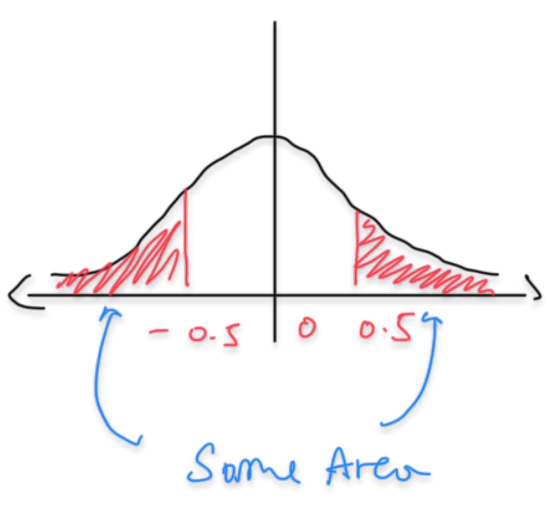


$$\Phi(-0.5) = P(Y \le -0.5) = P(Y \ge 0.5)$$
$$= 1 - P(Y \le 0.5)$$
$$= 1 - \Phi(0.5)$$

In general:

$$\boxed{\Phi(-y) = 1 - \Phi(y)}$$

Converting normal variables to standard normal

Let $X$ be a normal random variable with mean $\mu$ and variance $\sigma^2$. Define another random variable:

$$\boxed{Y = \frac{X-\mu}{\sigma}}$$

$Y$ is a linear function of $X$, hence it is also a Normal random variable.

$$E[Y] = E\left[\frac{X-\mu}{\sigma}\right] = \frac{1}{\sigma}\left[E[X] - \mu\right]$$
$$= \frac{1}{\sigma}(\mu - \mu) = 0 \Rightarrow \boxed{E[Y] = 0}$$

$$Var(Y) = Var\left(\frac{X-\mu}{\sigma}\right) = Var\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right)$$
$$= \frac{1}{\sigma^2} Var(X) = \frac{\cancel{\sigma^2}}{\cancel{\sigma^2}} = 1$$

$$\Rightarrow \boxed{Var(Y) = 1}$$

Thus, $Y$ is a standard normal random variable

Eg: The annual snowfall at a particular location is modeled as a normal random variable with mean 60 inches and standard deviation of $\sigma = 20$. What is the probability that this year's snowfall will be atleast 80 inches?

Let $X$ be the snowfall. It is given that

$\mu = 60$ & $\sigma = 20$

Converting to standard random variable:

$$Y = \frac{X-60}{20}$$

when $X = 80$
$Y = 1$

So, $P(X \ge 1) = 1 - P(X \le 1)$
$= 1 - 0.84 = 0.16$

Joint PDFs of multiple random variables:

Let $X$ & $Y$ be continuous random variables associated with the same experiment.

Then $X$ & $Y$ are jointly continuous if there is a non-ve func$^n$ $f_{X,Y}(x,y)$ such that

$$P(a \le X \le b, c \le Y \le d) = \int_c^d \int_a^b f_{X,Y}(x,y)\,dx\,dy$$

The function $f_{X,Y}(x,y)$ satisfies the property:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = 1.$$

This $f_{X,Y}(x,y)$ is a joint PDF of $X$ & $Y$.

Joint CDF

Let $X$ & $Y$ be two random variables associated with same experiment, their joint CDF is given by,

$$\boxed{F_{X,Y}(x,y) = P(X \le x, Y \le y)}$$

If $X$ & $Y$ are continuous:

$$\boxed{F_{X,Y}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(s,t)\,dt\,ds}$$

We have:

$$\boxed{\frac{\partial^2 F_{X,Y}(x,y)}{\partial x\,\partial y} = f_{X,Y}(x,y)}$$

The marginal PDFs of $X$ & $Y$ can be obtained from the joint PDF as follows:

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$
$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

Expectation: If $X$ & $Y$ are continuous random variables:

$$E[aX + bY + c] = aE[X] + bE[Y] + c$$

Conditioning one random variable on another:

Let $X$ & $Y$ be a continuous random variable with joint PDF $f_{X,Y}$. The conditional PDF of $X$ given $Y = y$ is defined as:

$$\boxed{f_{X/Y}(x/y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}}$$

Independence of two continuous random variables:

$$\boxed{f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) \ \forall x, y}$$

Note that,

$$\boxed{f_{X/Y}(x/y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = f_X(x)}$$

If random variables are independent

Relation b/w independence & CDF:

Two random variables $X$ & $Y$ are independent then:

$$\boxed{F_{X,Y}(x,y) = F_X(x) F_Y(y)}$$

Relation b/w expectation & independence If $X$ & $Y$ are independent, then

$$\boxed{E[XY] = E[X]\,E[Y]}$$

Independence & Variance:

$$\boxed{Var(X+Y) = Var(X) + Var(Y)}$$

Eg: The waiting time, in hours, b/w successive speeders spotted by a radar is a continuous random variable with CDF:

$$P(X < x) = F(x) = \begin{cases} 1 - e^{-8x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Find the probability of waiting less than 12 minutes b/w successive speeders.

1 hr = 60 min $\Rightarrow$ 12 min = $\frac{1}{60} \times 12$
12 min = $\frac{1}{5}$ hr

$$\Rightarrow P(X < 1/5) = F(1/5) = 1 - e^{-8/5}$$