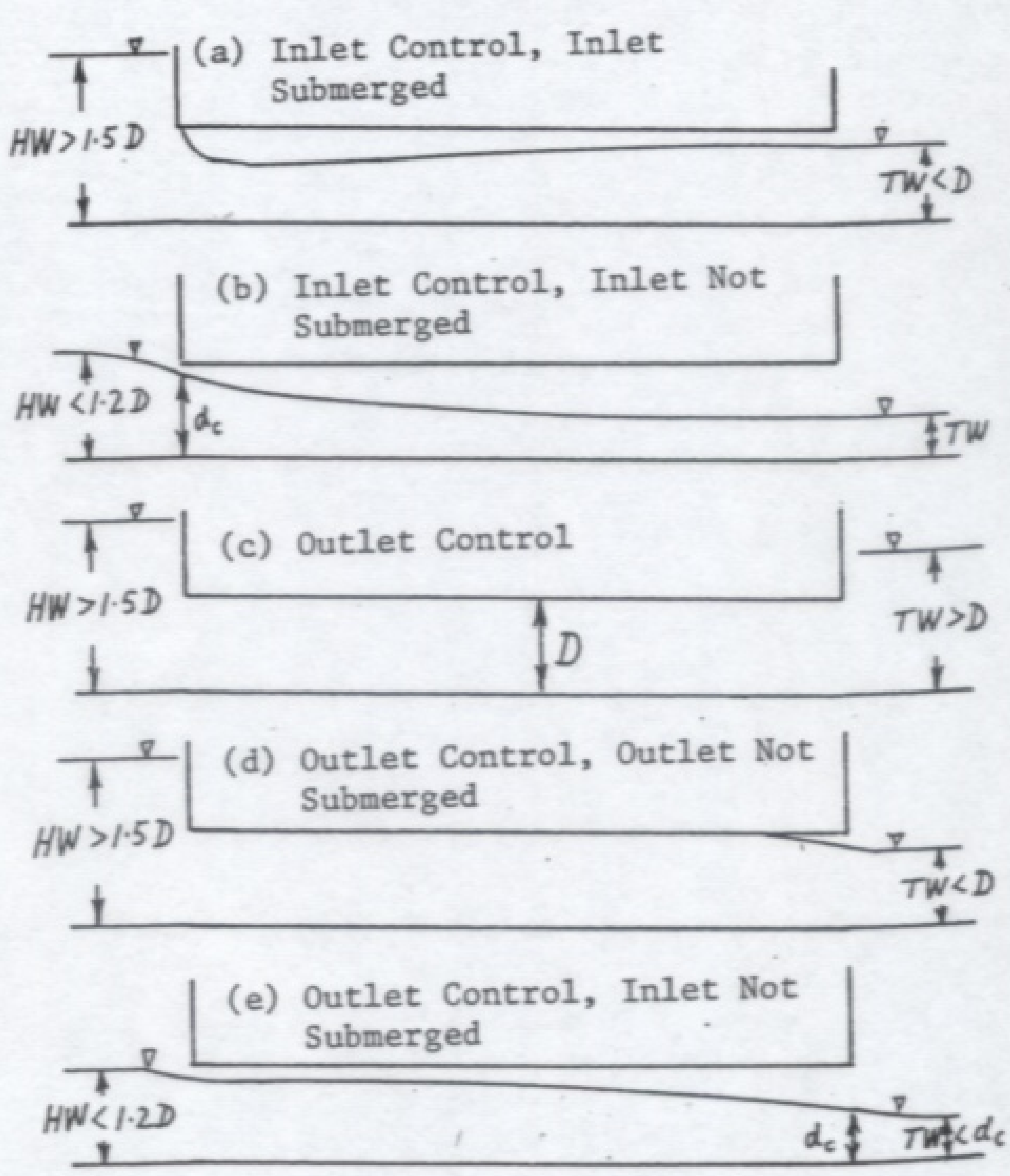


Figure 1 Types of Culvert Flow

grade, flow remains supercritical in the barrel. If the tailwater depth exceeds critical depth, a hydraulic jump can form near the outlet. Inlet control with both inlet and outlet not submerged (Figure 1b) passes through critical depth at the inlet, with supercritical flow in the barrel.

With outlet control and both inlet and outlet submerged (Figure 1c) the culvert flows full under pressure. The culvert can also flow full over part of its length, then part full at the outlet (Figure 1d). The point at which the water surface breaks away from the culvert crown depends on the tailwater depth and culvert grade, and can be determined using backwater calculation. If the culvert is laid at a flat grade, outlet control can occur with both inlet and outlet not submerged, and part full flow throughout the barrel (Figure 1e). In this case critical depth occurs at the outlet and the flow in the barrel is subcritical. Minor variations of these main types can occur, depending on the relative values of critical depth, normal depth, culvert height and tailwater depth.

A further variation is for the culvert to flow part full initially as in Figure 1a with the flow depth increasing until it touches the crown, after which full pipe flow occurs. This may occur in long culverts laid at flat grades. Varied flow calculations can be used to calculate the culvert length and grade at which this occurs, producing curves defining hydraulically long or short culverts, as given in Carter (1957) and reproduced in Chow (1959). These references also contain a full classification of flow types.

3 DISCHARGE WITH INLET CONTROL

3.1 Inlet control, Inlet not submerged

With culverts subject to inlet control, the important factors are the entrance conditions, including the entrance type, existence and angle of headwalls and wingwalls, projection of the culvert into the headwater pond, and the use of hood type inlets.

The head-discharge relation has two distinct regimes, for inlet submerged and not submerged. With the inlet not submerged, the flow passes through critical depth at the inlet and the flowrate can be estimated from this fact. For a box culvert of width B and height D with flowrate Q the critical depth is given by

$$d_c = (Q^2/gB^2)^{0.333} = 0.4672(Q/B)^{0.667} \tag{1}$$

The specific energy E_c, at critical flow in the box culvert is

$$Ec = d_c + V_c{}^2/2g = 1.5d_c = 0.7008(Q/B)^{0.667} \tag{2}$$

For a circular pipe culvert of diameter D (Figure 2a) critical depth occurs when the Froude number equals 1.0

$$\mathsf{F} = Q.B_c{}^{0.5}/g^{0.5}A_c{}^{1.5} = 1 \tag{3}$$

Values of B_c and A_c depend on the culvert diameter D and the critical depth d_c, and equation (3) can be re-arranged to the form

$$Q/g^{0.5}.D^{2.5} = \text{function } (d_c/D) \tag{4}$$

The specific energy at critical depth for the circular culvert is given by $E_c = d_c + V_c{}^2/2g$ and can be calculated for any given values Q, D and d_c. Figure 2(b) plots these relations. The fitted equations of Henderson (1966) describe well the specific energy relation, and the critical depth relation is also closely fitted by the equations shown. Thus for both box and circular pipe culverts the critical depth and specific energy can be calculated for any flowrate Q. If it is assumed that energy losses between the headwater pond and the entrance are negligible, then E_c corresponds to the energy line associated with the headwater. It is usually further assumed that the approach velocity head $V_1{}^2/2g$ is negligible and that the energy line corresponds to the headwater depth HW. This assumption is generally satisfactory, since headwater ponding usually occurs, but it should be noted that in some cases the headwater depth HW may be $V_1{}^2/2g$ less than the calculated value based on E_c.

3.2 Inlet Control, Inlet Submerged

In this case the inlet acts as a flow constriction and the head-discharge relation can be established by considering the energy equation, assuming no energy losses at the inlet.

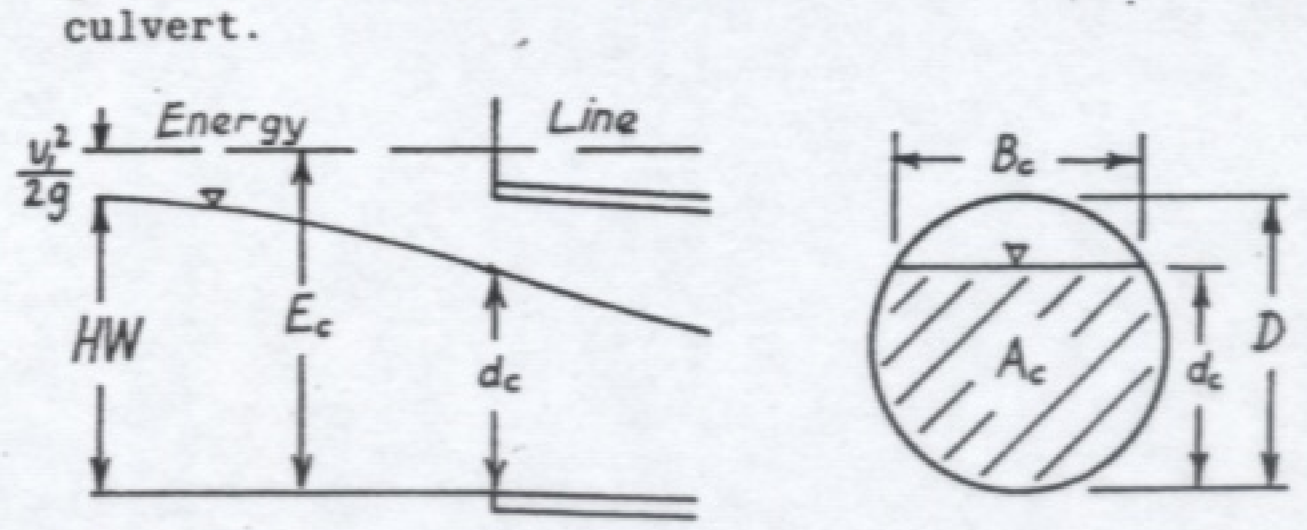
In a box culvert, the inlet can be considered to be a sluice gate (Henderson, 1966). Applying the Bernoulli equation between upstream ponded water and the vena contracta formed immediately downstream of the culvert entrance (Figure 3) gives

$$E = y_2 + V_2{}^2/2g = y_2 + Q^2/2gB^2y_2{}^2 \tag{5}$$

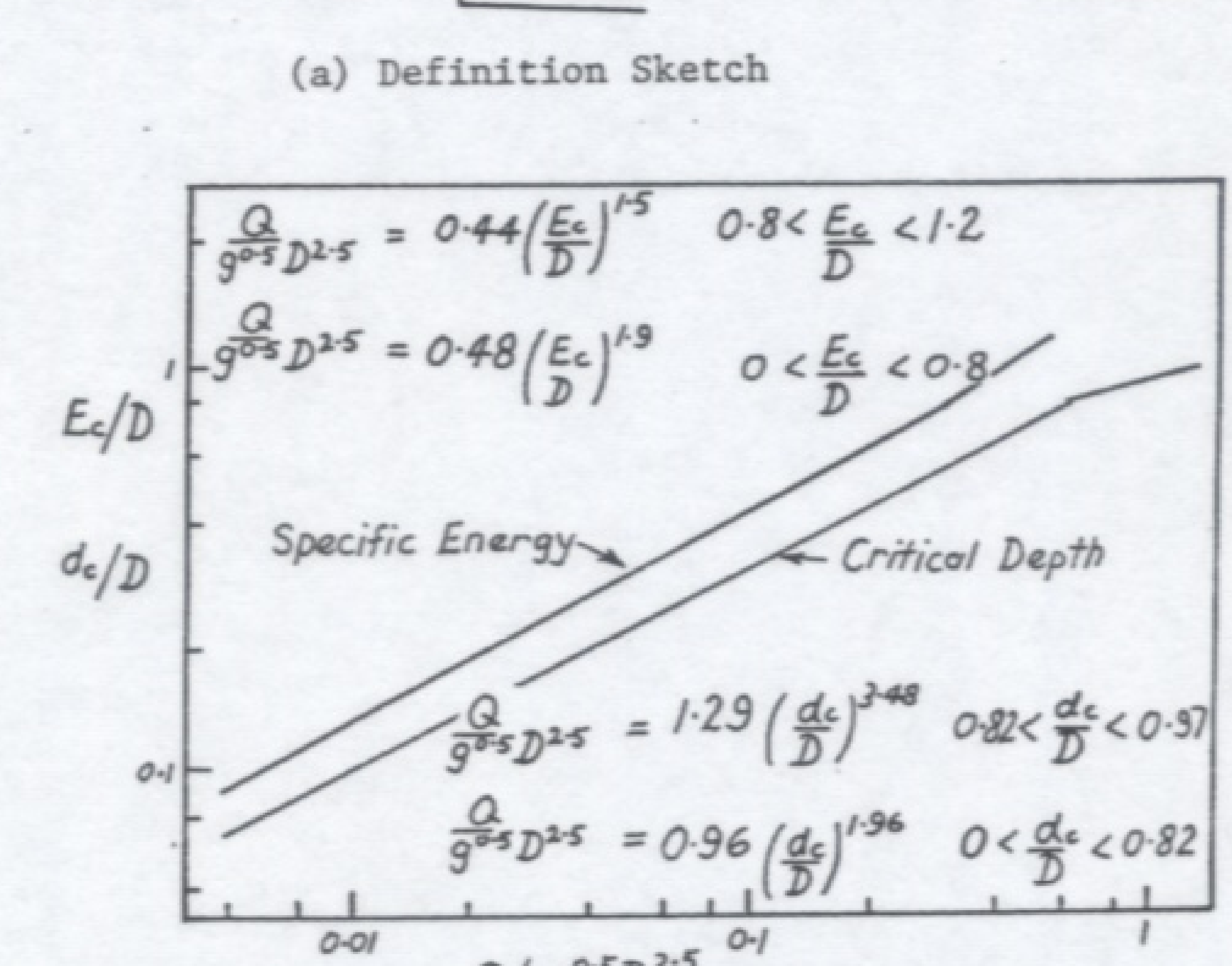
The flow depth at the vena contracta $y_2 = C_c.D$, where C_c is a contraction co-efficient which is close to 0.6 for a square edge entrance. Rearranging (5) gives

$$Q = C_c.B.D\ [2g(E - C_cD)]^{0.5} \tag{6}$$

If the approach velocity is negligible, E can be replaced by the headwater depth HW and equation (6) gives a head-discharge relation for the box culvert.



(a) Definition Sketch



(b) Critical Depth and Critical Energy

Figure 2 Critical Flow in Circular Culvert

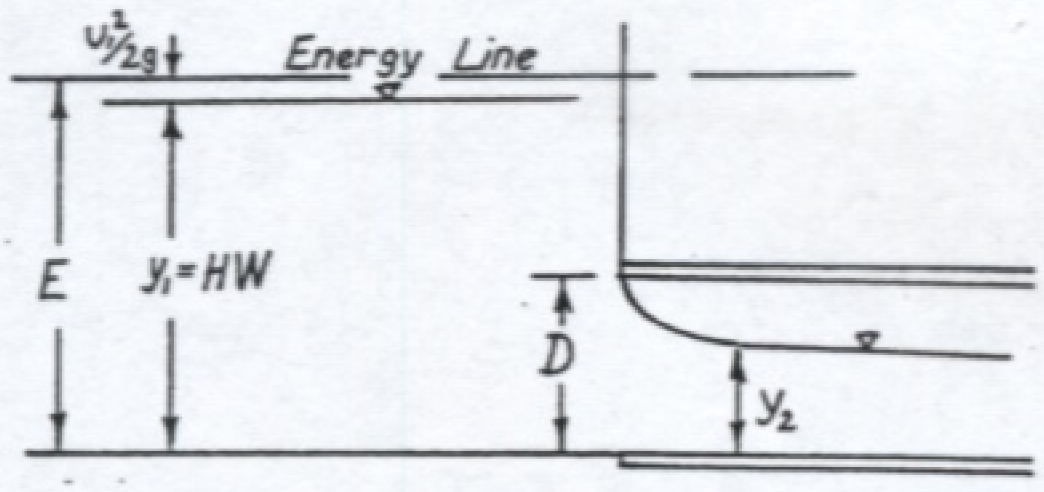


Figure 3 Inlet Control, Inlet Submerged

For a circular pipe culvert, the outlet acts as an orifice with a head-discharge relation.

$$Q = C.\ \frac{\pi D^2}{4}\ [2g(E - C_c D)]^{0.5} \qquad (7)$$

3.3 Experimental results for culverts with inlet control

Equations (6) and (7) for inlet control with submerged inlet and equation (2) and Figure 2b for inlet control with inlet not submerged indicate the form of the head-discharge relation. The actual relation depends on values of the contraction coefficient, depending on the type of inlet and wingwalls. In design practice, head-discharge relations for both box and circular pipe culverts with inlet control are based on the results of various experimental studies. Experimental results of Mavis, Blaisdell, French, Bossy, Straub and Morris, Yarrell et al and the U.S. Geological Survey are presented in references 2,10,7,3 and 4. The design data are generally presented in the form of graphs or nomographs.

Figure 4 shows the head-discharge relation for two box culverts using design graphs of Mavis (Chow, 1959) for square edge inlet, and the U.S. Department of Transportation (1965) for wingwall flare between 30^o and 75^o. All cases cover the full range of inlet submerged and inlet not submerged. Figure 4 also shows equation (2) for inlet not submerged and equation (6) for inlet submerged, with C_c = 0.6. All three sets of data agree closely. In all cases a change of relation occurs at a point in the range 1.2 < HW/D < 1.5 with discharge $Q \propto HW^{1.5}$ in the lower range and approaching $Q \propto HW^{0.6}$ in the upper range.

Figure 5 shows the head-discharge relation for two circular pipe culverts using design graphs of Mavis (Chow, 1959) and the U.S. Department of Transportation (1965). Figure 5 also shows equations presented by Henderson (1966) based on the critical flow E_c/D versus $Q/g^{0.5}D^{2.5}$ relation of Figure 2b. These equations apply only to inlet control with inlet not submerged. Henderson introduced a side contraction coefficient in terms of the culvert grade So, the final equations being

$$Q/g^{0.5}D^{2.5} = 0.48(So/0.4)^{0.05}(HW/D)^{1.9} \qquad 0 < HW/D < 0.8 \qquad (8)$$

$$Q/g^{0.5}D^{2.5} = 0.44(So/0.4)^{0.05}(HW/D)^{1.5} \qquad 0.8 < HW/D < 1.2 \qquad (9)$$

All sets of data agree well. The head-discharge relation changes at a point in the range 1.2 < HW/D < 1.5, with discharge $Q \propto HW^{1.63}$ in the lower range and $Q \propto HW^{0.64}$ in the upper range.

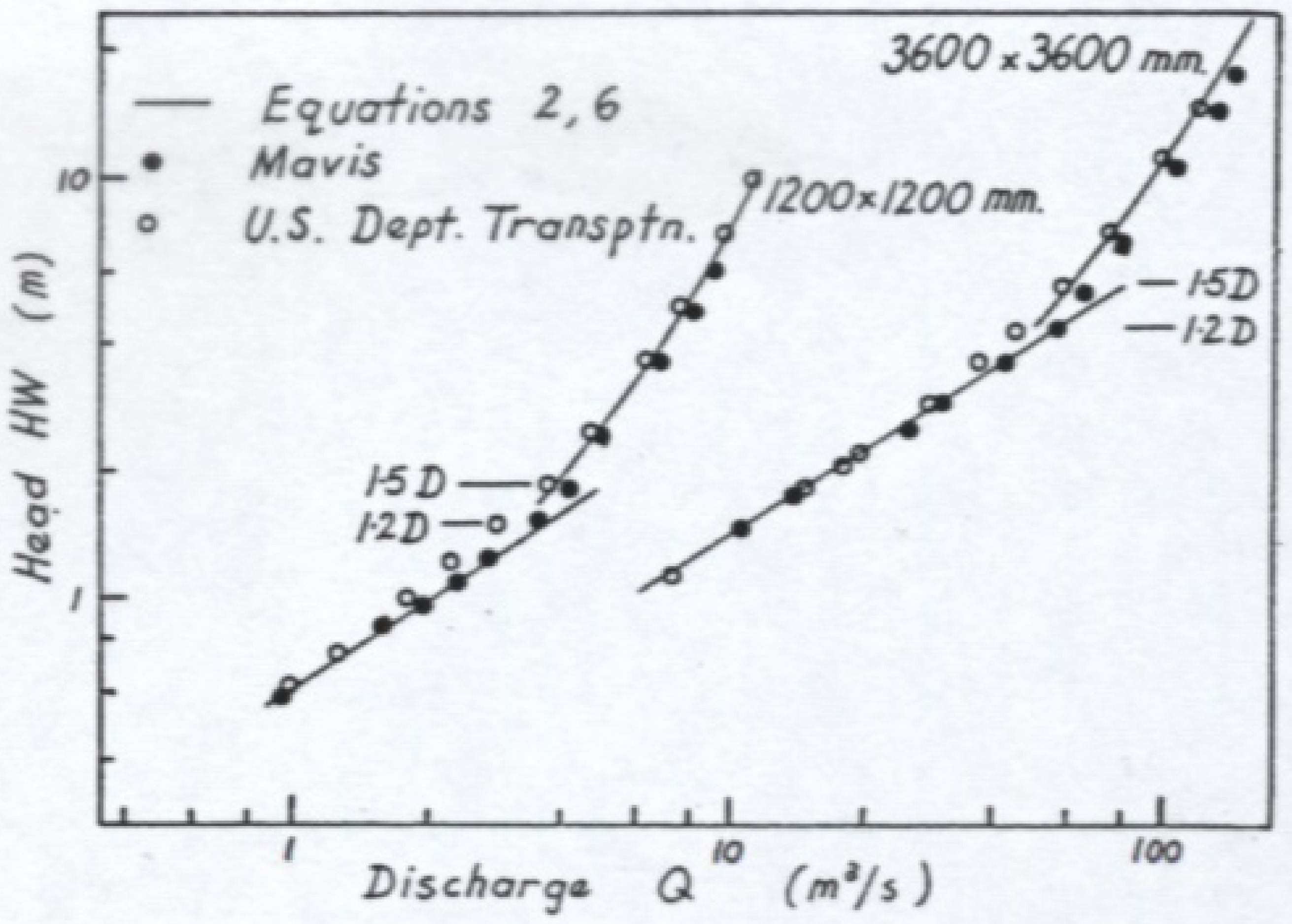


Figure 4 Box Culvert with Inlet Control

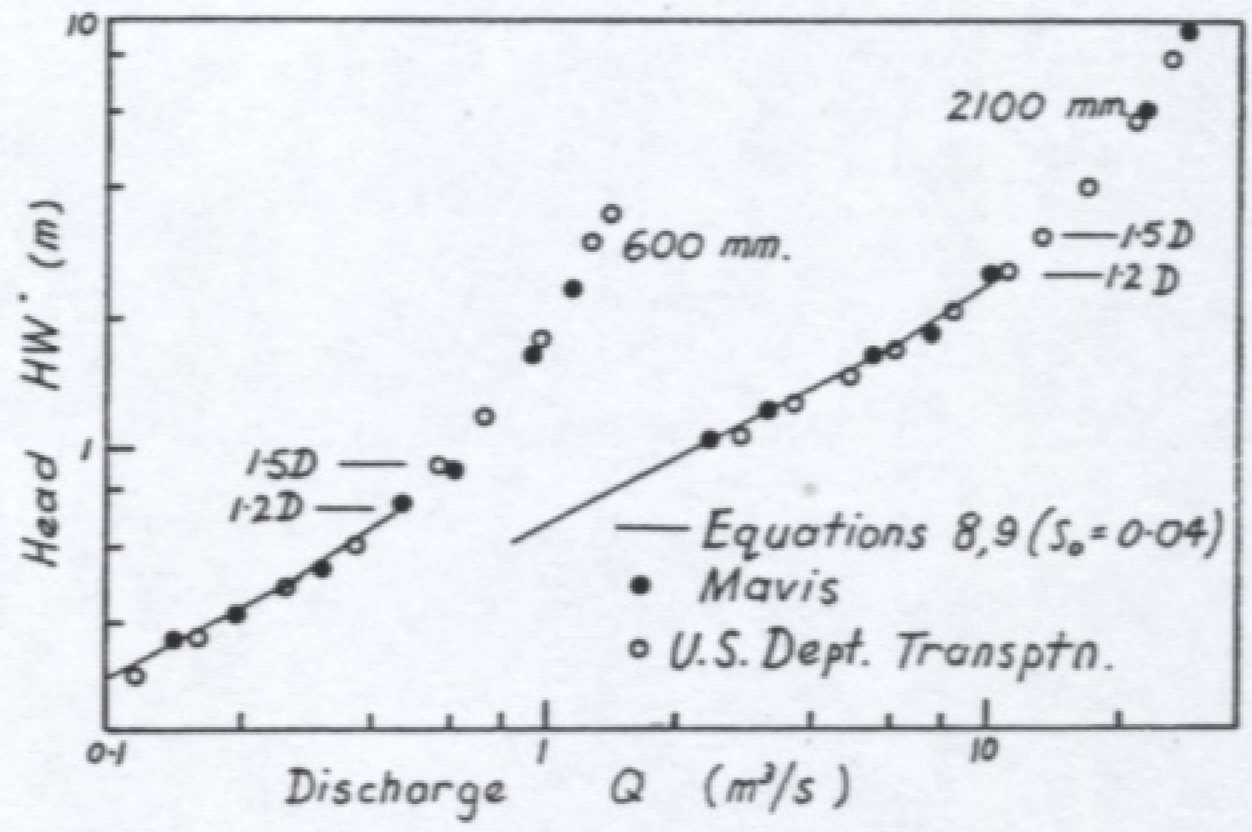


Figure 5 Circular Pipe Culvert, ~~Outlet~~ Inlet Control

4 DISCHARGE WITH OUTLET CONTROL

Culvert flow with outlet control may occur for high tailwater depths TW, and for culverts of large length L laid at flat grade S_o. Culvert discharges are then affected by entrance and exit losses, and friction losses in the barrel. For a culvert of total cross section area A and velocity in the barrel v = Q/A, the equation is derived from Figure 6a. Note that the hydraulic grade line (HGL) coincides with the water surface upstream and downstream of the culvert and is $v^2/2g$ below the energy line (EL) in the barrel.

$$HW = TW + V^2/2g + S_f.L + k_e.V^2/2g - S_o.L \qquad (10)$$

The energy line slope S_f is obtained from Mannings equation

$$S_f = V^2n^2/R_h{}^{1.333} \qquad (11)$$

where the hydraulic radius R_h = cross section area/wetted perimeter and Manning n is near to 0.011 for concrete. Equation (10) then becomes

$$HW = TW + \frac{v^2}{2g}\ (1 + \frac{n^2L.2g}{R_h{}^{1.333}} + k_e) - S_o.L \qquad (12)$$

The second term on the right hand side contains the exit, friction and entrance losses respectively and is called the total head loss H. Nomographs giving H in terms of flowrate Q, n, k_e culvert size

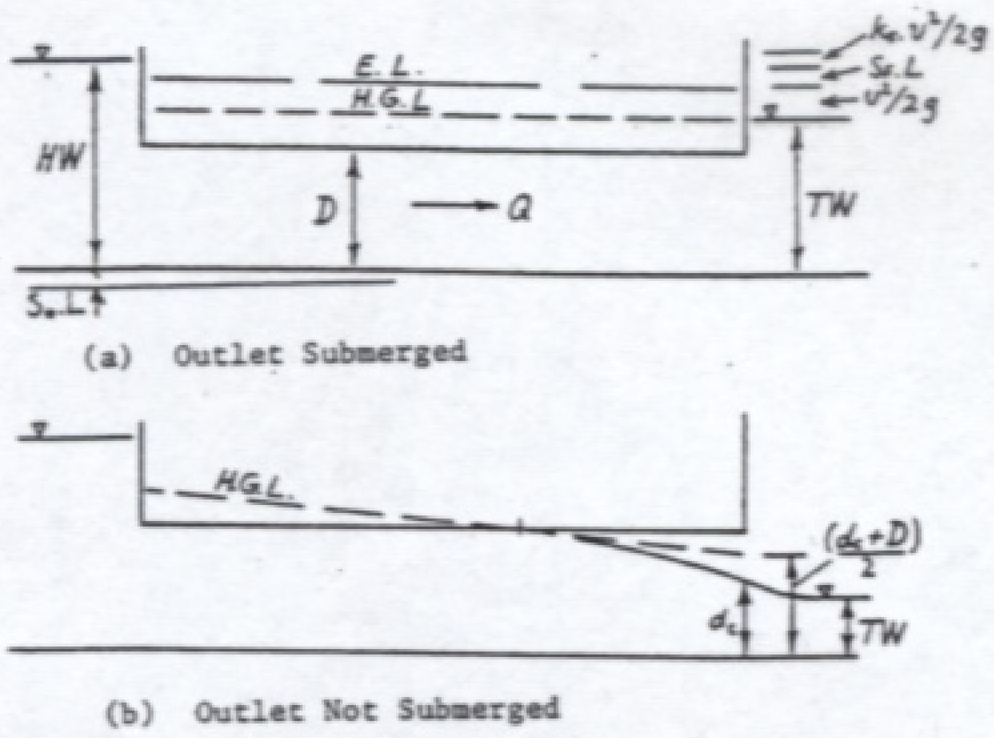


Figure 6 Culvert with Outlet Control

and L have been prepared by the U.S. Department of Transportation (1965). Equation (12) can then be used to calculate the headwater depth HW corresponding to the flowrate if outlet control occurs.

Outlet control may also occur with the outlet unsubmerged (Figure 6b). Two cases are possible. For $TW < d_c$, critical flow occurs at the outlet and the flow depth d_c can be obtained from equation (1) and Figure 2(b) for box and circular pipe culverts respectively. For $d_c < TW < D$ the depth at the outlet equals TW. For both of these cases the headwater depth is still calculated from equation (12) but with TW replaced by an equivalent hydraulic grade line level h_o at the outlet.

The equivalent hydraulic grade line h_o is taken to be the greater of $(dc + D)/2$ and TW, with a maximum value of $h_o = D$. This is based on the assumption that the equivalent hydraulic grade line continues to fall linearly after the water surface leaves the culvert crown, and this has been verified by backwater calculations (U.S. Department of Transportation, 1965).

If $HW < D + (1 + k_e)V^2/2g$ the hydraulic grade line inside the culvert inlet lies below the crown and part full flow occurs throughout the barrel. In the case the previous methods are accurate only for $HW > 0.75D$. For lower values, more accurate results can be obtained by backwater calculations, or from reference (14).

It should be noted that equations (10) and (12) refer to the upstream energy line, and that in some cases the actual headwater depth HW may be $V_1^2/2g$ below the value calculated using these equations.

5 DESIGN PROCEDURE

For a given flowrate and culvert size and type, the required headwater depth can be calculated using the methods in the preceding sections for inlet control and for outlet control. The governing flow control is set by the larger of these headwater depths. Conversely, for a given culvert size and type, and headwater depth, flow control is determined by the smaller of the two calculated flowrates.

The logical sequence of steps in the design process is given in flow chart form by the U.S. Department of Transportation (1965) (Figure 7). Of the various design procedures available, Hydrologic

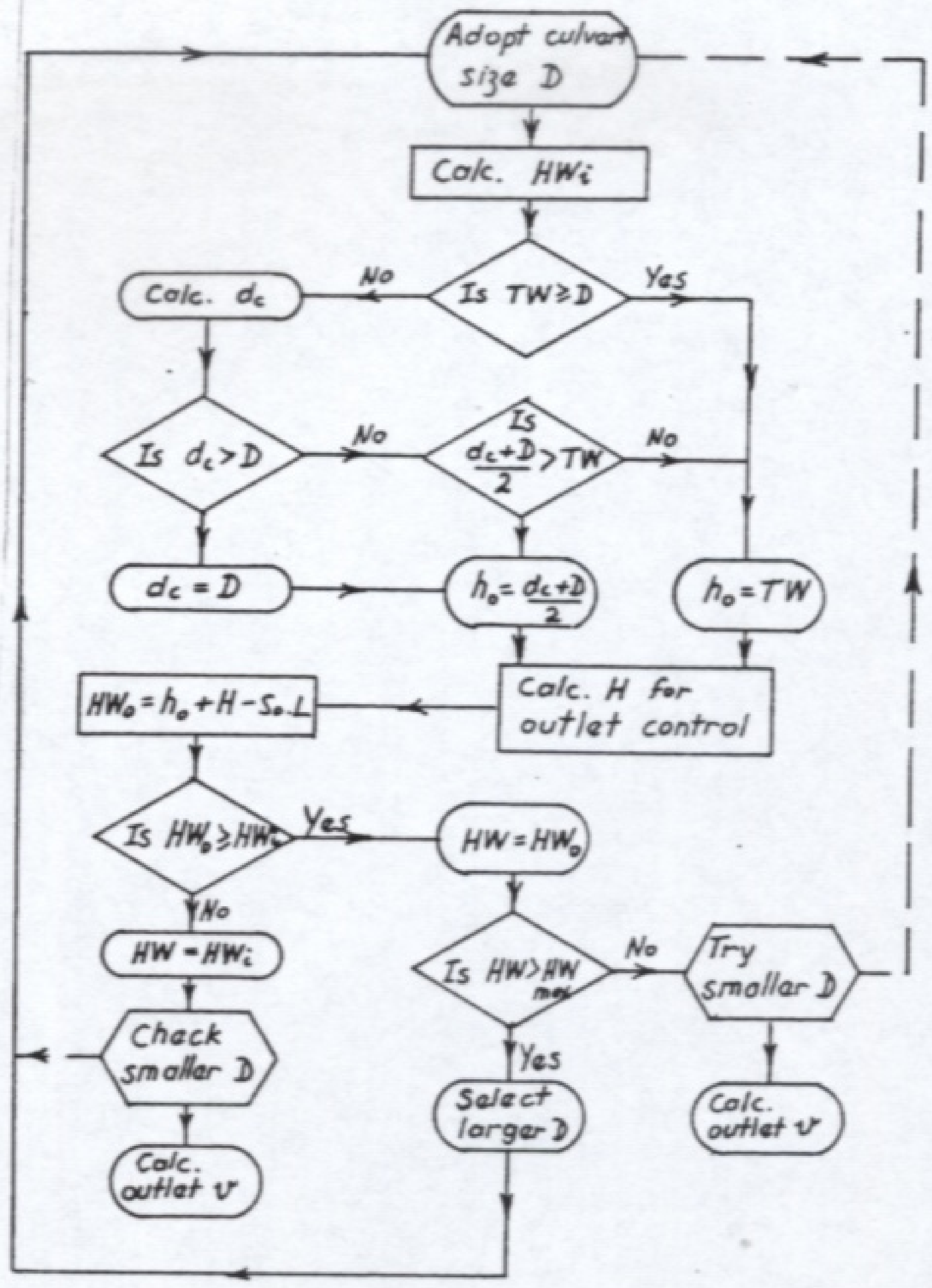


Figure 7 Design Flow Chart

Engineering Circular No.5 of the U.S. Department of Transportation (1965) has gained wide acceptance. The procedure is comprehensive and easy to use. It has been incorporated into design manuals of the Queensland and Western Australia Main Roads Departments and in manufactures guides (References 6, 9 and 12). References 1 and 6 contain extensive bibliographies.

It should be noted that all design charts and equations discussed in the preceeding sections are based on actual internal culvert sizes rather than nominal sizes, and manufacturer's brochures should be consulted.

Finally, it should be noted that inlet control governs in the great majority of cases. For these, generalised equations can be fitted to the HEC 5 nomographs. For box culverts with wingwall flare between 30^o and 75^o:

For $HW/D < 1.35$ $\quad Q = 1.70\ B.HW^{1.50}$ (13)

For $HW/D > 1.35$ $\quad Q = 2.20\ B.D^{0.89}HW^{0.61}$ (14)

For circular pipe culverts, square edge entrance with headwall:

For $HW/D < 1.2$ $\quad Q = 1.32\ D^{0.87}HW^{1.63}$ (15)

For $HW/D > 1.2$ $\quad Q = 1.62\ D^{1.87}HW^{0.63}$ (16)

Equation (13) is identical with the critical depth relation (equation 2) and applies to all culvert heights D. Figures (8) and (9) show that all four

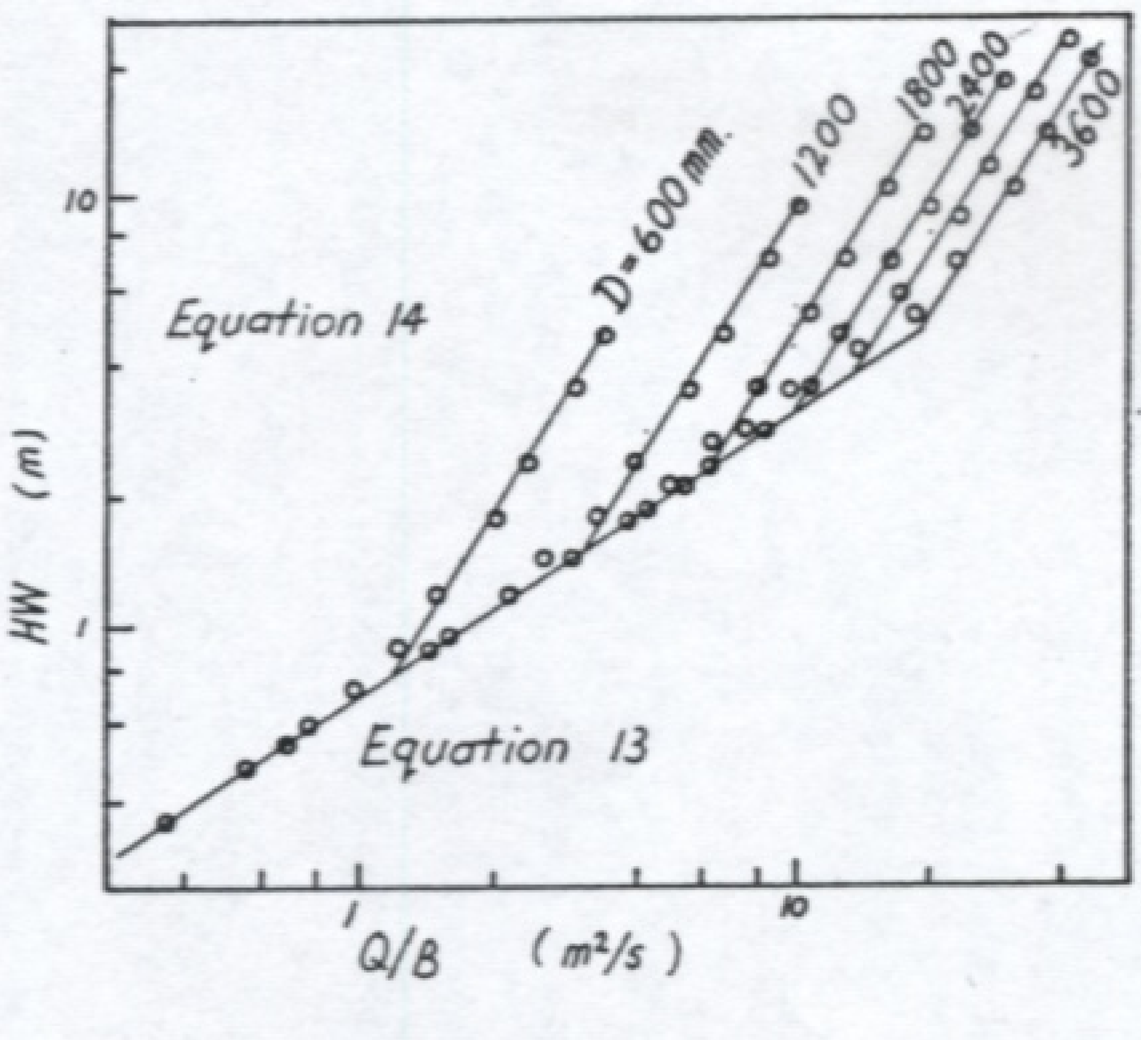


Figure 8 Equations for Box Culverts, Inlet Control

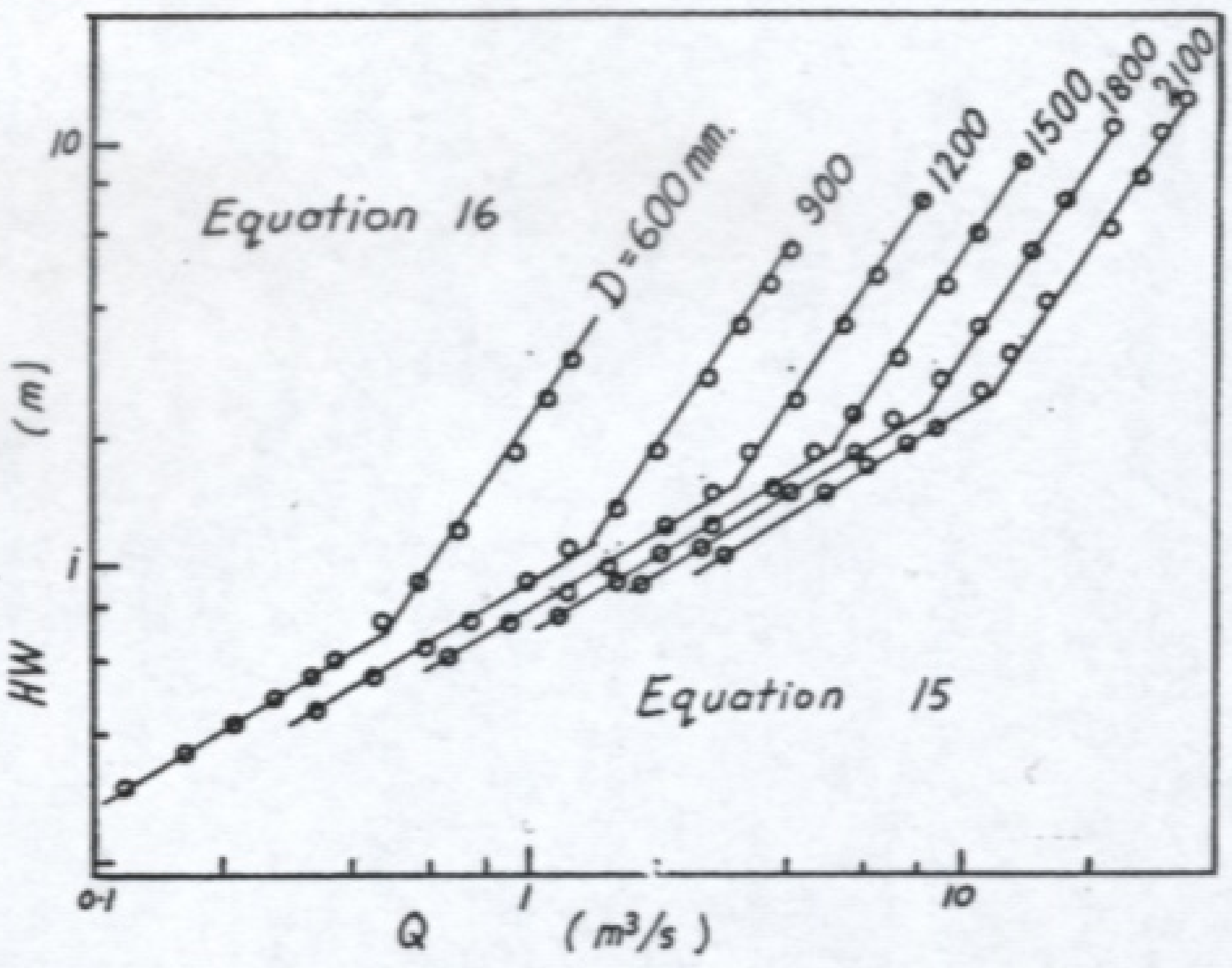


Figure 9 Equations for Circular Pipe Culverts, Inlet Control

equations fit the data well over the range $0.3 < HW/D < 8$, with only slight deviation at the change of relation.

7 CONCLUSIONS

The hydraulic behaviour of culverts is complicated, involving a range of quite different flow conditions. Fortunately, considerable experimental data are available to verify the theoretical analyses. Design procedures based on the experimental data are also available.

The major distinguishing feature in culvert hydraulics is whether the flow is subject to inlet control or to outlet control. For inlet control two distinct regimes exist, depending on whether the inlet is submerged or not submerged. Theoretical and experimental results are consistent and show a change in the head-discharge relation as the inlet becomes submerged. Outlet control is less common, occurring for long culverts, laid at flat grades and with high tailwater depths.

For the more common flow with inlet control, generalised head-discharge relations have been presented, based on the U.S. Department of Transportation (1965) nomographs.

6 ACKNOWLEDGEMENT

The assistance of the Department of Civil and Mining Enginering, University of Wollongong, in the preparation of this paper is acknowledged.

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