Exampl: Radar Detection:

If an aircraft is present in a certain area, the radar detects it. It sends an alarm with probability 0.99. If no aircraft is there, it generates an alarm (false alarm) with probability 0.1. It is assumed that an aircraft is present with probability 0.05.

A B C

P(no aircraft present and a false alarm) = ?

p(aircraft present and no detection) = ?

$$P\left(\frac{\text{alarm}}{\text{no aircraft}}\right) = 0.1 = \frac{P(\text{alarm} \cap \text{no aircraft})}{P(\text{no aircraft})}$$

$$P\left(\frac{\text{alarm}}{\text{aircraft}}\right) = 0.99$$

$$P(\text{alarm} \cap \text{no aircraft}) = 0.1 \times 0.95 = 0.095$$

$$P\left(\frac{\text{no alarm}}{\text{aircraft}}\right) = 0.01$$

$$P(\text{no alarm} \cap \text{aircraft}) = 0.01 \times P(\text{aircraft}) = 0.01 \times 0.05 = 5 \times 10^{-4}$$

Multiplicative Rule:- Suppose an event A occurs if and only if each one of several events $A_1, A_2, \ldots A_n$ has occured, i.e.

$$\boxed{A = A_1 \cap A_2 \cap A_3 \cdots \cap A_n}$$

Definition:

$$P(A) = P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 \cap A_2) \cdots P\left(A_n / \bigcap_{i=1}^{n-1} A_i\right)$$

Example: If 3 cards are drawn from a deck of 52 cards without replacement, what is the probability that none of these three cards is a "heart"?

52: ♡, clover, spade, diamond

Sol^n: Multiplicative rule will be used here:

Case 1: I^{st} card is not heart (A_1)

Case 2: II^{nd} card is not a heart (A_2)

Case 3: III^{rd} . . . . . . (A_3)

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 \cap A_2)$$

$$= \frac{39 \times 38 \times 37}{52 \times 51 \times 50}$$

$$P(A_1) = \frac{39}{52}$$

$$P(A_2/A_1) = \frac{38}{51}$$

$$P(A_3/A_1 \cap A_2) = \frac{37}{50}$$

$$P(\text{none are } ♡) = \frac{39 \times 38 \times 37}{52 \times 51 \times 50}$$

Total probability Theorem:

Let $A_1, A_2 \ldots A_n$ be disjoint events that form a partition of sample space. Assume that $P(A_i) > 0 \; \forall i$ then for any event B,

$$\boxed{P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + \cdots + P(A_n) \cdot P(B/A_n)}$$