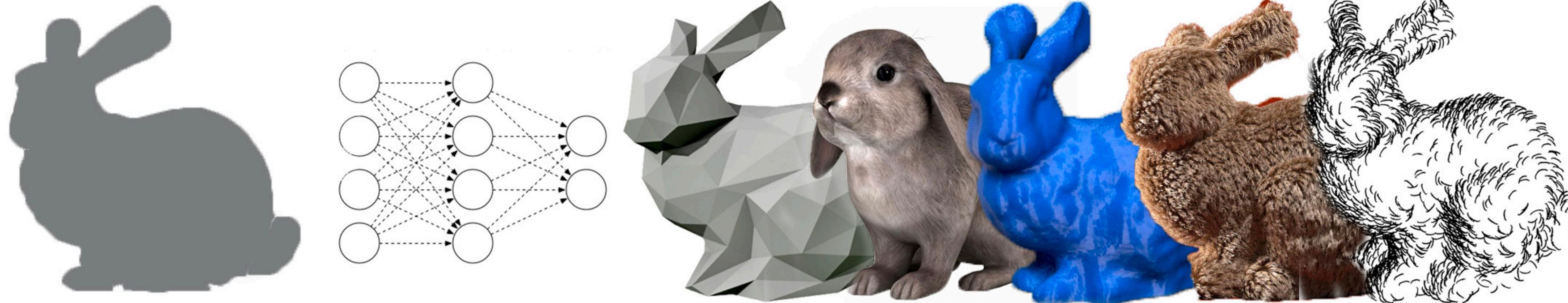


# **COMP0169: Machine Learning for Visual Computing**

## **Linear and Nonlinear Models**

### **(Optimization)**



# Lectures will be Recorded

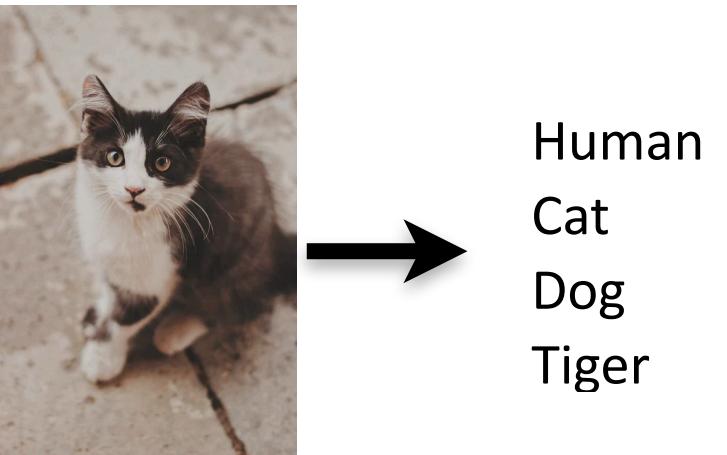
# Last Week

# Last Week

- Introduction and applications

# Last Week

- Introduction and applications
- Image classification



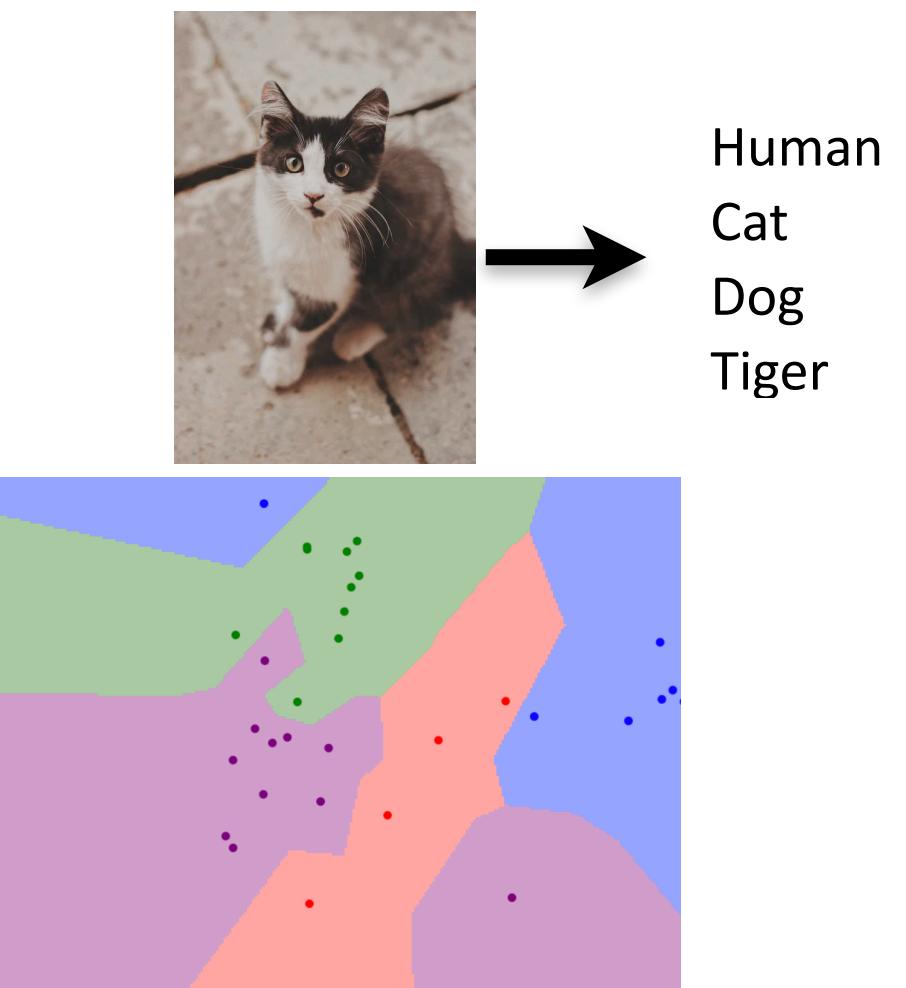
Human  
Cat  
Dog  
Tiger

# Last Week

- Introduction and applications

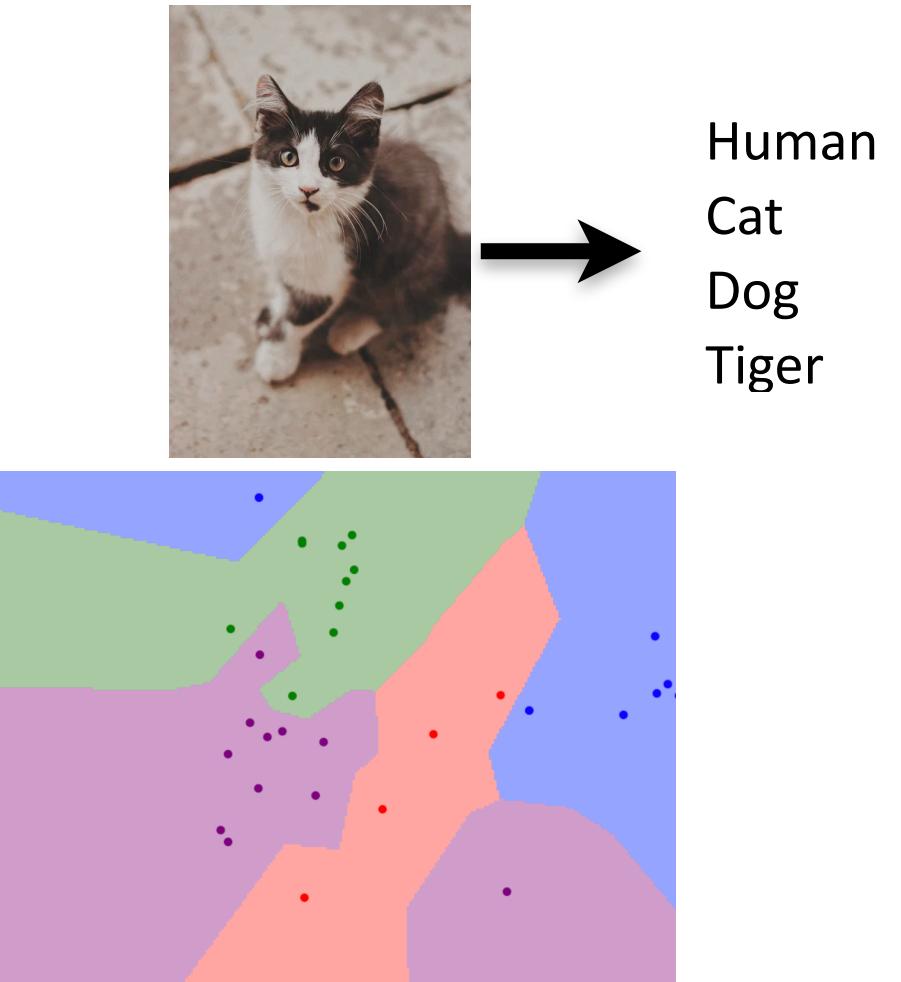
- Image classification

- kNN classifier



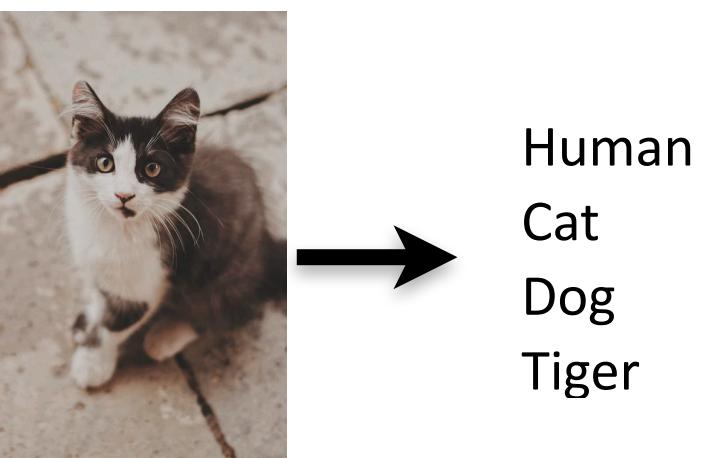
# Last Week

- Introduction and applications
- Image classification
- kNN classifier
- LS Fitting

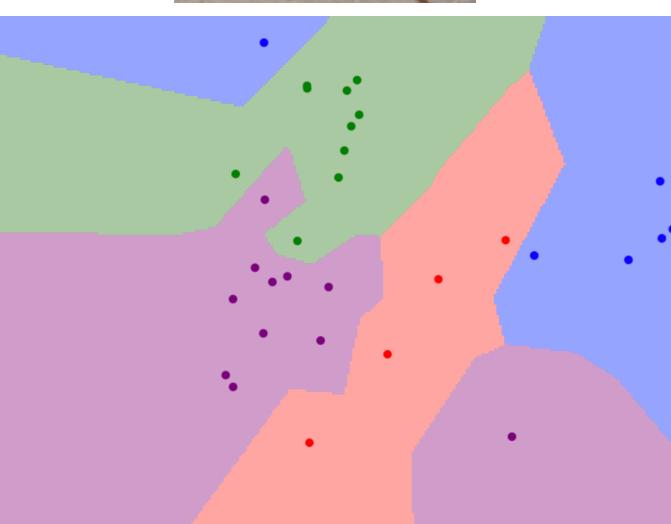


# Last Week

- Introduction and applications



- Image classification



- kNN classifier

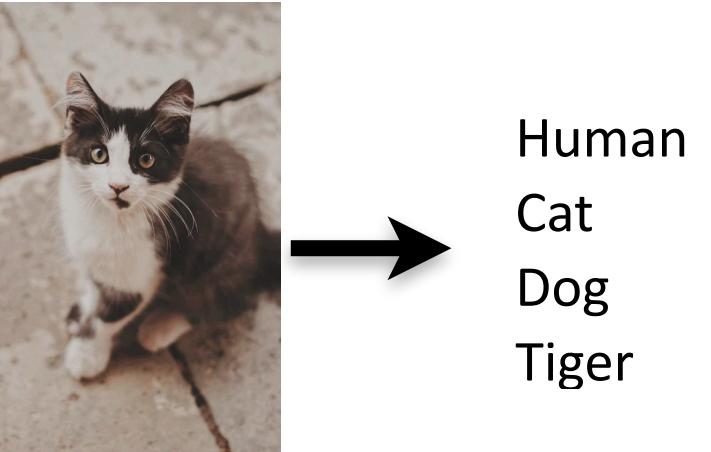
- LS Fitting

- Normal equation

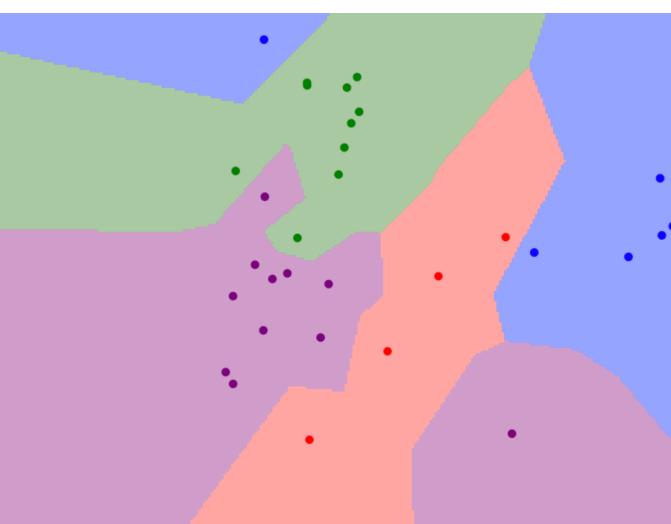
$$\mathbf{w} = (X^T X)^{-1} X^T y$$

# Last Week

- Introduction and applications



- Image classification



- kNN classifier

- LS Fitting

- Normal equation

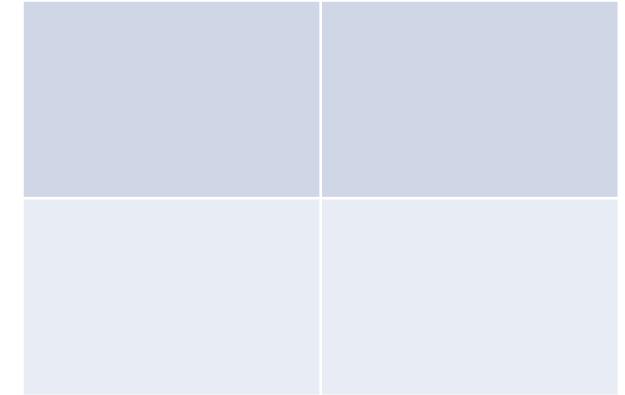
$$\mathbf{w} = (X^T X)^{-1} X^T y$$

- Hyperparameter search (k-fold validation)

# Image Classification

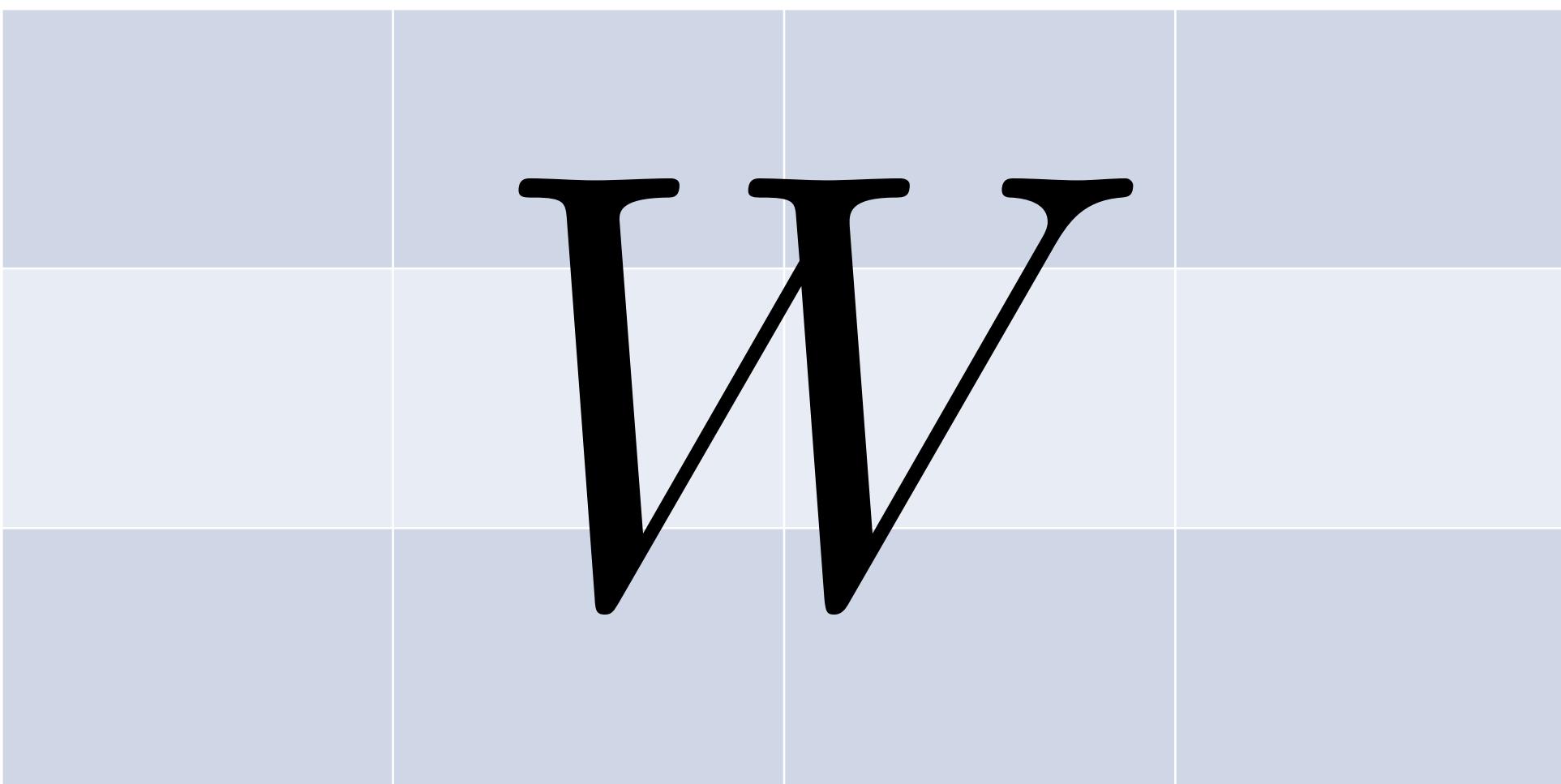
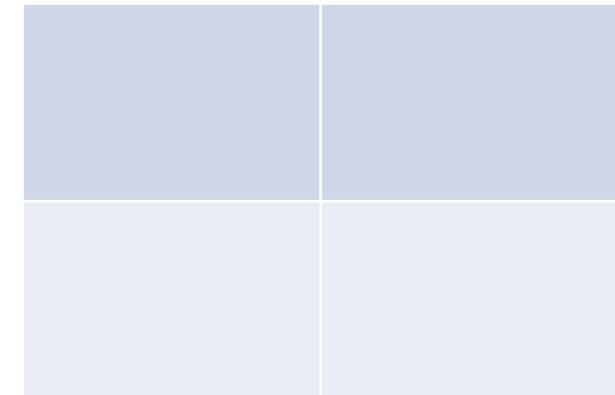
# Image Classification

$I$



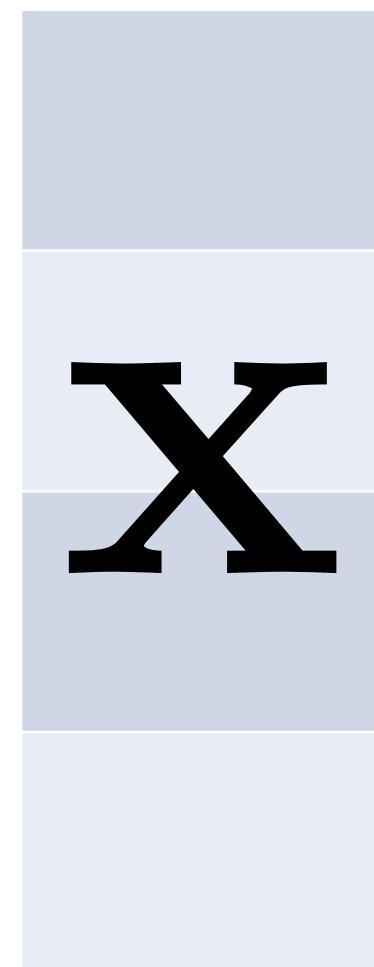
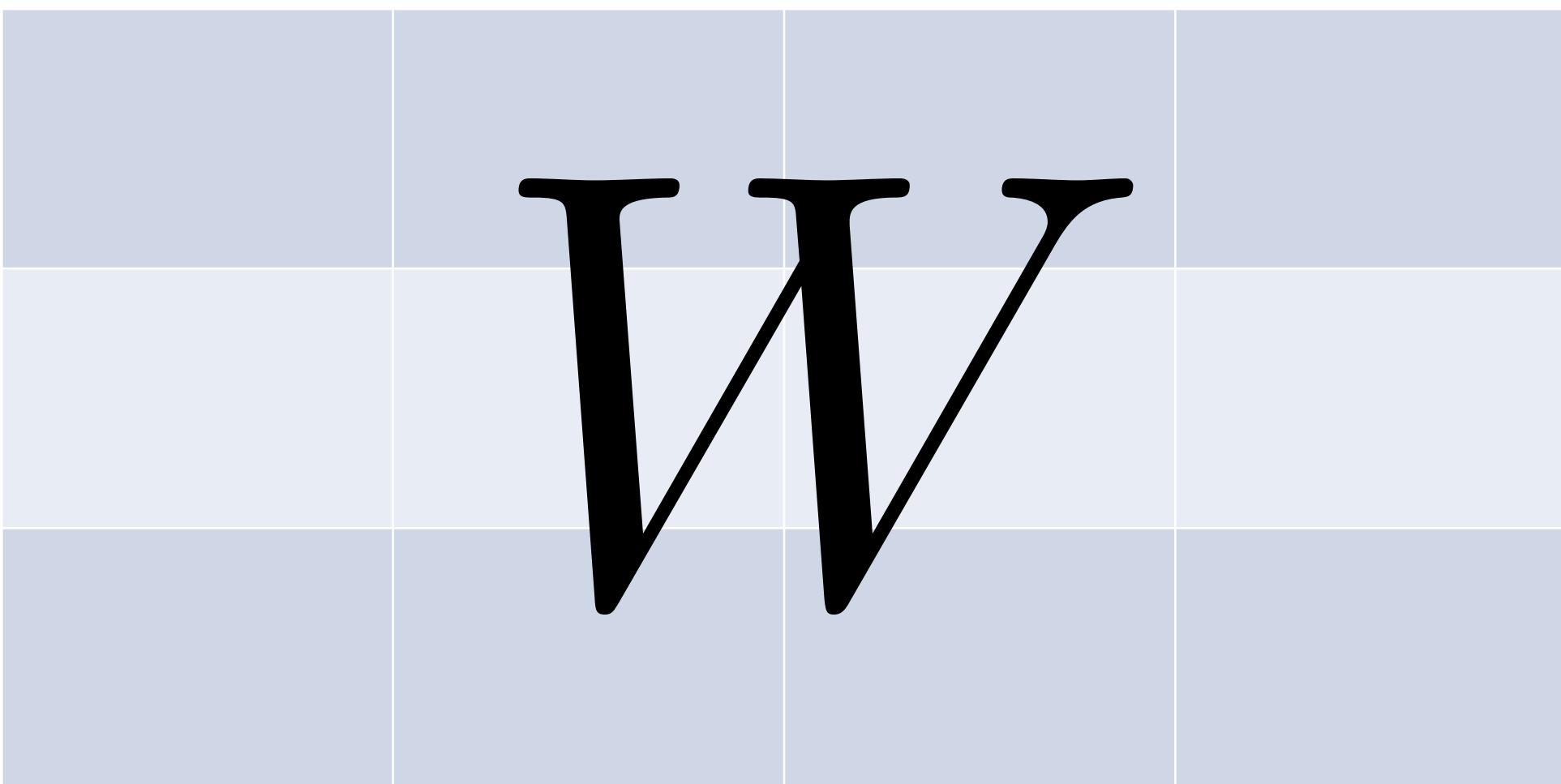
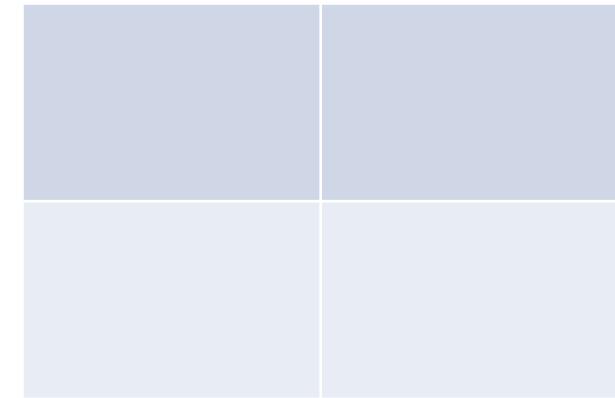
# Image Classification

$I$



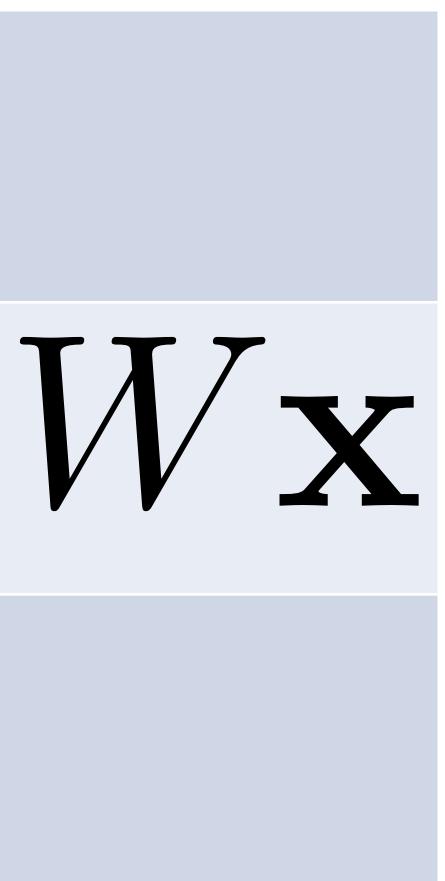
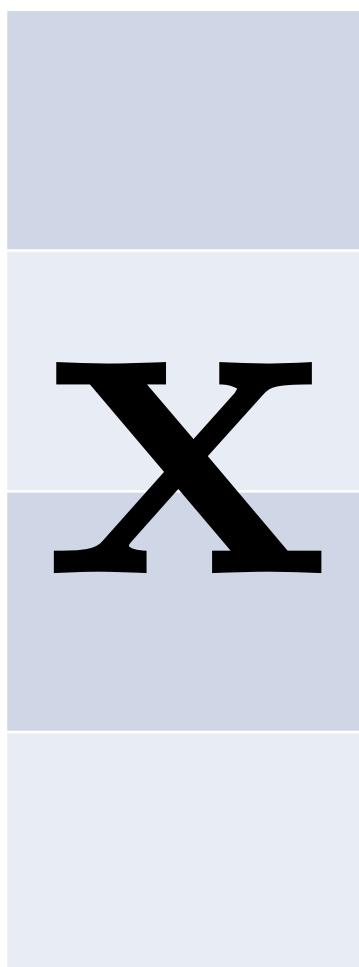
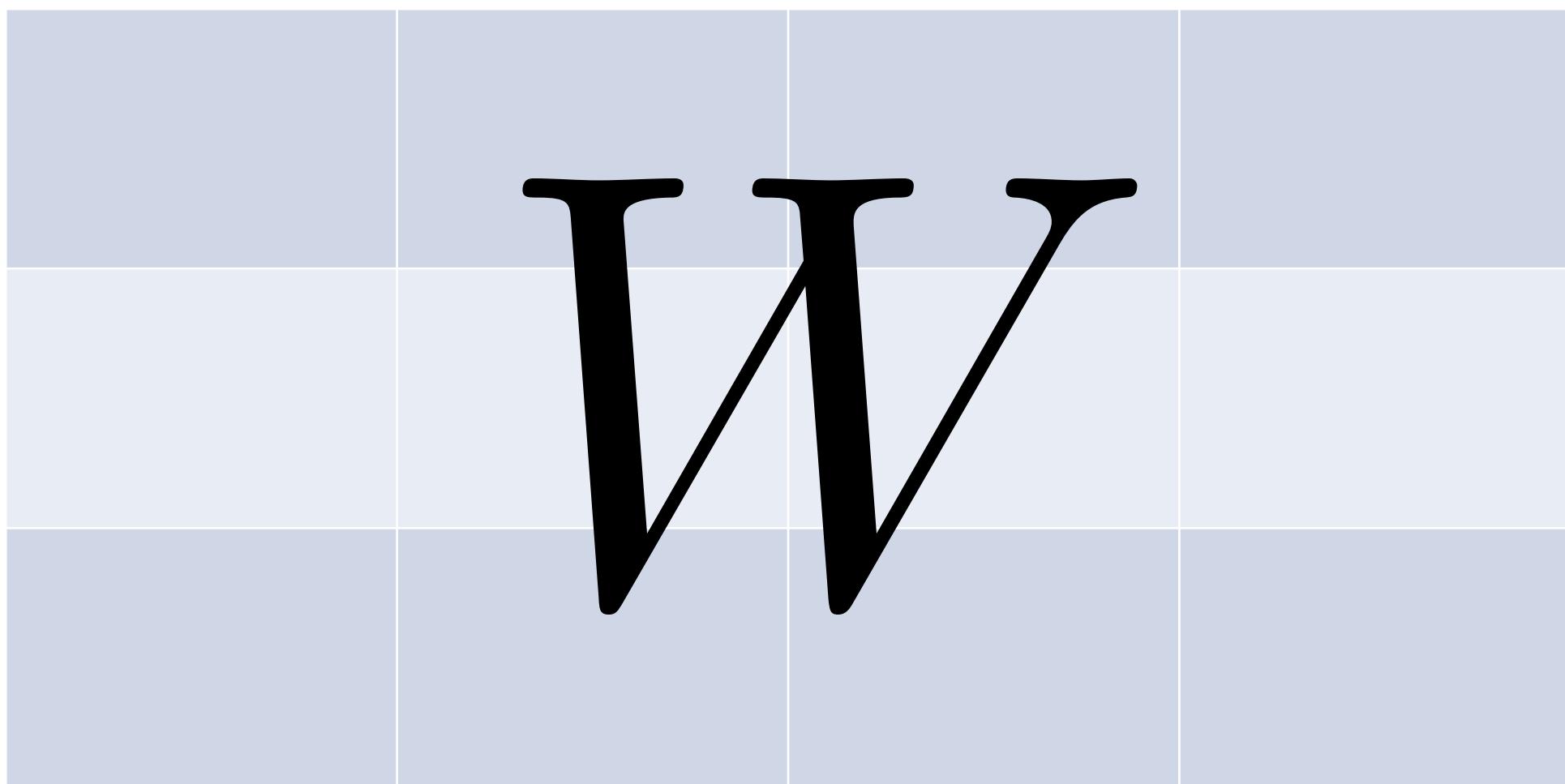
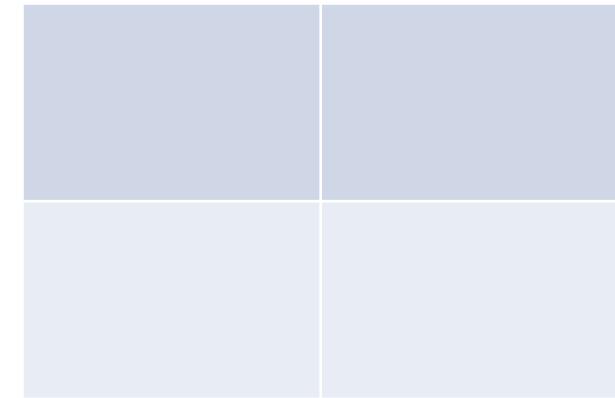
# Image Classification

$I$



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$I$



# Image Classifier

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- Data (training/validation/test)

$$\{\mathbf{X}_i, y_i\}$$

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- Data (training/validation/test)  $\{\mathbf{X}_i, y_i\}$
- Scoring function  
(probability of belonging to class k)  $f(\mathbf{x}_i, \mathbf{W})$

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- Loss function  $L_i(W) := h(f(\mathbf{x}_i, W), y_i)$   $L(W) := \frac{1}{N} \sum_i L_i(W)$
- Optimization  $\mathbf{W}^* := \arg \min L(W) = \arg \min L_i(W)/N$

# Linear versus Nonlinear Models

$$s = f(\mathbf{x}_i, W) = W\mathbf{x}_i$$

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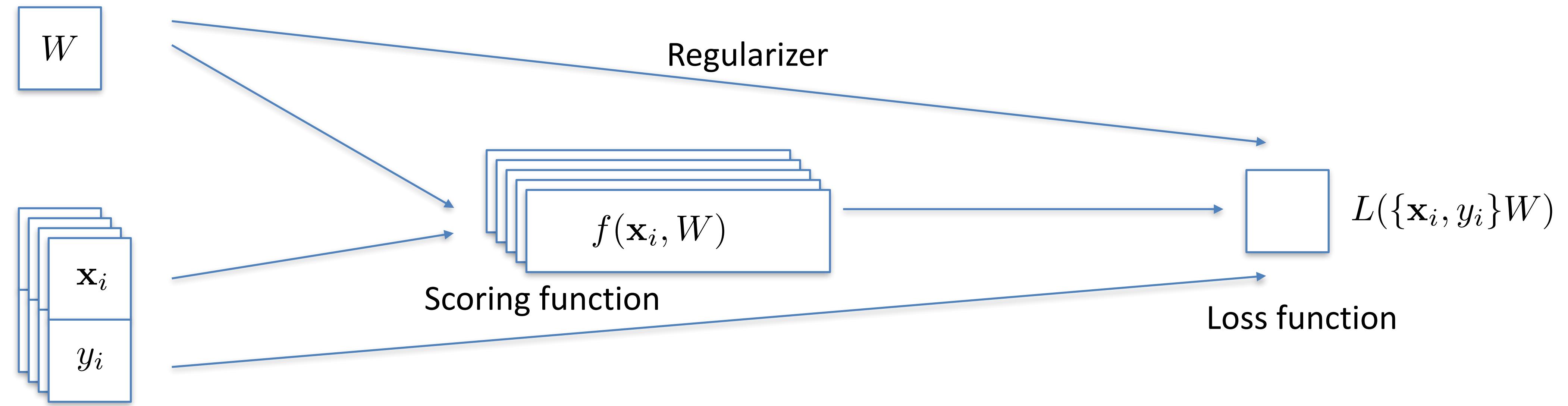
- Objective function #1  $L_i(W) := (Wx_i - y_i)^2$

# Linear versus Nonlinear Models

$$s = f(\mathbf{x}_i, W) = W \mathbf{x}_i$$

- Objective function #1  $L_i(W) := (W \mathbf{x}_i - y_i)^2$
- Objective function #2  $L_i(W) := -\log \left( \frac{e^{W \mathbf{x}_i y_i}}{\sum_j e^{W \mathbf{x}_j y_j}} \right) = -\log \left( \frac{e^{sy_i}}{\sum_j e^{sy_j}} \right)$

# Image Classifier



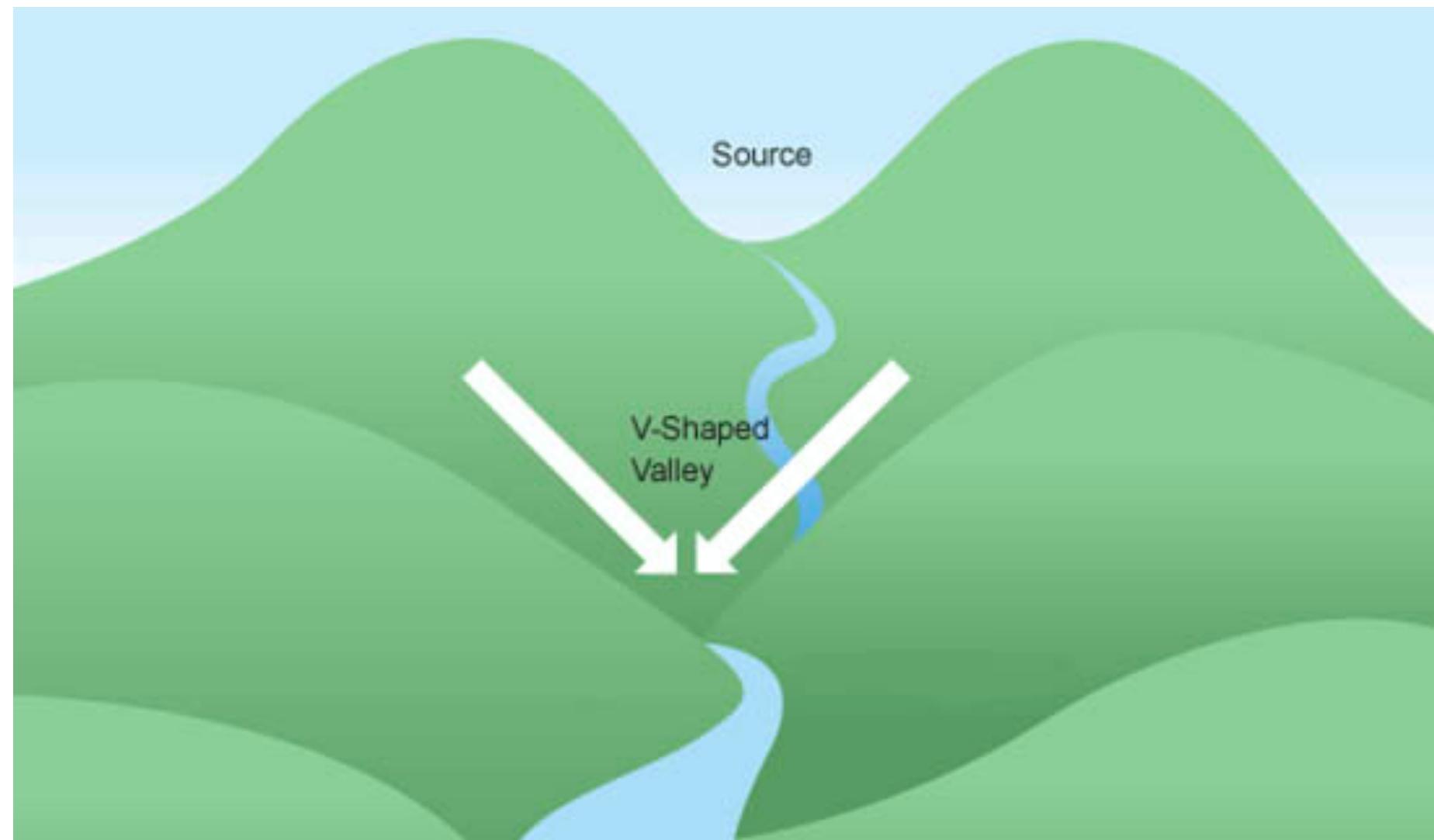
# Optimize

$$\mathbf{W}^* = \arg \min g(\mathbf{x}_i, W)$$

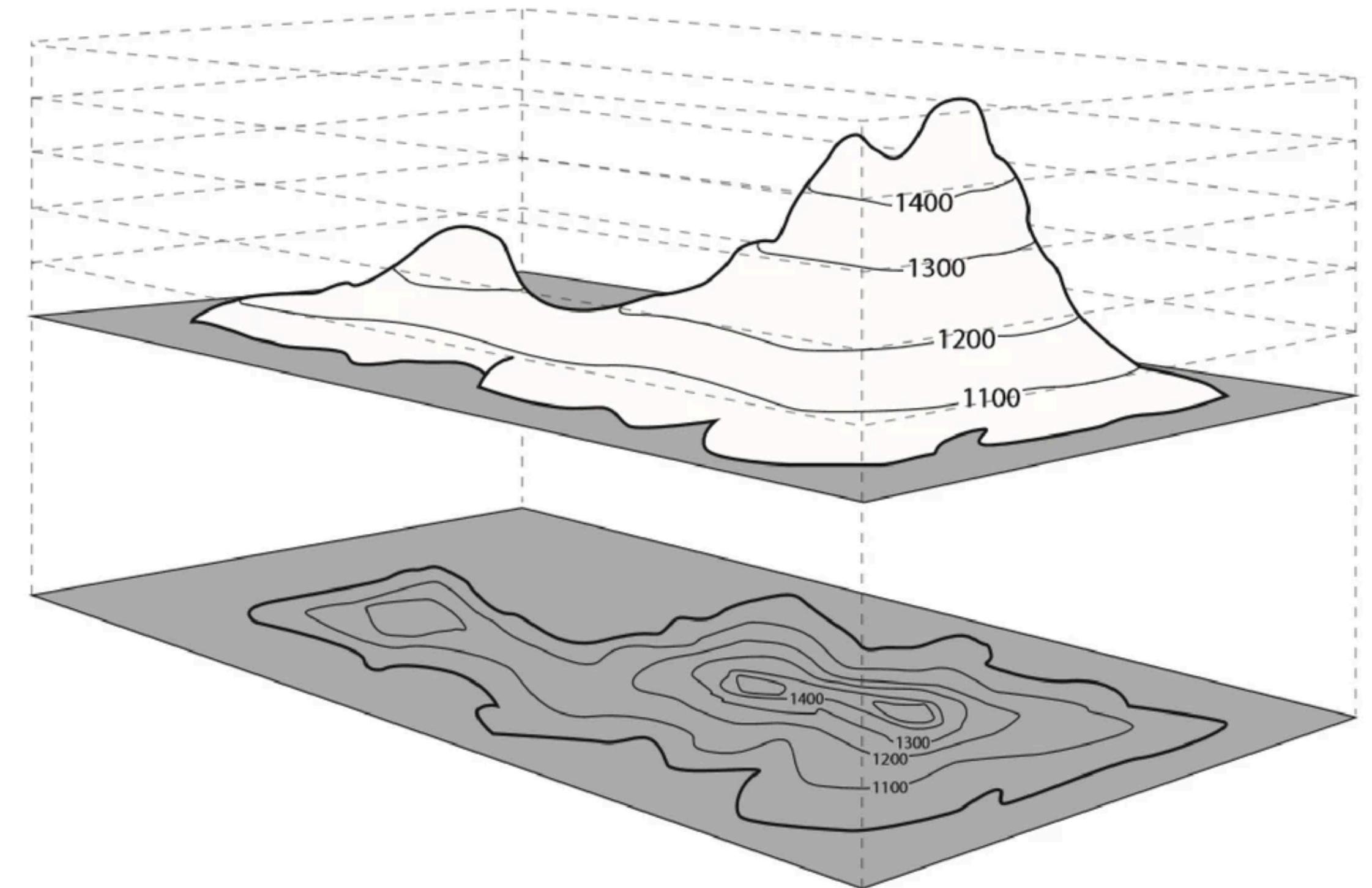
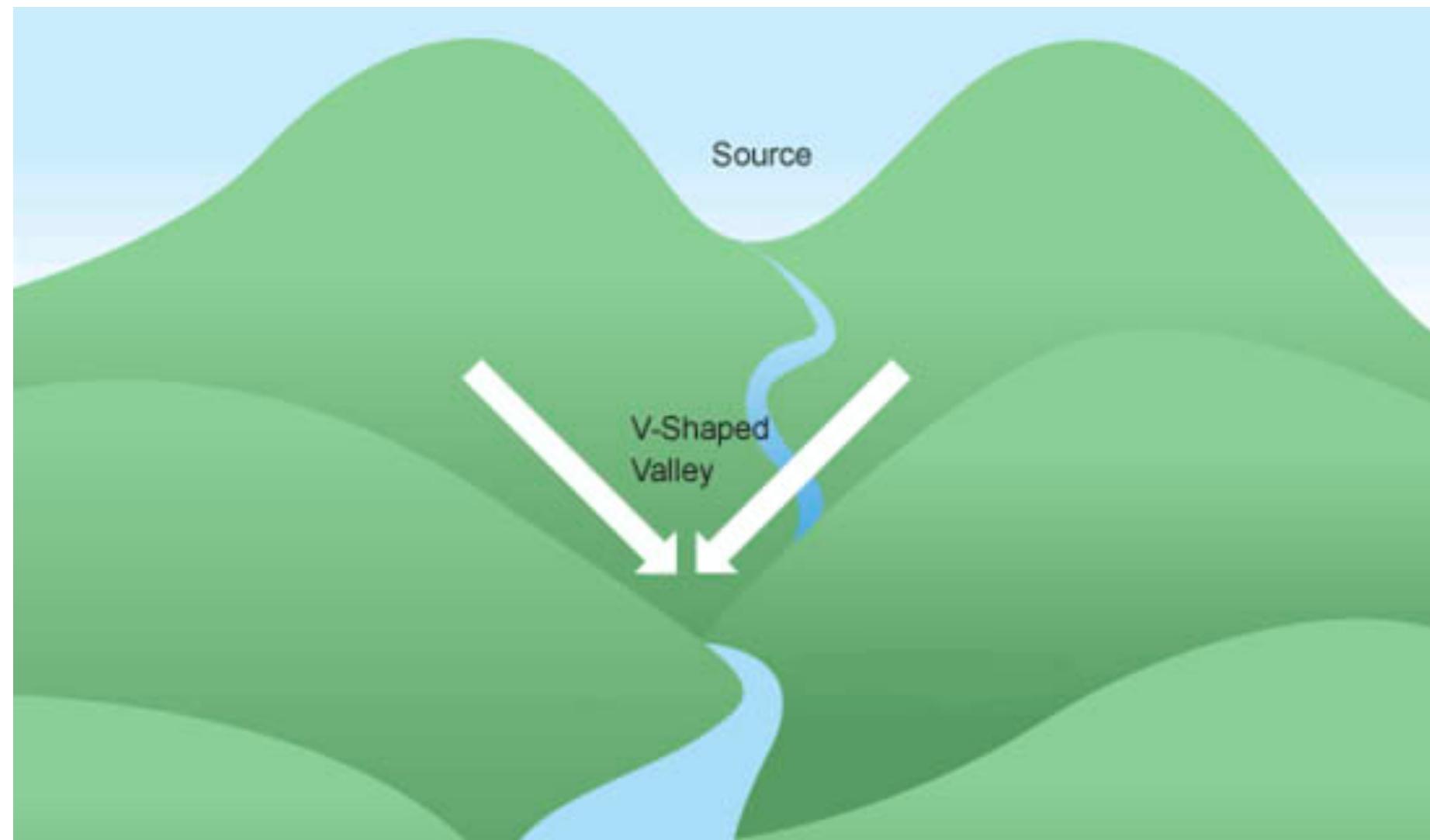
$$\nabla g(\mathbf{x}_i, W) = \left[ \frac{\partial g(\mathbf{x}_i, W)}{\partial w_i} = 0 \right]_{D \times 1}$$

# Error Landscape

# Error Landscape

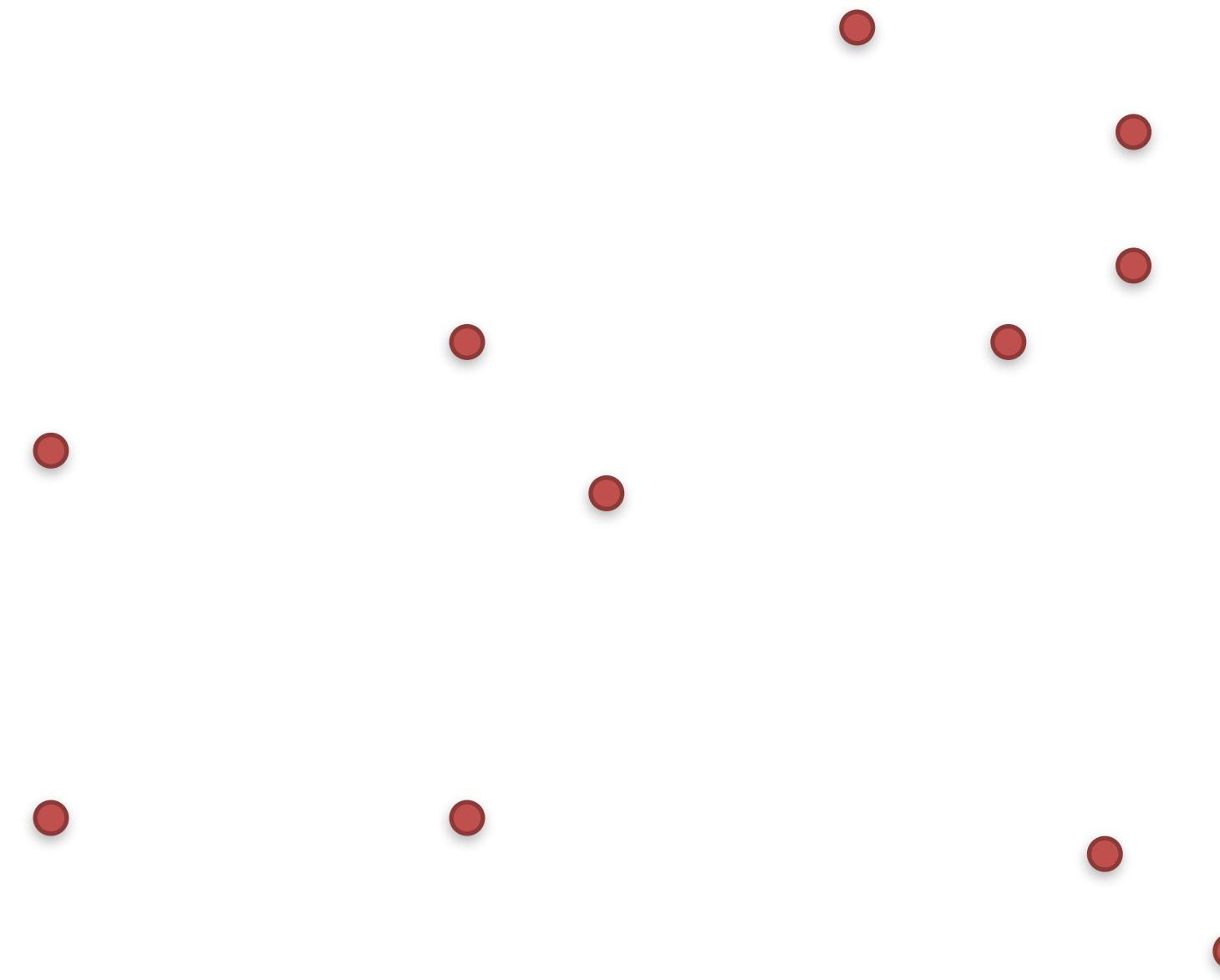


# Error Landscape



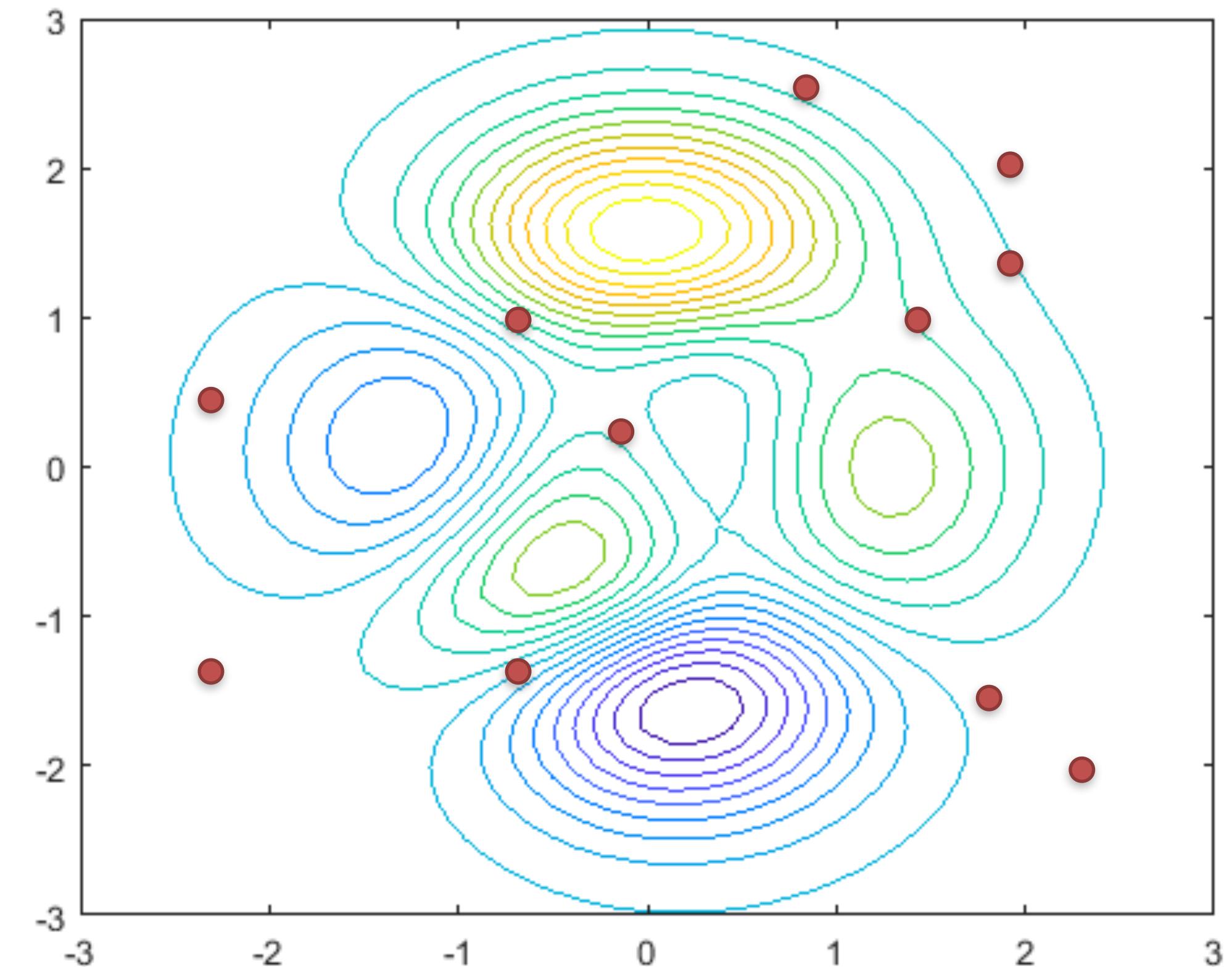
# Method 1a: Random search

- $\text{bestLoss} = \inf$
- Loop
  - Pick random  $W$
  - Compute loss
  - if  $\text{loss} < \text{bestLoss}$ 
    - $\text{bestLoss} = \text{loss}$
    - $\text{currW} = W$
  - endif
- endLoop



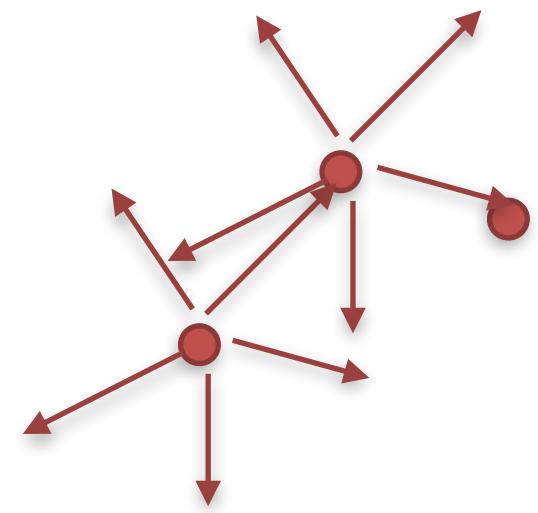
# Method 1a: Random search

- bestLoss = inf
- Loop
  - Pick random W
  - Compute loss
  - if loss < bestLoss
    - bestLoss = loss
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  - endif
- endLoop



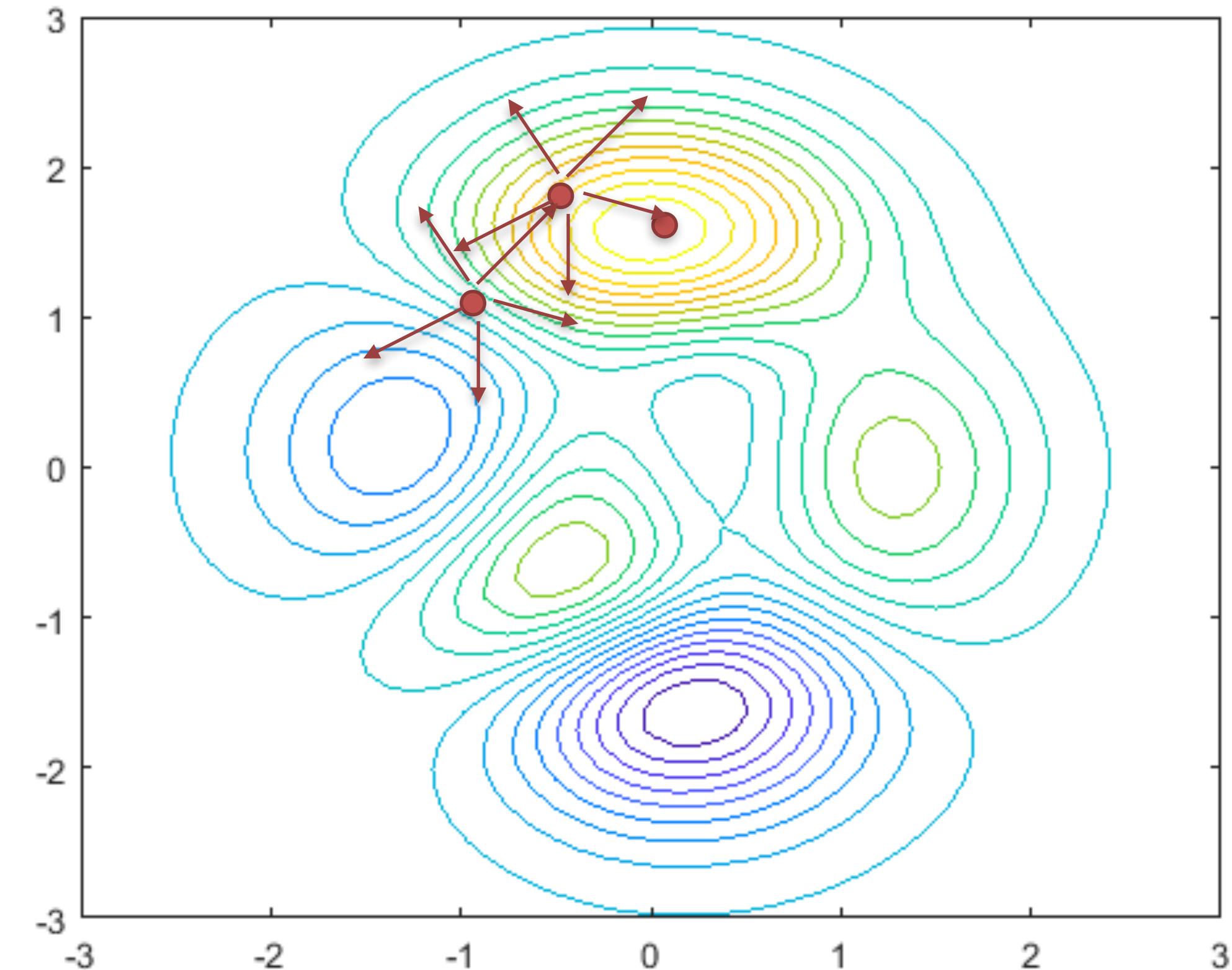
# Method 1b: Random search++

- bestLoss = inf
- $W = \text{setRand}$
- stepsize
- Loop
  - $W_{\text{local}} = W + \text{stepsize} * W_{\text{random}}$
  - Compute loss
  - if loss < bestLoss
    - bestLoss = loss
    - $W = W_{\text{local}}$
  - endif
- endLoop

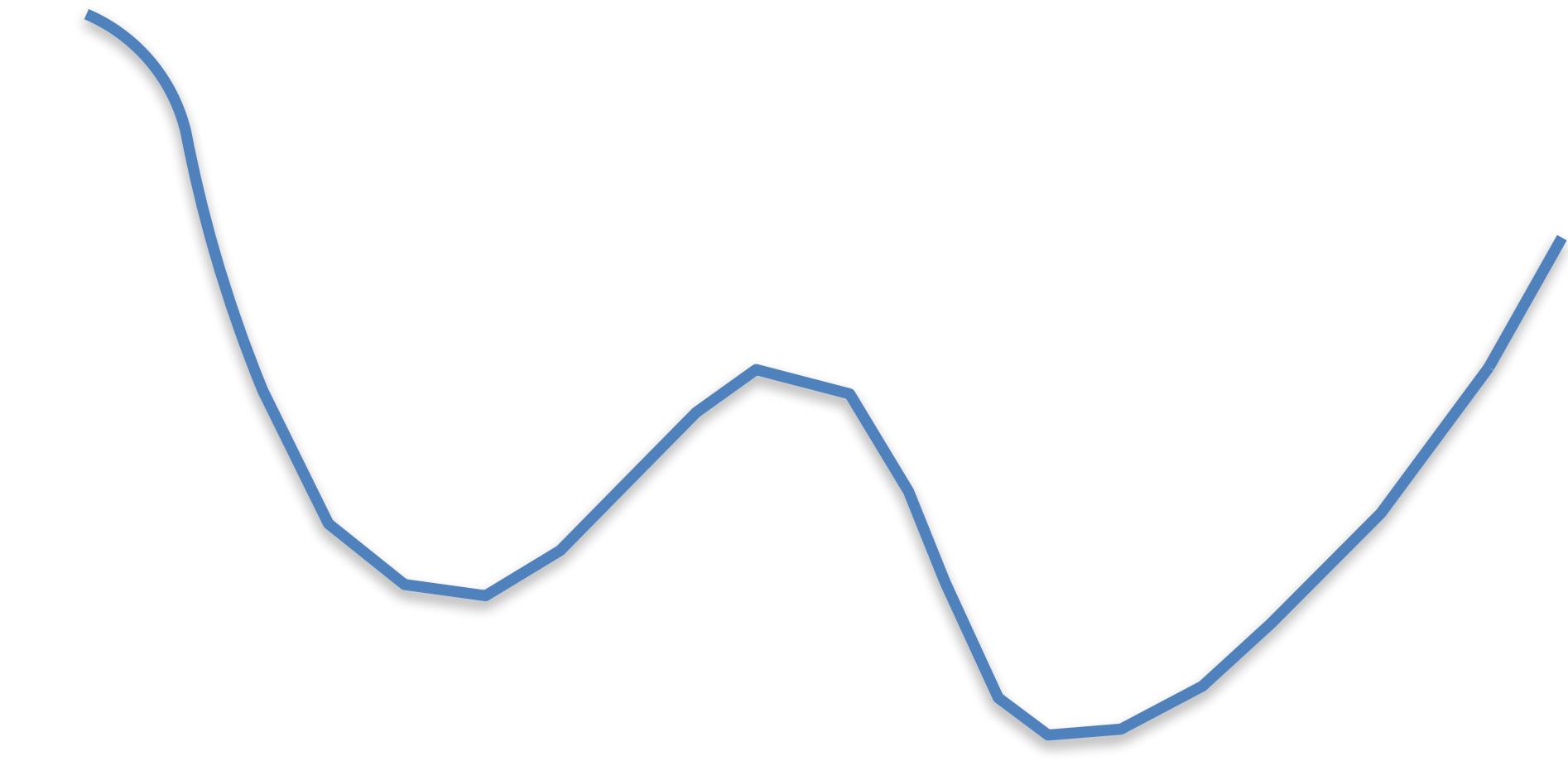


# Method 1b: Random search++

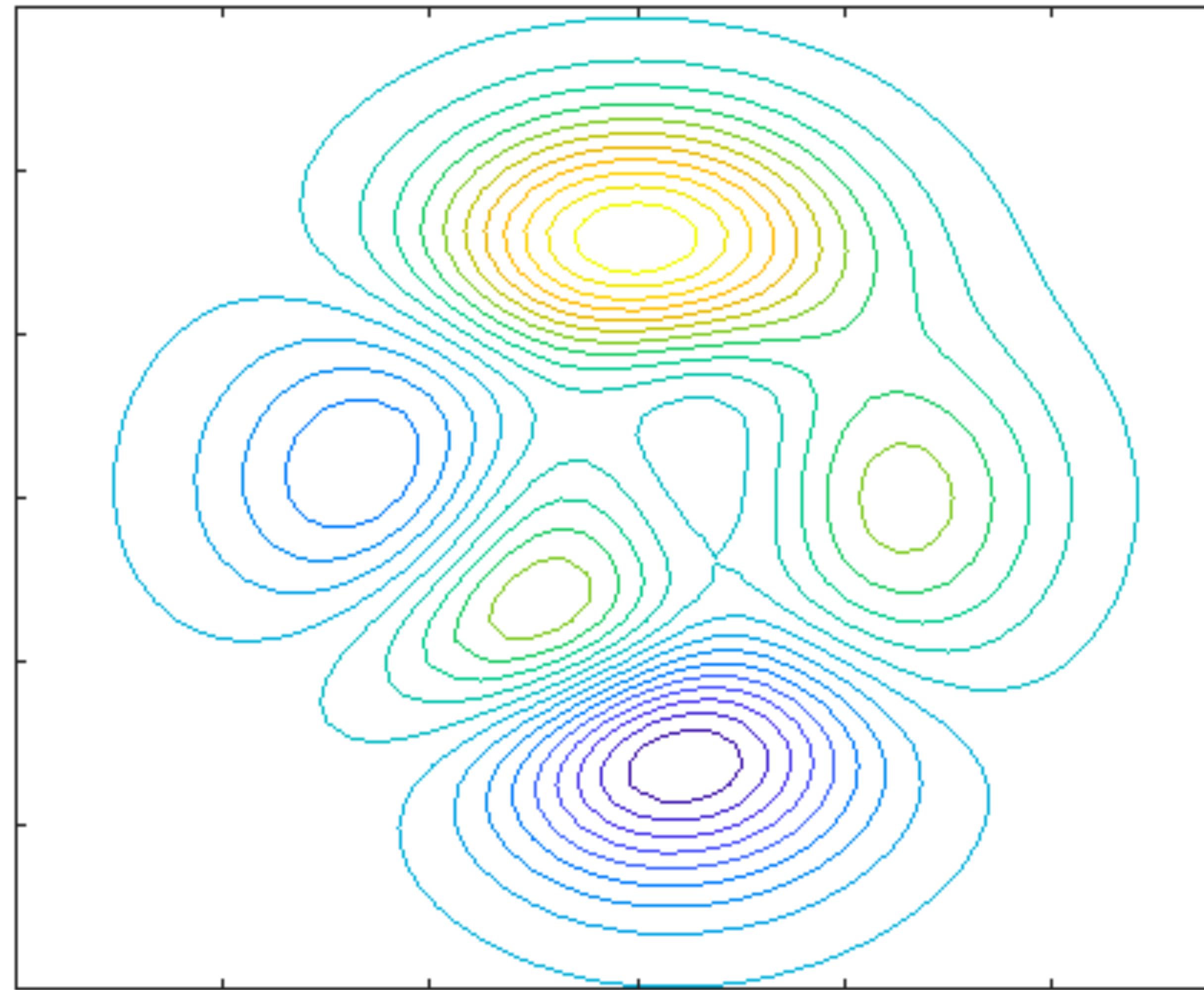
- bestLoss = inf
- $W = \text{setRand}$
- stepsize
- Loop
  - $W_{\text{local}} = W + \text{stepsize} * W_{\text{random}}$
  - Compute loss
  - if loss < bestLoss
    - bestLoss = loss
    - $W = W_{\text{local}}$
  - endif
- endLoop



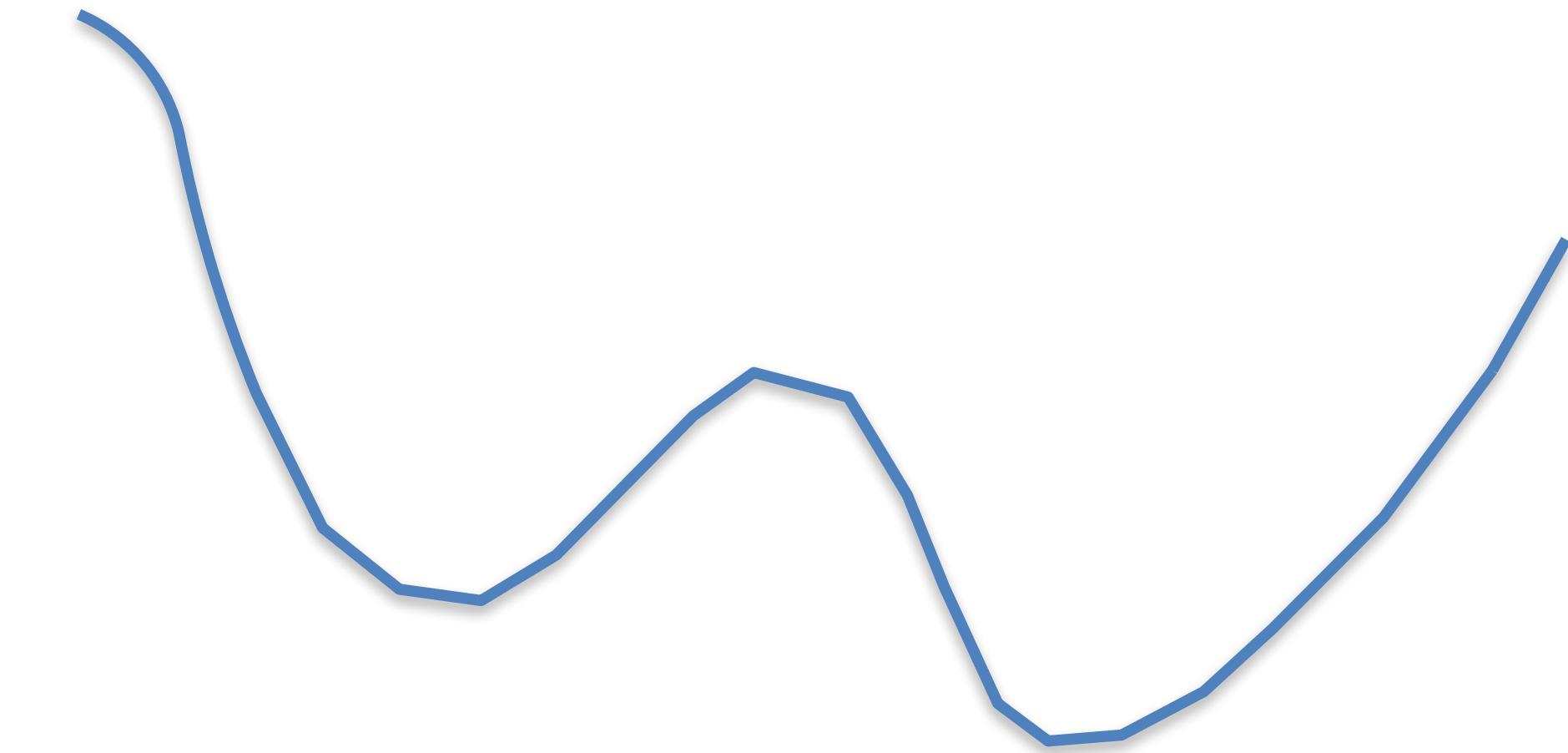
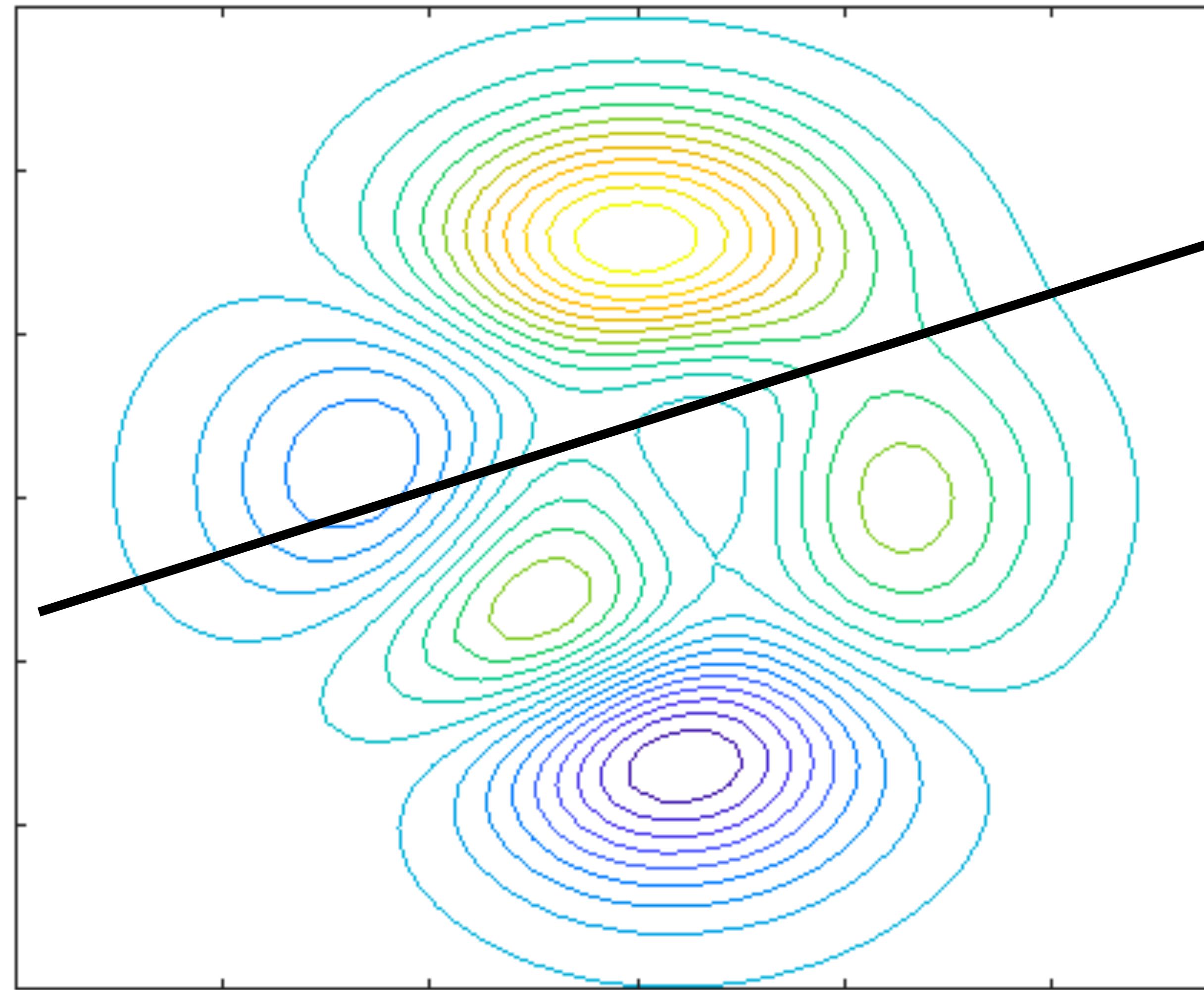
# Method 2: Follow Negative of Gradient



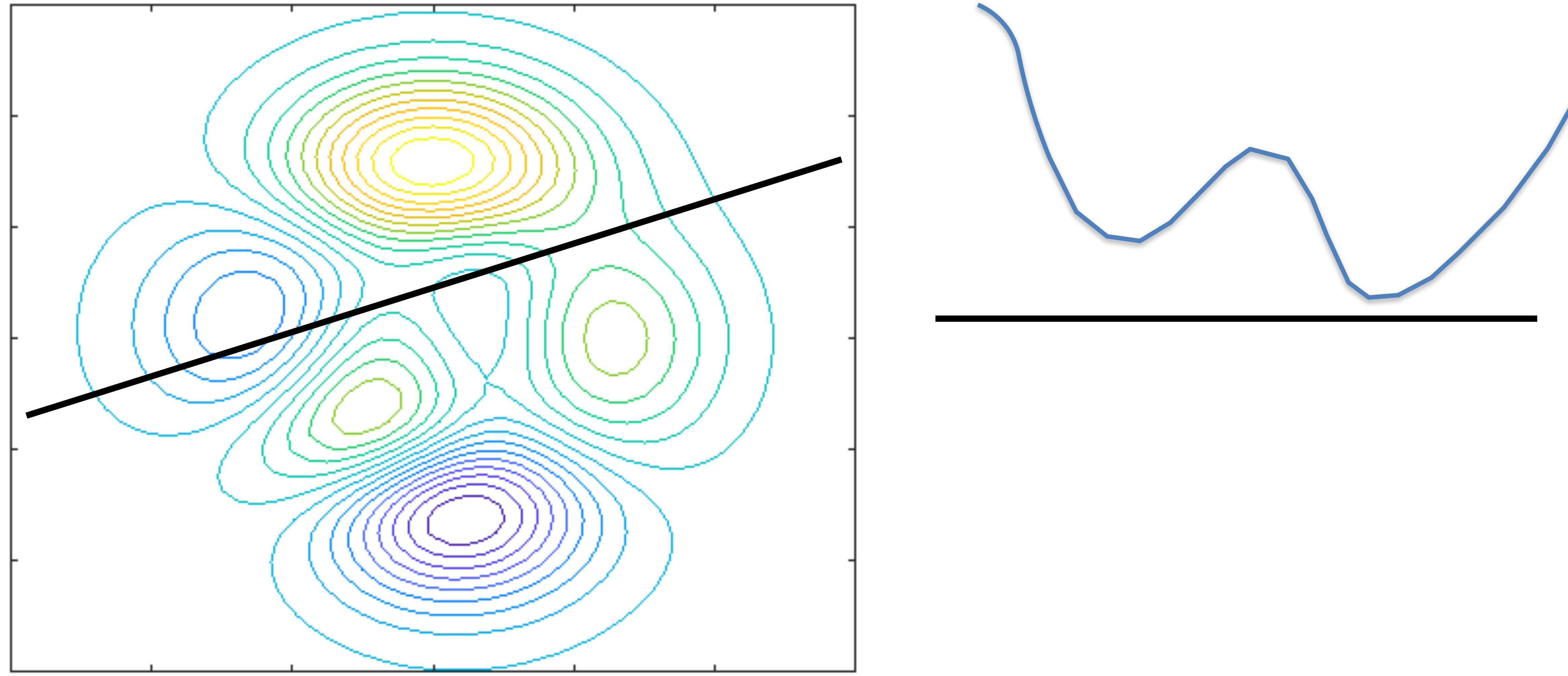
# Method 2: Follow Negative of Gradient



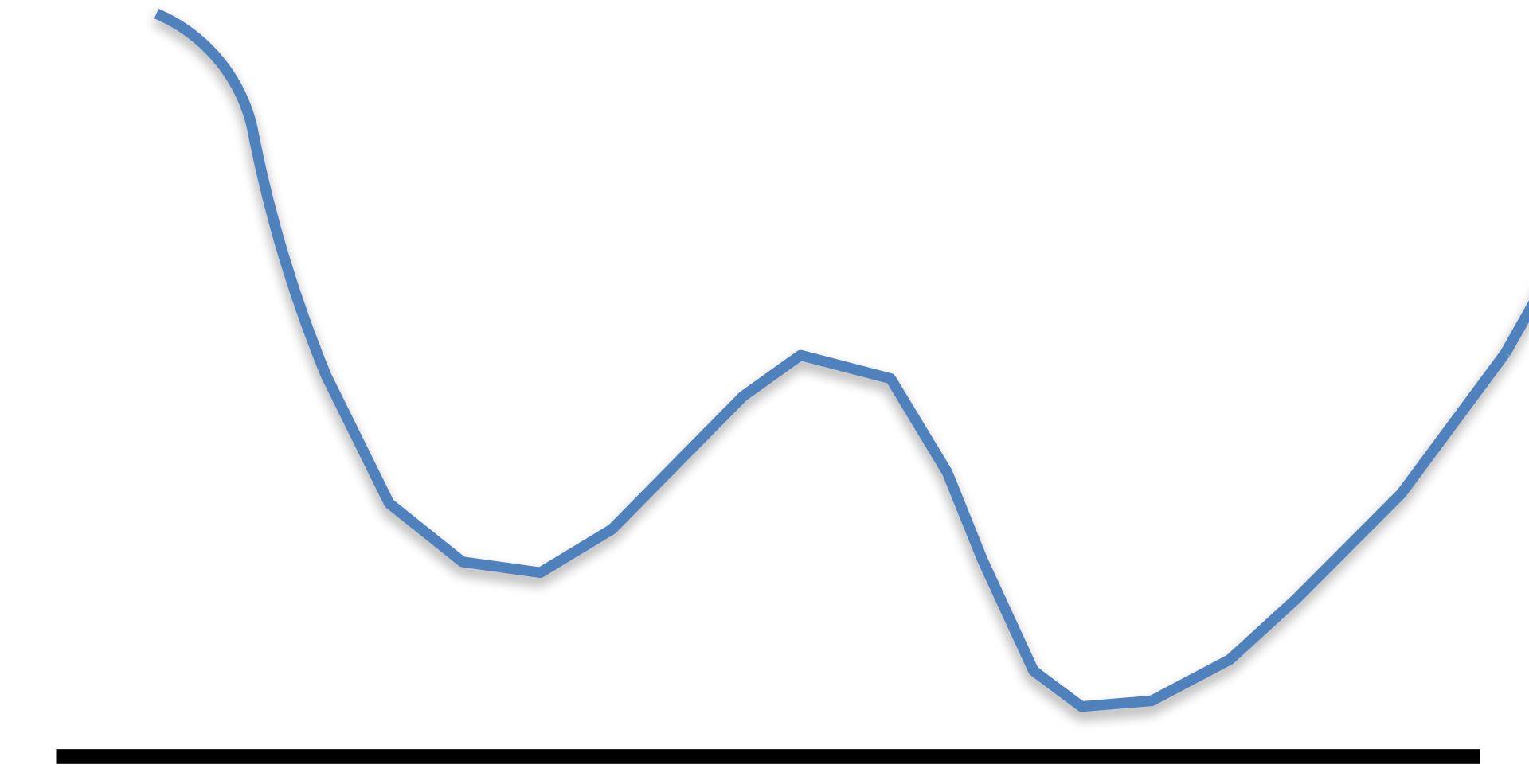
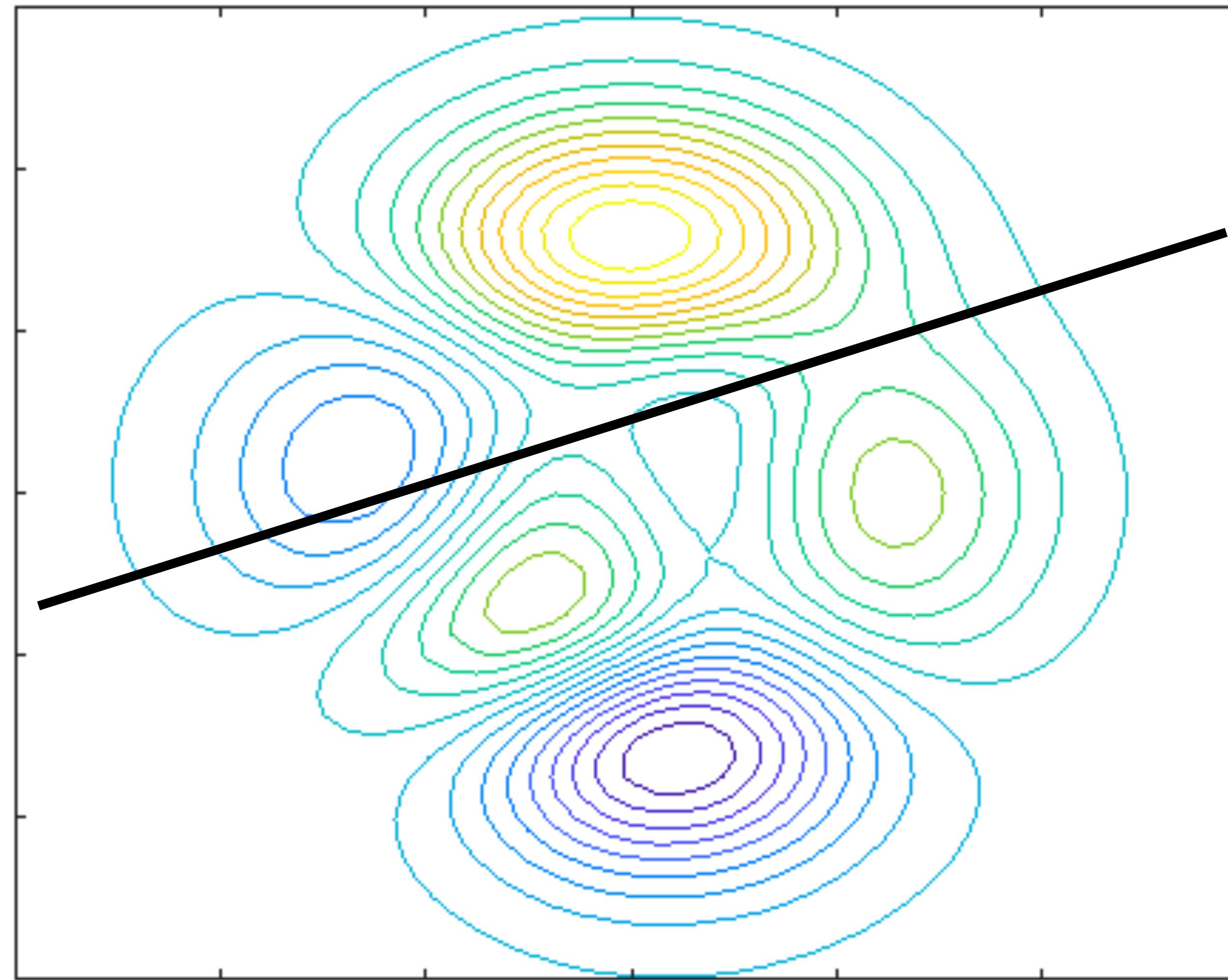
# Method 2: Follow Negative of Gradient



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# Method 2: Follow Negative of Gradient



$$\frac{df(x)}{dx}$$