Note 1 (Propositional Logic)

- $P \implies Q \equiv \neg P \vee Q$
- $P \implies Q \equiv \neg Q \implies \neg P$
- $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
- $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
- $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
- $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
- $\neg(\forall x P(x)) \equiv \exists x \neg P(x)$
- $\neg(\exists x P(x)) \equiv \forall x \neg P(x)$

Note 2/3 (Proofs)

- Direct proof
- Proof by contraposition
- Proof by cases
- Proof by induction
  - Base case (prove smallest case is true)
  - Inductive hypothesis (assume $n = k$ true for weak induction, assume $n \leq k$ true for strong induction)
  - Inductive step (prove $n = k + 1$ is true)
- Pigeonhole principle
  - Putting $n + m$ balls in n bins $\implies \geq 1$ bin has ≥ 2 balls

Note 4 (Sets)

- $\mathscr{P}(S)$ = powerset/set of all subsets; if $|S| = k$, $|\mathscr{P}(S)| = 2^k$
- One to one (injection); $f(x) = f(y) \implies x = y$
- Onto (surjection); $(\forall y \exists x)(f(x) = y)$; “hits" all of range
- Bijection: both injective and surjective

Note 5 (Countability & Computability)

- Countable if bijection to $\mathbb{N}$
- Cantor-Schroder-Bernstein Theorem: bijection between A and B if there exists injections $f : A \to B$ and $g : B \to A$
- Cantor diagonalization: to prove uncountability, list out possibilities, construct new possibility different from each listed at one place (ex. reals $\in (0, 1)$, infinite binary strings, etc)
- $A \subseteq B$ and B is countable $\implies A$ is countable
- $A \supseteq B$ and A is uncountable $\implies B$ is uncountable
- Infinite cartesian product sometimes countable ($\emptyset \times \emptyset \times \cdots$), sometimes uncountable ($\{0, 1\}^\infty$)
- Halting Problem: can’t determine for every program whether it halts (uncomputable)
- Reduction of `TestHalt(P, x)` to some task (here, `TestTask`)
  - define inner function that does the task if and only if `P(x)` halts
  - call `TestTask` on the inner function and return the result in `TestHalt`

Note 6 (Graph Theory)

- K_n has $\frac{n(n-1)}{2}$ edges
- Handshaking lemma: total degree = $2e$
- Trees: (all must be true)
  - connected & no cycles
  - connected & has $n - 1$ edges ($n = |V|$)
  - connected & removing an edge disconnects the graph
  - acyclic & adding an edge makes a cycle
- Hypercubes:
  - n-length bit strings, connected by an edge if differs by exactly 1 bit
  - n-dimensional hypercube has $n2^{n-1}$ edges, and is bipartite (even vs odd parity bitstring)
- Eulerian walk: visits each edge once; only possible if connected and all even degree or exactly 2 odd degree
- Eulerian tour: Eulerian walk but starts & ends at the same vertex; only possible if all even degree and connected
- Planar graphs
  - $v + f = e + 2$
  - $\sum_{i=1}^{f} s_i = 2e$ where s_i = number of sides of face i
  - $e \leq 3v - 6$ if planar (because $s_i \geq 3$)
  - $e \leq 2v - 4$ if planar for bipartite graphs (because $s_i \geq 4$)
  - nonplanar if and only if the graph contains K_5 or $K_{3,3}$
  - all planar graphs can be colored with ≤ 4 colors

Note 7 (Modular Artithmetic)

- x^{-1} (modular inverse) exists mod m if and only if $\gcd(x, m) = 1$
- Extended Euclidean Algorithm:

x	y	$\lfloor x/y \rfloor$	a	b	
35	12	2	$\boxed{-1}$	$\boxed{3}$	answer
12	11	1	1	-1	
11	1	11	$\boxed{0}$	$\boxed{1}$	
$\boxed{1}$	0				start
gcd					

  - new a = old b
  - new $b = a - b\left\lfloor \frac{x}{y} \right\rfloor$
  - if $\gcd(x, y) = 1$, then $a = x^{-1} \bmod y$, $b = y^{-1} \bmod x$
- Chinese Remainder Theorem:
  - find bases b_i that are $\equiv 1 \bmod m_i$ and $\equiv 0 \bmod m_j$ for $j \neq i$
    $\to$ $b_i = c_i(c_i^{-1} \bmod m_i)$ where $c_i = \prod_{i \neq j} m_j$
  - $x \equiv \sum a_i b_i \pmod{\prod m_i}$
  - solution is unique mod $\prod m_i$
  - m_i must be pairwise relatively prime in order to use CRT

Note 8 (RSA)

- Scheme: for primes p, q, find e coprime to $(p - 1)(q - 1)$
  - public key: $N = pq$ and e
  - private key: $d = e^{-1} \bmod (p - 1)(q - 1)$
  - encryption of message m: $m^e \pmod N = y$
  - decryption of encrypted message y: $y^d \pmod N = m$
- Fermat’s Little Theorem (FLT): $x^p \equiv x \pmod p$, or $x^{p-1} \equiv 1 \pmod p$ if x coprime to p
- Prime Number Theorem: $\pi(n) \geq \frac{n}{\ln n}$ for $n \geq 17$, where $\pi(n)$ = # of primes $\leq n$
- Breaking RSA if we know d:
  - we know $de - 1 = k(p - 1)(q - 1)$, where $k \leq e$ because $d < (p - 1)(q - 1)$
  - so $\frac{de-1}{k} = pq - p - q - 1$; $pq = N$, so we can find p, q because we know d, e, k

Note 9 (Polynomials)

- Property 1: nonzero polynomial of degree d has at most d roots
- Property 2: $d + 1$ pairs of points (x_i distinct) uniquely defines a polynomial of degree at most d
- Lagrange Interpolation:
  - $\Delta_i(x) = \prod_{i \neq j} \frac{x - x_j}{x_i - x_j}$
  - $P(x) = \sum_i y_i \Delta_i(x)$
- Secret Sharing (normally under $GF(p)$):
  - $P(0)$ = secret, $P(1), \ldots, P(n)$ given to all people
  - $P(x)$ = polynomial of degree $k - 1$, where k people are needed to get the secret
- Rational Root Theorem: for $P(x) = a_n x^n + \cdots + a_0$, the roots of $P(x)$ that are of the form $\frac{p}{q}$ must have $p \mid a_0$, $q \mid a_n$

Note 10 (Error Correcting Codes)

- Erasure Errors: k packets lost, message length n; need to send $n + k$ packets because $P(x)$ of degree $n - 1$ needs n points to define it
- General Errors: k packets corrupted, message length n; send $n + 2k$ packets
- Berlekamp Welch:
  - $P(x)$ encodes message (degree $n - 1$)
  - $E(x)$ constructed so that roots are where the errors are (degree k); coefficients unknown
  - $Q(x) = P(x)E(x)$ (degree $n + k - 1$)
  - substitute all (x_i, r_i) into $Q(x_i) = r_i E(x_i)$, make system of equations
  - solve for coefficients; $P(x) = \frac{Q(x)}{E(x)}$

Note 11 (Counting)

- 1st rule of counting: multiply # of ways for each choice
- 2nd rule of counting: count ordered arrangements, divide by # of ways to order to get unordered
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ = # ways to select k from n
- Stars and bars: n objects, k groups $\to$ n stars, $k-1$ bars $\to \binom{n+k-1}{k-1} = \binom{n+k-1}{n}$
- Zeroth rule of counting: if bijection between A and B, then $|A| = |B|$
- Binomial theorem: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$
- Hockey-stick identity: $\binom{n}{k+1} = \binom{n-1}{k} + \binom{n-2}{k} + \cdots + \binom{k}{k}$
- Derangements: $D_n = (n-1)(D_{n-1} + D_{n-2}) = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$
- Principle of Inclusion-Exclusion: $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$ More generally, alternate add/subtract all combinations
- Stirling's approximation: $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$

Note 12 (Probability Theory)

- Sample points = outcomes
- Sample space = Ω = all possible outcomes
- Probability space: $(\Omega, \mathbb{P}(\omega))$; (sample space, probability function)
- $0 \le \mathbb{P}(\omega) \le 1, \forall \omega \in \Omega$; $\sum_{\omega \in \Omega} \mathbb{P}(\omega) = 1$
- Uniform probability: $\mathbb{P}(\omega) = \frac{1}{|\Omega|}, \forall \omega \in \Omega$
- $\mathbb{P}(A) = \sum_{\omega \in A} \mathbb{P}(\omega)$ where A is an event
- If uniform: $\mathbb{P}(A) = \frac{\text{\# sample points in } A}{\text{\# sample points in } \Omega} = \frac{|A|}{|\Omega|}$
- $\mathbb{P}(\overline{A}) = 1 - \mathbb{P}(A)$

Note 13 (Conditional Probability)

- $\mathbb{P}(\omega \mid B) = \frac{\mathbb{P}(\omega)}{P(B)}$ for $\omega \in B$
- $\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \to \mathbb{P}(A \cap B) = \mathbb{P}(A \mid B)\,\mathbb{P}(B)$
- Bayes' Rule:
$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B \mid A)\,\mathbb{P}(A) + \mathbb{P}(B \mid \overline{A})\,\mathbb{P}(\overline{A})}.$$
- Total Probability Rule (denom of Bayes' Rule):
$$\mathbb{P}(B) = \sum_{i=1}^{n} \mathbb{P}(B \cap A_i) = \sum_{i=1}^{n} \mathbb{P}(B \mid A_i)\,\mathbb{P}(A_i)$$
for A_i partitioning Ω
- Independence: $\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$ or $\mathbb{P}(A \mid B) = \mathbb{P}(A)$
- Union bound: $\mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} \mathbb{P}(A_i)$

Note 14 (Random Variables)

- Bernoulli distribution: used as an indicator RV
- Binomial distribution: $\mathbb{P}(X = i) = i$ successes in n trials, success probability p
  - If $X \sim \text{Bin}(n,p)$, $Y \sim \text{Bin}(m,p)$ independent, $X+Y \sim \text{Bin}(n+m,p)$
- Hypergeometric distribution: $\mathbb{P}(X = k) = k$ successes in N draws w/o replacement from size N population with B objects (as successes)
- Joint distribution: $\mathbb{P}(X = a, Y = b)$
- Marginal distribution: $\mathbb{P}(X = a) = \sum_{b \in B} \mathbb{P}(X = a, Y = b)$
- Independence: $\mathbb{P}(X = a, Y = b) = \mathbb{P}(X = a)\,\mathbb{P}(Y = b)$
- Expectation: $\mathbb{E}[X] = \sum_{x \in \mathcal{X}} x\,\mathbb{P}(X = x)$
- LOTUS: $\mathbb{E}[g(X)] = \sum_{x \in \mathcal{X}} g(x)\,\mathbb{P}(X = x)$
- Linearity of expectation: $\mathbb{E}[aX + bY] = a\,\mathbb{E}[X] + b\,\mathbb{E}[Y]$
- X, Y independent: $\mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y]$

Note 15 (Variance/Covariance)

- Variance: $\text{Var}(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}\left[X^2\right] - \mathbb{E}[X]^2$
  - $\text{Var}(cX) = c^2\,\text{Var}(X)$, $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\,\text{Cov}(X,Y)$
  - if indep: $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$
- Covariance: $\text{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$
- Correlation: $\text{Corr}(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$, always in $[-1, 1]$
- Indep. implies uncorrelated (Cov = 0), but not other way around, ex.
$$X = \begin{cases} 1 & p = 0.5 \\ -1 & p = -0.5 \end{cases}, \quad Y = \begin{cases} 1 & X = -1, p = 0.5 \\ -1 & X = -1, p = 0.5 \\ 0 & X = 1 \end{cases}$$

Note 16 (Geometric/Poisson Distributions)

- Geometric distribution: $\mathbb{P}(X = i)$ = exactly i trials until success with probability p; use $X - 1$ for failures until success
  - Memoryless Property: $\mathbb{P}(X > a + b \mid X > a) = \mathbb{P}(X > b)$; i.e. waiting $> b$ units has same probability, no matter where we start
- Poisson distribution: λ = average # of successes in a unit of time
  - $X \sim \text{Pois}(\lambda)$, $Y \sim \text{Pois}(\mu)$ independent: $X + Y \sim \text{Pois}(\lambda + \mu)$
  - $X \sim \text{Bin}(n, \frac{\lambda}{n})$ where $\lambda > 0$ is constant, as $n \to \infty$, $X \to \text{Pois}(\lambda)$

Note 20 (Continuous Distributions)

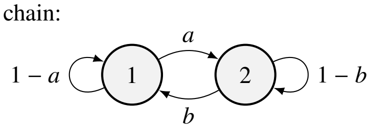
- Probability density function:
  - $f_X(x) \ge 0$ for $x \in \mathbb{R}$
  - $\int_{-\infty}^{\infty} f_X(x)\,\mathrm{d}x = 1$
- Cumulative density function: $F_X(x) = \mathbb{P}(X \le x) = \int_{-\infty}^{x} f_X(t)\,\mathrm{d}t$, $f_X(x) = \frac{\mathrm{d}}{\mathrm{d}x} F_X(x)$
- Expectation: $\mathbb{E}[X] = \int_{-\infty}^{\infty} x f_X(x)\,\mathrm{d}x$
- LOTUS: $\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\,\mathrm{d}x$
- Joint distribution: $\mathbb{P}(a \le X \le b, c \le Y \le d)$
  - $f_{XY}(x,y) \ge 0, \forall x, y \in \mathbb{R}$
  - $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1$
- Marginal distribution: $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,\mathrm{d}y$; integrate over all y
- Independence: $f_{XY}(x,y) = f_X(x) f_Y(y)$
- Conditional probability: $f_{X|A}(x) = \frac{f_X(x)}{\mathbb{P}(A)}$, $f_{X|Y}(x \mid y) = \frac{f_{XY}(x,y)}{f_Y(y)}$
- Exponential distribution: continuous analog to geometric distribution
  - Memoryless property: $\mathbb{P}(X > t + s \mid X > t) = \mathbb{P}(X > s)$
  - Additionally, $\mathbb{P}(X < Y \mid \min(X,Y) > t) = \mathbb{P}(X < Y)$
  - If $X \sim \text{Exp}(\lambda_X)$, $Y \sim \text{Exp}(\lambda_Y)$ independent, then $\min(X,Y) \sim \text{Exp}(\lambda_X + \lambda_Y)$ and $\mathbb{P}(X \le Y) = \frac{\lambda_X}{\lambda_X + \lambda_Y}$
- Normal distribution (Gaussian distribution)
  - If $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$, $Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$ independent: $Z = aX + bY \sim \mathcal{N}(a\mu_X + b\mu_Y, a^2\sigma_X^2 + b^2\sigma_Y^2)$
- Central Limit Theorem: if $S_n = \sum_{i=1}^{n} X_i$, all X_i iid with mean μ, variance σ^2,
$$\frac{S_n}{\sigma} \approx \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right); \quad \frac{S_n - n\mu}{\sigma\sqrt{n}} \approx \mathcal{N}(0,1).$$

Note 17 (Concentration Inequalities, LLN)

- Markov's Inequality: $\mathbb{P}(X \ge c) \le \frac{\mathbb{E}[X]}{c}$, if X nonnegative, $c > 0$
- Generalized Markov: $\mathbb{P}(|Y| \ge c) \le \frac{\mathbb{E}[|Y|^r]}{c^r}$ for $c, r > 0$
- Chebyshev's Inequality: $\mathbb{P}(|X - \mu| \ge c) \le \frac{\text{Var}(X)}{c^2}$ for $\mu = \mathbb{E}[X]$, $c > 0$
  - Corollary: $\mathbb{P}(|X - \mu| \ge k\sigma) \le \frac{1}{k^2}$ for $\sigma = \sqrt{\text{Var}(X)}$, $k > 0$
- Confidence intervals:
  - For proportions, $\mathbb{P}(|\hat{p} - p| \ge \varepsilon) \le \frac{\text{Var}(\hat{p})}{\varepsilon^2} \le \delta$, where δ is the confidence level (95% interval $\to \delta = 0.05$)
  - $\hat{p}$ = proportion of successes in n trials, $\text{Var}(\hat{p}) = \frac{p(1-p)}{n}$
  - $\implies n \ge \frac{1}{4\varepsilon^2\delta}$
  - For means, $\mathbb{P}\left(\left|\frac{1}{n}S_n - \mu\right| \ge \varepsilon\right) \le \frac{\sigma^2}{n\varepsilon^2} = \delta$
  - $S_n = \sum_{i=1}^{n} X_i$, all X_i's iid mean μ, variance σ^2
  - $\implies \varepsilon = \frac{\sigma}{\sqrt{n\delta}}$, interval $= S_n \pm \frac{\sigma}{\sqrt{n\delta}}$
  - With CLT, $\mathbb{P}(|A_n - \mu| \le \varepsilon) = \mathbb{P}\left(\left|\frac{(A_n - \mu)\sqrt{n}}{\sigma}\right| \le \frac{\varepsilon\sqrt{n}}{\sigma}\right) \approx 1 - 2\Phi\left(-\frac{\varepsilon\sqrt{n}}{\sigma}\right) = 1 - \delta.$ Here, $A_n = \frac{1}{n}S_n$ and CLT gives $A_n \approx \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$; use inverse cdf to get ε
- Law of large numbers: as $n \to \infty$, sample average of iid $X_1, \ldots X_n$ tends to population mean

Note 24 (Markov Chains)

- Markov chain = sequence of states
- X_n = state at time step n
- $\mathcal{X}$ = state space (finite)
- π_n = distribution of states at time step n
- Markov property (memoryless): $\mathbb{P}(X_{n+1} = i \mid X_n, X_{n-1}, \ldots, X_0) = \mathbb{P}(X_{n+1} = i \mid X_n)$
- Transition matrix ($\mathbf{P}$): transition probabilities, from row to column
- $\pi_n = \pi_0 \mathbf{P}^n$
- Invariant distribution: $\pi = \pi\mathbf{P}$; solve balance equations in addition to $\sum \pi(i) = 1$
- Irreducible Markov chain: any state can be reached from any other
- All irreducible Markov chains have a unique invariant distribution:
$$\pi(i) = \lim_{n\to\infty} \frac{1}{n} \sum_{m=0}^{n-1} \mathbf{1}\{X_m = i\}.$$
  That is, average time spent at state $i = \pi(i)$
- Periodicity: let $d(i) = \gcd\{n > 0 : \mathbb{P}(X_n = i \mid X_0 = i) > 0\}$
  - If $d(i) > 1$, periodic with period d
  - If $d(i) = 1$, aperiodic
  - If irreducible, all states have the same $d(i)$
- Aperiodic irreducible Markov chains will always converge to the invariant distribution
- Hitting time: # of steps before first reaching state i
  - Let $\beta(j)$ denote this value, starting at state j; set up system of equations and solve
- $\mathbb{P}(A \text{ before } B)$: similarly, let $\alpha(j)$ denote this probability, starting at state j; set up system of equations and solve (use TPR)
- Similar problems: let $f(i)$ denote # of steps or probability, starting at state i; set up equations and solve
- 2-state Markov chain:



$\mathbf{P} = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$ and $\pi = \begin{bmatrix} \frac{b}{a+b} & \frac{a}{a+b} \end{bmatrix}$

Other

- In CS70, naturals and whole numbers both include 0
- Sum of finite geometric series: $\frac{1-r^n}{1-r}a$ where r is ratio, a is first term, n is number of terms
- Memoryless property independence: if X, Y both memoryless (i.e. geometric or exponential), then $\min(X,Y)$ and $\max(X,Y) - \min(X,Y)$ are independent
- If $S_n = \sum_{i=1}^{n} X_i$, then (useful for variance of indicators)
$$\mathbb{E}\left[S_n^2\right] = \sum_{i=1}^{n} \mathbb{E}\left[X_i^2\right] + \sum_{i \neq j} \mathbb{E}\left[X_i X_j\right] = \sum_{i=1}^{n} \mathbb{E}\left[X_i^2\right] + 2\sum_{i<j} \mathbb{E}\left[X_i X_j\right]$$
- Coupon Collector Problem:
  - n distinct items, each with equal probability; X_i = # of tries before ith new item, given $i-1$ already
  - $S_n = \sum X_i$ = total tries before getting all items
  - We have $X_i \sim \text{Geom}(\frac{n-i+1}{n})$ because $i-1$ old items, so $n-i+1$ new items
  - Hence,
$$\mathbb{E}[S_n] = \sum_{i=1}^{n} \mathbb{E}[X_i] = \sum_{i=1}^{n} \frac{n}{n-i+1} = n\sum_{i=1}^{n} \frac{1}{i} \approx n(\ln n + 0.5772)$$

Table 1: Common Discrete Distributions

Distribution	Parameters	PMF ($\mathbb{P}(X = k)$)	CMF ($\mathbb{P}(X \leq k)$)	Expectation ($\mathbb{E}[X]$)	Variance ($\mathrm{Var}(X)$)	Support
Uniform	$\mathrm{Uniform}(a, b)$	$\frac{1}{b-a+1}$	$\frac{k-a+1}{b-a+1}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$X \in [a, b]$
Bernoulli	$\mathrm{Bernoulli}(p)$	$\begin{cases} 1 & p \\ 0 & 1-p \end{cases}$	—	p	$p(1-p)$	$X \in \{0, 1\}$
Binomial	$\mathrm{Bin}(n, p)$	$\binom{n}{k} p^k (1-p)^{n-k}$	—	np	$np(1-p)$	$X \in \mathbb{N}$
Geometric	$\mathrm{Geom}(p)$	$p(1-p)^{k-1}$	$1-(1-p)^k$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$X \in \mathbb{N}$
Poisson	$\mathrm{Pois}(\lambda)$	$\frac{\lambda^k e^{-\lambda}}{k!}$	—	λ	λ	$X \in \mathbb{N}$
Hypergeometric	$\mathrm{Hypergeometric}(N, K, n)$	$\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$	—	$n\frac{K}{N}$	$n\frac{K(N-K)(N-n)}{N^2(N-1)}$	$X \in \mathbb{N}$

Table 2: Common Continuous Distributions

Distribution	Parameters	PDF ($f_X(x)$)	CDF ($F_X(x) = \mathbb{P}(X \leq x)$)	Expectation ($\mathbb{E}[X]$)	Variance ($\mathrm{Var}(X)$)	Support
Uniform	$\mathrm{Uniform}(a, b)$	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$X \in [a, b]$
Exponential	$\mathrm{Exp}(\lambda)$	$\lambda e^{-\lambda x}$	$1 - e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$X \in [0, \infty)$
Normal/Gaussian	$\mathcal{N}(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$\Phi(x)$	μ	σ^2	$X \in \mathbb{R}$