1. Find the value of T(2) for the recurrence relation T(n) = 3T(n-1) + 12n, given that T(0)=5.

Ans= We are given the recurrence relation:

$T(n) = 3T(n-1) + 12n$ T(n)=3T(n−1)+12n

with the base case:

$T(0) = 5$ T(0)=5 We need to find $T(2)$ T(2).

Step 1: Compute $T(1)$ T(1)

Using the recurrence relation:

$T(1) = 3T(0) + 12(1)$ T(1)=3T(0)+12(1)

Substituting $T(0) = 5$ T(0)=5:

$T(1) = 3(5) + 12(1) = 15 + 12 = 27$ T(1)=3(5)+12(1)=15+12=27

Step 2: Compute $T(2)$ T(2)

Now, using the recurrence relation again:

$T(2) = 3T(1) + 12(2)$ T(2)=3T(1)+12(2)

Substituting $T(1) = 27$ T(1)=27:

$T(2) = 3(27) + 12(2) = 81 + 24 = 105$ T(2)=3(27)+12(2)=81+24=105

Final Answer: 105

2. Given a recurrence relation, solve it using the substitution method:

a. T(n) = T(n-1) + c

b. T(n) = 2T(n/2) + n

c. T(n) = 2T(n/2) + c

d. T(n) = T(n/2) + c

Ans==

1. (a) $T(n) = T(n-1) + c$ T(n)=T(n−1)+c

This is a simple arithmetic recurrence.

Expanding the recurrence:

$T(n) = T(n-1) + c$ T(n)=T(n−1)+c $T(n-1) = T(n-2) + c$ T(n−1)=T(n−2)+c $T(n-2) = T(n-3) + c$ T(n−2)=T(n−3)+c

Continuing this process, after $k$ k steps:

$T(n-k) = T(n-k-1) + c$ T(n−k)=T(n−

When $k = n$ k=n, we reach the base case $T($

$T(n) = T(0) + nc$ T(n)=T(0)+nc

Final Solution:

$T(n) = T(0) + nc$ T(n)=T(0)+nc

2. (b) $T(n) = 2T(n/2)$

$+ n$ T(n)=2T(n/2)+n This is a divide-and-conquer recurrence.

Expanding the recurrence:

$T(n) = 2T(n/2) + n$ T(n)=2T(n/2)+n $T(n/2) = 2T(n/4) + n/2$ T(n/2)=2T(n/4)+n/2

Substituting this into the first equation:

$T(n) = 2[2T(n/4) + n/2] + n$ T(n)=2[2T(n/4)+n/2]+n $= 4T(n/4) + 2(n/2) + n$ =4T(n/4)+2(n/2)+n $= 4T(n/4) + n + n$ =4T(n/4)+n+n $= 4T(n/4) + 2n$ =4T(n/4)+2n
Expanding further:

$T(n) = 8T(n/8) + 3n$ T(n)=8T(n/8)+3n After $k$ k steps, we get:

$T(n) = 2kT(n/2k) + kn$ T(n)=2 k T(n/2 k )+kn

Since the recurrence reduces the problem size by half each time, we set

$n/2k = 1$ n/2 k =1, so $k = \log\ 2n$ k=log 2n.

Substituting $k = \log\ 2n$ k=log 2n:

$T(n) = 2\log\ 2nT(1) + n\log\ 2n$ T(n)=2 log 2n T(1)+nlog 2n Since $2\log\ 2n = n$ 2 log 2n =n, we get:

$T(n) = nT(1) + n\log\ 2n$ T(n)=nT(1)+nlog 2n If $T(1)$ T(1) is a constant $c'$ c ' , then:

$T(n) = O(n\log\ n)$ T(n)=O(nlogn)

3. (c) $T(n) = 2T(n/2)$

$+ c$ T(n)=2T(n/2)+c

This is another divide-and-conquer recurrence, but with a constant term.

Expanding the recurrence:

$T(n) = 2T(n/2) + c$ T(n)=2T(n/2)+c $T(n/2) = 2T(n/4) + c$ T(n/2)=2T(n/4)+c

Substituting:

$T(n) = 2[2T(n/4) + c] + c$ T(n)=2[2T(n/4)+c]+c $= 4T(n/4) + 2c + c$ =4T(n/4)+2c+c $= 4T(n/4) + 3c$ =4T(n/4)+3c

Expanding further:

$T(n) = 8T(n/8) + 7c$ T(n)=8T(n/8)+7c

$T(n) = 2 k T(n / 2 k) + (2 k - 1) c$ T(n)=2 k T(n/2 k )+(2 k −1)c Setting $n / 2 k = 1$ n/2 k =1, so $k = \log\ 2 n$ k=log 2n:

$T(n) = 2 \log\ 2 n T(1) + (2 \log\ 2 n - 1) c$ T(n)=2 log 2n T(1)+(2 log 2n −1)c Since 2 $\log\ 2 n = n$ 2 log 2n =n, we get:

$T(n) = n T(1) + (n - 1) c$ T(n)=nT(1)+(n−1)c

For constant $T(1)$ T(1), this simplifies to:

$T(n) = O(n)$ T(n)=O(n)

4. (d) $T(n) = T(n / 2)$

$+ c$ T(n)=T(n/2)+c

This recurrence reduces the problem size by half in each step.

Expanding the recurrence:

$T(n) = T(n / 2) + c$ T(n)=T(n/2)+c $T(n / 2) = T(n / 4) + c$ T(n/2)=T(n/4)+c

Substituting:

$T(n) = [T(n / 4) + c] + c$ T(n)=[T(n/4)+c]+c $= T(n / 4) + 2 c$ =T(n/4)+2c

Expanding further:

$T(n) = T(n / 8) + 3 c$ T(n)=T(n/8)+3c After $k$ k steps:

$T(n) = T(n / 2 k) + k c$ T(n)=T(n/2 k )+kc Setting $n / 2 k = 1$ n/2 k =1, so $k = \log\ 2 n$ k=log 2n:

$T(n) = T(1) + c \log\ 2 n$ T(n)=T(1)+clog 2n For constant $T(1)$ T(1), we get:

$T(n) = O(\log\ n)$ T(n)=O(logn)

3. Given a recurrence relation, solve it using the recursive tree approach:

a. T(n) = 2T(n-1) +1

1. (a) $T(n) = 2 T(n - 1)$

$+ 1$ T(n)=2T(n−1)+1

Expanding the recurrence (recursive tree method)

Expanding the recurrence step by step:

$T(n) = 2 T(n - 1) + 1$ T(n)=2T(n−1)+1 $T(n - 1) = 2 T(n - 2) + 1$ T(n−1)=2T(n−2)+1

Substituting

$T(n - 1)$ T(n−1) into $T(n)$ T(n):

$T(n) = 2 [2 T(n - 2) + 1] + 1$ T(n)=2[2T(

=4T(n−2)+2+1=4T(n−2)+3 Expanding furt

$T(n - 2) = 2 T(n - 3) + 1$ T(n−2)=2T(n−3)+1

Substituting:

$T(n) = 8T(n-3) + 7$ T(n)=8T(n−3)+7 After $k$ k steps:

$T(n) = 2 k T(n-k) + (2 k - 1)$ T(n)=2 k T(n−k)+(2 k −1) Finding $k$ k We reach the base case $T(0)$ T(0) when $k = n$ k=n:

$T(n) = 2 n T(0) + (2 n - 1)$ T(n)=2 n T(0)+(2 n −1) Assuming $T(0) = c$ T(0)=c:

$T(n) = 2 n c + (2 n - 1)$ T(n)=2 n c+(2 n −1) For large $n$ n, the dominant term is $2 n$ 2 n , so:

$T(n) = O(2 n)$ T(n)=O(2 n )

b. T(n) = 2T(n/2) + n

Ans==(b)

$T(n) = 2T(n/2) + n$ T(n)=2T(n/2)+n

This is a divide-and-conquer recurrence.

Expanding the recurrence (recursive tree method)

$T(n) = 2T(n/2) + n$ T(n)=2T(n/2)+n

Expanding further:

$T(n/2) = 2T(n/4) + n/2$ T(n/2)=2T(n/4)+n/2

Substituting:

$T(n) = 2[2T(n/4) + n/2] + n$ T(n)=2[2T(n/4)+n/2]+n = $4T(n/4) + 2(n/2) + n$ =4T(n/4)+2(n/2)+n = $4T(n/4) + 2n$ =4T(n/4)+2n Expanding further:

$T(n) = 8T(n/8) + 3n$ T(n)=8T(n/8)+3n Continuing this expansion, after $k$ k levels:

$T(n) = 2 k T(n/2 k) + kn$ T(n)=2 k T(n/2 k )+kn Finding $k$ k We reach the base case when $n/2 k = 1$ n/2 k =1, so:

$2 k = n \Rightarrow k = \log 2 n$ 2 k =n⇒k=log 2n Substituting $k = \log 2 n$ k=log 2n:

$T(n) = 2 \log 2 n T(1) + n \log 2 n$ T(n)=2 log 2n T(1)+nlog 2n Since $2 \log 2 n = n$ 2 log 2n =n, we get:

$T(n) = nT(1) + n \log 2 n$ T(n)=nT(1)+nlog 2n For constant $T(1)$ T(1), this simplifies to:

$T(n) = O(n \log n)$ T(n)=O(nlogn)