Space M of all 3x3 matrices

Subspace of Symmetric 3x3

Subspace of 3x3 upper triangular matrices

Bases for M = all 3x3's:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

dim M = 9

dim S = 6

dim U = 6

$S \cap U$ = symm. & upper triangular

= diagonal 3x3's

$\dim(S \cap U) = 3$

$S \cup U \rightarrow$ not a subspace.

$S + U$ = any element of S + any element of U

= all 3x3 matrices.

$\dim(S+U) = 9$

$\Rightarrow \dim(S) + \dim(U) = \dim(S+U) + \dim(S \cap U)$

$6 + 6 \qquad\qquad 9 \qquad\qquad 3$

sol^n both

$y = \cos x$

$$\frac{d^2y}{dx^2} + y = 0$$

$y = \sin x$

Complete sol^n: $y = c_1 \cos x + c_2 \sin x$

$\dim\begin{pmatrix}\text{sol}^n \\ \text{space}\end{pmatrix} = 2$

Basis

$$A_{2\times 3} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix} \quad r = 1$$

$\dim C(A) = \text{rank} = \dim C(A^T)$

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$$

Rank 1 matrix:

$$\boxed{A = u v^T}$$

M = all 5×17 matrices

Subset of rank 1 matrices

not a subspace

In R^4 $\quad v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix}$ $\quad$ S = all v in R^4

with $v_1+v_2+v_3+v_4=0$

S = nullspace of $A = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$

↳ rank = r = 1

$\dim N(A) = n-r = 4-1 = 3$

Basis for S:

$$\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$C(A) = R^1$ $\qquad$ $N(A^T) = \{0\}$.

Graph : { nodes , edges }

set .