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Complex Systems

Master of Informatics Engineering

2022/2023

Projects

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1 Introduction

In this document we present the different projects to be done by the students of the course Complex Systems of the Master in Informatics Engineering of the University of Coimbra. Projects are to be performed by a group of **two** students[1]. Each group must indicate, no later than **15 of March**, the name and code of **five** projects, by order of preference (For example, 1- TP1 - Ising Model, 2- TP2 - Forest Fire, . . .). Your **preferences** should be send by email to Tiago Baptista (baptista@dei.uc.pt).

The assignment of the projects to the groups will be announced no later than **19 of March**. The deadline for delivering the work is **12 of May**. You must submit your work trough **InforEstudante**. The quality of the work will be judged taking into account the intrinsic value of what was achieved, namely:

- methodology;
- results;
- critical evaluation of the results;
- quality of the written report;
- quality of the public defense.

The chosen deadline do not preclude the possibility of **delivering the work before that date**. There will be a **public defense**.

The document may be written in Portuguese or in English, have a **maximal** length of 10 pages, and follow the Springer LNCS template (it is available on the support material of the course in **UCStudent** in Word and Latex). In special cases (e.g., lots of images and tables) this limit may be exceeded. Besides the document that describe the work done, it is expected that the student also deliver the code and a short document describing how to use it, so all the experiments can be replicated by the professor. To do this work the students are free to consult and use all sorts of elements, including code. If you use, even partially, code made by others, credit should be given **explicitly** to these authors in the delivered code. That said, **plagiarism** is not allowed and all sources must be clearly identified.

[1] If a student prefer to do the work alone he/she must ask for permission.

During the course we have studied and discussed systems, models and simulations. Our major goal is to understand the long term behavior os complex systems from the devised models, either by mathematical analysis or by computer simulations. The proposed projects raise different types of challenges, some being more theory oriented while others have an applicational flavor. In some of the cases we already proposed models in class, but most of the time they were unrealistic simplifications. The improvement of these models will be valued.

2 The themes

Bellow we will describe the projects. **Again**: send us your preferences, ordered, indicating the code (e.g., TP1) and the name (e.g., Ising Model). In the cases where we did not discuss the theme in class we may provide, upon request, some material on the topic. Nevertheless, we will valuate the autonomous search by the student. It is clear that for each theme more than one type of model may be used. If nothing is explicitly mentioned on that issue the students are free to use the approach they think is the best, giving a justification for the choice made.

2.1 TP1- Ising model

Brief description

We all know that water can be in one of three states: solid, liquid and gaseous. Being in one particular state depends on the temperature and the way it constraints the molecular organization of the water. We also know that a **phase transition**, i.e., a change of states, may occurs suddenly due to a small variation of the temperature. One of the most popular models to understand a phase transition is called the Ising Model. It was devised as a mathematical model in statistic mechanics, but it has been used in other contexts like financial markets and social systems.

Goals

The goal of this work is to implement and study the Ising Model in its original version. The easiest way to implement it is to consider a 2D cellular world where each cell can be in one of two states: -1 and 1. We can suppose that the change of state of each cell depends probabilistically of the state of its neighborhoods. Alternatively, we may use the global state of the grid, an approach known as **mean field theory**. Analise the system in the two approaches, studying the behavior of a system in time and in space. Identify the existence, or not, of phase transitions.

2.2 TP2- Forest Fire

Brief description

Cellular Automata are discrete mathematical models that evolve both in time and in space, which have been used as models of different kind os systems. To define a CA one have to identify/define its topology, the set of possible states, the neighborhood and the set of transition rules. In the standard version of a CA the transitions are deterministic and synchronous. There are several variants of CA, namely probabilistic and/or with asynchronous state update.

Goals

We are going to implement and study a CA probabilistic, 2D, and with rectangular symmetry. We are going to use this CA to study the dynamics of forest fires' propagation. In your simulations the three have an initial density. A tree can be in one of three possible state: (1) good; (2) burning; (3) burned. Do some experiments with different values of initial tree's density, different neighborhoods, different state transition rules. Study the conditions under which the fire can be controlled/limited to a certain region. Study the influence of the size and localization of the initial burning trees. Finally, Change your model so it may be possible for tree to (re)born. What are the consequences?

2.3 TP3- Spread of diseases

Brief description

It is a fact that the way we fight an infectious disease is fundamental to control and limit it propagation. We are going to do a simple study about the spread of a disease based on the SIR model, that suppose three possible states for an individual: (1) susceptible; (2) Infected; and (3) Recovered. There are several variants of the model, namely the SIS model and the SEIR model. In the former, a susceptible person can be infected (S $\rightarrow$ I), and an infected person can become susceptible again (I $\rightarrow$ S). There is no immunization. In the latter situation, a susceptible person (S) as to be exposed to the virus a certain time (E) before it gets infected (I) and, later, recover (R).

Goals

We want to study these models, based both on mathematics and by means of computer simulations. Analise different situations, i.e., different values of the parameters and different values of the environment. Draw your conclusions, in particular by comparing the mathematical and simulation results.

2.4 TP4- Iterated Prisoner's Dilemma

Brief description

The Prisoner's Dilemma is a zero-sum game involving two players that have been used to study the conditions under which cooperation may emerge. In its basic version, two thieves (A and B) are held in prison without the possibility to communicate with each other. To each of them a proposal is made: if they confess the crime they are accused of and testify agains his/her companion (Defect) they will be released while the companion will be sentenced to 5 years in prison, unless the companion behaves the same in which case both will be sentenced to 3 years in prison. If none of the prisoners confess and testify agains the other (Silent) they will be both sentenced to 1 year in prison for a minor crime. Table 1 illustrate the situation.

		B	
		Defect	Silent
A	Defect	(3,3)	(0,5)
	Silent	(5,0)	(1,1)

Table 1: Prisoner's Dilemma

When the game is repeated N times we call the problem of devising the best strategy for each player the Iterated Prisoner's Dilemma problem.

Goals

The goal is to study the IPD problem by defining a 2D grid where each cell corresponds to a player (i.e., a prisoner) that can be in one of two states: Silent or Defect. To start, we will study different behaviors for each player as a function of the state of its neighbors and for different initial configurations (i.e., different initial proportions of Silent and Cooperate agents.). Then we will make the problem more complex by supposing that each player keeps itself silent until it reaches the k-order game or until other (neighbor) player defects (whichever occurs first), and then defect until the end. Start with different values of k for each player, between 0 (always defect) and the number of games N (always silent). Try to identify the best value for k.

2.5 TP5- Host-Parasite System

Brief description

We know that there are many examples in nature where parasites live within a host, sometimes being the cause of its death due to infection, sometimes not. Suppose a 2D rectangular grid where each cell may be free, may have a healthy host or a host with a parasite. Suppose further that an infected agent has a certain probability of dying (either immediately or after a certain time), and that a healthy agent may, probabilistic, become infected due to the number of neighbors already infected.

Goals

We want to study the behavior of this system for (1) different initial configurations (i.e., the percentage of individuals healthy and infected), (2) different neighborhoods, (3) different probabilities of death and, (4) different probabilities of being infected. We want to visualize not only the time and space evolution of the system, but also to visualize in a plot the evolution of the number of infected agents over time.

2.6 TP6- Aggregates formation

Brief description

Suppose a world where there are agents that can be in one of two states: active or inactive. The active agents move randomly and if they find an agent in its neighborhood that is inactive they become inactive too for ever. Agents that try to move beyond the world border disappear.

Goals

Our goal is to study the behavior of the system in space and time. In particular, we want to observe what happen for different neighborhoods and different initial conditions (e.g., different initial inactive agents and respective positioning in the world; in the limit, just one in the center.).

2.7 TP7- Sand pile model

Brief description

The sand pile model was first proposed by Per Bak and colleagues as a model for phenomena like earthquakes, and it is the basis of the self-organizing criticality theory. In the model the world is a finite 2D grid where each cell can have a number of sand grains between 0 and 4. A cell with 4 grains of sand is said to be in a critical state and its 4 grains will be distributed by the four neighbors (north, south, west and east). As a consequence some of the neighbors may themselves enter the critical state and will also distributed their 4 grains to its neighbors. And the process continues until it stabilizes. This is called an avalanche. In the model, initially the number of grains per cell is defined randomly. Over time, we put one grain of sand in a particular cell (e.g., the one in the center) and the system evolves producing from time to time avalanches. During an avalanche the process of adding grains is stopped. If an avalanche occur in the limit of the world some grains will be lost, for the world is limited.

Goals

We want to study the phenomenon of avalanches. We will colors for each number of grains of sand, to have a clear visualization of the dynamics of the avalanches. We will test different initial values of sand's grains and study the dimension of the avalanches and plot a graph that relates the size of the avalanche with the number of times that size occurs.

2.8 TP8- Synchronization

Brief description

We all experienced the the phenomenon of synchronization when at the end of a spectacle the audience clap their hands. The mergence of synchronism is a phenomenon that occur in kind of systems, e.g., biological, chemical and social. Synchronism is the result of local interactions based on very simple rules. The flashing of fireflies is a well known example of the emergence of synchronism. We want to study that phenomenon by setting up a simple simulation of the flashing of fireflies. Each firefly flash at a natural frequency that may be slightly modified. After flashing there is a period of time where the firefly is inhibit of flashing.

Goals

Suppose that you have a 2D world with a certain number of fireflies each of them with its own flash frequency and latency time. When a firefly notices the flash of a neighbor firefly it has the tendency to anticipate its own flash time, shortening its period. On the other hand, if when it flash the majority of its neighbors fireflies doesn't flash, the next time it will flash later, thus increasing its period. Based on these ideas study the conditions under which the emergence and stabilization of synchronism takes place.

2.9 TP9- Traffic jam

Brief description

We know that we can simulate a simple traffic jam with a a 1D CA. This phenomenon is also true for computers networks, power grids or mobile communications. Let's study the problem now with a more realistic traffic network yet simple enough to have a manageable complexity. To that end, suppose you have a 2D world where each cell can be empty or have a car on it. Each car has a direction of movement associated (north, south, west, east).

Goals

Define the rules of the CA and study the condition under which you observe the emergence of a traffic congestion. Does you observe a phenomenon of phase transition (flow to congestion)? Does the type of update mechanism (synchronous, asynchronous) makes any difference? Finally, suppose you add traffic lights to your model. What are the implications for the dynamics of the system?

2.10 TP10- Picking objects

Brief description

Suppose a world where there are two types of things: ants and wood sticks. The ants may wonder in the world randomly. When they find a wood stick in their neighborhood they pick it, if they are not carrying already one, and continue their random walk. If they encounter a pile of wood sticks while carrying one they deposit it in the pile and, again, proceed with their random walk.

Goals

We want to study the macro behavior of this kind of world, analyzing it in the time and in the space dimensions. We want to know the influence of the initial density of wood sticks and of ants. Also, we intent to clarify the importance of the type of neighborhood considered (i.e., Moore, Von Neumann or other). Suppose that you introduce a limit for the capacity of each cell to keep the wood sticks. What is the result? Suppose that we may initialize a cell with more than on wood stick, and that there is a minimum size for a pile so you can add another stick to it. What are the consequences?

2.11 TP11- Gene regulatory network

Brief description

Genetic expression os a two phases process: (1) the DNA is transcribed into RNA and, (2) the RNA is translated into Proteins. This process is not linear for some proteins (called transcription factors) may activate (positive influence) or inhibit (negative influence) the expression of certain genes of the DNA. In the end what we have is a Gene Regulatory Network (GRN). A GRN may be described by a system of differential equations, that we may use to understand the long term behavior of the network: how the concentration of genes and proteins change over time, the existence and stability of fixed points, the importance of the value of certain parameters for the dynamics (steady state, oscillation,...).

Goals

The goal od this project is to study a particular GRN, called **repressilator**, that is present in bacteria, and that is defined by a system of six differential equations. It is a network involving only three genes and three proteins engaged ion a process of negative circular regulation (see figure 1).

The system of differential equations has one equation for each gene and one equation for each protein. On the other hand, each equation is composed by a positive term (the rate of production) and a negative one (the rate of degradation). A possible model for each of the three pairs of equations is the following:

$$\dot{g}_i = \frac{dg_i}{dt} = -g_i + \frac{\alpha}{1 + p_j^n} + \alpha_0$$

$$\dot{p}_i = \frac{dp_i}{dt} = -\beta\left(p_i - g_i\right)$$

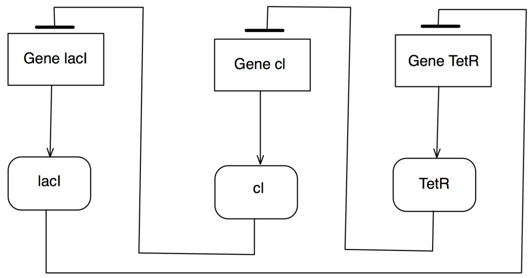

Figure 1: A rede *Repressilator*

In this model thera are some implicit assumptions and there are several parameters $(\alpha, \alpha_0, \beta, n)$. We want to study the long term behavior of this system for different seeds and parameters. For what values do you observe and oscillatory behavior or the convergence for a steady state? If you change the assumption that the parameters are all equal irrespective of the pair gene/protein, what are the consequences for the dynamics of the system?

2.12 TP12- HIV

Brief description

We know that virus propagate at high speed. In the case of HIV we can identify three phases: (1) acute phase, with short duration and high concentration of the virus; (2) no symptoms phase, of long duration and low concentration of the virus; (3) final phase, when the immune systems breaks and the patient dies due to some infection. The fact that the the immune system deteriorates over time make the application of standard models of spreading of infections non applicable.

Goals

Our main goal is to study a model based on differential equations, which describes the relationship between the virus and the immune system. It includes the fact that the virus may mutate and that the immune system can face these mutations. A first model is as follows (the subscript i denotes the mutant and the corresponding response of the immune system).

$$
\begin{aligned}
\dot{v}_i &= \frac{dv_i}{dt} = -v_i \left(r - px_i - qz\right) \\
\dot{x}_i &= \frac{dx_i}{dt} = -cv_i - bx_i) \\
\dot{z} &= \frac{dz}{dt} = kv - bz
\end{aligned}
$$

Here v_i denotes the mutant i of a virus, x_i reflects the dimension of the response of the immune system, and Z the dimension of the response of the immune system against all the mutants (i.e., $v = \sum_{i=1}^{n} v_i$). This model is not specific of the HIV. If we want to model the HIC case we have to add the deterioration of the immune system:

$$
\begin{aligned}
\dot{v}_i &= \frac{dv_i}{dt} = -v_i \left(r - px_i - qz\right) \\
\dot{x}_i &= \frac{dx_i}{dt} = -cv_i - bx_i - uvx_i \\
\dot{z} &= \frac{dz}{dt} = kv - bz - uvz
\end{aligned}
$$

If we compare the two system we see that the terms $-uvx_i$ and $-uvz$ describe how the mutants deteriorate the immune system. The goal is to study this last system in order to understand its long term behavior as a function of the parameters (r, p, q, c, u, k). In terms of the **relationship** among these parameters we may have three situations: (1) immediate disease, i.e., there is no long duration phase without symptoms; (2) chronic infection without disease; (3) chronic infection with disease. To simplify, lets suppose that there are only one type of virus, i.e., $i = 1$. Later, if you want, you may increase i.

2.13 TP13- Excitable Medium

Brief description

An excitable medium is a kind of non-linear systems characterized by the fact that each of its elements or cells can be in one of three states: (1) rest, (2) excited and (3) recovery. An element in the rest state can have its state changed to excited due to a certain perturbation, staying in that state for a certain amount of time. An excited element enter the recovery time when its excited time is over, and stays in that state for a particular amount of time before comeback to the rest state. A cell in the rest state may change to the excited one if a certain number of its neighbors are in the excited state. These kind of systems are present, for example, in a forest fire or in certain heart or brain conditions. These systems can be modeled by a 2D grid where each cell can be in one of the three above mentioned possible states.

Goals

We want to study these system identifying the possible behavior as a function of:

- the size and type of the of the neighborhood;
- the minimum number of neighbors excited required to a rest cell to enter the excited state;
- the time that a cell remain in the excited state;
- the time a cell remain in the recovery state.

It is clear that the behavior also depends on the initial conditions, i.e., the number and position of the cells in rest, excited or recovery state. Try to find the initial conditions that guarantee the appearance of different types of waves.

2.14 TP14- General Chaos Game

Brief description

We want to study a generalization of the chaos game. In our simulation we will have a list of N, bi-dimensional, points (A_x, A_y) and a constant $\epsilon \in [0, 1]$. The game proceed by choosing a random initial point (x_0, y_0) and then iterating the process:

- choose one of the N points as target (A_x, B_y)
- update x and y:

$$x_{t+1} = x_t + \epsilon(A_x - x_t)$$
$$y_{t+1} = y_t + \epsilon(A_y - y_t)$$

Goals

Study the system as a function of the number of points and the value o ϵ. Use different N colors to visualize the system. What kind of conclusions can you draw?

2.15 TP15- Ecological System

Brief description

Suppose an ecological system with two species that eat the same food. If we use x_1, x_2, y to correspond to the concentrations of species 1, 2 and the food, respectively, we may describe our system by the following system of differential equations:

$$\begin{aligned}
\dot{x_1} &= (b_1 y - a_1)x_1 \\
\dot{x_2} &= (b_2 y - a_2)x_2 \\
\dot{y} &= q - (c_1 x_1 + c_2 x_2)y
\end{aligned}$$

where we suppose that all constants are non-negative and q is the, constant rate, supply of food. Moreover, we may have notice that the reproduction rate of each species is proportional to the food concentration.

Goals

Study the dynamics of the system, including its long term behavior for different values of the parameters and try to answer the question about the possible coexistence of the two species. If you assume that the supply of food fluctuates in time, i.e., $q = q(t)$, what are the consequences for the behavior of the system?

2.16 TP16- Advertising

Brief description

We may see advertising as the spread of a virus. The potential buyers x are *infected* trough contact with advertisement with a brand and becomes an user y. Here is a possible model.

$$\begin{aligned}
\dot{x} &= k - \alpha x y + \beta y \\
\dot{y} &= \alpha x y - (\beta + \epsilon)y
\end{aligned}$$

From the model it is clear that when a potential buyer becomes an user x decreases and y increases. We use the parameter to take into account those people that switch to a rival product. We also include a small factor, ϵ, to measure the effect of disappearing people.

Goals

Study the dynamics of the system, including its long term behavior for different values of the parameters, includes zeroing the parameters β and ϵ. Simulate enough trajectories (orbits) so you can describe the phase portrait of the system.

2.17 TP17- Predator - Preys (I)

Brief description

In class we studied a simple one predator - one prey model. Suppose now that we have not one but two preys. A very simple model of this system can be:

$$\begin{aligned}\dot{p} &= -p\,\eta_p + p\,w_a a + p\,w_b b\\ \dot{a} &= \alpha_1\,a - \alpha_2\,p\,a\\ \dot{b} &= \beta_1\,b - \beta_2\,p\,b\end{aligned}$$

where p is the predator, a and b are the preys. All parameters are positive.

Goals

Our goal is to study the behavior of the system for different values of the parameters, including the time evolution of the species and the existence of different class of dynamics.

2.18 TP18- Predators - Prey (II)

Brief description

In class we studied a simple one predator - one prey model. Suppose now that we have two competing species of predators(x and y) and just one prey (z). A simple model of the system is:

$$\begin{aligned}\dot{x} &= r_1\,x(1-\frac{x}{k_1}) - b_1\,x\,y + \beta_1\,x\,z\\ \dot{y} &= r_2\,y(1-\frac{y}{k_2}) - b_2\,x\,y + \beta_2\,y\,z\\ \dot{z} &= a\,z(1-\frac{z}{A}) - \eta_1\,x\,z - \eta_2\,y\,z\end{aligned}$$

where x and y are the predators and z is the prey. All parameters are positive.

Goals

Our goal is to study the behavior of the system for different values of the parameters, including the time evolution of the species and the existence of different class of dynamics.

2.19 TP19- Delayed Logistic Equation

Brief description

In class we studied the logistic equation:

$$\dot{x} = r\, x(1 - x)$$

There is a modified version that includes a time delay:

$$\dot{x}(t) = r\, x(t)(1 - x(t - \tau))$$

The system's behavior now depends on two parameters, r and τ.

Goals

Our goal is to study the behavior of the system for different values of the parameters, including the time evolution of x and the existence of different class of dynamics.

2.20 TP20 - Brusselator

Brief description

Certain types of chemical reaction can be described by the following system of differential equations:

$$\begin{aligned}\dot{x} &= a - (b + 1)\, x + x^2\, y \\ \dot{y} &= b\, x - x^2\, y\end{aligned}$$

where x and y are concentrations and a and b are parameters strictly positive (i.e., > 0).

Goals

Our goal is to study the behavior of the system for different values of the parameters, including the time evolution of x and y. It is possible for the system to have an oscillatory behavior?