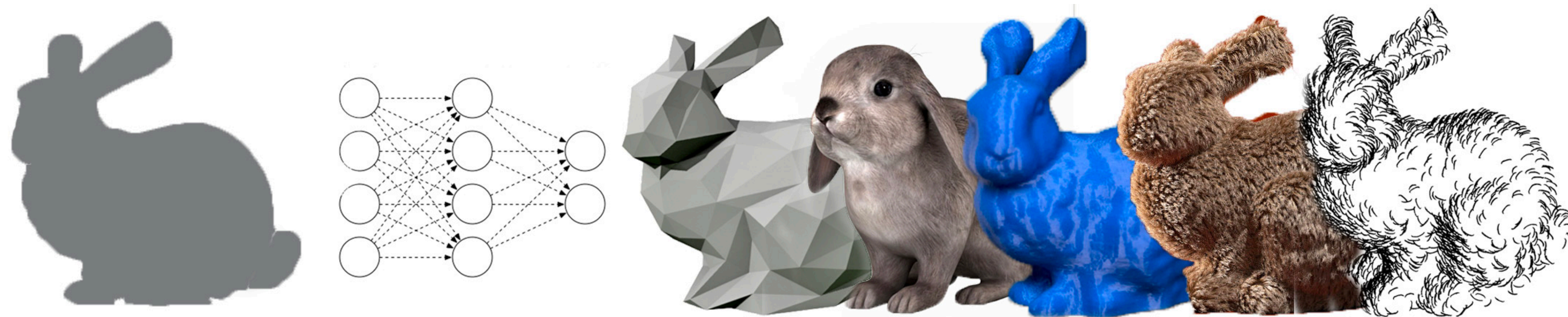


# COMP0169: Machine Learning for Visual Computing

## Linear and Nonlinear Models

### (Optimization)



# Lectures will be Recorded

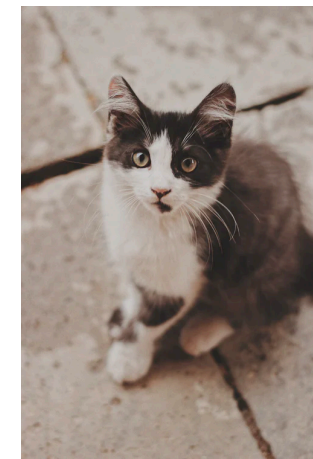
# Last Week

# Last Week

- Introduction and applications

# Last Week

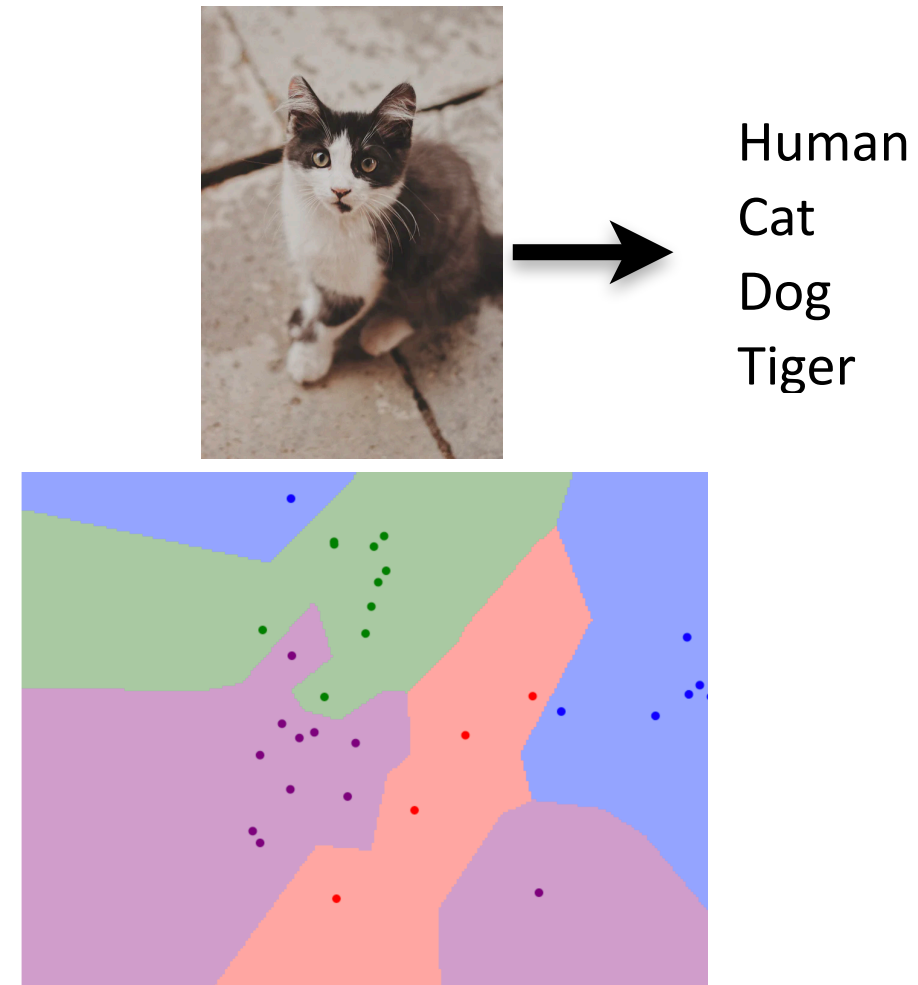
- Introduction and applications
- Image classification



Human  
Cat  
Dog  
Tiger

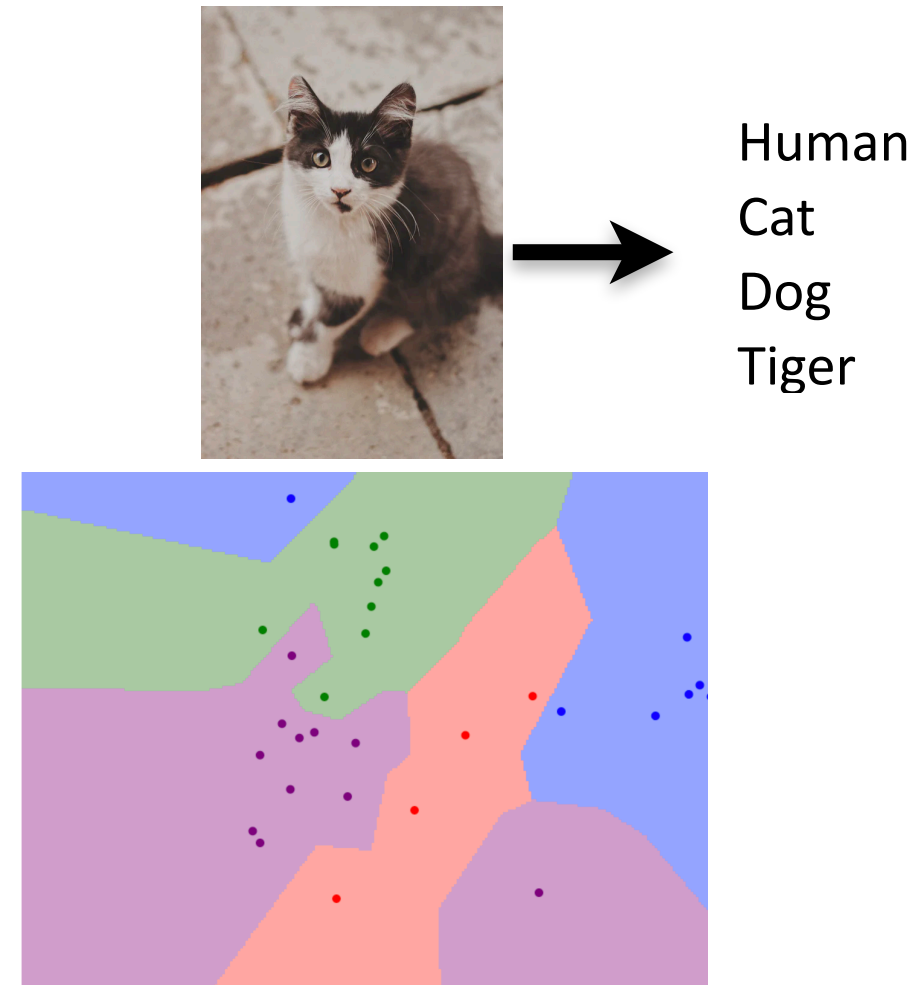
# Last Week

- Introduction and applications
- Image classification
- kNN classifier



# Last Week

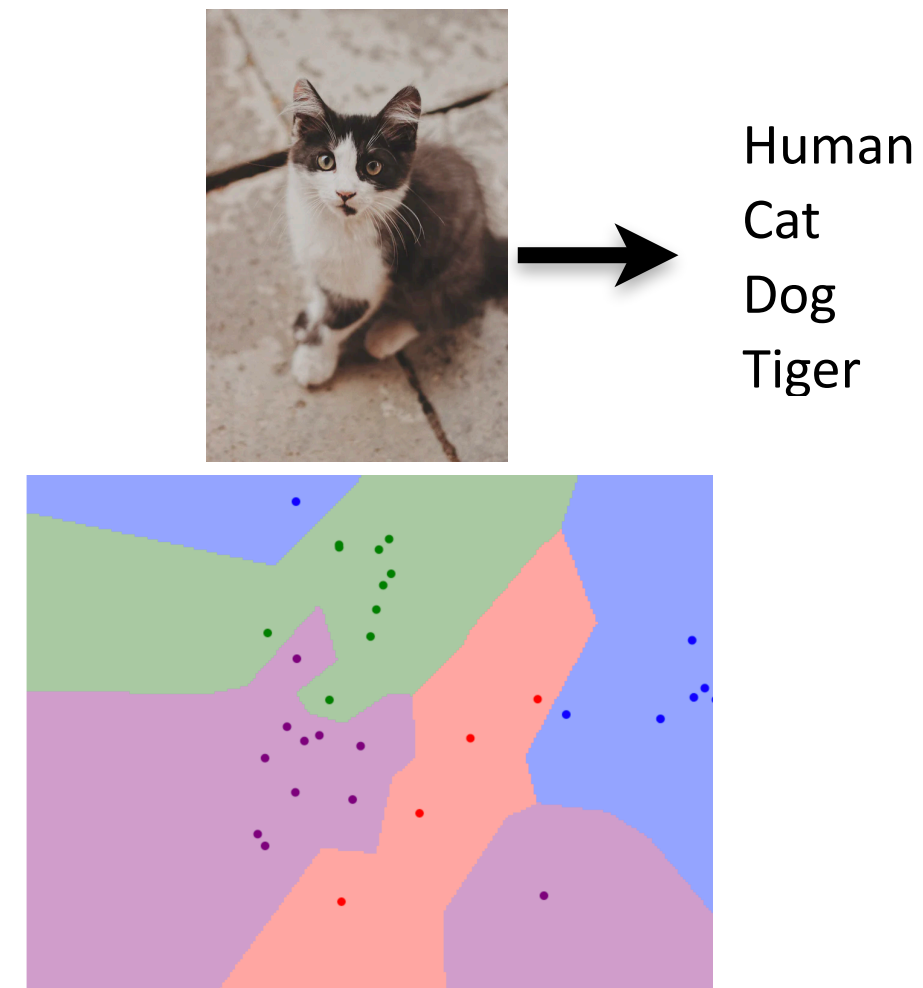
- Introduction and applications
- Image classification
- kNN classifier
- LS Fitting





# Last Week

- Introduction and applications
- Image classification
- kNN classifier
- LS Fitting
- Normal equation

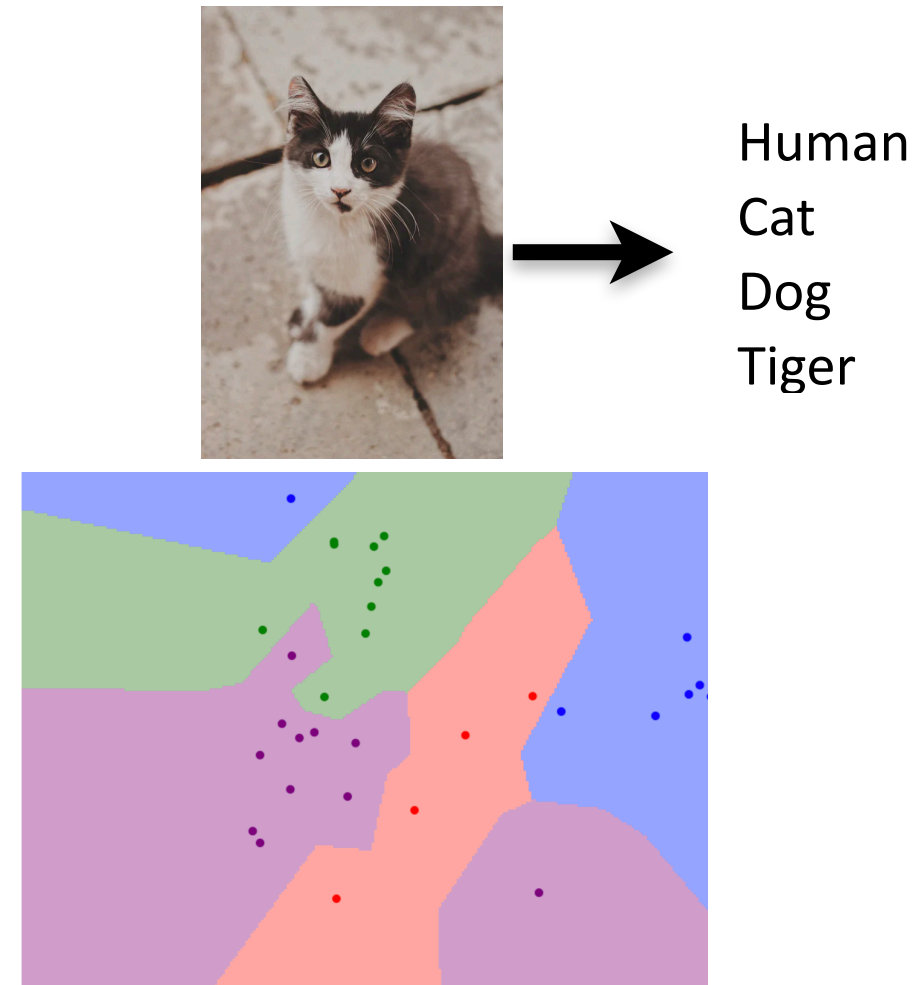


$$\mathbf{w} = (X^T X)^{-1} X^T y$$



# Last Week

- Introduction and applications
- Image classification
- kNN classifier
- LS Fitting
- Normal equation
- Hyperparameter search (k-fold validation)

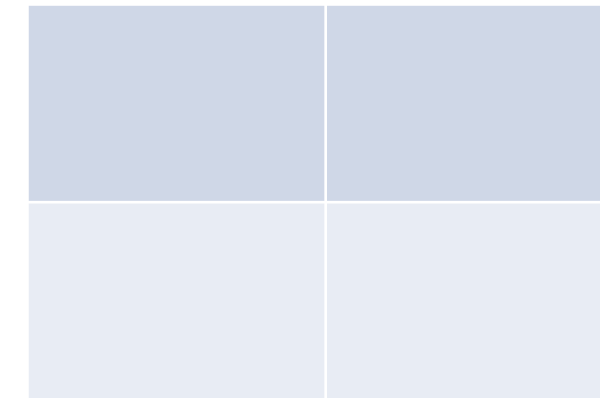


$$\mathbf{w} = (X^T X)^{-1} X^T y$$

# Image Classification

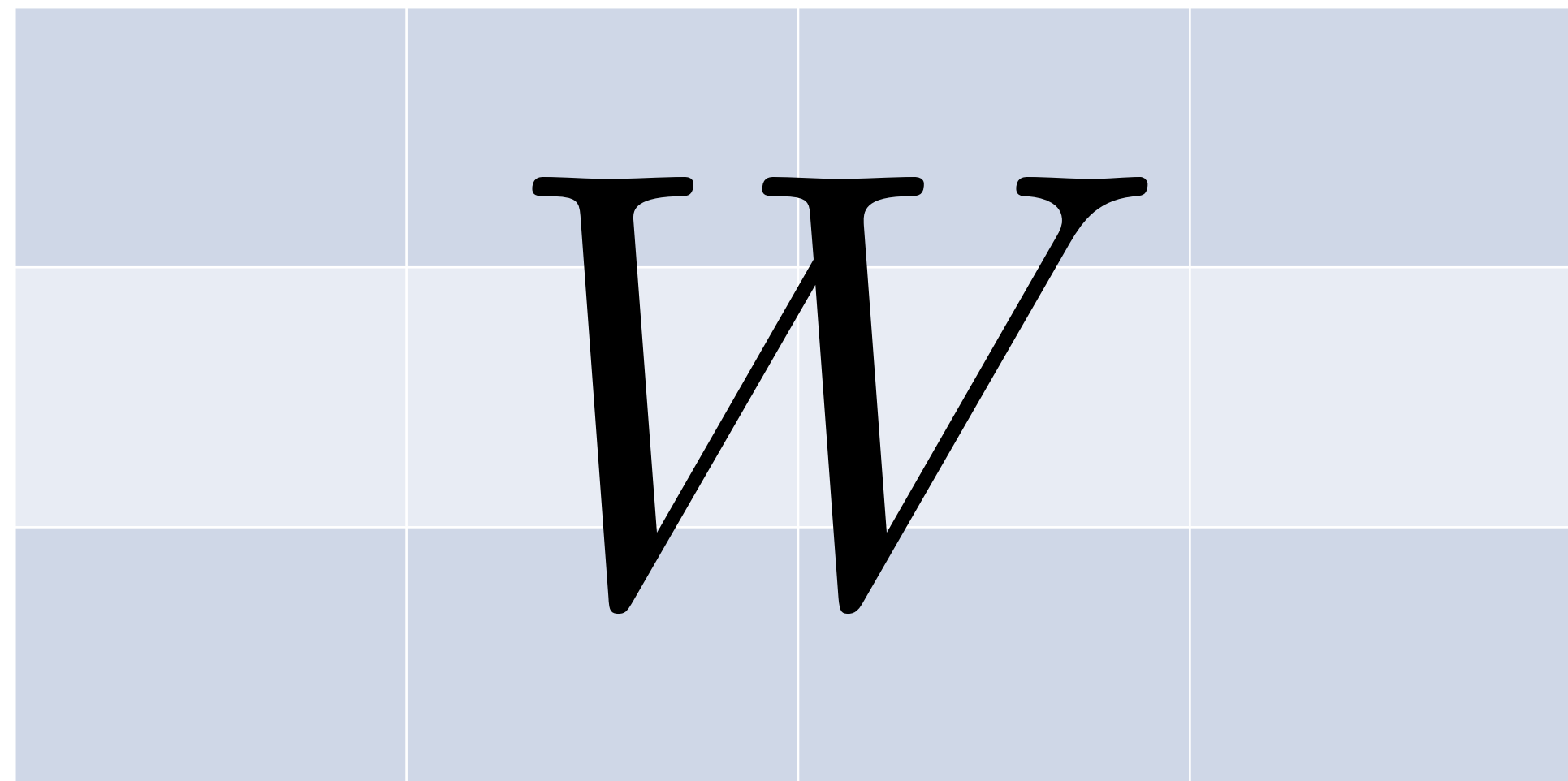
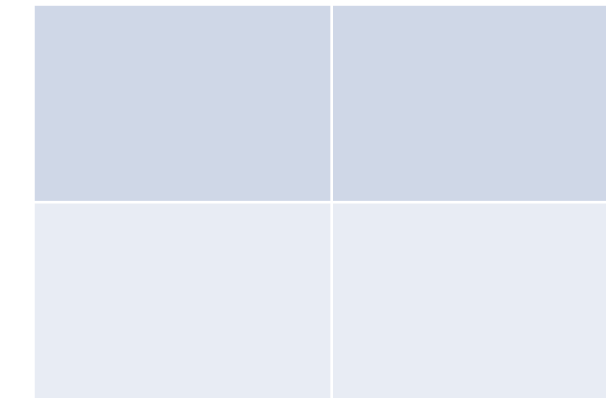
# Image Classification

$I$



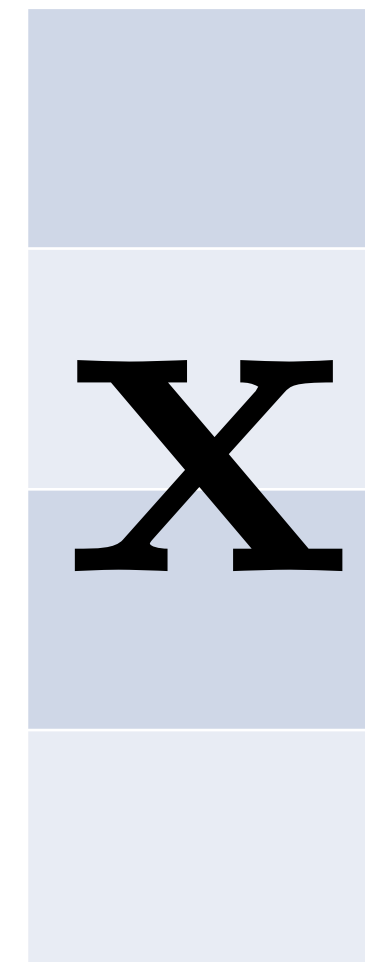
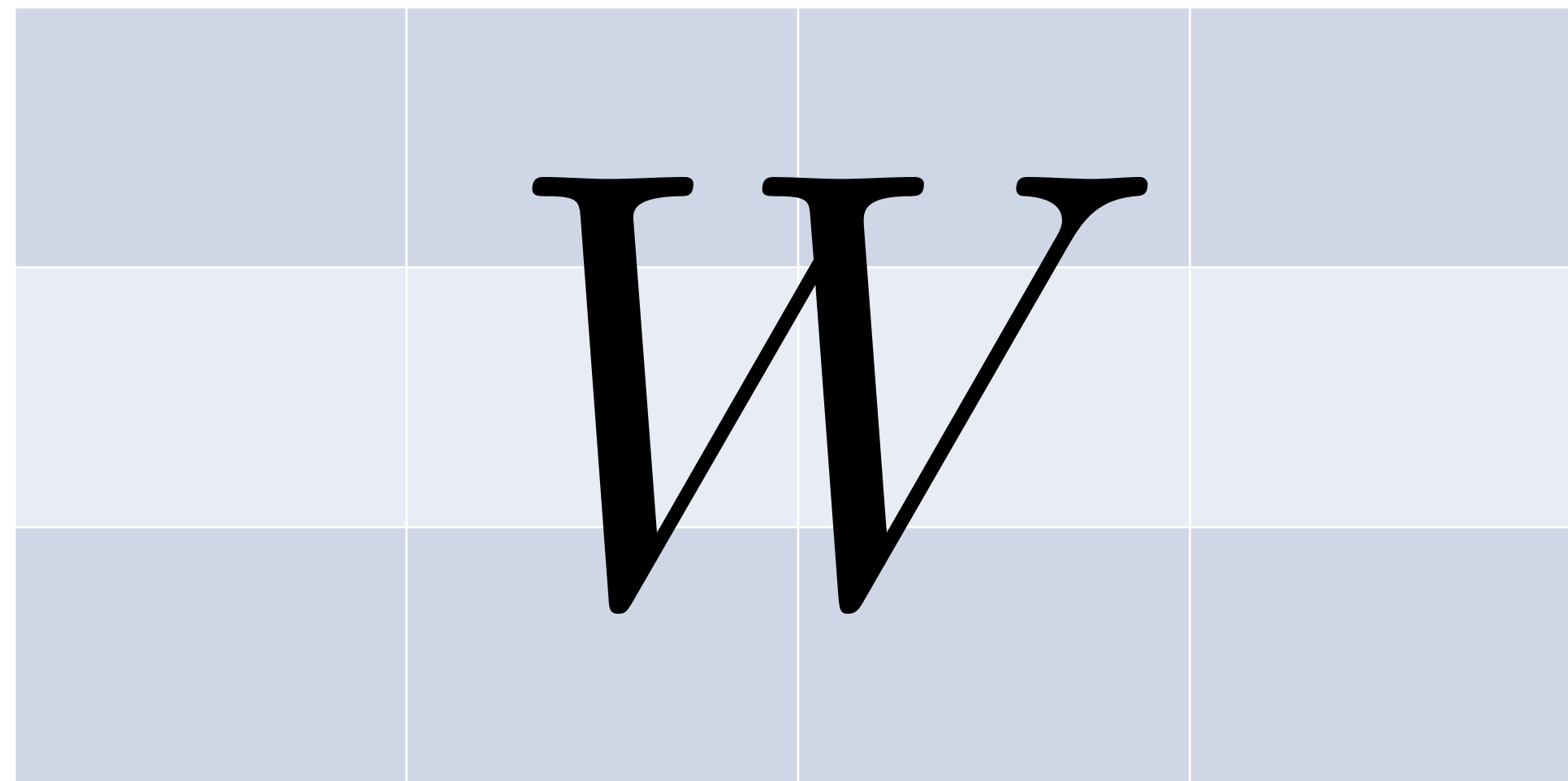
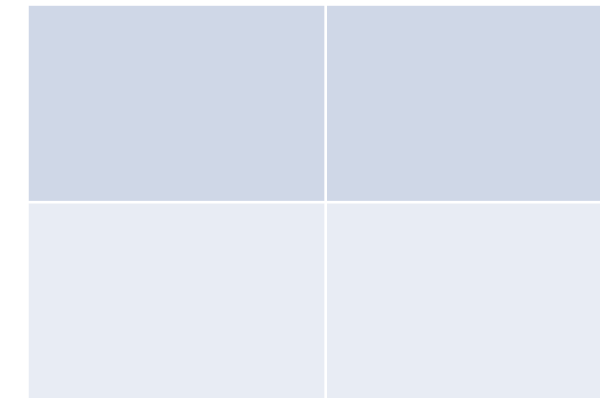
# Image Classification

$I$



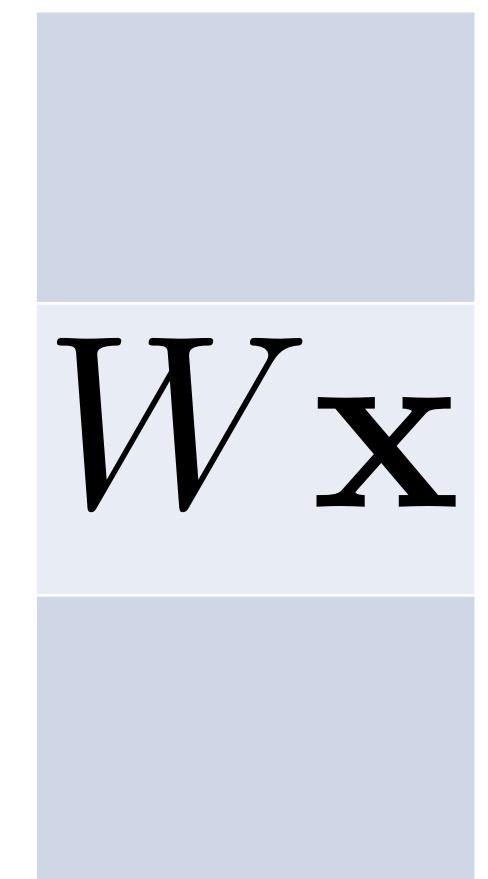
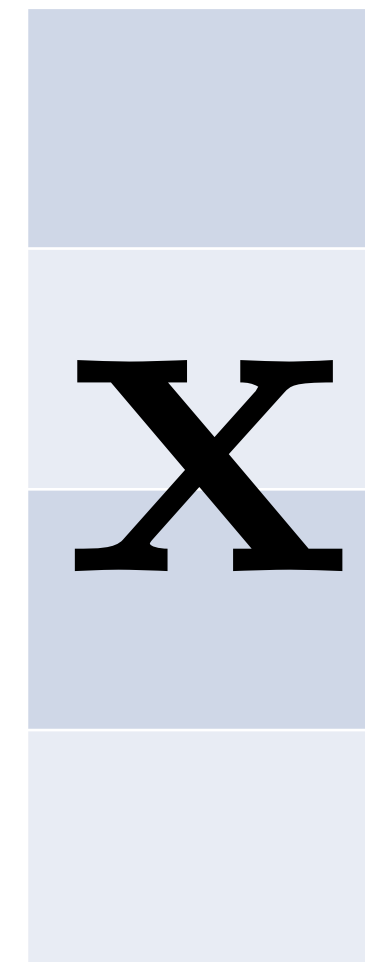
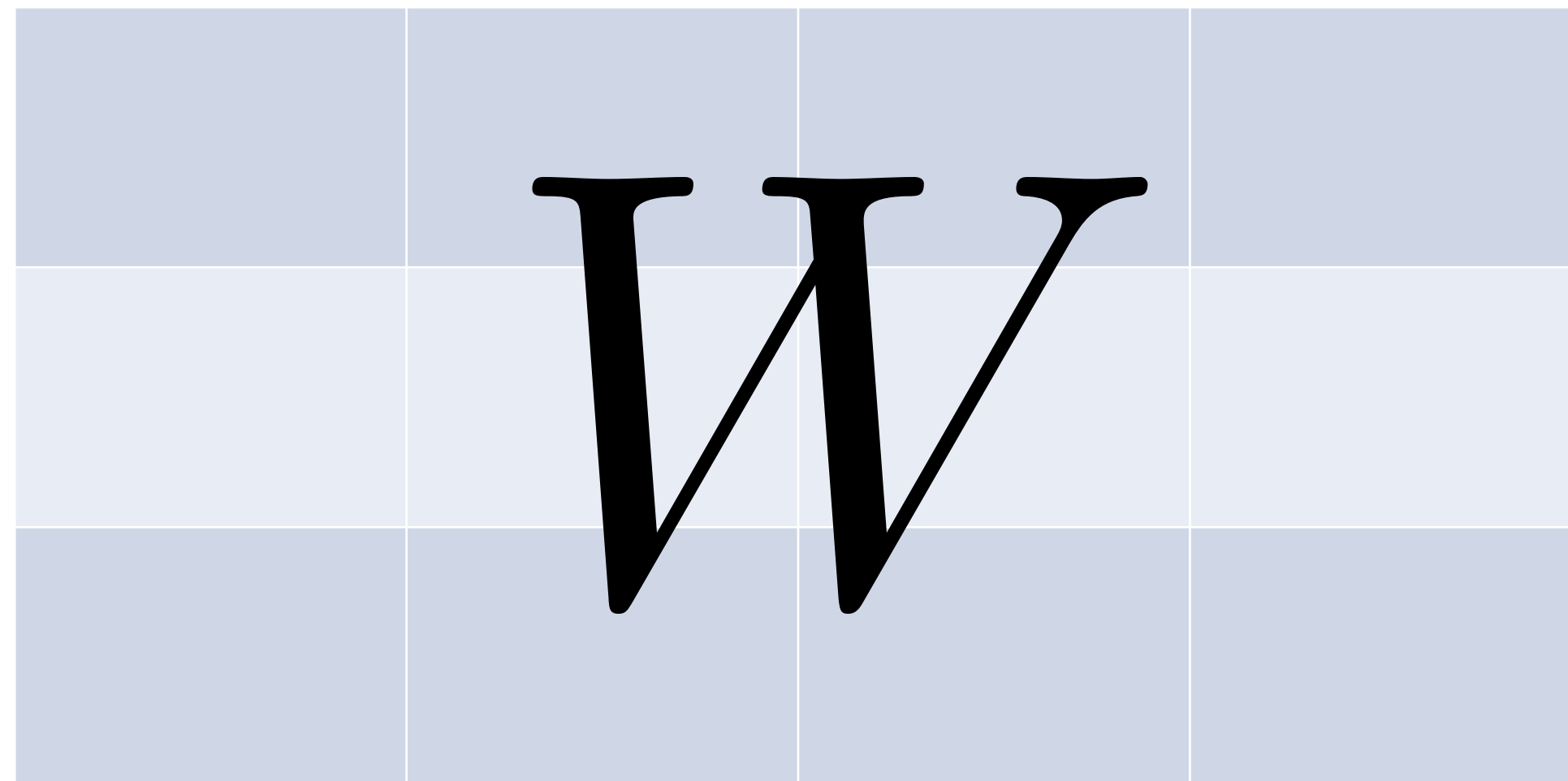
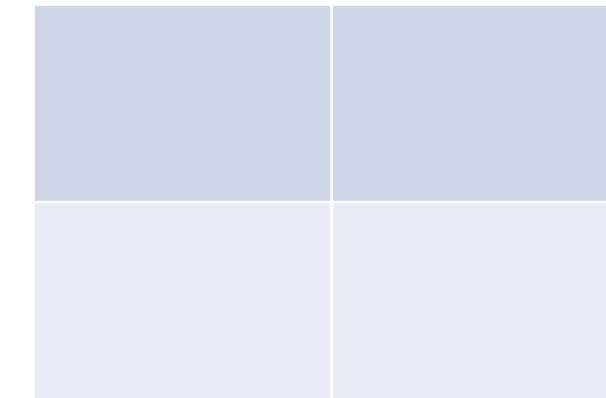
# Image Classification

$I$



# Image Classification

$I$



# Image Classifier



# Image Classifier

- Data (training/validation/test)

$$\{\mathbf{x}_i, y_i\}$$

# Image Classifier

- Data (training/validation/test)  $\{\mathbf{x}_i, y_i\}$
- Scoring function  
(probability of belonging to class k)  $f(\mathbf{x}_i, \mathbf{W})$

# Image Classifier

- Data (training/validation/test)  $\{\mathbf{x}_i, y_i\}$
- Scoring function  
(probability of belonging to class k)  $f(\mathbf{x}_i, \mathbf{W})$
- Loss function  $L_i(W) := h(f(\mathbf{x}_i, W), y_i)$   $L(W) := \frac{1}{N} \sum_i L_i(W)$

# Image Classifier

- Data (training/validation/test)  $\{\mathbf{x}_i, y_i\}$
- Scoring function  
(probability of belonging to class k)  $f(\mathbf{x}_i, \mathbf{W})$
- Loss function  $L_i(W) := h(f(\mathbf{x}_i, W), y_i)$   $L(W) := \frac{1}{N} \sum_i L_i(W)$
- Optimization  $\mathbf{W}^* := \arg \min L(W) = \arg \min L_i(W)/N$

# Linear versus Nonlinear Models

$$s = f(\mathbf{x}_i, W) = W\mathbf{x}_i$$

# Linear versus Nonlinear Models

$$s = f(\mathbf{x}_i, W) = W\mathbf{x}_i$$

- Objective function #1  $L_i(W) := (Wx_i - y_i)^2$

# Linear versus Nonlinear Models

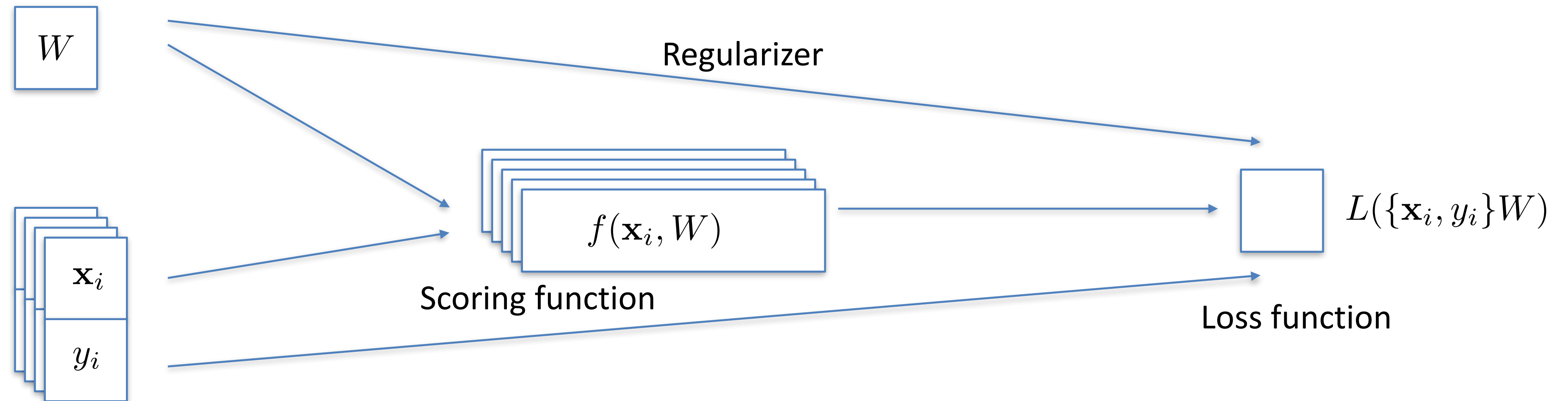
$$s = f(\mathbf{x}_i, W) = W\mathbf{x}_i$$

- Objective function #1  $L_i(W) := (Wx_i - y_i)^2$

- Objective function #2  $L_i(W) := -\log \left( \frac{e^{Wx_i y_i}}{\sum_j e^{Wx_j y_j}} \right) = -\log \left( \frac{e^{sy_i}}{\sum_j e^{sy_j}} \right)$



# Image Classifier



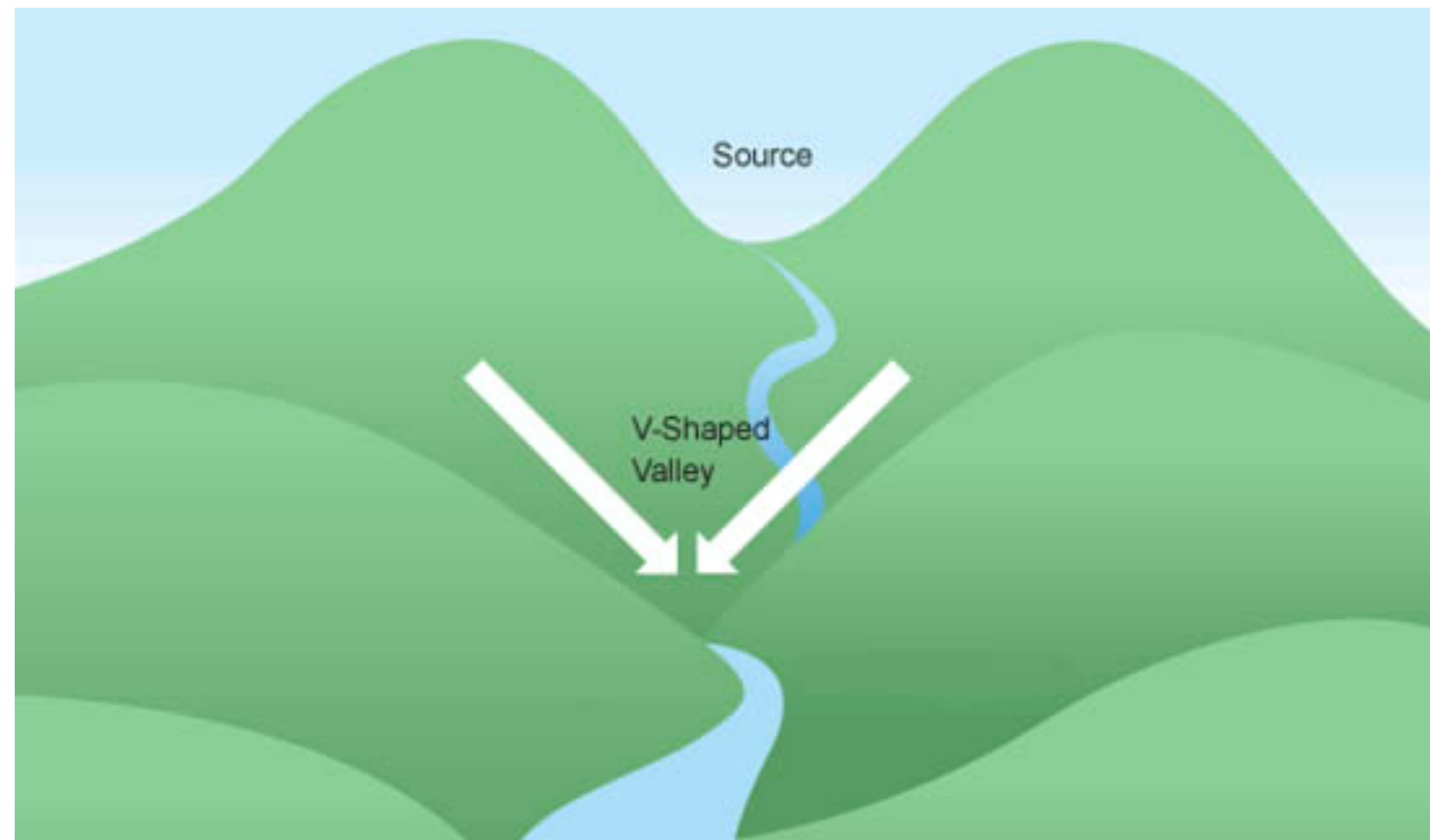
# Optimize

$$\mathbf{W}^* = \arg \min g(\mathbf{x}_i, W)$$

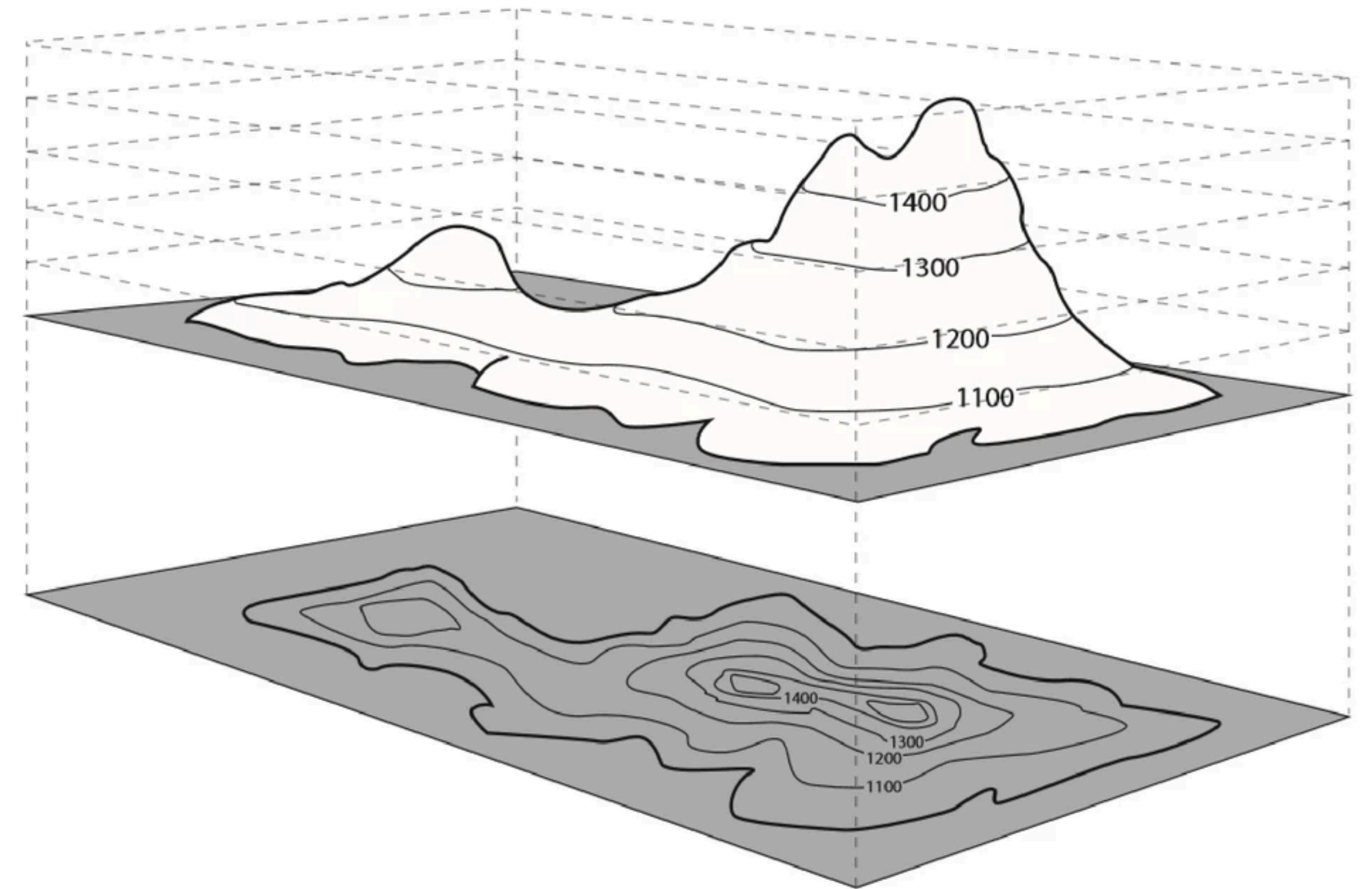
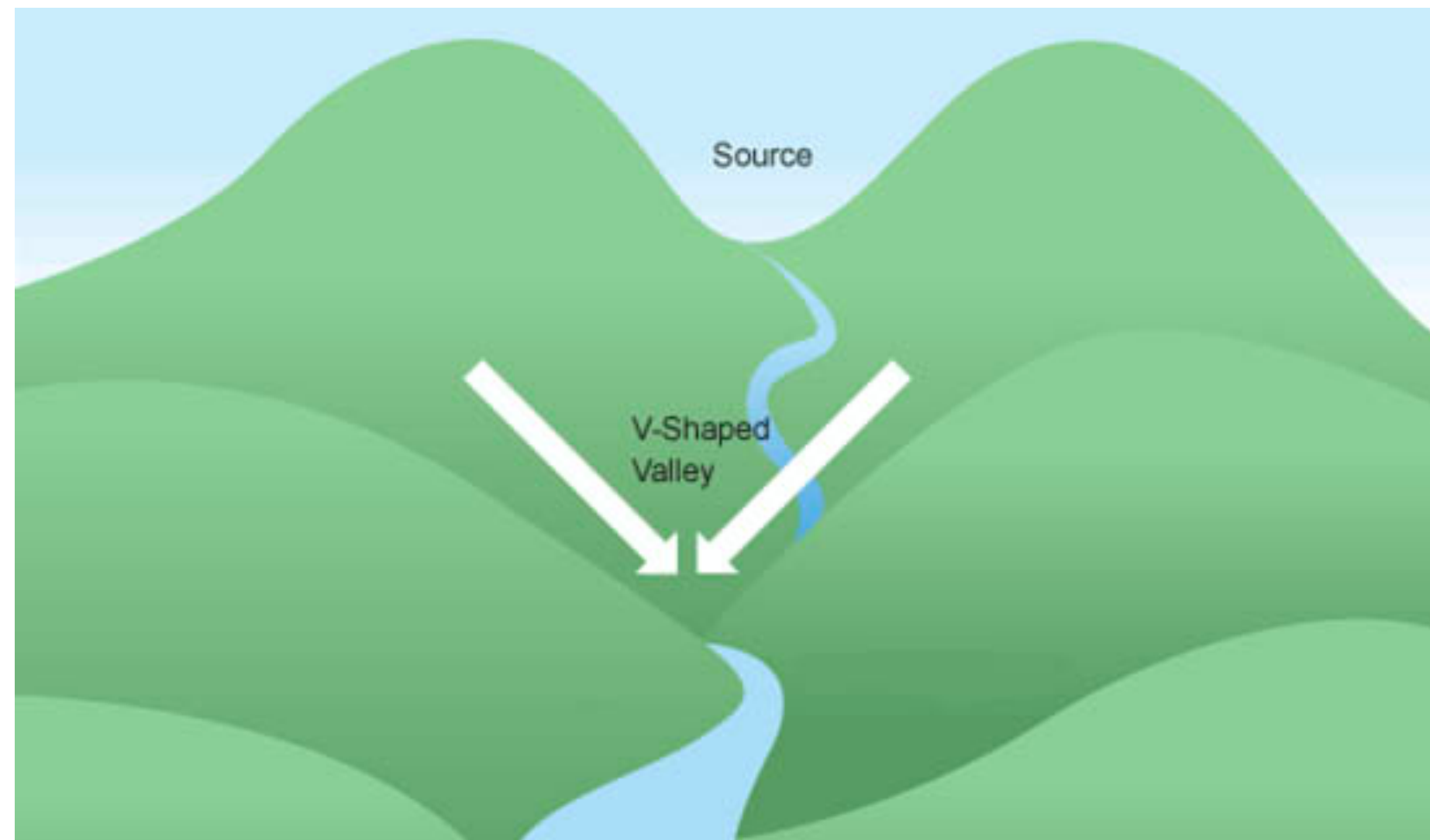
$$\nabla g(\mathbf{x}_i, W) = \left[ \frac{\partial g(\mathbf{x}_i, W)}{w_i} = 0 \right]_{D \times 1}$$

# Error Landscape

# Error Landscape

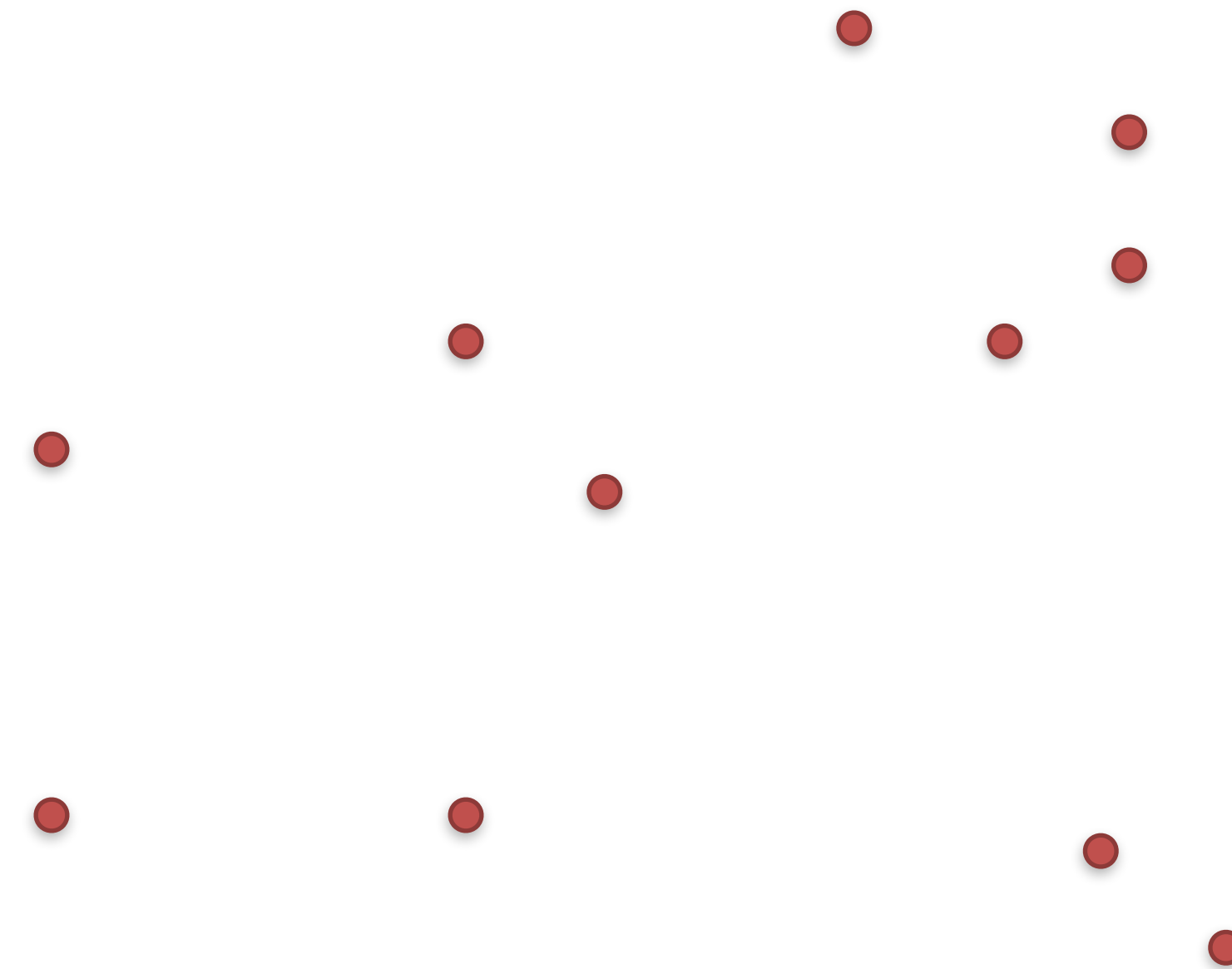


# Error Landscape



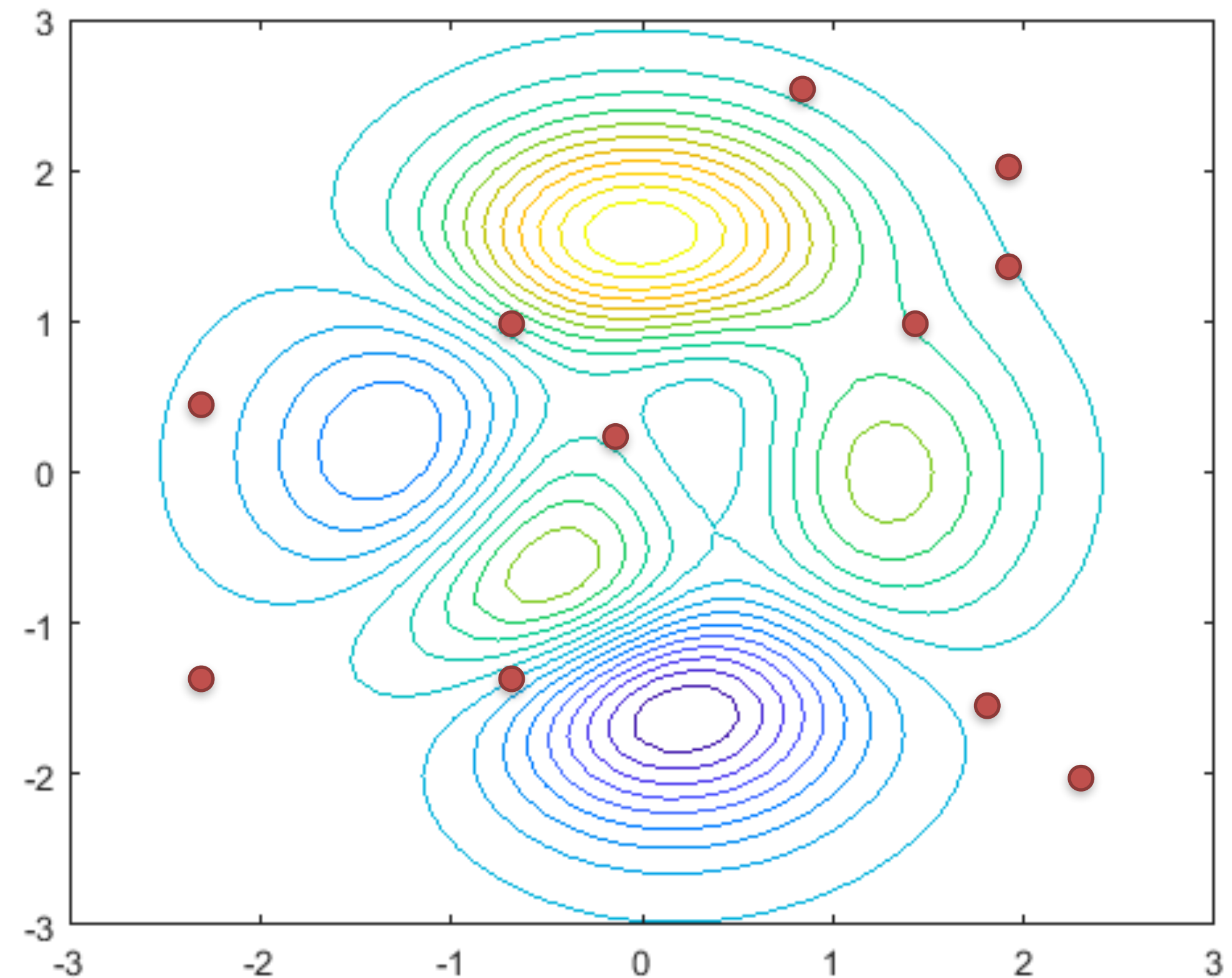
# Method 1a: Random search

- `bestLoss = inf`
- Loop
  - Pick random  $W$
  - Compute loss
  - if `loss < bestLoss`
    - `bestLoss = loss`
    - `currW = W`
  - endif
- endLoop



# Method 1a: Random search

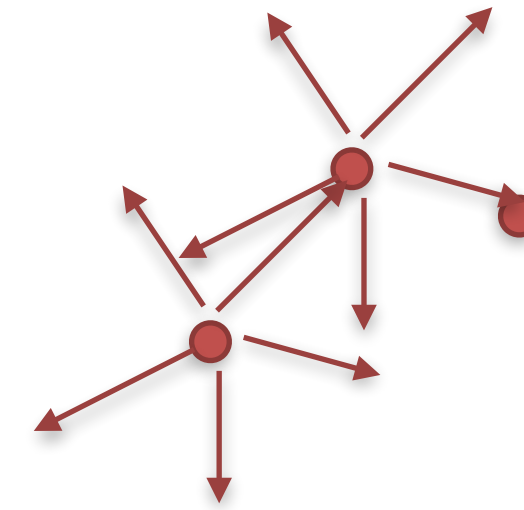
- $\text{bestLoss} = \text{inf}$
- Loop
  - Pick random  $W$
  - Compute loss
  - if  $\text{loss} < \text{bestLoss}$ 
    - $\text{bestLoss} = \text{loss}$
    - $\text{currW} = W$
  - endif
- endLoop





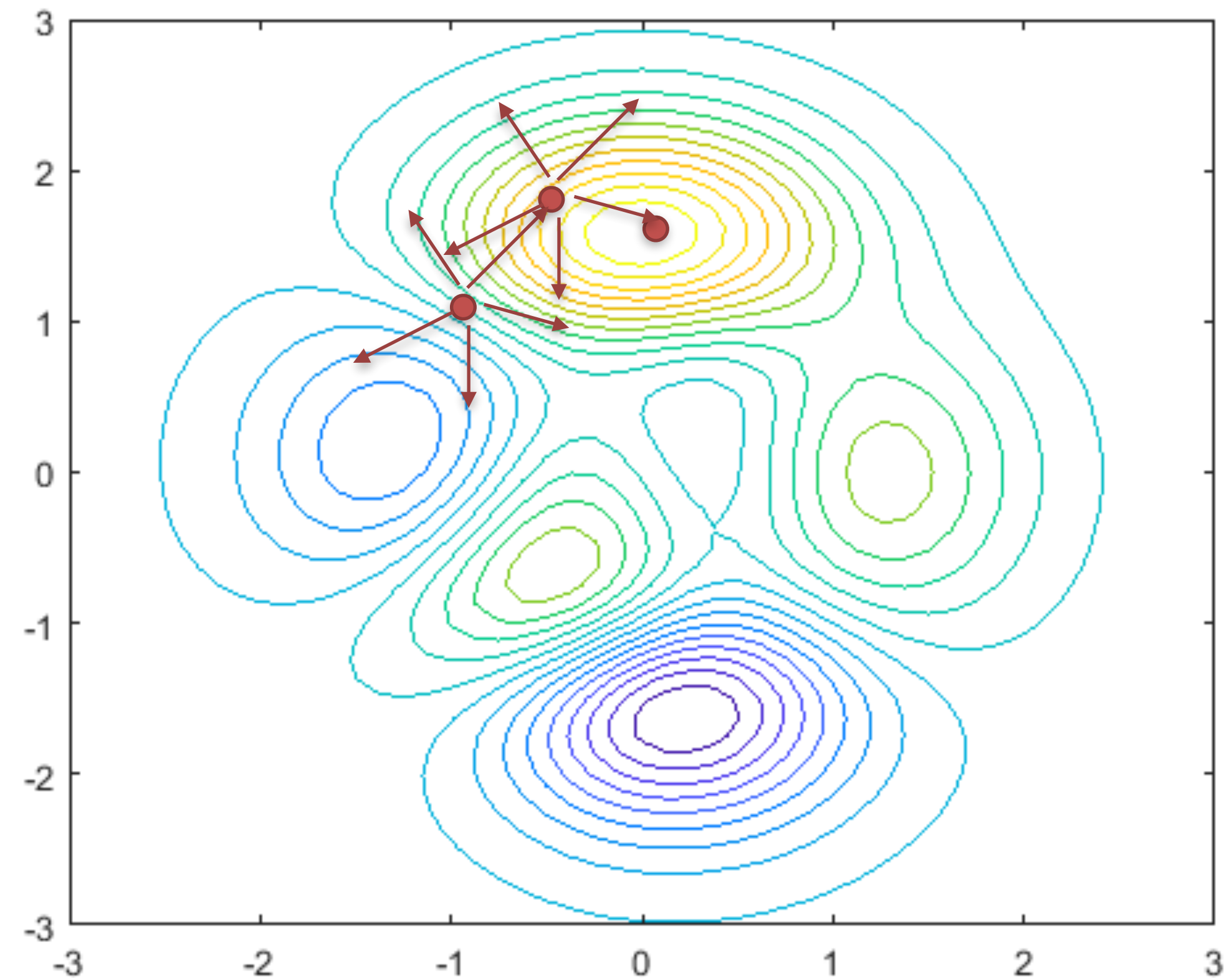
# Method 1b: Random search++

- $\text{bestLoss} = \text{inf}$
- $W = \text{setRand}$
- stepsize
- Loop
  - $W_{\text{local}} = W + \text{stepsize} * W_{\text{random}}$
  - Compute loss
  - if  $\text{loss} < \text{bestLoss}$ 
    - $\text{bestLoss} = \text{loss}$
    - $W = W_{\text{local}}$
  - endif
- endLoop



# Method 1b: Random search++

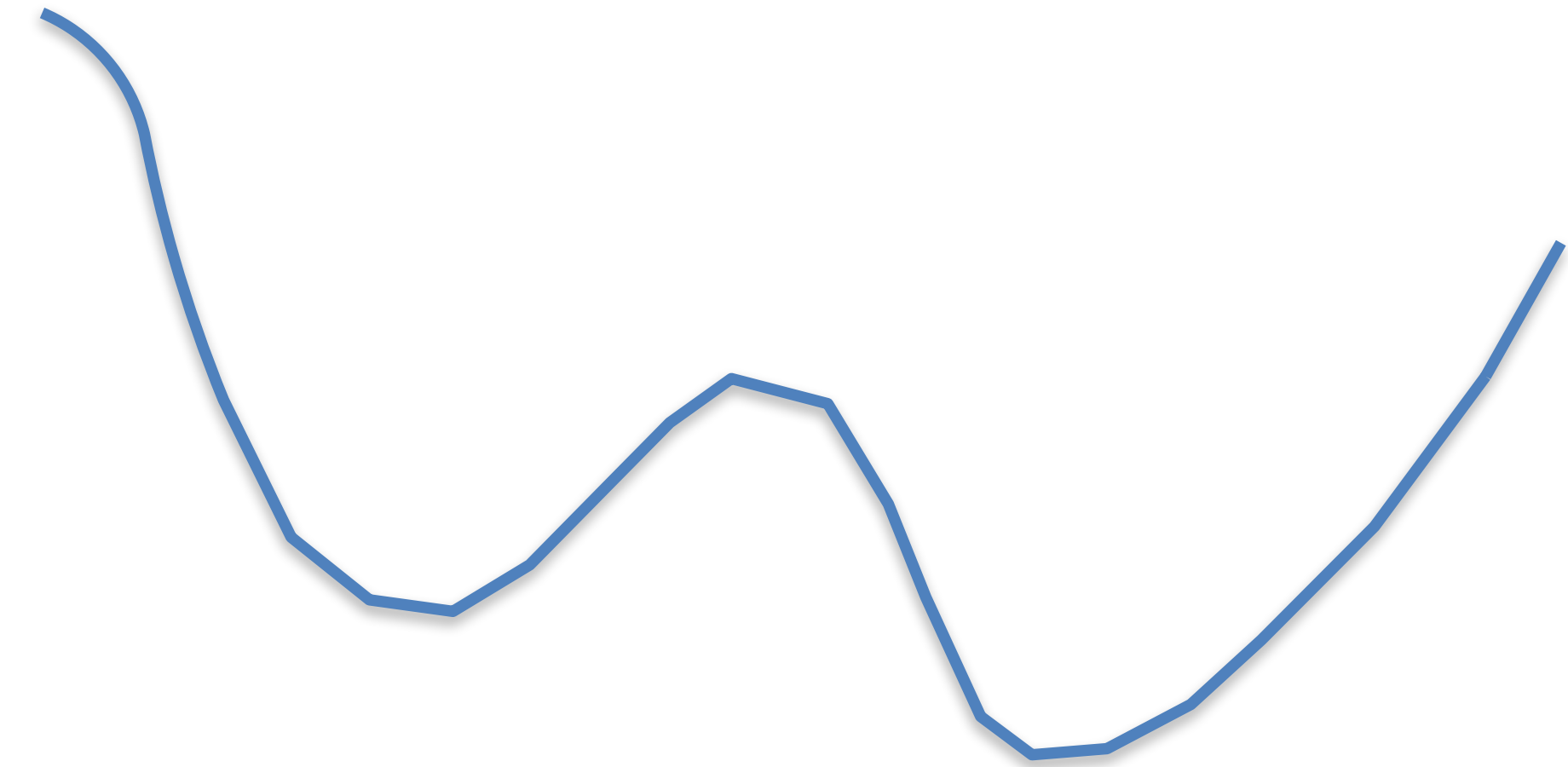
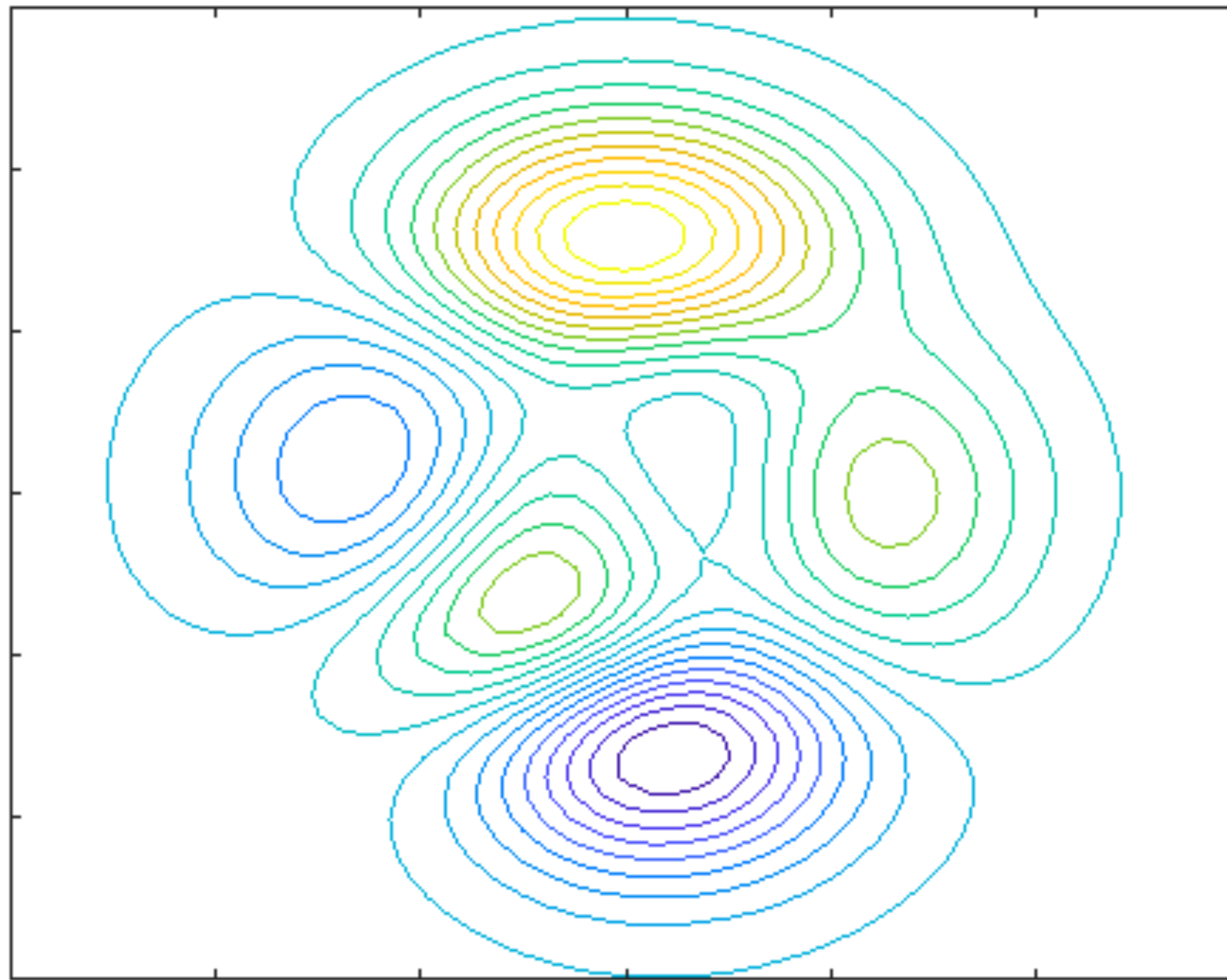
- $\text{bestLoss} = \text{inf}$
- $W = \text{setRand}$
- stepsize
- Loop
  - $W_{\text{local}} = W + \text{stepsize} * W_{\text{random}}$
  - Compute loss
  - if  $\text{loss} < \text{bestLoss}$ 
    - $\text{bestLoss} = \text{loss}$
    - $W = W_{\text{local}}$
  - endif
- endLoop



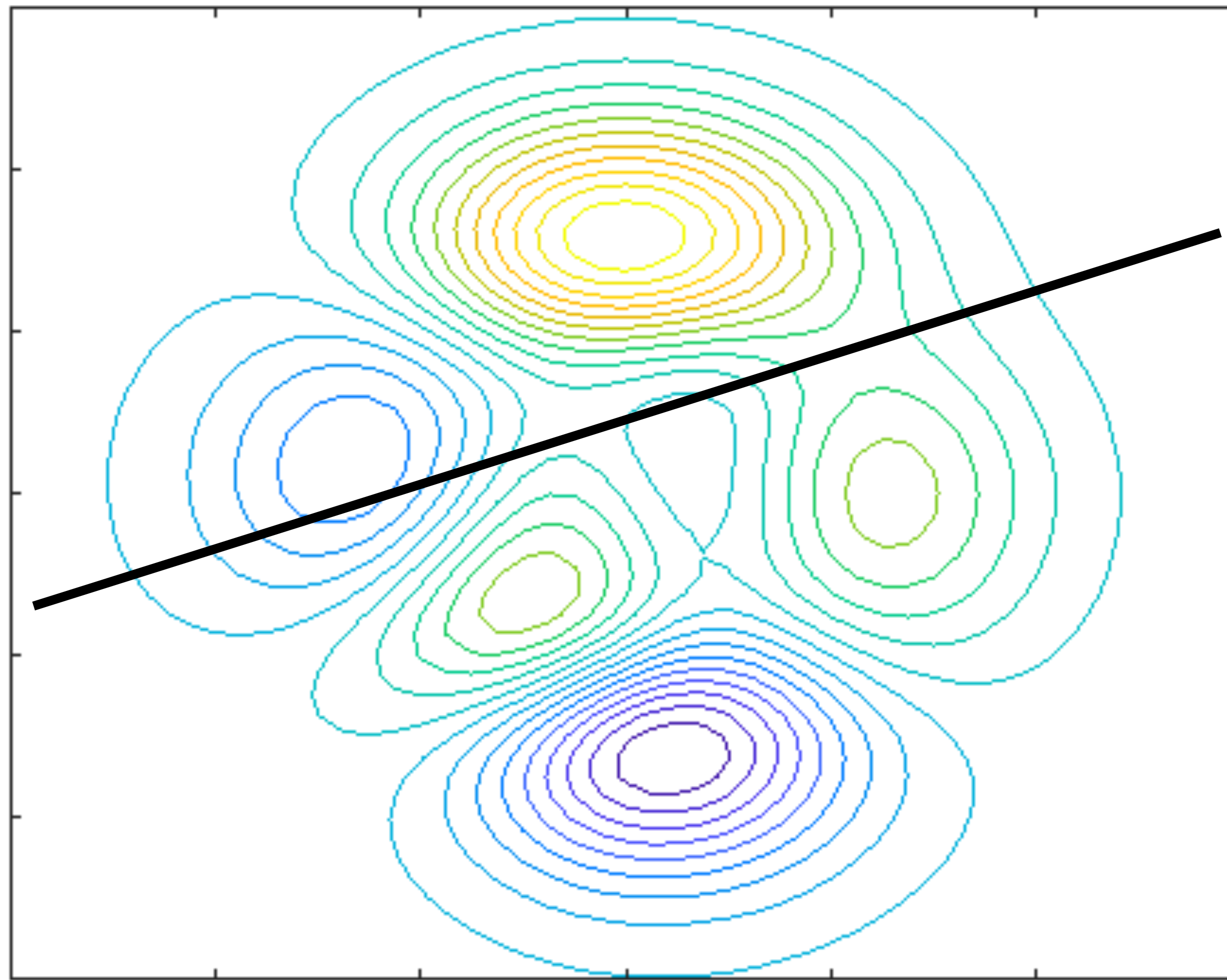
# Method 2: Follow Negative of Gradient



# Method 2: Follow Negative of Gradient

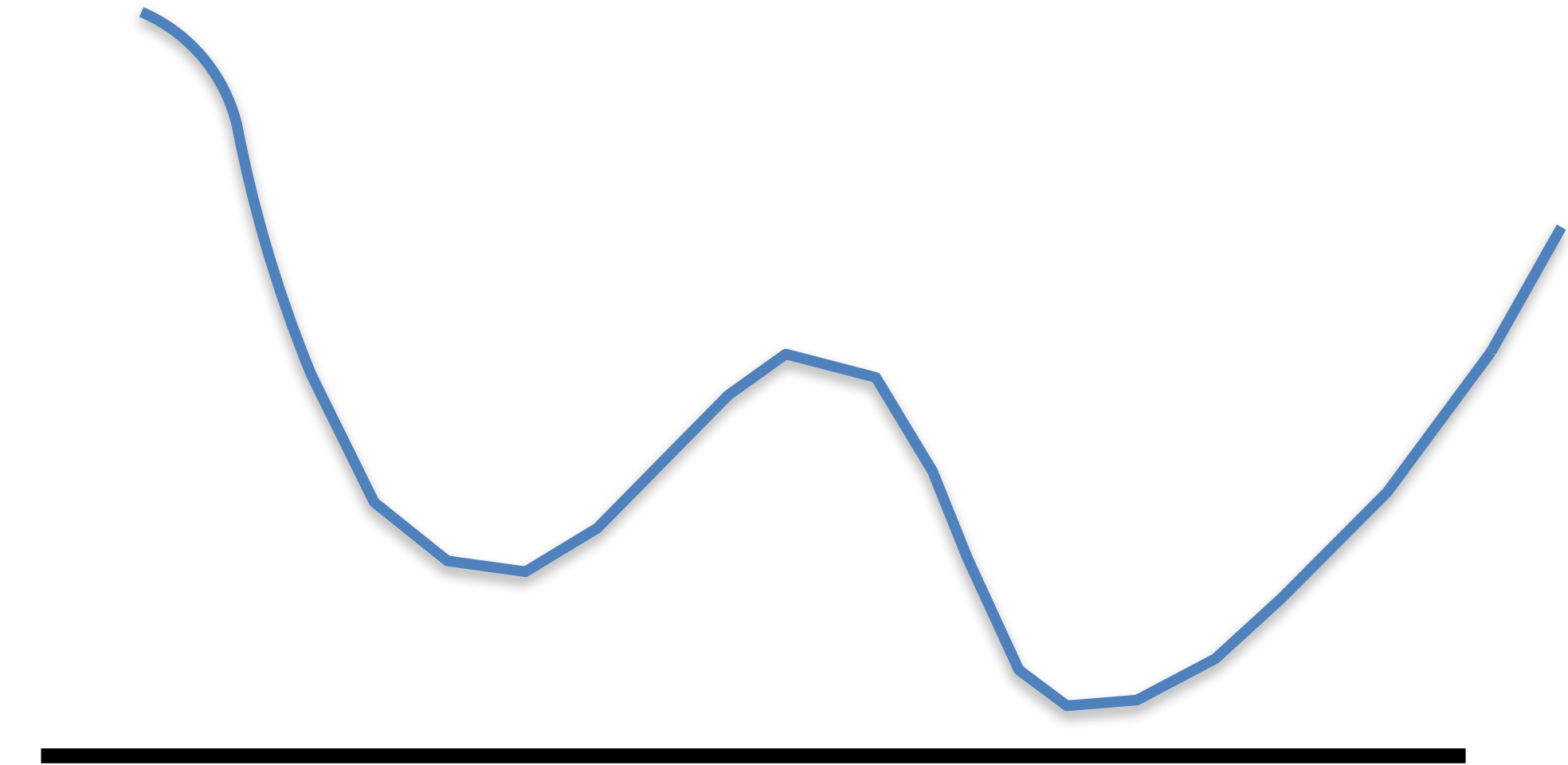
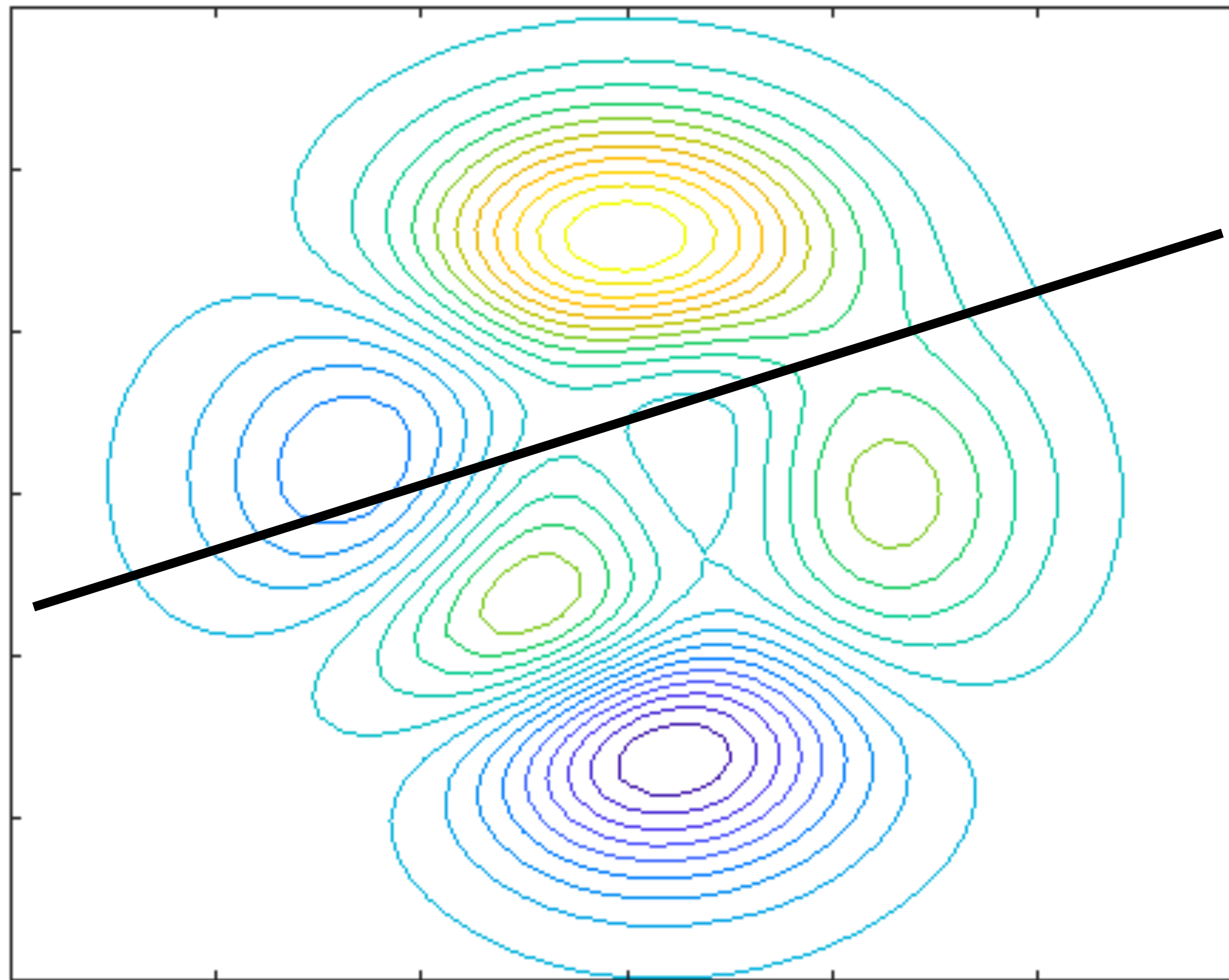


# Method 2: Follow Negative of Gradient

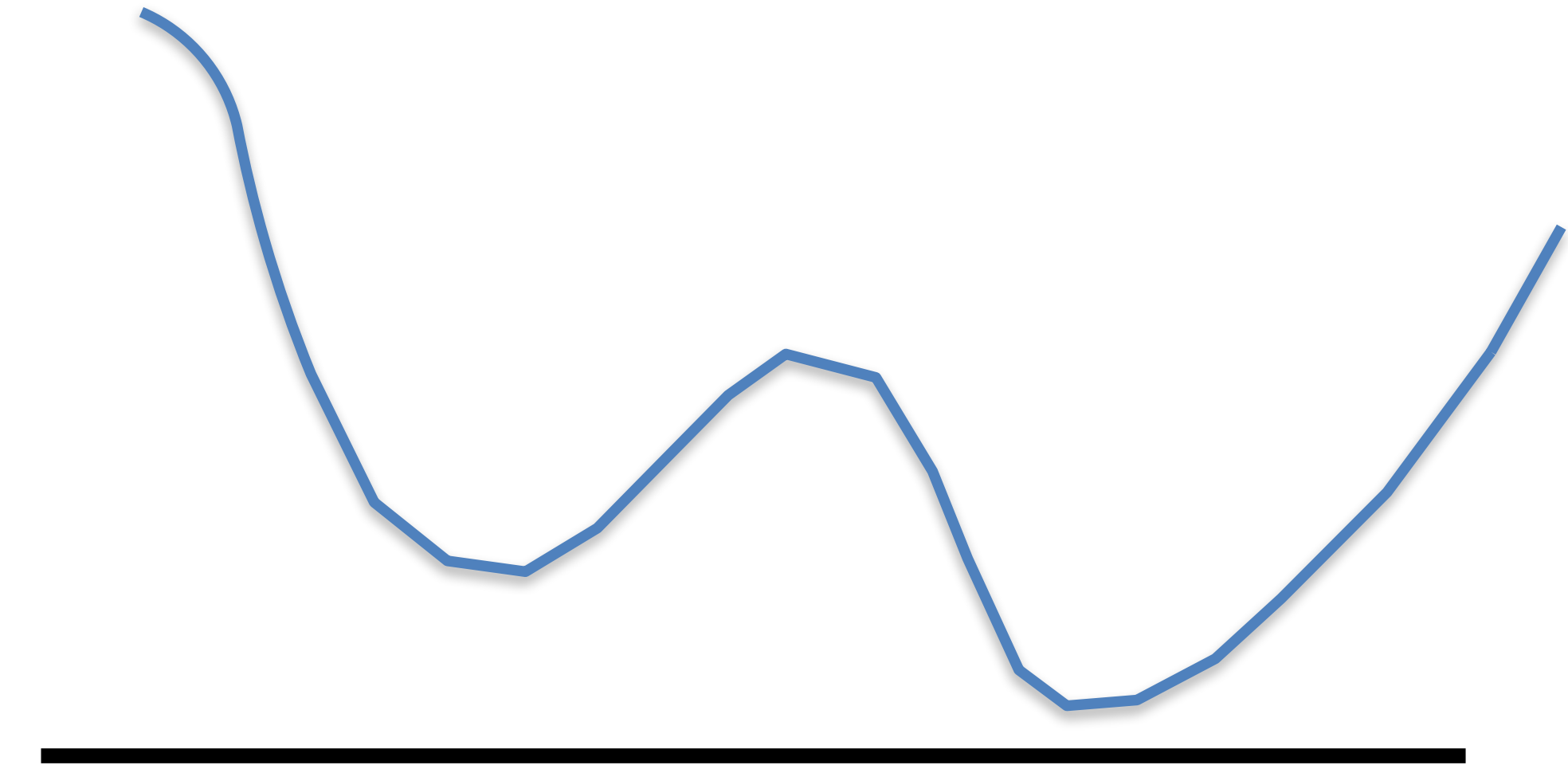
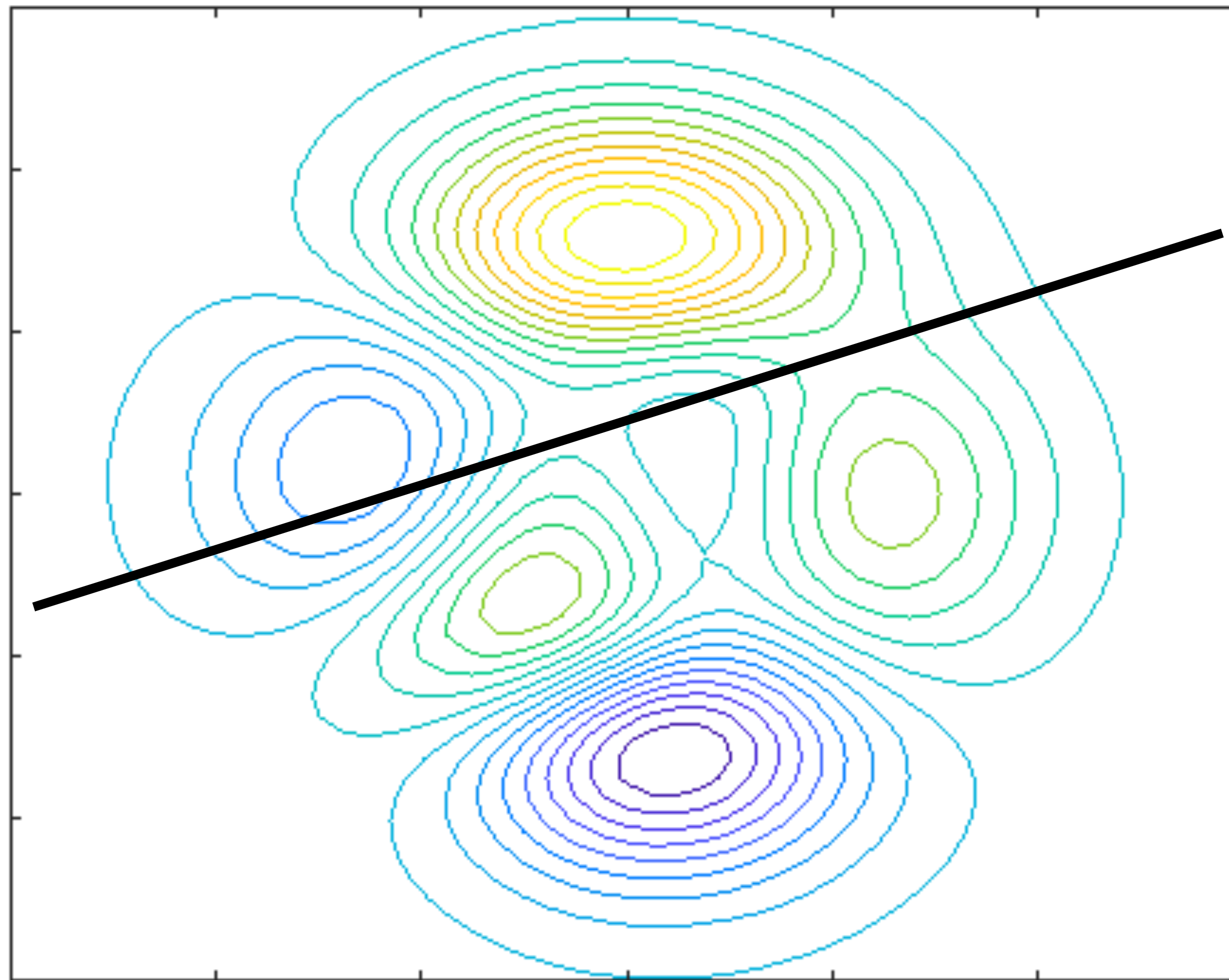




# Method 2: Follow Negative of Gradient



# Method 2: Follow Negative of Gradient



$$\frac{df(x)}{dx}$$