Localized Weighted Sum Method for Many-Objective Optimization

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***Abstract*—Decomposition via scalarization is a basic concept for multiobjective optimization. The weighted sum (WS) method, a frequently used scalarizing method in decomposition-based evolutionary multiobjective (EMO) algorithms, has good features such as computationally easy and high search efficiency, compared to other scalarizing methods. However, it is often criticized by the loss of effect on nonconvex problems. This paper seeks to utilize advantages of the WS method, without suffering from its disadvantage, to solve many-objective problems. A novel decomposition-based EMO algorithm called multiobjective evolutionary algorithm based on decomposition LWS (MOEA/D-LWS) is proposed in which the WS method is applied in a local manner. That is, for each search direction, the optimal solution is selected only amongst its neighboring solutions. The neighborhood is defined using a hypercone. The apex angle of a hypervcone is determined automatically in *a priori*. The effectiveness of MOEA/D-LWS is demonstrated by comparing it against three variants of MOEA/D, i.e., MOEA/D using Chebyshev method, MOEA/D with an adaptive use of WS and Chebyshev method, MOEA/D with a simultaneous use of WS and Chebyshev method, and four state-of-the-art many-objective EMO algorithms, i.e., preference-inspired co-evolutionary algorithm, hypervolume-based evolutionary, θ-dominance-based algorithm, and SPEA2+SDE for the WFG benchmark problems with up to seven conflicting objectives. Experimental results show that MOEA/D-LWS outperforms the comparison algorithms for most of test problems, and is a competitive algorithm for many-objective optimization.**

***Index Terms*—Decomposition, evolutionary computation, local, multiobjective evolutionary algorithm based on decomposition (MOEA/D), multiobjective optimization, weighted sum (WS).**

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I. Introduction

Multiobjective problems (MOPs) arise regularly in real life, where multiple objectives have to be optimized simultaneously. Typically, an MOP can be written as follows:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})) \\ \text{subject to} \quad & \mathbf{x} \in \boldsymbol{\Omega} \end{aligned} \tag{1}$$

where $\mathbf{x}$ is a decision vector in $\boldsymbol{\Omega}$ (which refers to a feasible search space), $\mathbb{R}^m$ refers to the objective space. $\mathbf{F} : \boldsymbol{\Omega} \rightarrow \mathbb{R}^m$ consists of m real-valued objective functions. Since objectives in an MOP are often conflicting with one another, the optimal solution set is not a single solution but a set of tradeoff solutions, namely, Pareto optimal solutions. The Pareto optimal front (PF), the image set of all the Pareto optimal solutions in the objective space, is of practical interest to a decision maker.

Evolutionary multiobjective (EMO) algorithms have demonstrated their effectiveness in solving MOPs. Their population-based nature enables an approximation of the PF to be obtained in a single run, and they tend to be robust to underlying cost function characteristics [1, pp. 5–7]. It has been accepted that Pareto-dominance-based EMO algorithms, multiobjective genetic algorithm (MOGA) [2], NSGA-II [3], though perform well on MOPs with two and three objectives, often have difficulty on handling MOPs with more than three objectives (termed many-objective problems) [4]. Up to now a number of EMO algorithms have been proposed to handle many-objective problems, which can be loosely classified as follows [5]: 1) modified Pareto-dominance or density estimation based, e.g., ϵ-dominance-based algorithm (DEA) [6] shift-density-based evolutionary algorithm (SDEA) [7], grid-based evolutionary algorithm (GrEA) [8] and 2) Performance indicator based, e.g., the indicator-based evolutionary algorithm (IBEA) [9], approximated hypervolume (HV)-based evolutionary algorithm (HypE) [10]; preferences-based, e.g., preference-inspired co-evolutionary algorithms (PICEA-g) [11]–[13] and PICEA-w [14], [15]; decomposition based, e.g., cellular MOGA [16], multiobjective evolutionary algorithm based on decomposition (MOEA/D) [17] and its variants MOEA/D-DD [18], MOEA/D-DU [19], NSGA-III [20], and θ-DEA [21]. Also, there are some other promising approaches such as the bi-goal evolutionary algorithm [22], the knee point-driven evolutionary algorithm [23], the improved two-archive algorithm (Two_Arch2) [24] and [25]–[28]. Readers are referred to [29] and [30] for a detailed survey. Amongst the above mentioned algorithms, decomposition-based EMO algorithms have drawn more and more attentions.

Decomposition via scalarization is a basic technique in traditional multiobjective optimization. A typical decomposition-based EMO algorithm, e.g., MOEA/D, decomposes an MOP into a number of sub-problems (which are often single-objective)[1] defined by a scalarizing method using different weights, and solve these sub-problems using a population-based search in a collaborative manner. The optimal solution of each single-objective problem corresponds to a Pareto optimal solution of the MOP [32, pp. 98–99]. Diversified solutions are obtained by employing different weights.

More recent studies have identified that the specification of weights [33] and/or scalarizing methods [34] has a crucial impact on the performance of MOEA/D and other decomposition-based EMO algorithms.

1) The specification of weights mainly impacts the distribution of the approximated PF. For example, evenly distributed weights might not lead to evenly distributed solutions for an MOP whose PF shape is nonlinear (i.e., convex or concave) [35]. Effectively when the PF is known *a priori*, the studies [33], [36] investigate how to compute an optimal distribution of weights for various L_p scalarizing methods. When the PF is unknown, a few effective methods are proposed, e.g., co-evolving weights with solutions [15], [37]; and using Pareto adaptive weights [38]–[42].
2) The specification of scalarizing methods mainly impacts the search efficiency. As discussed in [32, p. 79] that the weighted Chevbshev method can find solutions in both convex and nonconvex PF regions whereas the weighted sum (WS) cannot; and in [43]–[45] that the WS method (or other L_p scalarizing methods) generally leads to a better convergence performance than the weighted Chebyshev method. Moreover, it is recently reported in [36] and [46] that as p increases (from 1 to ∞), the L_p scalarizing method becomes more robust on PF geometries and less effective in terms of the search efficiency. An optimal p setting exists for problems having certain PF shapes [36].

The simultaneous optimization of objectives over three (termed many-objective optimization) remains challenging in terms of obtaining a full and satisfactory approximation of the PF. These challenges include: 1) inefficiency of Pareto-dominance relation; 2) increased conflict between convergence and diversity; 3) difficulty of the calculation of some performance metrics; 4) inefficiency for offspring generation; and 5) the representation and visualization of tradeoff surface.

In order to handle many-objective optimization, this paper proposes an LWS method which utilizes the high search efficiency of the weighted method, meanwhile avoids its disadvantage (inefficiency on nonconvex problems). Major contributions are as follows.

1) An LWS method is proposed, i.e., using the WS method in a local manner. The LWS method enables a decomposition-based EMO algorithm to find solutions in both convex and nonconvex PF regions. The neighborhood size of an LWS, namely, its working space, is defined using a hypercone. The apex angle of the hypervcone equals to the average angle to its nearest m neighboring weights. The apex of the hypercone is automatically determined in an *a priori*.
2) An instantiation of decomposition-based algorithms using the LWS method, denoted as MOEA/D-LWS, is proposed. MOEA/D-LWS introduces no additional parameter, and is demonstrated to outperform MOEA/D using the Chebyshev method, MOEA/D with an adaptive use of WS and Chebyshev method [45] and MOEA/D with a simultaneous use of WS and Chebyshev method [47] on most of the WFG benchmark problems with up to seven conflicting objectives.
3) MOEA/D-LWS is compared against four many-objective optimizers, i.e., PICEA-g, HypE, θ-DEA, and SPEA2+SDE and is shown as competitive.

The remainder of this paper is organized as follows. Section II provides some background knowledge. Section III elaborates the LWS method and the algorithm MOEA/D-LWS. Experimental descriptions, results and related discussions are presented in Sections IV and V, respectively. Finally, Section VII concludes this paper and identifies some future studies.

II. Background

A. Basic Definitions

1) Pareto-Dominance: $\mathbf{x}$ is said to Pareto dominate $\mathbf{y}$, denoted by $\mathbf{x} \preceq \mathbf{y}$, if and only if $\forall i \in \{1, 2, \ldots, m\}, f_i(\mathbf{x}) \leq f_i(\mathbf{y})$ and $f_j(\mathbf{x}) < f_j(\mathbf{y})$ for at least one index $j \in \{1, 2, \ldots, m\}$.

2) Pareto Optimal Solution: A solution $\mathbf{x}^* \in \mathbf{\Omega}$ is said to be Pareto optimal if and only if $\nexists \mathbf{x} \in \mathbf{\Omega}$ such that $\mathbf{x} \preceq \mathbf{x}^*$. The set of all Pareto optimal solutions is called the Pareto optimal set (PS). The set of all Pareto optimal vectors, PF $= \{\mathbf{F}(\mathbf{x}) \in \mathbb{R}^m | \mathbf{x} \in \text{PS}\}$, is called the PF.

3) Ideal Point: An objective vector $\mathbf{z}^* = (z_1^*, \ldots, z_m^*)$ where z_i^* is the infimum of $f_i(\mathbf{x})$ for every $i \in \{1, 2, \ldots, m\}$.

a) Utopian point: An infeasible $\mathbf{z}^u$ whose component can be formed by $z_i^u = z_i^* - \epsilon_i, \ i = 1, 2, \ldots, m$, where z_i^* is the component of the *ideal* objective vector, and $\epsilon_i > 0$ is a relatively small but computationally significant scalar.

b) Nadir point: An objective vector $\mathbf{z}^{\text{nad}} = (z_1^{\text{nad}}, \ldots, z_m^{\text{nad}})$ where z_i^{nad} is the supremum of $f_i(\mathbf{x}), \mathbf{x} \in \text{PS}$ for every $i \in \{1, 2, \ldots, m\}$.

B. Decomposition Approaches: L_p Scalarizing Method

Though a number of scalarizing methods are available for decomposing an MOP, we focus on the family of L_p scalarizing methods due to its simpleness and popularity.[2] Mathematically,

[1]Note that in some variants of MOEA/D sub-problems are multiobjective [31].

[2]There are other scalarizing methods such as the penalty-based boundary intersection (PBI) [17] and the normal boundary intersection (NBI) [48].

a weighted L_p scalarizing method can be written as

$$g^{wd}(\mathbf{x}|\mathbf{w}, p) = \left(\sum_{i=1}^{m} \lambda_i \big(f_i(\mathbf{x}) - z_i^u\big)^p\right)^{\frac{1}{p}}, \; p \geq 1$$
$$\lambda_i = \left(\frac{1}{w_i}\right) \tag{2}$$

where $\mathbf{z}^u = (z_1^u, z_2^u, \dots, z_m^u)$ is the *utopian* point, $\mathbf{w} = (w_1, w_2, \dots, w_m)$ is a weighting vector and $\sum_{i=1}^{m} w_i = 1$, $w_i > 0$.

The two frequently used scalarizing methods, the WS and Chebyshev, can be derived by setting $p = 1$ and $p = \infty$, respectively. That is

$$g^{\text{ws}}(\mathbf{x}|\mathbf{w}) = \sum_{i=1}^{m} \big(\lambda_i \big(f_i(\mathbf{x}) - z_i^u\big)\big)$$
$$g^{\text{ch}}(\mathbf{x}|\mathbf{w}) = \max_{i=1}^{m} \big(\lambda_i \big(f_i(\mathbf{x}) - z_i^u\big)\big). \tag{3}$$

By optimizing a scalarizing method with different weights, a set of Pareto optimal solutions could be obtained. If necessary, the following normalization procedure should be applied during the search:

$$\bar{f}_i = \frac{f_i - z_i^*}{z_i^{\text{nad}} - z_i^*}. \tag{4}$$

When the z_i^* and z_i^{nad} are not available, we could use the smallest and largest f_i of all nondominated solutions found so far as approximations of z_i^* and z_i^{nad} as in many other studies (e.g., [17], [25], and [49]). However, this approximation is not always very accurate especially in early generations. If necessary, more advanced methods can be applied [50].

C. Weighted Sum Method in Multiobjective Optimization

As a common concept in multiobjective optimization, the WS method has been discussed prominently in [32], [51], and [52] since its introduction by Zadeh [53]. The method linearly aggregates all the individual objective functions in an MOP into one objective by using a weight vector.

Before EMO algorithms get popularized, the WS method is mainly used in an *a priori* and *interactive* way, that is, a weight vector is predefined before the search, or changed during the search progressively. For instance, in [54] the WS method is applied to multiobjective structure optimization. Weights are predefined, multiple Pareto optimal solutions are obtained by a systematic change in weights in different algorithm runs. Also, in [55] the WS method is applied to topology optimization. Weights are altered to yield different Pareto optimal solutions. Within an EMO algorithm, the WS method, embedded with a set of predefined weights, is applied to search a set of Pareto optimal solutions in a single run. For example, in [56] the WS method embedded with random weights is used for selecting good solutions in an MOGA. In MSOPS [57] and MOEA/D [17], the WS method (as one of the proposed methods), embedded with evenly distributed weights is employed for multiobjective optimization.

Optimizing a WS could constitute either an independent method or a component of other methods. In [58] and [59], a simulated annealing algorithm is proposed wherein the WS method is applied as an acceptance criterion. In [60], the WS method is combined with tabu search for solving bi-objective 0-1 knapsack problems. In the two-phase local search (TPLS) and double-TPLS [61] and the multi-objective genetic local search [62]–[64] the WS method is also used to guide the local search.

Despite its wide applications, many studies demonstrate its inability on capture Pareto optimal solutions in nonconvex regions [35], [65], [66]. This deficiency is often answered with alternative scalarizing methods, e.g., the Chebyshev method, the NBI method. Noticeably, Wang *et al.* [36], [46] propose to use Pareto adaptive L_p scalarizing methods, that is, choosing a suitable L_p scalarizing method based on the estimated PF shape on line. Such alternatives can be effective and valuable, whereas they are usually independent of the WS method. From another aspect, there are studies investigating the incorporation of tricks into the WS method. For example, Jin *et al.* [67], [68] proposed that by using an archive and periodically altered weights, the WS method-based search is able to obtain solutions on nonconvex PF. Kim and de Weck [69], [70] proposed to first use the WS method to quickly obtain an approximation of the PF, then construct a mesh of Pareto front patches. The patches are further refined by imposing additional equality constraints which connect the *nadir* point and the expected Pareto optimal solutions on a piecewise planar surface in the objective space. This method is reported as effective. However, constructing the mesh of Pareto front patches becomes rather complex on many-objective problems. In [71], a bi-level WS method is proposed for multiobjective optimization. However, the idea is still based on the use of Pareto front patches.

In addition to the above-mentioned approaches Ishibuchi *et al.* [45], [47] proposed a mixed use of the WS and Chebyshev method in attempt to harness benefits of both methods. Specifically, in [45] the WS is used to guide the search unless a concave PF region is detected in which case the Chebyshev method is used instead. In [47], the WS and Chebyshev method are simultaneously applied to evaluate candidate solutions. Experimental results show that both the methods can find solutions in nonconvex PF regions. Also, their performance, especially the convergence, is much better than a single use of the Chebyshev method.

Overall the literature has repeatedly demonstrated that: 1) the WS method cannot find solutions on nonconvex PF regions and 2) the WS method has high search efficiency than the Chebyshev method. In order to harness its benefits and avoid its drawbacks, this paper proposes to use it in a local manner, that is, the working space of a WS method is restricted within a hypercone. Given the high search efficiency of the WS method, decomposition-based EMO algorithms using the LWS method are expected to have good performance on many-objective problems.

III. MOEA/D-LWS

A. Motivation: High Search Efficiency of the Weighted Sum Method

High search efficiency of the WS method is the main motivation for promoting its application in decomposition-based

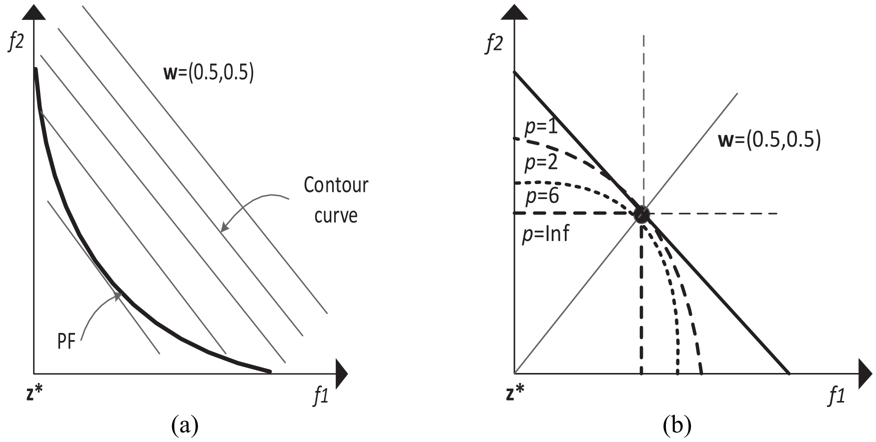

Fig. 1. Illustration of contour curves of (a) WS method and other (b) L_p scalarizing methods.

EMO algorithms. It is well known that Pareto dominance-based fitness evaluation mechanisms do not work well on many-objective problems [4], [72]. As explained in [73], "*Pareto-dominance methods and the Chebyshev scalarizing function are equivalent, in the sense that neither method in itself, has better probability to find superior solutions. In fact the aforementioned probabilities are the same*." Thus it is likely that the Chebyshev method does not work well on many-objective functions. Actually, [45] and [73] have shown that better results are obtained by the WS than the Chebyshev method for some many-objective problems. That is, it has been reported that much efficient search is realized by the WS than the Chebyshev method. However, the WS has its inherent disadvantage: it cannot appropriately handle nonconvex PFs. In this paper, we propose an idea to remedy this disadvantage in order to utilize the high search ability of the WS for many-objective problem in decomposition-based algorithms. Next, analyis of the search efficiency of the WS and Chebyshev method, inspired by [73], is presented.

The contour curve of the WS method is shown in Fig. 1(a), which is a straight line. The contour curve divides the objective space into two subspaces. Solutions in one subspace are better than those on the contour curve while solutions in the other subspace are worse. Solutions lie in the same contour curve have the identical scalar value. That is, the size of a superior region is loosely $(1/2)$, and is regardless of the number of objectives. Compared with other L_p scalarizing methods, we can observe from their contour curves, as shown in Fig. 1(b), that the size of superior region is smaller than $(1/2)$, e.g., $(1/2^m)$ for the Chebyshev method. Moreover, such value decreases significantly as m, the number of objectives, increases. This therefore indicates that: 1) the probability of finding a better solution (measured by the chosen scalarizing method) is $(1/2)$ for the WS method and is smaller than $(1/2)$ for other L_p scalarizing methods and 2) the probability keeps unchanged for the WS method while it becomes remarkably small for other L_p scalarizing methods when m increases. As a result, it is not likely that the Chebyshev method is very efficient for many-objective problems in comparison with the WS method [73].

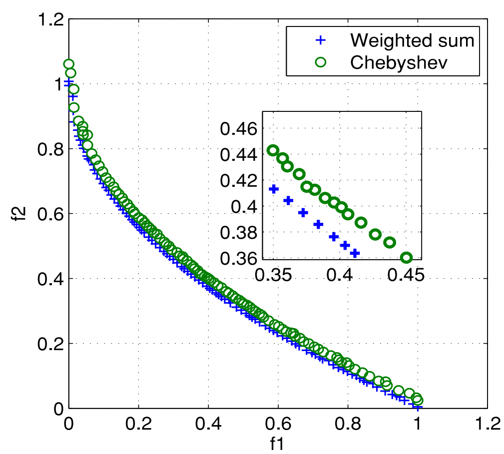

Fig. 2. PFs obtained by MOEA/D with the WS and the Chebyshev method.

Empirically, MOEA/D embedded with the WS method and the Chebyshev method are compared for the ZDT1 benchmark [74] (ZDT is for Zitzler-Deb-Thiele). Both algorithms are run for 31 times, and each run with 250 generations. The population size is set to 100. Other settings are the same as in [75]. The PFs obtained by each algorithm, corresponding to the median HV value over 31 runs, are shown in Fig. 2. From the results, it is evident that the WS method offers a better performance. The obtained solutions are closer to the true PF than those obtained by the Chebyshev method.

Therefore it is desirable to apply the WS method to guide the evolutionary search. Of course an effective strategy that enables the WS method to find solutions in nonconvex PF regions is required.

B. Methodology: The Localized Weighted Sum Method

This section presents our idea—the LWS method, denoted as LWS. The LWS applies the WS method in a local manner. That is, each WS method $g^{\text{ws}}(\mathbf{x}|\mathbf{w}^i)$ is restricted to work only within a defined hypercone. The center line of the hypercone is along the weight $\mathbf{w}^i$, and its apex angle is Θ_i, see Fig. 3 the shaded region. The main reason for setting the working

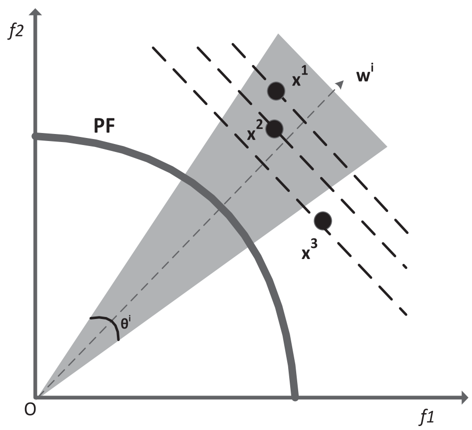

Fig. 3. Illustration of the hypercone region for weight vector $\mathbf{w}^i$.

space as a hypercone (rather than a hypercylinder) is as follows. Assuming that N weight vectors: $\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^N$ are employed, and the PF is divided into N parts (which essentially is the way a decomposition-based algorithm does). Each WS method with a different weight vector tries to find a Pareto optimal solution. Given the shape of the hypercone (wide at the bottom and narrow at the top, see Fig. 3), its application, compared with the use of cylinder, implicitly enables a larger space to be explored (i.e., more solutions to be evolved) at the early stage of the search along each search direction. This is obviously beneficial in finding a global optimum. In another aspect, as the search progresses solutions gradually approach to the PF. The solution found by each WS method is expected to be within the associated region of PF. Thus, the working space should narrow down. The region of the hypercone suits exactly such a process, and therefore is employed.

A WS method $g^{\text{ws}}(\mathbf{x}|\mathbf{w}^i)$ only measures solutions that are inside the hypercone of $\mathbf{w}^i$. For solutions that are outside the hypercone, their WS values are set to ∞. For a m-objective problem, the apex angle of the hypervcone, Θ_i is defined as follows:

$$\Theta_i = \frac{\sum_{j=1}^{j=m} \theta_{ij}^{\text{ww}}}{m} \tag{5}$$

where θ_{ij}^{ww} is the angle of the jth closest weight vector to weight $\mathbf{w}^i$. Note that the dot product $\mathbf{w}^i \cdot \mathbf{w}^j$ provides the cosine of their angle. It is worth mentioning that given N evenly distributed weight vectors, Θ_i is almost identical for different hypercones. Seen from Fig. 3, by the standard WS method solution $\mathbf{x}^3$ is the best, whereas by the LWS method solution $\mathbf{x}^2$ is considered as the best since $\mathbf{x}^3$ is outside of the hypercone.

Overall, by the localized strategy, the PF is divided into a number of small sub-PFs. Each LWS method corresponds to a sub-PF (which is bounded by a hypercone), and tends to find a Pareto optimal solution within this hypercone. For a convex sub-PF, the obtained solution might be along (or near) the search direction (which is sub-PF shape dependent). For a nonconvex sub-PF, the obtained solution is at the boundary of the sub-PF. Since the center line of the hypercone is along the search direction $\mathbf{w}^i$, and its apex angle is Θ, the offfset between the obtained solution and $\mathbf{w}^i$ is maximally $(\Theta_i/2)$. Given that evenly distributed weights are employed, the size of hypecones [determined by (5)] for different LWS methods is almost the same, which therefore naturally leads to a set of diversified solutions. A further discussion with respect to the uniformity of solutions obtained by LWS is given in Section VI-A.

It is worth mentioning that the very recent inspiring work MOEA/D-DU by Yuan *et al.* [19] also proposed to update solutions that are only within the neighborhood of the newly generated solution. Differently, the neighborhood is defined with Euclidean distances. Moreover, an additional parameter k has to be appropriately defined before the search. In addition, effectiveness of the cone-based neighborhood has been reported in [76] and [77]. Besides, in MOEA/D-DU, the Chebyshev method is used whereas this paper utilizes the high convergence ability of the LWS.

C. Incorporation of the LWS Method Into MOEA/D

This section elaborates our proposed algorithm MOEA/D-LWS. The Pseudo-code is presented in Algorithm 1. Prior to the evolution (lines 1–5), the following operations are conducted. N evenly distributed weights $W \leftarrow \{\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^N\}$ are generated (as will be described later), and the same size of solutions $S \leftarrow \{\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^N\}$ are initialized. Each $\mathbf{w}$ is randomly paired up with an $\mathbf{x}$. For a weight $\mathbf{w}^i$, its T neighboring weights $B(\mathbf{w}^i)$ are identified. The neighboring relation is determined in terms of θ_{ij}^{ww}. Also the associated neighboring solutions $B(\mathbf{x}^i)$ of $\mathbf{x}^i$ are identified. Lastly, the apex angle of the hypercone Θ_i for $\mathbf{w}^i$ is calculated by (5). All solutions are then evolved for *maxGen* generations.

1) *Lines 6–16:* New offspring solutions are reproduced (as will be described below). For a Parent solution $\mathbf{x}^i$, its offspring is generated based the neighbors of $\mathbf{x}^i$, $B(\mathbf{x}^i)$ with a probability 0.8, and based on the whole population S with a probability 0.2.
2) *Lines 17–19:* All parent and offspring solutions are combined. Their objective values are computed, which are then applied to update the *ideal* and *nadir* points.
3) *Line 20:* Calculate the angle θ_{ij}^{sw} between $\mathbf{F}(\mathbf{x})^{\mathbf{i}}$ and $\mathbf{w}^j$.
4) *Lines 21 and 22:* Calculate the WS value $g^{\text{ws}}(\mathbf{x}^i|\mathbf{w}^j)$, denoted as C_{ij}. Set C_{ij} as ∞ if θ_{ij}^{sw} is larger than Θ_j.
5) *Lines 23–27:* For each weight, find $\mathbf{x}$, the solution in the joint population with the smallest $g^{wd}(\mathbf{x}|\mathbf{w}^i)$ value.
6) *Line 28:* Update the offline archive *archiveS* with newly obtained solutions S based on the Pareto-dominance.

Lastly, one can further obtain evenly distributed solutions by choosing the nearest solution (measured by angle) for each weight from *archiveS*.

1) Generation of Evenly Distributed Weights: First evenly distributed N points on a hypersphere $f_1^2 + f_2^2 + \ldots + f_m^2 = 1$ are obtained via minimizing a metric V defined in (6). These points are then converted into weights by $w_i = (x_i/\sum_{i=1}^m x_i)$, where x_i is the ith component of a point

$$V = \max_{i=1}^{N} \max_{j=1, j\neq i}^{N} \left(\mathbf{x}^i \cdot \mathbf{x}^j\right). \tag{6}$$

The metric V measures the worst-case angle of two nearest neighbors. The inner maximization finds the nearest two neighbors in terms of the angle between them. The

Algorithm 1: MOEA/D Using LWS Scalarizing Method

Input: N evenly distributed weights, $W \leftarrow \{\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^N\}$, N candidate solutions, $S \leftarrow \{\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^N\}$, selection neighborhood size, T, maximum generation index, *maxGen*

Output: S, *archiveS*

1 Assign each weight, $\mathbf{w}^i$ with a randomly selected candidate solution, $\mathbf{x}^i$;
2 Find T neighboring weights $B(\mathbf{w}^i)$ of $\mathbf{w}^i$ in terms of θ_{ij}^{ww};
3 Identify the associated neighboring solutions $B(\mathbf{x}^i)$ of $\mathbf{x}^i$;
4 Calculate the apex angle Θ_i for each weight $\mathbf{w}^i$ by Eq. (5);
5 Set the archive $archiveS = \emptyset$, the mating pool $Q = \emptyset$ and the probability of mating restriction $\delta = 0.8$;
6 **for** $gen \leq maxGen$ **do**
7   **for** $i \leftarrow 1$ **to** N **do**
8   Set a temporary solution set $Sc = \emptyset$;
9   **if** $rand < \delta$ **then**
10             $Q \leftarrow B(\mathbf{x}^i)$;
11   **else**
12             $Q \leftarrow S$;
13   **end**
14   Generate a new solution $\mathbf{x}'$ by applying SBX and PM operators to solutions selected from Q;
15         $Sc \leftarrow Sc \uplus \mathbf{x}'$;
16   **end**
17     $JointS \leftarrow S \uplus Sc$;
18   Compute the objective function values *JointF* of all candidate solutions in *JointS*;
19   Update *ideal* and *nadir* vectors, and normalize *JointF* into [0,1];
20   Calculate the angle θ_{ij}^{sw} between $\mathbf{F}(\mathbf{x})^{\mathbf{i}}$ and weight $\mathbf{w}^j$, obtaining an angle matrix θ^{sw};
21   For each solution, e.g., $\mathbf{x}^i$ and its associated weight, e.g., $\mathbf{w}^j$, if $\theta_{ij}^{sw} \leq \Theta_j$ compute $g^{ws}(\mathbf{x}^i|\mathbf{w}^j)$, denoted as C_{ij};
22   Otherwise, set $C_{ij} \leftarrow \infty$;
23     $S \leftarrow \emptyset$;
24   **for** $i \leftarrow 1$ **to** N **do**
25         $\mathbf{x}^i \leftarrow \arg\min_{\mathbf{x} \in JointS} C_{ij}$;
26         $S \leftarrow S \cup \mathbf{x}^i$;
27   **end**
28   Update *archiveS* with S using Pareto-dominance relation;
29 **end**

outer maximum operator finds the largest angle between two nearest neighbors. The optimal set of weights is produced when the outer maximum is minimized. More details about this method is available in [57]. Compared with the simplex-lattice design method introduced in [17], this method is able to generate any number of evenly distributed weights.

2) Generation of New Offspring Solutions: The simulated binary crossover (SBX) and polynomial mutation (PM) operators are applied to generate an offspring population Sc. As recommended in [4] and [13], the SBX control parameters p_c and η_c are set to 1 and 30, respectively. The PM control parameters p_m and η are set to $1/n$ and 20 where n is the number of decision variables.

MOEA/D-LWS effectively is within a $(\mu+\lambda)$ elitism framework where $\mu = \lambda = N$. N parent solutions and their offspring solutions of the size N are pooled together. The size of the offspring population can also be different from the parent population size. Then new parents of the size N are selected from the joint population. Specifically, in MOEA/D-LWS the neighborhood structure $B(\bullet)$ is used only for choosing a pair of parents. It is not used for solution update. The generated solution $\mathbf{x}'$ for $\mathbf{w}^i$ is not necessarily compared with $\mathbf{x}^i$ for solution update. Solutions that are close to each weight vector (i.e., neighbors of a weight vector) in the merged population are compared to generate the new parent population. Also, a small number of weight vectors can share the same solution (i.e., the same solution can be selected for different weights in line 25).

With respect to the time complexity, the calculation of the WS values of all solutions on all weights runs at $\mathcal{O}(N \times N)$. Identification of the minimal WS value among $2N$ values at line 25 runs at $\mathcal{O}(N)$. Therefore, the overall time complexity of the algorithm is $\mathcal{O}(N^2)$.

IV. Experiment Description

A. Test Problems

Test problems 2–9 from the WFG test suite [78], invoked in two-, four-, and seven-objective instances, are applied to benchmark the considered algorithm performance.[3] In each case the WFG position parameter (k) and the distance parameter (l) are set to 6 and 94, respectively, providing a constant number of decision variables ($n = 100$) for each problem instance. The reason for setting $k = 6$ is that WFG problems require k to be divisible by $m - 1$. Choosing $n = 100$ is simply to better demonstrate the superiority of the WS method. A large l creates difficulty on the convergence of an algorithm. Attributes of these problems include separability or nonseparability, unimodality or multimodality, unbiased or biased parameters, and convex or concave geometries. The *ideal* and *nadir* points for these problems are $[0, 0, \ldots, 0]$ and $[2, 4, \ldots, 2m]$, respectively. Hereafter, we use WFGx-m to denote the problem WFGx with m objectives.

B. Competitor Algorithms and Parameter Settings

From the literature we select two effective methods by Ishibuchi *et al.* [45], [47] as the competitor algorithms.

1) In [45], the WS and the Chebyshev methods are adaptively chosen to guide the search. More specifically, the WS is used as the scalarizing method unless a nonconvex region is detected in which case the Chebyshev method

[3] The WFG1 is not used since our preliminary experiments show that even for 1e+8 function evaluations no algorithm can approximate its PF. The reason might be the employed search operators lacks the ability for exploiting solutions with high precision. Understanding this issue may well unlock further understanding of MOEA performance. Besides, test problems 2–9 can cover most of the problem attributes.

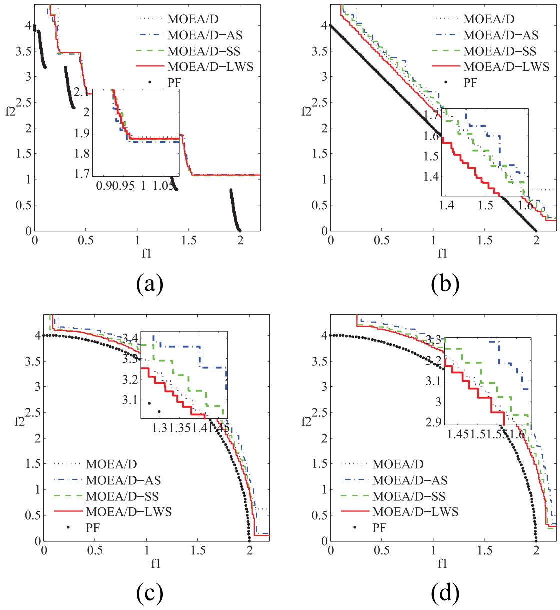

Fig. 4. Attainment surfaces for (a) WFG2-2, (b) WFG3-2, (c) WFG4-2, and (d) WFG5-2.

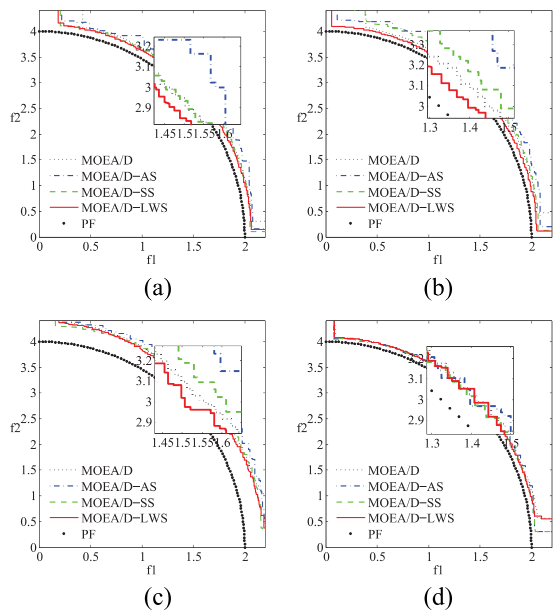

Fig. 5. Attainment surfaces for WFG6-2 to WFG9-2. (a) WFG6-2. (b) WFG7-2. (c) WFG8-2. (d) WFG9-2.

is used. Note that when the WS value of a solution is found to be better than all its neighbors for multiple weights, we say the solution is in a nonconvex PF region. The neighborhood size is set to T, though it is a user defined value.

2) In [47], the WS and Chebyshev methods are simultaneously used to guide the search. The authors provide two implementations. One is to use both scalarizing methods on each search direction. The other is to alternately assign a scalarizing method to each search direction. No additional parameter is required. The second implementation is used in this paper since the first implementation

TABLE I

SETTINGS OF GENETIC OPERATORS AND OTHER PARAMETER

Parameters	Settings
Simulated Binary Crossover (SBX)	$p_c = 1, \eta = 30$
Polynomial Mutation (PM)	$p_m = 1/n, \eta = 20$
MOEA/D selection neighborhood size	T = 10% of the N
MOEA/D mating restriction probability	$\delta = 0.8$
MOEA/D replacement size	nr = 10% of the T

doubles the population size which may result in an unfair comparison.

For clarity, MOEA/D using the *adaptive* strategy is denoted as MOEA/D-AS, and MOEA/D using the *simultaneous* strategy is denoted as MOEA/D-SS. The standard MOEA/D [75] using the Chebyshev method is used as a base line algorithm. Note that the aim of the comparison in this section is to examine the effect of using the WS in MOEA/D. Thus, the proposed method is compared with other ideas of scalarization in MOEA/D. Comparison with other EMO algorithms will be shown later in Section VI.

For all test problems, each algorithm is run for 31 runs subjected to a statistical analysis, and each run for maxGen = 250 generations. The population size N is set to 100, 200, and 700 for two-, four-, and seven-objective problems, respectively. The genetic operators and other parameters are listed in Table I, and are fixed across all algorithm runs.

C. Performance Assessment

Algorithm performance is assessed by the median attainment surface (i.e., 50% attainment surface), the HV, the generational distance (GD), and the coverage of two sets (C metric) [79], [80]. The median attainment surface allows a visual inspection of both proximity and diversity performance. The HV metric measures the volume of the space enclosed by the Pareto approximation set and a given reference point. Both high proximity and large diversity are needed for obtaining a large HV value. The GD metric returns the average distance of the obtained solutions to their nearest neighbor on the PF. The smaller the GD the better the convergence. The C metric measures the proximity performance of one set over another. $C(A, B)$ refers to the fraction of solutions in set B that are dominated at least by one solution in set A. $C(A, B) > C(B, A)$ indicates a better convergence of set A. These performance metrics are calculated using all nondominated solutions found during the search.[4] Prior to the calculation, all solutions are normalized by the *ideal* and *nadir* points. The reference point used for the HV calculation is set to $(1.1, 1.1, \ldots, 1.1)$.

V. EXPERIMENTAL RESULTS

A. Median Attainment Surface Results

The median attainment surfaces across the 31 runs of each algorithm are plotted in Figs. 4 and 5. The PF of each problem serves as a reference. It is observed from the results that MOEA/D-LWS appears to have comparable diversity performance as the other algorithms on all problems. However,

[4]Readers can refer to [81] for discussions about the choice of solution set for algorithm comparison.

TABLE II
HV METRIC (MEAN/STD) COMPARISON RESULTS. THE SYMBOL "−," =, OR "+" MEANS THE CONSIDERED ALGORITHM IS STATISTICALLY WORSE THAN, COMPARABLE TO OR BETTER THAN MOEA/D-LWS

		MOEA/D-LWS	MOEA/D	MOEA/D-AS	MOEA/D-SS
$m = 2$	WFG2	0.6098(0.0109)	0.5937(0.0218)$^-$	0.6088(0.0092)$^=$	0.6156(0.0099)$^=$
	WFG3	0.6026(0.0130)	0.5582(0.0165)$^-$	0.5642(0.0108)$^-$	0.5676(0.0121)$^-$
	WFG4	0.3726(0.0013)	0.3572(0.0079)$^-$	0.3323(0.0040)$^-$	0.3597(0.0040)$^-$
	WFG5	0.3081(0.0103)	0.3027(0.0040)$^-$	0.2550(0.0084)$^-$	0.2941(0.0033)$^-$
	WFG6	0.3600(0.0044)	0.3348(0.0056)$^-$	0.3237(0.0122)$^-$	0.3497(0.0083)$^-$
	WFG7	0.3767(0.0063)	0.3420(0.0034)$^-$	0.3002(0.0179)$^-$	0.3232(0.0090)$^-$
	WFG8	0.2822(0.0017)	0.2653(0.0069)$^-$	0.2379(0.0091)$^-$	0.2691(0.0048)$^-$
	WFG9	0.3759(0.0250)	0.3598(0.0014)$^-$	0.3669(0.0007)$^-$	0.3728(0.0071)$^=$
$m = 4$	WFG2	1.2990(0.0083)	1.1748(0.0082)$^-$	1.2120(0.0107)$^-$	1.1828(0.0852)$^-$
	WFG3	1.0638(0.0166)	0.9622(0.0213)$^-$	0.9765(0.0183)$^-$	1.0130(0.0309)$^-$
	WFG4	0.9320(0.0099)	0.7791(0.0158)$^-$	0.7673(0.0288)$^-$	0.8342(0.0301)$^-$
	WFG5	0.8518(0.0070)	0.6790(0.0116)$^-$	0.6730(0.0156)$^-$	0.7722(0.0064)$^-$
	WFG6	0.9397(0.0086)	0.8411(0.0086)$^-$	0.8426(0.0131)$^-$	0.8969(0.0170)$^-$
	WFG7	1.0157(0.0040)	0.8853(0.0289)$^-$	0.8688(0.0261)$^-$	0.9415(0.0172)$^-$
	WFG8	0.7733(0.0121)	0.6667(0.0192)$^-$	0.6756(0.0068)$^-$	0.7128(0.0244)$^-$
	WFG9	0.8802(0.0294)	0.8097(0.0617)$^-$	0.8183(0.0239)$^-$	0.8449(0.0165)$^-$
$m = 7$	WFG2	1.8152(0.0232)	1.7623(0.0240)$^-$	1.7455(0.0516)$^-$	1.7524(0.0233)$^-$
	WFG3	1.6772(0.0238)	1.6298(0.0315)$^-$	1.6104(0.0111)$^-$	1.5603(0.0219)$^-$
	WFG4	1.5916(0.0127)	1.2684(0.0605)$^-$	1.2761(0.0844)$^-$	1.2856(0.0751)$^-$
	WFG5	1.3588(0.0035)	1.1186(0.0441)$^-$	1.1166(0.0495)$^-$	1.1634(0.0561)$^-$
	WFG6	1.5901(0.0131)	1.2189(0.0998)$^-$	1.2012(0.1112)$^-$	1.2371(0.1159)$^-$
	WFG7	1.6898(0.0044)	1.4322(0.0696)$^-$	1.4081(0.0800)$^-$	1.4060(0.0853)$^-$
	WFG8	1.2724(0.0246)	0.9147(0.0718)$^-$	0.9538(0.1177)$^-$	0.8474(0.1014)$^-$
	WFG9	1.5244(0.0299)	1.0180(0.1637)$^-$	1.0972(0.0915)$^-$	1.1466(0.0884)$^-$
$\sharp. - / = /+$			24/0/0	23/1/0	22/1/0

its convergence performance is better than the others on all problems except for WFG2-2 and WFG9-2. It should be pointed out that no algorithm has completely converged to the PF. This is because all the WFG problems are configured with $n = 100$ decision variables among which 94 are distance variables ($l = 94$). As pointed out in [78], the increase of the number of WFG distance variables creates more difficulties for algorithms to converge to the PF. These results also suggest that large-scale global optimization [82] is challenging for MOPs.

B. HV and C Metric Results

Tables II and III show the comparison results in terms of the HV and C metrics, respectively. The nonparametric Wilcoxon-Ranksum two-sided method at the 95% confidence level is applied to test whether the results are statistically different.

From Table II the following results are observed.

1) Comparing MOEA/D-LWS with MOEA/D, MOEA/D-LWS is better for all problems.
2) Comparing MOEA/D-LWS with MOEA/D-AS, MOEA/D-LWS is better for 23 out of the 24 problems. On WFG2-2 the two algorithms perform comparably.
3) Comparing MOEA/D-LWS with MOEA/D-SS, MOEA/D-LWS is also better for 22 out of the 24 problems. On WFG2-2 and WFG9-2, the two algorithms show comparable performance.

Convergence comparison results observed from Table III are as follows.

1) Comparing MOEA/D-LWS with MOEA/D, MOEA/D-LWS is better for 22 out of the 24 problems. On WFG2-2 and WFG9-2, the two algorithms show comparable convergence performance.
2) Comparing MOEA/D-LWS with MOEA/D-AS, MOEA/D-LWS is better for 20 out of the 24 problems. On WFG2-2, WFG9-2, WFG2-4, and WFG8-4 the two algorithms perform comparably.
3) Comparing MOEA/D-LWS with MOEA/D-SS, MOEA/D-LWS is also better for 18 out of the 24 problems. For the three 2-objective problems (WFG2, WFG5, and WFG9) and the three 4-objective problems (WFG3, WFG8, and WFG9), the two algorithms are found to be comparable.

Upon closer examination, the following results are observed.

1) On WFG2-2 MOEA/D-LWS, MOEA/D-AS, and MOEA/D-SS perform comparably. We find that during the search MOEA/D-AS applies the weighted Chebyshev method only for a few times at the early stage of the search. This indicates that MOEA/D-AS has successfully identified the PF shape of WFG2-2, and thus using the WS method for most of time which leads to a comparable performance as MOEA/D-LWS. With respect to MOEA/D-SS, the WS method is naturally embedded. Thus, its performance could be comparable

TABLE III

C METRIC (MEAN/STD) COMPARISON RESULTS. THE SYMBOL "$<$," "$=$," OR "$>$" MEANS THAT THE CONSIDERED ALGORITHM IS STATISTICALLY WORSE, COMPARABLE OR BETTER THAN MOEA/D-LWS. LWS REFERS TO MOEA/D-LWS, AS REFERS TO MOEA/D-AS, SS REFERS TO MOEA/D-SS

	C(MOEA/D,LWS)		C(LWS,MOEA/D)	C(AS,LWS)		C(LWS,AS)	C(SS,LWS)		C(LWS,SS)
				2-objective problems					
WFG2	0.1564(0.3070)	$<$	0.7282(0.4289)	0.2485(0.3405)	$=$	0.6006(0.5135)	0.4165(0.3609)	$=$	0.3167(0.3980)
WFG3	0.2455(0.3364)	$=$	0.6020(0.5450)	0.0021(0.0048)	$<$	0.8092(0.1852)	0(0)	$<$	0.9493(0.0817)
WFG4	0.0537(0.0404)	$<$	0.6999(0.2846)	0.0022(0.0050)	$<$	0.9676(0.0245)	0.0768(0.0371)	$<$	0.7308(0.0648)
WFG5	0.2353(0.4523)	$=$	0.2234(0.4371)	0(0)	$<$	0.9841(0.0240)	0.2774(0.2515)	$=$	0.3117(0.4145)
WFG6	0.0266(0.0470)	$<$	0.8663(0.1646)	0(0)	$<$	0.9487(0.0477)	0.0953(0.0687)	$<$	0.6539(0.1817)
WFG7	0.0292(0.0653)	$<$	0.8840(0.2324)	0(0)	$<$	0.9601(0.0464)	0(0)	$<$	0.9468(0.0817)
WFG8	0.0360(0.0478)	$<$	0.7931(0.2188)	0.0148(0.0331)	$<$	0.8543(0.1104)	0.1179(0.1474)	$<$	0.7342(0.2213)
WFG9	0.2374(0.4099)	$<$	0.6566(0.4474)	0.3663(0.3657)	$=$	0.4268(0.4278)	0.4285(0.4829)	$=$	0.3326(0.4095)
				4-objective problems					
WFG2	0(0)	$<$	0.3201(0.2905)	0.0050(0.0082)	$=$	0.1074(0.1553)	0.0037(0.0084)	$<$	0.2171(0.1064)
WFG3	0.0043(0.0096)	$<$	0.1558(0.1074)	0(0)	$<$	0.1024(0.0774)	0.0100(0.0096)	$=$	0.0582(0.0916)
WFG4	0.0012(0.0027)	$<$	0.3478(0.0643)	0.0012(0.0027)	$<$	0.4647(0.1095)	0.0073(0.0051)	$<$	0.1899(0.0871)
WFG5	0(0)	$<$	0.3321(0.0774)	0(0)	$<$	0.3553(0.1023)	0.0011(0.0026)	$<$	0.1334(0.0303)
WFG6	0.0063(0.0109)	$<$	0.2745(0.1046)	0.0091(0.0057)	$<$	0.1826(0.0627)	0.0397(0.0195)	$=$	0.0194(0.0435)
WFG7	0.0013(0.0029)	$<$	0.2585(0.0790)	0(0)	$<$	0.3640(0.1674)	0.0075(0.0053)	$<$	0.1109(0.0499)
WFG8	0.0065(0.0066)	$<$	0.1079(0.0361)	0.0146(0.0088)	$=$	0.1029(0.0731)	0.0372(0.0116)	$=$	0.0338(0.0407)
WFG9	0.0137(0.0189)	$<$	0.3154(0.2562)	0.0197(0.0193)	$<$	0.0740(0.0763)	0.0164(0.0102)	$=$	0.0141(0.0154)
				7-objective problems					
WFG2	0(0)	$<$	0.0805(0.0853)	0.0007(0.0022)	$<$	0.0740(0.0436)	0.0009(0.0029)	$<$	0.1466(0.1366)
WFG3	0.0026(0.0016)	$<$	0.0232(0.0350)	0.0020(0.0045)	$<$	0.0476(0.0423)	0(0)	$<$	0.1984(0.0602)
WFG4	0.0043(0.0018)	$<$	0.1010(0.0646)	0.0043(0.0027)	$<$	0.1019(0.0473)	0.0057(0.0018)	$<$	0.0544(0.0337)
WFG5	0.0006(0.0010)	$<$	0.0584(0.0300)	0.0002(0.0006)	$<$	0.0397(0.0159)	0.0023(0.0024)	$<$	0.0480(0.0146)
WFG6	0.0027(0.0030)	$<$	0.1225(0.0273)	0.0025(0.0109)	$<$	0.1372(0.0556)	0.0054(0.0055)	$<$	0.1492(0.0928)
WFG7	0.0002(0.0007)	$<$	0.1155(0.0284)	0.0006(0.0029)	$<$	0.1206(0.0264)	0.0009(0.0012)	$<$	0.1723(0.0542)
WFG8	0.0150(0.0056)	$<$	0.0754(0.0413)	0.0071(0.0066)	$<$	0.0477(0.0365)	0.0157(0.0048)	$<$	0.0920(0.0472)
WFG9	0(0)	$<$	0.3214(0.1718)	0(0)	$<$	0.1840(0.1306)	0.0009(0.0025)	$<$	0.1854(0.1868)
	♯ </=/>	22/2/0		♯ </=/>	20/4/0		♯ </=/>	18/6/0	

with MOEA/D-LWS for WFG2-2. For the other comparisons on 2-objective problems (having nonconvex PF shape), MOEA/D-LWS shows better performance than the other algorithms mainly because of the high search efficiency of LWS. In other words, on these problems MOEA/D-AS chooses to use the Chebyshev method for most of time. Regarding MOEA/D-SS, half of the weights takes no effect.

2) As the number of objectives increases, the superiority of MOEA/D-LWS becomes more evident. Taking the 7-objective problems as an example, MOEA/D-LWS shows better convergence performance than any of the three algorithms for all problems. Its overall performance (measured by the HV metric) is also better than the other three algorithms for almost all problems (22 out of the 24 comparisons). The reason for the inferior performance of MOEA/D is that the algorithm applies the Chebyshev method during the whole search whose search efficiency is not as good as the WS. The same reason, together with the incapability of the WS to handle the nonconvex region, apply to MOEA/D-AS and MOEA/D-SS. Noticeably, on WFG2-4 and WFG2-7 MOEA/D-AS is not comparable to MOEA/D-LWS. Possibly because that MOEA/D-AS cannot detect the concave region successfully with the chosen neighborhood size T.

To further compare the performance of MOEA/D-LWS and its competitor algorithms, the changes of HV and GD metrics over generations (till 1000 generations) are examined. Fig. 6 illustrates the results on WFG4-4 for instance. From the results we can observe that MOEA/D-LWS continuously exhibits better overall performance (measured by HV metric) and convergence performance (measured by GD metric) than the other three competitor MOEAs. Similar results are also obtained for most of other WFG problems.

In conclusion the LWS method is effective which makes MOEA/D-LWS perform better than the considered competitors. In particular the LWS enables MOEA/D-LWS to find solutions on both convex and nonconvex PF regions. In addition the superiority of MOEA/D-LWS becomes more evident as the number of objectives increases. We can therefore make a strong claim to use MOEA/D-LWS for many-objective optimization.

VI. EXPERIMENT DISCUSSION

This section studies three further issues as part of a wider discussion for the effect of the LWS method: 1) uniformity performance of obtained solutions; 2) comparison with other three evolutionary many-objective optimizers; and 3) comparison with its variants.

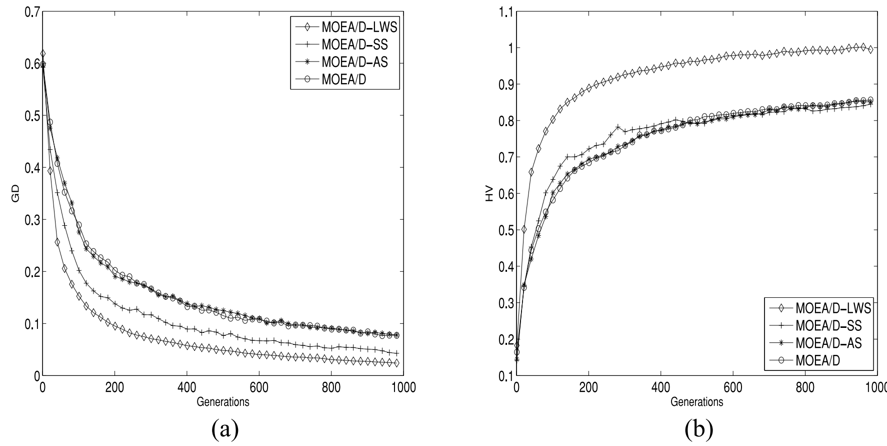

Fig. 6. Performance of algorithms over generations in terms of HV and GD metrics for WFG4-4 problems. (a) GD values over generations. (b) HV values over generations.

A. Solutions Obtained by MOEA/D-LWS With Respect to Uniformity

Experimental results have demonstrated the effectiveness of MOEA/D-LWS on dealing with many-objective problems as well as nonconvex PFs. This section discusses the uniformity of solutions obtained by MOEA/D-LWS.

As mentioned earlier, each LWS attempts to find a Pareto optimal solution within its associated hypercone. Given a set of N hypercones, diversified solutions could be found by MOEA/D-LWS. However, it should be mentioned that the uniformity of obtained solutions might not be as good as solutions obtained by MOEA/D with the Chebyshev method, in particular, for problems with nonconvex PFs.

Fig. 7 illustrates two sets of solutions that are possibly obtained by MOEA/D-LWS using the same weights (i.e., $\mathbf{w}^1$, $\mathbf{w}^2$, and $\mathbf{w}^3$). The solutions obtained by MOEA/D with Chebyshev method severe as references. It shows that in the ideal case solutions obtained by the use of Chebyshev method (black circles) are constantly distributed along each search direction. However, for the LWS method the obtained solutions (black points) might be distributed differently, see the difference between Fig. 7(a) and (b). This is because that for a local nonconvex PF there are more than one boundary solutions could be seen as the candidate optimal solution. The obtained solutions might not consistently sit at the same side of the weight. In other words, the uniformity of obtained solutions is not guaranteed. However, given that a number of weights are employed in MOEA/D-LWS, and each LWS is associated with a different hypercone, the diversity performance of MOEA/D-LWS would not be much deteriorated because of this limitation. As an illustration, solutions obtained by MOEA/D-LWS for 3-objective WFG problems are plotted (see the supplementary file) from which we can observe that MOEA/D-LWS achieves a relatively good diversity performance, though solutions are not uniformly distributed.

In addition, for a convex PF the LWS can maintain a good uniformity performance. As an illustration, Fig. 8 shows the performance of MOEA/D-LWS on a convex MOP, namely, ZDT1. By comparing against MOEA/D with the Chebyshev method, we can observe that MOEA/D-LWS achieves a good uniformity performance. The reason is that, as mentioned earlier, for a convex PF, the obtained solution have more chances to be along (or near) the search direction.

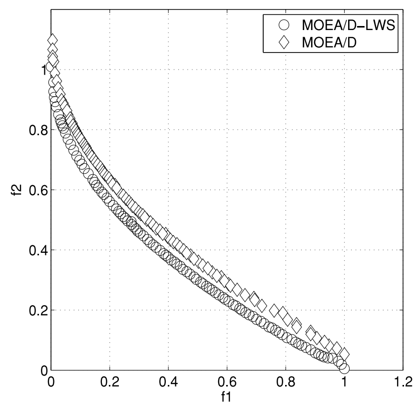

Fig. 7. Illustration of the performance of MOEA/D-LWS and MOEA/D with the Chebyshev method on ZDT1 with a convex PF.

However, it is worth mentioning that a perfect uniformity as the Chebyshev method offers is still not guaranteed. This is because that the optimal solution for each LWS method is not always along the search direction exactly due to the contour lines of the LWS as well as the PF shape.

Overall, though the LWS method is not as good as the Chebyshev method with respect to the uniformity of obtained solutions, it is much better than the WS method, being less affected by the PF shape (in particular, being able to find diversified solutions even for non-PFs). Also, the number of obtained solutions increases as the number of employed weights increases. Moreover, the LWS is clearly better than the Chebyshev method in terms of convergence property (as shown in our previous experiments).

B. Comparison With PICEA-g, HypE, θ-DEA, and SPEA2+SDE

This section further investigates the performance of MOEA/D-LWS on many-objective problems by comparing

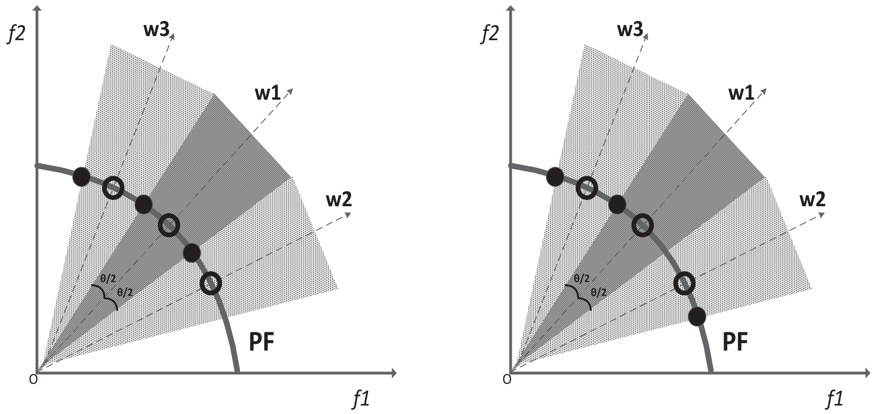

Fig. 8. Illustration of uniformity performance of MOEA/D-LWS. Solutions (black points) are obtained by the LWS method; solutions (black circles) are obtained by the Chebyshev method. (a) Uniformity performance of MOEA/D-LWS case 1. (b) Uniformity performance of MOEA/D-LWS case 2.

it against four state-of-the-art many-objective algorithms (representing different classes), PICEA-g [11], HypE [10], θ-DEA [21], and SPEA2+SDE [7].

1) PICEA-g is one realization of the preference-inspired co-evolutionary algorithms. The ingenuity in PICEAs lies in the simple recognition that decision-maker preferences bring additional comparability between solutions that is otherwise missing from the standard Pareto-dominance relation used in traditional EMO algorithms. What follows from this realization is the hypothesis that, by maintaining a family of preferences during search, then there may be sufficient comparability to find the Pareto optimal set. The working principle of PICEA is, in essence, a generalization of the known decomposition approaches. PICEA generalizes the decomposition concept to any type of preference information (e.g., goals and weights) and also provides a means of adapting the set of preferences (i.e., co-evolution) to drive the search effectively in the absence of a known PF shape. In PICEA-g goal vectors are taken as preferences. Candidate solutions gain high fitness by dominating as many goals in objective space as possible. Goal vectors only gain fitness by being dominated by a candidate solution, however, the fitness is reduced if other solutions also dominate the same goal vector. In this paper, the number of goal vectors N_{goal} is set as identical to the population size N.
2) HypE is an effective performance IBEA. The IBEA transforms the optimization of an MOP into the optimization of an indicator. In HypE the HV is used as the indicator. What makes the HypE different from other HV-based algorithms is that it employs a Monte Carlo method to approximate the HV value. This effectively reduces the computational effort required for the exact HV calculation [83]. In this paper, the exact HV is used for 2-objective problems. For 4- and 7-objective problems, the Monte Carlo method with $N_{\text{sp}} = 2N \times m$ sampling points in objective space is applied to approximate the HV value.
3) θ-DEA is a very recently proposed many-objective optimizer. The algorithm is implemented within a $(\mu + \lambda)$ elitist framework, and uses a new dominance relation, namely, θ-dominance to rank solutions in the environmental selection phase. The θ-dominance, adopting the same expression of PBI scalarizing function, can effectively balance both convergence and diversity when a suitable θ is specified. In θ-DEA first the nondominated sorting-based on the Pareto-dominance is applied to sort solutions in different front levels. Then, similar to the crowding distance used in NGSA-II, the θ-dominance is further applied to select solutions. Before applying the θ-dominance, the clustering operation is conducted, i.e., each solution is assigned to its closest reference direction. The reference directions are evenly distributed. In each cluster, the θ optimal solution is selected. θ-DEA is reported to perform well on the DTLZ [84] and WFG benchmarks. Also, it is compared favorably with state-of-the-art many-objective optimizers such as NSGA-III and GrEA. In this paper, the same set of weights as MOEA/D-LWS is used in θ-DEA, and parameter $\theta = 5$ [21] is adopted.
4) SPEA2+SDE applies a shift-based density estimation (SDE) strategy to the well-known SPEA2 [85]. The SDE strategy concerns both the distribution and convergence information of individuals. That is, it shifts the position of solutions based on their convergence information when estimating their densities. Numerical studies have demonstrated the efficiency of the SDE strategy. It can significantly improve performance of Pareto-dominance-based EMO algorithms for many-objective problems. It represents an important class of many-objective optimizers, i.e., modified density-based class.

In addition, in order make a fair comparison the same mating restriction strategy (Algorithm 1 lines 9–13) as MOEA/D-LWS is applied to all the three algorithms.

The HV comparison results of the three algorithms are shown in Table IV. To avoid repetitions HV results of MOEA/D-LWS are not shown. The same parameter settings in Table I are adopted. From Table IV it is observed that MOEA/D-LWS is not as good as HypE for 2-objective problems. However, its performance is better than HypE for all

TABLE IV
MEAN HV RESULTS OF PICEA-G, HYPE, θ-DEA, AND SPEA2+SDE. THE SYMBOL "$-$," $=$, OR $+$ MEANS THE CONSIDERED ALGORITHM IS STATISTICALLY WORSE THAN, COMPARABLE TO OR BETTER THAN MOEA/D-LWS

	$M=2$				$M=4$				$M=7$			
	PICEA-g	HypE	θ-DEA	SDE	PICEA-g	HypE	θ-DEA	SDE	PICEA-g	HypE	θ-DEA	SDE
WFG2	0.5964^{-}	0.6327^{+}	0.6281^{+}	0.6859^{+}	1.2796^{-}	1.2554^{-}	1.1955^{-}	1.2936^{-}	1.8345^{+}	1.8006^{-}	1.6166^{-}	1.8697^{+}
WFG3	0.5708^{-}	0.6153^{+}	$0.5988^{=}$	0.6376^{+}	1.0458^{-}	1.0446^{-}	0.8079^{-}	1.0890^{+}	$1.6676^{=}$	1.6209^{-}	0.9540^{-}	1.1236^{-}
WFG4	0.3290^{-}	0.3759^{+}	0.3650^{-}	0.3978^{+}	0.8097^{-}	0.8041^{-}	0.7307^{-}	0.8825^{-}	1.2729^{-}	1.1899^{-}	0.9080^{-}	1.3343^{-}
WFG5	0.3058^{-}	0.3354^{+}	0.3269^{+}	0.3666^{+}	0.7625^{-}	0.7456^{-}	0.6495^{-}	0.8229^{-}	1.1951^{-}	1.0766^{-}	0.7380^{-}	1.2972^{-}
WFG6	0.3223^{-}	0.3532^{-}	0.3473^{-}	0.3885^{+}	0.7904^{-}	0.7622^{-}	0.6676^{-}	0.8844^{-}	1.2519^{-}	1.0863^{-}	0.7197^{-}	1.3410^{-}
WFG7	0.3418^{-}	0.3871^{+}	$0.3769^{=}$	0.4104^{+}	0.9079^{-}	0.9015^{-}	0.8082^{-}	0.9775^{-}	1.5696^{-}	1.4849^{-}	1.0797^{-}	1.4867^{-}
WFG8	0.2487^{-}	0.2761^{+}	$0.2808^{=}$	0.3486^{+}	0.6866^{-}	0.6963^{-}	0.6575^{-}	0.7642^{-}	1.1001^{-}	1.0099^{-}	0.7940^{-}	1.2631^{-}
WFG9	0.3177^{-}	$0.3631^{=}$	0.3523^{-}	0.3923^{+}	0.7975^{-}	0.8047^{-}	0.7042^{-}	$0.8646^{=}$	1.3760^{-}	1.3153^{-}	0.8568^{-}	1.3158^{-}
	no. of –/=/+				no. of –/=/+				no. of –/=/+			
	8/0/0	1/1/6	2/3/2	0/0/8	8/0/0	8/0/0	8/0/0	5/2/1	6/1/1	8/0/0	8/0/0	7/0/1

the 4- and 7-objective problems. Compared with PICEA-g, MOEA/D-LWS is better for most of the test problems. With respect to θ-DEA, MOEA/D-LWS is better on two 2-objective problems, and all the 4- and 7-objective problems. Compared with SPEA2+SDE, though MOEA/D-LWS is inferior for all 2-objective problems it exhibits better performance for five 4-objective problems and seven 7-objective problems. Such results suggest that MOEA/D-LWS is a very competitive many-objective optimizer. In addition, PICEA-g is inferior to HypE and θ-DEA for 2-objective problems. However, as the number of objectives increases, PICEA-g appears to be superior to HypE and θ-DEA.

Whereas the four competitor algorithms are found as inferior to MOEA/D-LWS for many-objective problems in this experiment, it does not mean that these algorithms are ineffective. As is known the performance of an algorithm is often impacted by its associated parameter settings. We have found that using fine-tuned parameter settings, PICEA-g and HypE could have comparable performance with MOEA/D-LWS. For example, by setting a larger N_{goal} (e.g., 4000) and a larger N_{sp} (e.g., 20 000) the three algorithms perform comparably on WFG4-4. Certainly, this is computationally more expensive. Whereas, θ-DEA is reported to perform well on WFG problems (with $n = 24$ decision variables) [21], good results are not obtained here. The reason might be that in this experiment the WFG problems are set with $n = 100$ decision variables which creates difficulty for the θ-dominance. Actually, we performed computational experiments by specifying the number of decision variables as $n = 24$, which is the same setting as in [21], In this case, comparable results are obtained from θ-DEA and MOEA/D-LWS. Besides, according to the no-free-lunch theory [86], none of algorithms can perform well on all types of problems [21], [87]. There must be some problems that θ-DEA works the best [21]. Therefore, identifying suitable algorithms for different types of problems deserves more studies.

Algorithm 2: Replacement Procedure

$S \leftarrow \emptyset$;
$J \leftarrow \{1, 2, 3, \ldots, N\}$;
Shuffle J randomly;
$Count[i] \leftarrow 0,\ i = 1, 2, \ldots, N$;
foreach $j \in J$ **do**
  Remove j from J;
  $i \leftarrow \arg\min_{i=1,2,\ldots,2N} C_{ij}$;
  $Count[i] \leftarrow Count[i] + 1$;
  $temp \leftarrow Count[i]$;
  if $temp \leq nr$ **then**
    $S \leftarrow S \cup \mathbf{x}^i$;
  else
    while $temp > nr$ **do**
      set $C_{ij,j=1,2,\ldots,N}$ to ∞;
      $i' \leftarrow \arg\min_{j=1 \text{ to } 2N} C_{ij}$;
      $Count[i'] \leftarrow Count[i'] + 1$;
      $temp \leftarrow Count[i']$;
    end
    $S \leftarrow S \cup \mathbf{x}^{i'}$;
  end
end

C. MOEA/D-LWS Versus Its Variant

MOEA/D-LWS allows multiple selections of a solution, see lines 23–27. This might decrease the algorithm performance, e.g., lack of solution diversity. In order to study this, the performance of its variants, denoted as MOEA/D-LWSnr, is examined. In MOEA/D-LWSnr a solution is selected to survive for maximally nr times. Specifically, lines 23–27 in MOEA/D-LWS are replaced by Algorithm 2.

The variants of MOEA/D-LWS, i.e., MOEA/D-LWS1 ($nr = 1$) and MOEA/D-LWS2 ($nr = 2$) are examined for the same test problems as in our previous computational experiments. The HV results are shown in Table V. It is found that none of algorithms can beat its competitors for most of the problems. Specifically, for 2-objective problems MOEA/D-LWS1 performs better than MOEA/D-LWS for three problems, and performs worse than MOEA/D-LWS for only one problem. MOEA/D-LWS2 performs better than MOEA/D-LWS for four problems, and performs worse than MOEA/D-LWS for two problem. For 4-objective problems

TABLE V
HV RESULTS (MEAN/STD) OF MOEA/D-LWS1 ($nr = 1$) AND MOEA/D-LWS2 ($nr = 2$). THE SYMBOL −, =, OR + MEANS THE CONSIDERED ALGORITHM IS STATISTICALLY WORSE THAN, COMPARABLE TO OR BETTER THAN MOEA/D-LWS

	$nr = 1$	$nr = 2$		$nr = 1$	$nr = 2$		$nr = 1$	$nr = 2$
WFG2-2	0.6308(0.0060)$^+$	0.6223(0.0115)$^+$	WFG2-4	1.2988(0.0089)$^=$	1.2991(0.0071)$^=$	WFG2-7	1.8624(0.0082)$^+$	1.8779(0.0040)$^+$
WFG3-2	0.6131(0.0006)$^+$	0.5920(0.0234)$^=$	WFG3-4	1.1030(0.0117)$^+$	1.0841(0.0157)$^+$	WFG3-7	1.7562(0.0132)$^+$	1.7621(0.0138)$^+$
WFG4-2	0.3720(0.0041)$^=$	0.3696(0.0031)$^-$	WFG4-4	0.9303(0.0110)$^=$	0.9353(0.0084)$^=$	WFG4-7	1.5207(0.0176)$^-$	1.5960(0.0162)$^=$
WFG5-2	0.3288(0.0046)$^+$	0.3075(0.0037)$^=$	WFG5-4	0.8538(0.0051)$^-$	0.8500(0.0088)$^-$	WFG5-7	1.3204(0.0109)$^-$	1.3451(0.0072)$^-$
WFG6-2	0.3552(0.0055)$^-$	0.3632(0.0020)$^+$	WFG6-4	0.8962(0.0124)$^=$	0.9271(0.0131)$^=$	WFG6-7	1.4533(0.0263)$^-$	1.5582(0.0150)$^-$
WFG7-2	0.3765(0.0110)$^=$	0.3664(0.0057)$^-$	WFG7-4	1.0033(0.0056)$^-$	1.0143(0.0029)$^=$	WFG7-7	1.6824(0.0050)$^-$	1.6821(0.0060)$^-$
WFG8-2	0.2880(0.0087)$^=$	0.2850(0.0046)$^+$	WFG8-4	0.7789(0.0106)$^=$	0.7837(0.0094)$^+$	WFG8-7	1.2245(0.0148)$^-$	1.2502(0.0145)$^-$
WFG9-2	0.3810(0.0133)$^=$	0.3687(0.0026)$^+$	WFG9-4	0.8729(0.0333)$^=$	0.8676(0.0295)$^=$	WFG9-7	1.4383(0.0206)$^-$	1.5154(0.0233)$^=$
♯ −/=/+	1/4/3	2/2/4		2/5/1	1/5/2		6/0/2	4/2/2

MOEA/D-LWS and its variants perform comparably for a majority of problems. This is demonstrated by the fact that the number of nondominated solutions (without counting the repeated ones) in the obtained Pareto approximation set by the three algorithms is comparable for most of test problems. For example, the number of nondominated solutions in the final generation for WFG4-4 obtained by MOEA/D-LWS, MOEA/D-LWS1, and MOEA/D-LWS2 are 168, 179, and 172 (averaged over 31 runs), respectively. For 7-objective problems, MOEA/D-LWS performs better than its variants.

Overall restricting the number of replacements is helpful in maintaining diversity of solutions. This leads to superior performance of MOEA/D-LWS1 and MOEA/D-LWS2 for 2-objective problems. However, convergence is often considered as more challenging than diversity for many-objective optimization [4]. As the number of objectives increases the superiority of MOEA/D-LWS1 and MOEA/D-LWS2 diminishes. In another aspect, this diversity promotion strategy leads to a detrimental effect on the convergence performance. Therefore, for many-objective problems MOEA/D-LWS appears to be better than its variants.

VII. CONCLUSION

Decomposition-based EMO algorithms have been repeated demonstrated as effective for addressing multiobjective and many-objective problems. Different from the Pareto-dominance relation, the use of weighted scalarizing methods brings additional comparability between candidate solutions, and therefore can guide candidate solutions toward the PF effectively. It is observed that different scalarizing methods exhibit different search efficiency. Despite the incapability of handling nonconvex regions, the WS method has the best search efficiency amongst all L_p scalarizing methods. This paper proposes an LWS method by which the drawback of the WS method is overcame while its high search efficiency is retained. Based on the LWS method, a novel decomposition-based algorithm, MOEA/D-LWS is proposed. It is compared against three related methods, i.e., MOEA/D, MOEA/D with an adaptive use of the WS and Chebyshev methods and MOEA/D with their simultaneous use for the well-known WFG benchmarks with upto seven objectives. Experimental results show that MOEA/D-LWS is much better than the competitor algorithms. Moreover, MOEA/D-LWS is compared against three many-objective optimizers: 1) PICEA-g; 2) HypE; and 3) θ-DEA, and is shown as competitive.

Regarding future studies, first we would like to assess MOEA/D-LWS on other problem types, e.g., multiobjective combinatorial problems, some real-world problems, e.g., the size of renewable energy systems [88], [89], project scheduling [90]–[92]. Second, we would like to extend MOEA/D-LWS to a hybridized evolutionary multicriteria decision making approach so as to assist the decision maker to find his/her preferred solutions [12], [93]. Third, as shown in the experiments, the algorithm performance often degrades as the number of decision variables increases. It is therefore worthwhile to investigate the scalability of MOEA/D-LWS for large-scale problems [94]. Fourth, a steady-state version of MOEA/D-LWS could be designed given to the merits of steady-state selection scheme. Fifthly, in MOEA/D-LWS some isolated inferior (or dominated) solutions might survive during the evolution in particular when the PF is disconnected or has a large hole. These isolated solutions might have a negative effect in terms of convergence. However, they are, to some extent good to diversity. How to effectively handle these solutions deserves further studies, e.g., using adaptive hypercone regions, adaptive weights. Lastly, decomposition-based EMO algorithms sometime struggle in maintain the uniformity of the solutions, a combination of Pareto-based and decomposition-based methods has shown promise for handling this issue, and thus requires further investigation [18], [95].

Source code of MOEA/D-LWS is available at http://ruiwangnudt.gotoip3.com/optimization.html.

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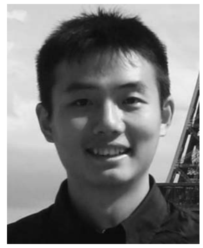

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