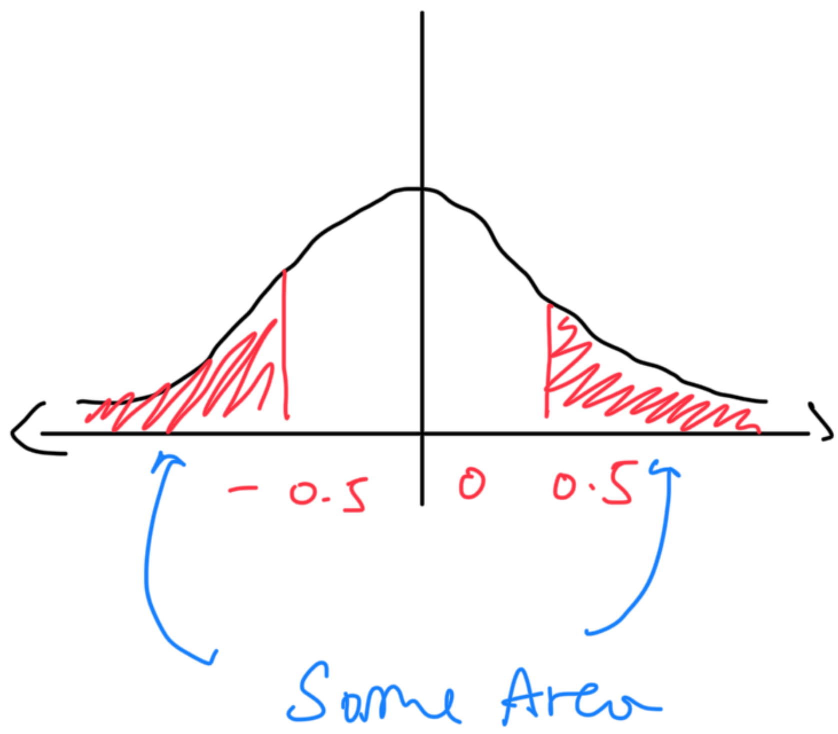
Standard normal random Variable



$$\Phi(-0.5) = P(Y \leq -0.5) = P(Y \geq 0.5)$$
$$= 1 - P(Y \leq 0.5)$$
$$= 1 - \Phi(0.5)$$

In general:

$$\boxed{\Phi(-y) = 1 - \Phi(y)}$$

Converting normal variables to standard normal

Let X be a normal random variable with mean μ and variance σ^2. Define another random variable:

$$\boxed{Y = \frac{X-\mu}{\sigma}}$$

Y is a linear function of X, hence it is also a <u>Normal random Variable</u>.

$$E[Y] = E\left[\frac{X-\mu}{\sigma}\right] = \frac{1}{\sigma}\left[E[X]-\mu\right]$$

$$= \frac{1}{\sigma}(\mu - \mu) = 0 \Rightarrow \boxed{E[Y] = 0}$$

$$Var(Y) = Var\left(\frac{X-\mu}{\sigma}\right) = Var\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right)$$

$$= \frac{1}{\sigma^2} Var(X) = \frac{\cancel{\sigma^2}}{\cancel{\sigma^2}} = 1$$

$$\Rightarrow \boxed{Var(Y) = 1}$$

Thus, Y is a <u>standard normal random variable</u>

Eg: The annual snowfall at a particular location is modeled as a normal random variable with mean 60 inches and standard deviation of $\sigma = 20$. What is the probability that this year's snowfall will be atleast 80 inches?

Let X be the snowfall. It is given that

$\mu = 60$ & $\sigma = 20$

Converting to standard random variable:

$$Y = \frac{X - 60}{20}$$

when $X = 80$

$Y = 1$

So, $p(X \geq 1) = 1 - p(X \leq 1)$

$= 1 - 0.84 = \underline{0.16}$

Joint PDFs of multiple random Variables:

Let X & Y be continuous random variables associated with the same experiment.

Then X & Y are jointly continuous if there is a non-ve func^n $f_{X,Y}(x,y)$ such that

$$P(a \le X \le b,\ c \le Y \le d) = \int_c^d \int_a^b f_{X,Y}(x,y)\,dx\,dy$$

The function $f_{X,Y}(x,y)$ satisfies the property:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = 1.$$

This $f_{X,Y}(x,y)$ is a joint PDF of X & Y.

Joint CDF

Let X & Y be two random variables associated with same experiment, their joint CDF is given by,

$$\boxed{F_{X,Y}(x,y) = P(X \le x,\ Y \le y)}$$

If X & Y are continuous:

$$\boxed{F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(s,t)\,dt\,ds}$$

We have:

$$\boxed{\partial^2 F_{X,Y}(x,y) = f_{X,Y}(x,y)}$$

$\overline{\partial x \partial y}$ $\quad {}_{X,Y}$

The marginal PDFs of X & Y can be obtained from the joint PDF as follows:

$$\boxed{\begin{aligned} f_X(x) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy \\ f_Y(y) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx \end{aligned}}$$

Expectation: If X & Y are continuous random variables:

$$E[aX + bY + c] = aE[X] + bE[Y] + c$$

Conditioning one random variable on another:

Let X & Y be a continuous random variable with joint PDF $f_{X,Y}$. The conditional PDF of X given Y=y is defined as:

$$\boxed{f_{X/Y}(x/y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}}$$

Independence of two continuous random variables :

$$\boxed{f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) \; \forall x,y}$$

Note that,

$$\boxed{f_{X/Y}(x/y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = f_x(x)}$$

If random variables are independent

Relation b/w independence & CDF:

Two random variables X & Y are independent then :

$$\boxed{F_{X,Y}(x,y) = F_X(x) F_Y(y)}$$

Relation b/w expectation & independence If X & Y are independent, then

$$\boxed{E[XY] = E[X] E[Y]}$$

Independence & Variance:

$$\boxed{Var(X+Y) = Var(X) + Var(Y)}$$

Eg: The waiting time, in hours, b/w successive speeders spotted by a radar is a continuous random variable with CDF:

$$P(X < x) = F(x) = \begin{cases} 1 - e^{-8x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find the probability of waiting less than 12 minutes b/w successive speeders.

$1 \text{ hr} = 60 \text{ min} \Rightarrow 12 \text{ min} = \frac{1}{\cancel{60}_{5}} \times \frac{\cancel{12}^{1}}{1}$

$12 \text{ min} = \frac{1}{5} \text{ hr}$

$\Rightarrow P(X < 1/5) = F(1/5) = 1 - e^{-8/5}$