Q1) When we throw a dart on a target, the probability of hitting the target is $1/4$. If dart is thrown three times, what is the probability of obtaining atleast one hit?

$P(S)$: probability of hitting

$P(F)$: . . . . . . missing

$$p(x=1) = {}^3C_1 (1/4)^1 (3/4)^2$$

$$p(x=2) = {}^3C_2 (1/4)^2 (3/4)^1$$

$$p(x=3) = {}^3C_3 (1/4)^3 (3/4)^0$$

Sum: $p(x=1)+p(x=2)+p(x=3) = 3 \times \frac{3^2}{4^3} + 3 \times \frac{3}{4^3} + \left(\frac{1}{4}\right)^3$

$$= \frac{1}{4^3}(27+9+1) = \frac{37}{64}$$

$$\boxed{p(S) = \frac{37}{64}}$$

Q2) Manufacturer X produces PC at two different locations, Delhi & Bombay. 15% of PC produced at Delhi are defective & 5% of PC produced at Bombay are defective. If 10,00,000 PCs are produced at Delhi & 1,50,000 are produced at Bombay in a year, find the probability of purchasing a defective PC

$P(\text{Defective}) \equiv (\ )\ P\ A$

Total PCs = 11,50,000

$$P(D) = \frac{10,00,000}{11,50,000} = \frac{100}{115} = \frac{20}{23}$$

$$P(B) = \frac{1,50,000}{11,50,000} = \frac{3}{23}$$

$$P(A) = P(D) \cdot P(A/D) + P(B) \cdot P(B/D)$$

$$= \frac{20}{23} \times \frac{3}{20} + \frac{1}{20} \times \frac{3}{23} = \frac{3}{23}\left(1 + \frac{1}{20}\right)$$

$$= \frac{3 \times 21}{23 \times 20} = \frac{63}{460} =$$

$$\boxed{P(A) = \frac{63}{460}}$$

Q) Consider two boxes:

A box is selected at random and a marble is drawn at random from that box. What is the probability that marble is Black?

$$P(B) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{2}{3} = \frac{1}{4} + \frac{1}{3}$$

$$= 0.25 + 0.33 = 0.58$$

$$P(B) = 0.58$$

Suppose the marble drawn was red. What is the probability that 1st box was selected.

$$P(A_1/R) = \frac{1/2 \times 1/2}{1/2 \times \frac{1}{2} + \frac{1}{3} \times 1/2} = \frac{1/2}{1/2 + 1/3}$$

$$= \frac{1}{1 + \frac{2}{3}} = \frac{3}{5}$$

$$\boxed{P(A_1/R) = 3/5}$$

Indepence: Let A & B be two events. The interesting case is, when the occurance of B provides no information & does not alter the probability of occurence of A

$$\boxed{P(A/B) = P(A)}$$

$$\Rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B)}$$

(This eqn is adopted as the definition of two independent events because it can be used even when $P(B)=0$, in which case $P(A/B)$ is undefined.

Q Are disjoint events independent?

Definition of disjoint events, $A \cap B = \phi$

$\Rightarrow P(A \cap B) = 0$, $P(A)$ & $P(B) > 0 \Rightarrow$

$$P(A) \cdot P(B) > 0 \Rightarrow P(A) \cdot P(B) \neq P(A \cap B)$$

Q Consider two, four sided die.

$$\Omega = \begin{Bmatrix} (1,1) & \cdots & (1,4) \\ \vdots & & \vdots \\ (4,1) & \cdots & (4,4) \end{Bmatrix}$$

Consider the following two events:

$A = \{1^{st}$ roll is $1\}$ $\quad B = \{$Sum of two rolls is $5\}$

Are A & B independent?

$P(A) = \frac{1}{4}$ $\quad B = \{(1,4), (4,1), (2,3), (3,2)\}$

$B = \frac{1}{4}$

$P(A \cap B) = \frac{1}{16}$

$P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$, so the two events are independent.