Infinite Symbol Inscribed in Square for UWB Anchors Calibration

Introduction

the infinite-shaped trajectory is widely used in calibration processes such as the one found in compasses integrated in mobile devices. This happens because the infinite shape shows several interesting and desirable properties such as:

1. **Continuous, Smooth Motion:** The infinity symbol consists of two connected loops, allowing for a smooth, uninterrupted motion path. This ensures there are no abrupt stops or directional changes, making it ideal for testing the fluidity and consistency of a tracking system.

2. **Multi-Directional Movement:** The symbol naturally incorporates movement in multiple directions: horizontal, vertical, and diagonal. This provides a comprehensive test of a spatial tracking system's ability to handle motion across different planes and axes.

3. **Repetition and Symmetry:** Its symmetrical nature ensures balanced movement, which can help detect inconsistencies in spatial recognition or calibration on either side of the tracked area. Repeated traversal over the same path allows for verifying accuracy and detecting drift or cumulative errors.

4. **Ease of Execution:** It is easy for humans or automated systems to follow the infinity pattern, whether using a hand-held tool, a stylus, or in freehand motion. This simplicity makes it accessible for both manual and automated calibration processes.

5. **Covers a Broad Area:** The extended loops of the infinity symbol allow for testing over a wide spatial area, helping to calibrate systems intended for larger working spaces.

6. **Applications Across Dimensions:** In 2D calibration, the symbol can test coverage in X and Y axes. In 3D systems, incorporating depth (Z-axis) into the infinity motion allows for evaluating depth perception and spatial alignment.

Using an infinity symbol for spatial calibration ensures a comprehensive, efficient, and user-friendly process for testing and improving the accuracy of spatial systems, especially for UWB anchor calibration.

The Square

Let the square have:

- Side length: L
- Center at the origin: $(0,0)$

The square's boundaries are:

$$x \in \left[-\frac{L}{2}, \frac{L}{2}\right], \quad y \in \left[-\frac{L}{2}, \frac{L}{2}\right].$$

Infinity Symbol

The infinity symbol is defined parametrically:

$$x(t) = a\cos(t), \quad y(t) = b\sin(2t),$$

where $t \in [0, 2\pi]$ is the parameter. To ensure the symbol touches the square's edges:

$$a = \frac{L}{2}, \quad b = \frac{L}{2}.$$

Key Points

The 8 key points of the infinity symbol are determined by specific values of t:

$$t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}.$$

Substituting these values into the parametric equations gives the coordinates:

$$\begin{aligned}
P_1 &= \left(\frac{L}{2}, 0\right), \\
P_2 &= \left(\frac{\sqrt{2}L}{4}, \frac{\sqrt{2}L}{4}\right), \\
P_3 &= \left(0, \frac{L}{2}\right), \\
P_4 &= \left(-\frac{\sqrt{2}L}{4}, \frac{\sqrt{2}L}{4}\right), \\
P_5 &= \left(-\frac{L}{2}, 0\right), \\
P_6 &= \left(-\frac{\sqrt{2}L}{4}, -\frac{\sqrt{2}L}{4}\right), \\
P_7 &= \left(0, -\frac{L}{2}\right), \\
P_8 &= \left(\frac{\sqrt{2}L}{4}, -\frac{\sqrt{2}L}{4}\right).
\end{aligned}$$

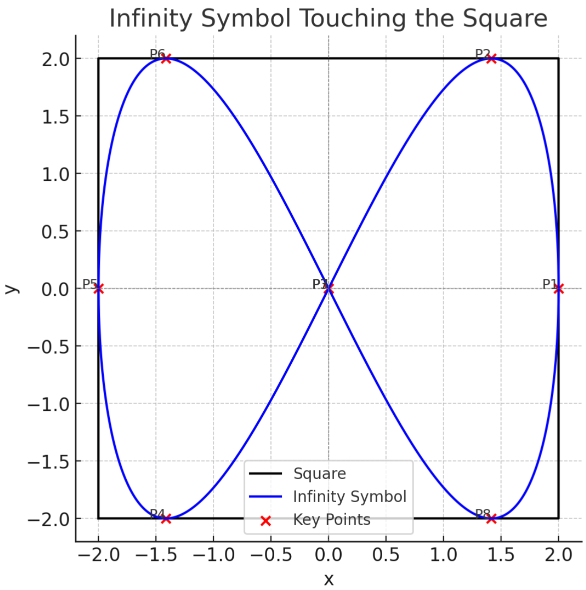

Figure 1: Infinite Symbol Inscribed in Square

Diagram

The diagram below illustrates the infinity symbol inscribed in the square, with key points labeled.

Conclusion

This method ensures the infinity symbol fits perfectly within the square, with key points positioned symmetrically along its edges.