Designation: E2022 – 16

Standard Practice for
Calculation of Weighting Factors for Tristimulus Integration[1]

This standard is issued under the fixed designation E2022; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice describes the method to be used for calculating tables of weighting factors for tristimulus integration using custom spectral power distributions of illuminants or sources, or custom color-matching functions.

1.2 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to its use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E284 Terminology of Appearance

E308 Practice for Computing the Colors of Objects by Using the CIE System

E2729 Practice for Rectification of Spectrophotometric Bandpass Differences

2.2 *CIE Standard:*

CIE Standard S 002 Colorimetric Observers[3]

3. Terminology

3.1 *Definitions*—Appearance terms in this practice are in accordance with Terminology E284.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *illuminant, n*—real or ideal radiant flux, specified by its spectral distribution over the wavelengths that, in illuminating objects, can affect their perceived colors.

3.2.2 *source, n*—an object that produces light or other radiant flux, or the spectral power distribution of that light.

3.2.2.1 *Discussion*—A source is an emitter of visible radiation. An illuminant is a table of agreed spectral power distribution that may represent a source; thus, Illuminant A is a standard spectral power distribution and Source A is the physical representation of that distribution. Illuminant D65 is a standard illuminant that represents average north sky daylight but has no representative source.

3.2.3 *spectral power distribution, SPD, S(λ), n*—specification of an illuminant by the spectral composition of a radiometric quantity, such as radiance or radiant flux, as a function of wavelength.

4. Summary of Practice

4.1 CIE color-matching functions are standardized at 1-nm wavelength intervals. Tristimulus integration by multiplication of abridged spectral data into sets of weighting factors occurs at larger intervals, typically 10-nm; therefore, intermediate 1-nm interval spectral data are missing, but needed.

4.2 Lagrange interpolating coefficients are calculated for the missing wavelengths. The Lagrange coefficients, when multiplied into the appropriate measured spectral data, interpolate the abridged spectrum to 1-nm interval. The 1-nm interval spectrum is then multiplied into the CIE 1-nm color-matching data, and into the source spectral power distribution. Each separate term of this multiplication is collected into a value associated with a measured spectral wavelength, thus forming weighting factors for tristimulus integration.

5. Significance and Use

5.1 This practice is intended to provide a method that will yield uniformity of calculations used in making, matching, or controlling colors of objects. This uniformity is accomplished by providing a method for calculation of weighting factors for tristimulus integration consistent with the methods utilized to obtain the weighting factors for common illuminant-observer combinations contained in Practice E308.

5.2 This practice should be utilized by persons desiring to calculate a set of weighting factors for tristimulus integration who have custom source, or illuminant spectral power distributions, or custom observer response functions.

[1] This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.04 on Color and Appearance Analysis.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from CIE (International Commission on Illumination), http://www.cie.co.at or http://www.techstreet.com.

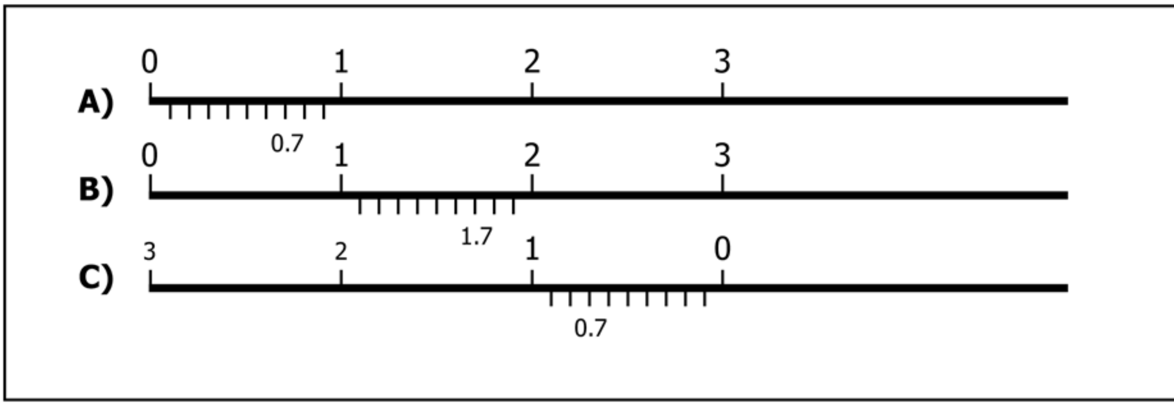

FIG. 1 The Values of *i* in Eq 1 are Plotted Above the Abscissa and the Values of *r* are Plotted Below for A) the First Measurement Interval; B) the Intermediate Measurement Intervals; and, C) the Last Measurement Interval Being Interpolated

6. Procedure

6.1 *Calculation of Lagrange Coefficients*—Obtain by calculation, or by table look-up, a set of Lagrange interpolating coefficients for each of the missing wavelengths.[4]

6.1.1 The coefficients should be quadratic (three-point) in the first and last missing interval, and cubic (four-point) in all intervals between the first and the last missing interval.

6.1.2 *Generalized Lagrange Coefficients*—Lagrange coefficients may be calculated for any interval and number of missing wavelengths by Eq 1:

$$L_j(r) = \prod_{i=0\, i\neq j}^{n} \frac{(r - r_i)}{(r_j - r_i)}, \text{ for } j = 0,1,\ldots n \tag{1}$$

where:

n = degree of coefficients being calculated,[5]
i and j = indices denoting the location along the abscissa,
π = repetitive multiplication of the terms in the numerator and the denominator, and
indices of the interpolant, r = chosen on the same scale as the values i and j.

6.1.2.1 Fig. 1 assist the user in selecting the values of *i, j*, and *r* for these calculations.

6.1.2.2 Eq 1 is general and is applicable to any measurement interval or interpolation interval, regular or irregular.

6.1.3 *10-nm Lagrange Coefficients*—Where the measured spectral data have a regular or constant interval, the equation reduces to the following:

$$L_0 = \frac{(r-1)(r-2)(r-3)}{-6} \tag{2}$$

$$L_1 = \frac{(r)(r-2)(r-3)}{2} \tag{3}$$

$$L_2 = \frac{(r-1)(r)(r-3)}{-2} \tag{4}$$

$$L_3 = \frac{(r-1)(r-2)(r)}{6} \tag{5}$$

for the cubic case, and to

$$L_0 = \frac{(r-1)(r-2)}{2} \tag{6}$$

$$L_1 = \frac{(r)(r-2)}{-1} \tag{7}$$

$$L_2 = \frac{(r-1)(r)}{2} \tag{8}$$

for the quadratic case. In each of the above equations, as many or as few values of *r* as required are chosen to generate the necessary coefficients.

6.1.3.1 Eq 2-8 are applicable when the spectral data are abridged at 10-nm intervals, and the interpolated interval is regular with respect to the measurement interval, presumably 1-nm.

6.1.4 Tables 1 and 2 provide both quadratic and cubic Lagrange coefficients for 10-nm intervals.

6.2 With the Lagrange coefficients provided, the intermediate missing spectral data may be predicted as follows:

$$P(\lambda) = \sum_{i=0}^{n} L_i m_i \tag{9}$$

where:

P = the value being interpolated at interval λ,
L = the Lagrange coefficients, and
m = the measured abridged spectral values.

TABLE 1 The Lagrange Quadratic Interpolation Coefficients Applicable to the First and Last Missing Interval for Calculation of 10-nm Weighting Factors for Tristimulus Integration

Index of Missing Wavelength	L_0	L_1	L_2
1	0.855	0.190	–0.045
2	0.720	0.360	–0.080
3	0.595	0.510	–0.105
4	0.480	0.640	–0.120
5	0.375	0.750	–0.125
6	0.280	0.840	–0.120
7	0.195	0.910	–0.105
8	0.120	0.960	–0.080
9	0.055	0.990	–0.045

[4] Hildebrand, F. B., *Introduction to Numerical Analysis*, Second Edition, Dover, New York, 1974, Chapter 3.

[5] Fairman, H. S., "The Calculation of Weight Factors for Tristimulus Integration," *Color Research and Application*, Vol 10, 1985, pp. 199–203.

TABLE 2 The Lagrange Cubic Interpolation Coefficients Applicable to the Interior Missing Intervals for Calculation of 10-nm Weighting Factors for Tristimulus Integration

Index of Missing Wavelength	L_0	L_1	L_2	L_3
1	–0.0285	0.9405	0.1045	–0.0165
2	–0.0480	0.8640	0.2160	–0.0320
3	–0.0595	0.7735	0.3315	–0.0455
4	–0.0640	0.6720	0.4480	–0.0560
5	–0.0625	0.5625	0.5625	–0.0625
6	–0.0560	0.4480	0.6720	–0.0640
7	–0.0455	0.3315	0.7735	–0.0595
8	–0.0320	0.2160	0.8640	–0.0480
9	–0.0165	0.1045	0.9405	–0.0285

Because the measured spectral values are as yet unknown, it may be best to consider this equation in its expanded form:

$$P(\lambda) = L_0 m_0 + L_1 m_1 + L_2 m_2 + L_3 m_3 \quad (10)$$

6.3 Multiply each $P(\lambda)$ by the 1-nm interval relative spectral power of the source or illuminant being considered.

6.3.1 It may be necessary to interpolate missing values of the source spectral power distribution $S(\lambda)$, if the source has been measured at other than 1-nm intervals.

6.3.2 Doing so results in the following equation:

$$S(\lambda)P(\lambda) = S(\lambda)L_0 m_0 + S(\lambda)L_1 m_1 + S(\lambda)L_2 m_2 + S(\lambda)L_3 m_3 \quad (11)$$

6.4 Multiply the weighted power at each 1-nm wavelength by the appropriate custom color-matching function value for that wavelength. Using the CIE color-matching functions as an example, obtain the CIE 1-nm data from CIE Standard S 002, Colorimetric Observers. Doing so results in the following equation:

$$\bar{x}(\lambda)S(\lambda)P(\lambda) = [\bar{x}(\lambda)S(\lambda)L_0]m_0 + [\bar{x}(\lambda)S(\lambda)L_1]m_1 + [\bar{x}(\lambda)S(\lambda)L_2]m_2 + [\bar{x}(\lambda)S(\lambda)L_3]m_3 \quad (12)$$

where:

$\bar{x}(\lambda)$ = the value of the CIE X color-matching function at wavelength λ, and the calculations are carried out for each of the three CIE color-matching functions, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$.

6.5 In the four terms on the right-hand side of this equation, the numerical values of the three factors in the brackets are known and should be multiplied into a single coefficient. The fourth factor, m_i, in each of the four additive terms is associated with a different measured wavelength.

6.6 Add all multiplicative coefficients dependent upon each different measured wavelength into a single coefficient applicable to that wavelength. This results in a single set of weighting factors that then will contain one value for each measured wavelength in each of three color-matching functions. The partial contribution to the tristimulus value at wavelength m_0 is:

$$[(\bar{x}(\lambda_0)S(\lambda_0)L_0) + (\bar{x}(\lambda_1)S(\lambda_1)L_0) + \ldots]m_0 = wt_0 m_0 \quad (13)$$

6.7 Normalize the weighting factors by calculating the following normalizing coefficient:

$$k = \frac{100}{\sum S(\lambda)\bar{y}(\lambda)} \quad (14)$$

where:

k = the normalizing coefficient,
$S(\lambda)$ = the power in the 1-nm spectrum, and
$y(\lambda)$ = the CIE Y color-matching function.

6.8 Multiply the weighting factors by k to normalize the set to $Y = 100$ for the perfect reflecting diffuser.

6.9 Beginning in January of 2010, rectification of bandpass differences is no longer accomplished by building the correction factors into a weight set for tristimulus integration. This is because to do so fails to correct the spectrum itself and corrects only the tristimulus values. Bandpass rectification is now under the jurisdiction of Practice E2729.

7. Precision

7.1 The precision of the practice is limited only by the precision of the data provided for the source spectral power distribution. The CIE color-matching functions are precise to six digits by definition. The Lagrange coefficients are precise to seven digits.

8. Keywords

8.1 color-matching functions; illuminant; illuminant-observer weights; source; tristimulus weighting factors

APPENDIXES

(Nonmandatory Information)

X1. EXAMPLE OF THE CALCULATIONS

X1.1 Table X1.1 gives the spectral power distribution (SPD) of a typical 3-band fluorescent lamp with a correlated color temperature of about 3000K. The first step is to multiply each value of the SPD by the appropriate CIE color matching function ($\bar{y}$ in this case), wavelength by wavelength, which is shown inTable X1.2 for three spectral regions: near 360 nm, 560 nm, and 830 nm. Table X1.3 shows a typical interpolation of a measured reflectance curve from a 10-nm reported interval to the 1-nm interval that matches the SPD-$\bar{y}$ product in the same three spectral regions. Tables X1.4-X1.6 illustrate how the same measured data, used to interpolate the missing reflectance data in several different intervals, can be combined with the illuminant-color matching function product to form a single weight at a single measurement point. Finally, Table X1.7 shows the resulting weight set for this 3000K source and the 1964 10° color matching functions. Table X1.7 is compatible with Tables 5 in Practice E308.

TABLE X1.1 Spectral Power Distribution of Typical 3-Band Fluorescent Lamp with Correlated Color Temperature of 3000 K (1-nm measurement interval)

λ	SPD	λ	SPD	λ	SPD	λ	SPD	λ	SPD	λ	SPD
360	0.004880	450	0.014870	540	0.162400	630	0.111200	720	0.004410	810	0.000000
361	0.004595	451	0.015040	541	0.277600	631	0.102900	721	0.003505	811	0.000000
362	0.004310	452	0.015210	542	0.392800	632	0.094620	722	0.002600	812	0.000000
363	0.020290	453	0.014980	543	0.353900	633	0.062350	723	0.002470	813	0.000000
364	0.036270	454	0.014750	544	0.315100	634	0.030080	724	0.002340	814	0.000000
365	0.047350	455	0.014370	545	0.429800	635	0.027420	725	0.002375	815	0.000000
366	0.058440	456	0.014000	546	0.544600	636	0.024770	726	0.002410	816	0.000000
367	0.031870	457	0.014060	547	0.383500	637	0.023050	727	0.002450	817	0.000000
368	0.005300	458	0.014110	548	0.222500	638	0.021330	728	0.002490	818	0.000000
369	0.004700	459	0.013930	549	0.182100	639	0.020750	729	0.001795	819	0.000000
370	0.004100	460	0.013760	550	0.141700	640	0.020170	730	0.001100	820	0.000000
371	0.003785	461	0.013470	551	0.113500	641	0.019920	731	0.001120	821	0.000000
372	0.003470	462	0.013180	552	0.085290	642	0.019660	732	0.001140	822	0.000000
373	0.003540	463	0.013470	553	0.070050	643	0.019740	733	0.001750	823	0.000000
374	0.003610	464	0.013750	554	0.054810	644	0.019810	734	0.002360	824	0.000000
375	0.003615	465	0.014000	555	0.046030	645	0.019550	735	0.002190	825	0.000000
376	0.003620	466	0.014250	556	0.037250	646	0.019280	736	0.002020	826	0.000000
377	0.004210	467	0.013810	557	0.034310	647	0.019080	737	0.003930	827	0.000000
378	0.004800	468	0.013370	558	0.031360	648	0.018880	738	0.005840	828	0.000000
379	0.005170	469	0.012870	559	0.030480	649	0.030460	739	0.003355	829	0.000000
380	0.005540	470	0.012370	560	0.029590	650	0.042050	740	0.000870	830	0.000000
381	0.005240	471	0.012640	561	0.029650	651	0.034870	741	0.002235		
382	0.004940	472	0.012900	562	0.029700	652	0.027690	742	0.003600		
383	0.004615	473	0.012640	563	0.029530	653	0.024990	743	0.002500		
384	0.004290	474	0.012380	564	0.029360	654	0.022290	744	0.001400		
385	0.003750	475	0.011680	565	0.029200	655	0.020120	745	0.002155		
386	0.003210	476	0.010970	566	0.029040	656	0.017950	746	0.002910		
387	0.003050	477	0.011050	567	0.029500	657	0.019130	747	0.002970		
388	0.002890	478	0.011130	568	0.029960	658	0.020320	748	0.003030		
389	0.002980	479	0.012680	569	0.029480	659	0.017400	749	0.003615		
390	0.003070	480	0.014240	570	0.029000	660	0.014470	750	0.004200		
391	0.002795	481	0.019080	571	0.029140	661	0.020750	751	0.003470		
392	0.002520	482	0.023910	572	0.029280	662	0.027030	752	0.002740		
393	0.002395	483	0.035600	573	0.029390	663	0.022910	753	0.002225		
394	0.002270	484	0.047290	574	0.029500	664	0.018790	754	0.001710		
395	0.002285	485	0.064030	575	0.040510	665	0.015270	755	0.000855		
396	0.002300	486	0.080770	576	0.051530	666	0.011740	756	0.000000		
397	0.002420	487	0.082540	577	0.060840	667	0.012890	757	0.000310		
398	0.002540	488	0.084310	578	0.070160	668	0.014040	758	0.000620		
399	0.002640	489	0.073870	579	0.079050	669	0.013040	759	0.000310		
400	0.002740	490	0.063440	580	0.087930	670	0.012030	760	0.000000		
401	0.002845	491	0.059500	581	0.090370	671	0.012230	761	0.000000		
402	0.002950	492	0.055560	582	0.092820	672	0.012430	762	0.000000		
403	0.062430	493	0.049350	583	0.098470	673	0.011550	763	0.000000		
404	0.121900	494	0.043140	584	0.104100	674	0.010680	764	0.000000		
405	0.085640	495	0.038320	585	0.102800	675	0.010140	765	0.000000		
406	0.049360	496	0.033490	586	0.101400	676	0.009600	766	0.000000		
407	0.032040	497	0.030100	587	0.113700	677	0.009705	767	0.000000		
408	0.014720	498	0.026710	588	0.126000	678	0.009810	768	0.000000		
409	0.009680	499	0.023390	589	0.097210	679	0.010690	769	0.000000		
410	0.004640	500	0.020080	590	0.068430	680	0.011560	770	0.000000		
411	0.005120	501	0.017300	591	0.085320	681	0.010990	771	0.000000		
412	0.005600	502	0.014520	592	0.102200	682	0.010420	772	0.000000		
413	0.005835	503	0.012700	593	0.103800	683	0.010040	773	0.000000		
414	0.006070	504	0.010870	594	0.105400	684	0.009650	774	0.000000		
415	0.006515	505	0.009670	595	0.083490	685	0.012730	775	0.000000		
416	0.006960	506	0.008470	596	0.061600	686	0.015810	776	0.000000		
417	0.007105	507	0.008350	597	0.064520	687	0.021660	777	0.000000		
418	0.007250	508	0.008230	598	0.067430	688	0.027500	778	0.000000		
419	0.007345	509	0.007905	599	0.077740	689	0.018370	779	0.000000		
420	0.007440	510	0.007580	600	0.088050	690	0.009240	780	0.000000		
421	0.007790	511	0.007370	601	0.068570	691	0.008135	781	0.000000		
422	0.008140	512	0.007160	602	0.049080	692	0.007030	782	0.000000		
423	0.008565	513	0.006895	603	0.047100	693	0.013520	783	0.000000		
424	0.008990	514	0.006630	604	0.045120	694	0.020020	784	0.000000		
425	0.009260	515	0.006435	605	0.048080	695	0.013810	785	0.000000		
426	0.009530	516	0.006240	606	0.051040	696	0.007600	786	0.000000		
427	0.009820	517	0.006200	607	0.065430	697	0.005805	787	0.000000		
428	0.010110	518	0.006160	608	0.079820	698	0.004010	788	0.000000		
429	0.010520	519	0.006355	609	0.231200	699	0.003575	789	0.000000		
430	0.010930	520	0.006550	610	0.382600	700	0.003140	790	0.000000		
431	0.011280	521	0.006560	611	0.600400	701	0.005040	791	0.000000		
432	0.011630	522	0.006570	612	0.818300	702	0.006940	792	0.000000		

TABLE X1.1 *Continued*

λ	SPD	λ	SPD	λ	SPD	λ	SPD	λ	SPD	λ	SPD
433	0.020610	523	0.006590	613	0.558200	703	0.008540	793	0.000000		
434	0.029590	524	0.006610	614	0.298100	704	0.010140	794	0.000000		
435	0.241400	525	0.007150	615	0.223100	705	0.024700	795	0.000000		
436	0.453200	526	0.007690	616	0.148200	706	0.039250	796	0.000000		
437	0.233900	527	0.008285	617	0.112500	707	0.047360	797	0.000000		
438	0.014620	528	0.008880	618	0.076780	708	0.055470	798	0.000000		
439	0.014530	529	0.009030	619	0.074490	709	0.047700	799	0.000000		
440	0.014450	530	0.009180	620	0.072200	710	0.039920	800	0.000000		
441	0.014400	531	0.011460	621	0.075760	711	0.047550	801	0.000000		
442	0.014340	532	0.013750	622	0.079320	712	0.055180	802	0.000000		
443	0.014430	533	0.018810	623	0.084640	713	0.033360	803	0.000000		
444	0.014510	534	0.023880	624	0.089950	714	0.011550	804	0.000000		
445	0.014490	535	0.024380	625	0.090240	715	0.007855	805	0.000000		
446	0.014470	536	0.024890	626	0.090530	716	0.004160	806	0.000000		
447	0.014650	537	0.044580	627	0.085950	717	0.002845	807	0.000000		
448	0.014820	538	0.064270	628	0.081370	718	0.001530	808	0.000000		
449	0.014850	539	0.113300	629	0.096260	719	0.002970	809	0.000000		

TABLE X1.2 Product of the SPD Values with a CIE Standard Observer Function (1-nm interval)

λ	$S(\lambda) \times \bar{y}$	λ	$S(\lambda) \times \bar{y}$	λ	$S(\lambda) \times \bar{y}$
360	0.004880 × 0.00000001340	540	0.162400 × 0.96198800000	790	0.000000 × .00000701280
361	0.004595 × 0.00000002029	541	0.277600 × 0.96754000000	791	0.000000 × .00000658580
362	0.004310 × 0.00000003056	542	0.392800 × 0.97223000000	792	0.000000 × .00000618570
363	0.020290 × 0.00000004574	543	0.353900 × 0.97617000000	793	0.000000 × .00000581070
364	0.036270 × 0.00000006805	544	0.315100 × 0.97946000000	794	0.000000 × .00000545900
365	0.047350 × 0.00000010065	545	0.429800 × 0.98220000000	795	0.000000 × .00000512980
366	0.058440 × 0.00000014798	546	0.544600 × 0.98452000000	796	0.000000 × .00000482060
367	0.031870 × 0.00000021627	547	0.383500 × 0.98652000000	797	0.000000 × .00000453120
368	0.005300 × 0.00000031420	548	0.222500 × 0.98832000000	798	0.000000 × .00000425910
369	0.004700 × 0.00000045370	549	0.182100 × 0.99002000000	799	0.000000 × .00000400420
370	0.004100 × 0.00000065110	550	0.141700 × 0.99176100000	800	0.000000 × .00000376473
371	0.003785 × 0.00000092880	551	0.113500 × 0.99353000000	801	0.000000 × .00000353995
372	0.003470 × 0.00000131750	552	0.085290 × 0.99523000000	802	0.000000 × .00000332914
373	0.003540 × 0.00000185720	553	0.070050 × 0.99677000000	803	0.000000 × .00000313115
374	0.003610 × 0.00000260200	554	0.054810 × 0.99809000000	804	0.000000 × .00000294529
375	0.003615 × 0.00000362500	555	0.046030 × 0.99911000000	805	0.000000 × .00000277081
376	0.003620 × 0.00000501900	556	0.037250 × 0.99977000000	806	0.000000 × .00000260705
377	0.004210 × 0.00000690700	557	0.034310 × 1.00000000000	807	0.000000 × .00000245329
378	0.004800 × 0.00000944900	558	0.031360 × 0.99971000000	808	0.000000 × .00000230894
379	0.005170 × 0.00001284800	559	0.030480 × 0.99885000000	809	0.000000 × .00000217338
380	0.005540 × 0.00001736400	560	0.029590 × 0.99734000000	810	0.000000 × .00000204613
381	0.005240 × 0.00002332700	561	0.029650 × 0.99526000000	811	0.000000 × .00000192662
382	0.004940 × 0.00003115000	562	0.029700 × 0.99274000000	812	0.000000 × .00000181440
383	0.004615 × 0.00004135000	563	0.029530 × 0.98975000000	813	0.000000 × .00000170895
384	0.004290 × 0.00005456000	564	0.029360 × 0.98630000000	814	0.000000 × .00000160988
385	0.003750 × 0.00007156000	565	0.029200 × 0.98238000000	815	0.000000 × .00000151677
386	0.003210 × 0.00009330000	566	0.029040 × 0.97798000000	816	0.000000 × .00000142921
387	0.003050 × 0.00012087000	567	0.029500 × 0.97311000000	817	0.000000 × .00000134686
388	0.002890 × 0.00015564000	568	0.029960 × 0.96774000000	818	0.000000 × .00000126945
389	0.002980 × 0.00019920000	569	0.029480 × 0.96189000000	819	0.000000 × .00000119662
390	0.003070 × 0.00025340000	570	0.029000 × 0.95555200000	820	0.000000 × .00000112809
391	0.002795 × 0.00032020000	571	0.029140 × 0.94860100000	821	0.000000 × .00000106368
392	0.002520 × 0.00040240000	572	0.029280 × 0.94098100000	822	0.000000 × .00000100313
393	0.002395 × 0.00050230000	573	0.029390 × 0.93279800000	823	0.000000 × .00000094622
394	0.002270 × 0.00062320000	574	0.029500 × 0.92415800000	824	0.000000 × .00000089263
395	0.002285 × 0.00076850000	575	0.040510 × 0.91517500000	825	0.000000 × .00000084216
396	0.002300 × 0.00094170000	576	0.051530 × 0.90595400000	826	0.000000 × .00000079464
397	0.002420 × 0.00114780000	577	0.060840 × 0.89660800000	827	0.000000 × .00000074978
398	0.002540 × 0.00139030000	578	0.070160 × 0.88724900000	828	0.000000 × .00000070744
399	0.002640 × 0.00167400000	579	0.079050 × 0.87798600000	829	0.000000 × .00000066748
400	0.002740 × 0.00200440000	580	0.087930 × 0.86893400000	830	0.000000 × .00000062970

TABLE X1.3 Interpolation of Measured Reflectance Factor from a 10-nm Measurement Interval to a 1-nm Interval for the First 10 nm interval (360 nm to 370 nm), an Intermediate Interval (550 nm to 560 nm), and for the Last Intermediate Interval (820 nm to 830 nm)

λ	Reflectance Factor	λ	Reflectance Factor	λ	Reflectance Factor
360	R0	540	R0	790	R4
361	0.855 × R0 +0.190 × R1 –0.045 × R2	550	R1	800	R3
362	0.720 × R0 +0.360 × R1 –0.080 × R2	551	–0.029 × R0 +0.941 × R1 +0.105 × R2 –0.016 × R3	810	R2
363	0.595 × R0 +0.510 × R1 –0.105 × R2	552	–0.048 × R0 +0.864 × R1 +0.216 × R2 –0.032 × R3	820	R1
364	0.480 × R0 +0.640 × R1 –0.120 × R2	553	–0.060 × R0 +0.774 × R1 +0.332 × R2 –0.046 × R3	821	0.055 × R0 +0.990 × R1 –0.045 × R2
365	0.375 × R0 +0.750 × R1 –0.125 × R2	554	–0.064 × R0 +0.672 × R1 +0.448 × R2 –0.056 × R3	822	0.120 × R0 +0.960 × R1 –0.080 × R2
366	0.280 × R0 +0.840 × R1 –0.120 × R2	555	–0.063 × R0 +0.563 × R1 +0.563 × R2 –0.063 × R3	823	0.195 × R0 +0.910 × R1 –0.105 × R2
367	0.195 × R0 +0.910 × R1 –0.105 × R2	556	–0.056 × R0 +0.448 × R1 +0.672 × R2 –0.064 × R3	824	0.280 × R0 +0.840 × R1 –0.120 × R2
368	0.120 × R0 +0.960 × R1 –0.080 × R2	557	–0.045 × R0 +0.331 × R1 +0.774 × R2 –0.060 × R3	825	0.375 × R0 +0.750 × R1 –0.125 × R2
369	0.055 × R0 +0.990 × R1 –0.045 × R2	558	–0.032 × R0 +0.216 × R1 +0.864 × R2 –0.048 × R3	826	0.480 × R0 +0.640 × R1 –0.120 × R2
370	R1	559	–0.016 × R0 +0.105 × R1 +0.941 × R2 –0.029 × R3	827	0.595 × R0 +0.510 × R1 –0.105 × R2
380	R2	560	R2	828	0.720 × R0 +0.360 × R1 –0.080 × R2
390	R3	570	R3	829	0.855 × R0 +0.190 × R1 –0.045 × R2
400	R4	580	R4	830	R0

TABLE X1.4 Formation of the CIE Triple Product (Interpolated Reflectance Factor) X (Illuminant) X (Standard Observer Function) Shown for the First 10-nm Interval (360 nm to 370 nm)

λ	Reflectance Factor × *S*(λ) × ȳ × First 10-nm Interval
360	(0.004880 × 0.00000001340) × R0
361	(0.855 × 0.004595 × 0.00000002029) × R0 + (0.190 × 0.004595 × 0.00000002029) × R1 + (–0.045 × 0.004595 × 0.00000002029) × R2
362	(0.720 × 0.004310 × 0.00000003056) × R0 + (0.360 × 0.004310 × 0.00000003056) × R1 + (–0.080 × 0.004310 × 0.00000003056) × R2
363	(0.595 × 0.020290 × 0.00000004574) × R0 + (0.510 × 0.020290 × 0.00000004574) × R1 + (–0.105 × 0.020290 × 0.00000004574) × R2
364	(0.480 × 0.036270 × 0.00000006805) × R0 + (0.640 × 0.036270 × 0.00000006805) × R1 + (–0.120 × 0.036270 × 0.00000006805) × R2
365	(0.375 × 0.047350 × 0.00000010065) × R0 + (0.750 × 0.047350 × 0.00000010065) × R1 + (–0.125 × 0.047350 × 0.00000010065) × R2
366	(0.280 × 0.058440 × 0.00000014798) × R0 + (0.840 × 0.058440 × 0.00000014798) × R1 + (–0.120 × 0.058440 × 0.00000014798) × R2
367	(0.195 × 0.031870 × 0.00000021627) × R0 + (0.910 × 0.031870 × 0.00000021627) × R1 + (–0.105 × 0.031870 × 0.00000021627) × R2
368	(0.120 × 0.005300 × 0.00000031420) × R0 + (0.960 × 0.005300 × 0.00000031420) × R1 + (–0.080 × 0.005300 × 0.00000031420) × R2
369	(0.055 × 0.004700 × 0.00000045370) × R0 + (0.990 × 0.004700 × 0.00000045370) × R1 + (–0.045 × 0.004700 × 0.00000045370) × R2
370	(0.004100 × 0.00000065110) × R1
380	(0.005540 × 0.00001736400) × R2
390	(0.003070 × 0.00025340000) × R3
400	(0.002740 × 0.00200440000) × R4

TABLE X1.5 Formation of the CIE Triple Product (Interpolated Reflectance Factor) X (Illuminant) X (Standard Observer Function) Shown for an Intermediate 10-nm Interval (550 nm to 560 nm)

λ	Reflectance Factor × $S(\lambda)$ × $\bar{y}$ × Interior 10-nm Intervals[A]
540	(0.162400 × 0.96198800000) × R0
550	(0.141700 × 0.99176100000) × R1
551	(–0.029 × 0.113500 × 0.99353000000) × R0 + (0.941 × 0.113500 × 0.99353000000) × R1 + (0.105 × 0.113500 × 0.99353000000) × R2 + (–0.016 × 0.113500 × 0.99353000000) × R3
552	(–0.048 × 0.085290 × 0.99523000000) × R0 + (0.864 × 0.085290 × 0.99523000000) × R1 + (0.216 × 0.085290 × 0.99523000000) × R2 + (–0.032 × 0.085290 × 0.99523000000) × R3
553	(–0.060 × 0.070050 × 0.99677000000) × R0 + (0.774 × 0.070050 × 0.99677000000) × R1 + (0.331 × 0.070050 × 0.99677000000) × R2 + (–0.045 × 0.070050 × 0.99677000000) × R3
554	(–0.064 × 0.054810 × 0.99809000000) × R0 + (0.672 × 0.054810 × 0.99809000000) × R1 + (0.448 × 0.054810 × 0.99809000000) × R2 + (–0.056 × 0.054810 × 0.99809000000) × R3
555	(–0.063 × 0.046030 × 0.99911000000) × R0 + (0.563 × 0.046030 × 0.99911000000) × R1 + (0.563 × 0.046030 × 0.99911000000) × R2 + (–0.063 × 0.046030 × 0.99911000000) × R3
556	(–0.056 × 0.037250 × 0.99977000000) × R0 + (0.448 × 0.037250 × 0.99977000000) × R1 + (0.672 × 0.037250 × 0.99977000000) × R2 + (–0.064 × 0.037250 × 0.99977000000) × R3
557	(–0.045 × 0.034310 × 1.00000000000) × R0 + (0.331 × 0.034310 × 1.00000000000) × R1 + (0.774 × 0.034310 × 1.00000000000) × R2 + (–0.060 × 0.034310 × 1.00000000000) × R3
558	(–0.032 × 0.031360 × 0.99971000000) × R0 + (0.216 × 0.031360 × 0.99971000000) × R1 + (0.864 × 0.031360 × 0.99971000000) × R2 + (–0.048 × 0.031360 × 0.99971000000) × R3
559	(–0.016 × 0.030480 × 0.99885000000) × R0 + (0.105 × 0.030480 × 0.99885000000) × R1 + (0.941 × 0.030480 × 0.99885000000) × R2 + (–0.029 × 0.030480 × 0.99885000000) × R3
560	(0.029590 × 0.99734000000) × R2
570	(0.029000 × 0.95555200000) × R3
580	(0.087930 × 0.86893400000) × R4

[A] Stearns, E. I and Stearns, R. E., “Influence of Spectophotometer Slits on Tristimulus Calculations,” *Color Research and Application*, Vol 13, 1988, pp. 257–259.

TABLE X1.6 Formation of the CIE Triple Product (Interpolated Reflectance Factor) X (Illuminant) X (Standard Observer Function) Shown for the Last 10-nm Interval (820 nm to 830 nm)

λ	Reflectance Factor × $S(\lambda)$ × $\bar{y}$ × Last 10-nm Intervals
790	(0.000000 × .00000701280) × R4
800	(0.000000 × .00000376473) × R3
810	(0.000000 × .00000204613) × R2
820	(0.000000 × .00000112809) × R1
821	(0.055 × 0.000000 × .00000106368) × R0 + (0.990 × 0.000000 × .00000106368) × R1 + (–0.045 × 0.000000 × .00000106368) × R2
822	(0.120 × 0.000000 × .00000100313) × R0 + (0.960 × 0.000000 × .00000100313) × R1 + (–0.080 × 0.000000 × .00000100313) × R2
823	(0.195 × 0.000000 × .0000094622) × R0 + (0.910 × 0.000000 × .00000094622) × R1 + (–0.105 × 0.000000 × .00000094622) × R2
824	(0.280 × 0.000000 × .00000089263) × R0 + (0.840 × 0.000000 × .00000089263) × R1 + (–0.120 × 0.000000 × .00000089263) × R2
825	(0.375 × 0.000000 × .00000084216) × R0 + (0.750 × 0.000000 × .00000084216) × R1 + (–0.125 × 0.000000 × .00000084216) × R2
826	(0.480 × 0.000000 × .00000079464) × R0 + (0.640 × 0.000000 × .00000079464) × R1 + (–0.120 × 0.000000 × .00000079464) × R2
827	(0.595 × 0.000000 × .00000074978) × R0 + (0.510 × 0.000000 × .00000074978) × R1 + (–0.105 × 0.000000 × .00000074978) × R2
828	(0.720 × 0.000000 × .00000070744) × R0 + (0.360 × 0.000000 × .00000070744) × R1 + (–0.080 × 0.000000 × .00000070744) × R2
829	(0.855 × 0.000000 × .00000066748) × R0 + (0.190 × 0.000000 × .00000066748) × R1 + (–0.045 × 0.000000 × .00000066748) × R2
830	(0.000000 × .00000062970) × R0

TABLE X1.7 Final Table of Weights Summing all Coefficients of Each 10-nm Intervals of the Measured Reflectance Factor

λ	*Wx*	*Wy*	*Wz*
360.0	0.000	0.000	0.001
370.0	0.001	0.000	0.003
380.0	0.001	0.000	0.003
390.0	–0.004	–0.000	–0.019
400.0	0.053	0.001	0.250
410.0	0.072	0.002	0.343
420.0	–0.071	–0.007	–0.370
430.0	1.868	0.096	9.226
440.0	2.765	0.156	13.733
450.0	0.340	0.051	1.858
460.0	0.445	0.092	2.555
470.0	0.269	0.093	1.715
480.0	0.228	0.461	2.132
490.0	0.244	2.392	3.304
500.0	0.013	0.743	0.676
510.0	0.005	0.399	0.098
520.0	0.023	0.419	0.051
530.0	–0.232	–0.036	0.039
540.0	7.759	22.891	0.392
550.0	9.035	22.674	0.262
560.0	1.680	2.402	–0.003
570.0	2.480	3.111	0.006
580.0	8.184	7.581	0.014
590.0	11.055	8.260	0.013
600.0	6.821	4.203	0.006
610.0	31.663	15.566	0.010
620.0	14.219	6.566	0.003
630.0	4.856	1.953	0.000
640.0	1.102	0.427	0.000
650.0	0.742	0.278	0.000
660.0	0.363	0.134	–0.000
670.0	0.126	0.046	0.000
680.0	0.053	0.019	0.000
690.0	0.036	0.013	0.000
700.0	0.012	0.004	0.000
710.0	0.023	0.008	0.000
720.0	0.002	0.001	0.000
730.0	0.000	0.000	0.000
740.0	0.000	0.000	0.000
750.0	0.000	0.000	0.000
760.0	0.000	0.000	0.000
770.0	–0.000	–0.000	0.000
780.0	0.000	0.000	0.000
790.0	0.000	0.000	0.000
800.0	0.000	0.000	0.000
810.0	0.000	0.000	0.000
820.0	0.000	0.000	0.000
830.0	0.000	0.000	0.000
Sums:	106.229	100.000	36.301

X2. PREVIOUS PRACTICE WITH RESPECT TO BANDPASS CORRECTION

X2.1 Prior to January 2010, rectification of spectral bandpass error was handled by Practice E308. At that time control of bandpass correction was transferred to Practice E2729. Both Practice E308 and this present practice utilized a Stearns' correction[6] with Venable coefficients. Interior passbands were corrected by

$$R_{c,i} = -0.083 \cdot R_{m,i-1} + 1.166 \cdot R_{m,i} - 0.083 \cdot R_{m,i+1} \quad (X2.1)$$

where R is a reflectance value at an indexed passband, c indicates a bandpass-corrected reflectance, and m indicates a measured reflectance. The index i varies from the second-measured passband to the next-to-last measured passband. The first and last passbands were corrected by

$$R_{c,i} = 1.083 \cdot R_{m,i} - 0.083 \cdot R_{m,i \pm 1} \quad (X2.2)$$

where the symbols are the same as Eq X2.1 and the index i and ± refer to the first and last measured passbands, respectively.

X2.1.1 By substituting weights appropriately for reflectances in the above equations, one could build the bandpass correction of the spectrum into the weight set, and it was the practice to do this until the advent of Practice E2729. See 6.9.

X2.2 In research that led to Practice E2729, a Task Group in the committee having jurisdiction over this practice found that the three-point formula was not the optimal correction, and a five-point formula was standardized in Practice E2729. Further the Task Group found that the Venable coefficients that had been in use were not even the best available set of coefficients for a three-point formula. The coefficients of Stearns and Stearns[6], which are

$$R_{c,i} = -0.10 \cdot R_{m,i-1} + 1.20 \cdot R_{m,i} - 0.10 \cdot R_{m,i+1}, \quad (X2.3)$$

give superior performance to the previously used coefficients.

[6] Stearns, E. I. and Stearns, R. E., "Influence of Spectrophotometer Slits on Tristimulus Calculations," *Color Research and Application*, Vol 13, 1988, pp. 257–259.