Q) A software manufacturer knows that 1 out of 10 software games that the company markets will be a financial success. The manufacturer selects 10 new games to market. What is the probability that exactly one game will be a financial success. What is the probability that atleast 2 games will be a success?

$p(s) = 1/10$

$^{10}C_1 (1/10)^1 (9/10)^9 = p\{\text{exactly one game is a success}\}$

X = no. of successful games

$P(\text{atleast 2 success}): \ ^{10}C_2 (1/10)^2 (9/10)^8 + {}^{10}C_3 (1/10)^3 (9/10)^7$

$+ {}^{10}C_4 (1/10)^4 (9/10)^6 + \ldots$

$1 - {}^{10}C_0 (9/10)^{10} - {}^{10}C_1 (1/10)(9/10)^9$

$= 1 - \frac{9^{10}}{10^{10}} - \frac{9^9}{10^9} \Rightarrow 1 - \frac{9^9}{10^9}\left(\frac{9}{10} + 1\right) = 1 - \frac{9^9 \times 19}{10^{10}}$

$P(\text{atleast 2 successes}) = 1 - \frac{9^9 \times 19}{10^9} = 0.26$

Q) In pulse code modulation (PCM), a PCM word consists of a sequence of binary digits of 1s and 0s.

(a) Suppose the PCM word length is n-bits long. How many distinct words are there?

2^n

(1) If each PCM word, 3 bits long is equally likely to occur what is the probability of a word with exactly two 1s occuring?

Using binomial Trials:

X = no. of 1s $= 2$

$P(X=2) = {}^3C_2 (1/2)^3 = \frac{3}{8}$

Q) A balanced coin is tossed nine times

(a) $P(\text{exactly 3 heads}) = \binom{9}{3}\left(\frac{1}{2}\right)^9$

(b) $p(\text{atleast 3 heads}) = 1 - {}^9C_0\left(\frac{1}{2}\right)^9 - {}^9C_1\left(\frac{1}{2}\right)^9 - {}^9C_2\left(\frac{1}{2}\right)^9$

$1 - \left(\frac{1}{2}\right)^9 \left(1 + 9 + \frac{9 \times 8}{2}\right)$

$1 - \left(\frac{1}{2}\right)^9 (1 + 9 + 36) = 1 - \left(\frac{1}{2}\right)^9 (46)$

≈ 0.91

Expectation or Mean of a random variable

We define the expected value of a random variable X as follows:

$$\boxed{E[X] = \sum_x x P_X(x)}$$

Where P_X is PMF of X

Ex: Let X be a random variable with PMF

$$P_X(x) = \begin{cases} \frac{1}{9} & x \text{ is an integer and } x \in [-4, 4] \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \sum_{-4}^{4} x P_X(x) = 0$$

Ex: A coin is tossed two times, each with a probability 3/4 for head $E[X] = ?$

X = no. of heads

$X \in \{0, 1, 2\}$

$E[X] = \sum_{x=0}^{2} x P_X(x) = 0 + 1 \times {}^2C_1 (3/4)(1/4) + 2 \times {}^2C_2 (3/4)^2$

$= \cancel{2} \times \frac{3}{\cancel{16}\,8} + \cancel{2} \times \frac{9}{\cancel{16}\,8} = \frac{3}{8}(1+3)$

$= \frac{3}{2}$

Remark: $\text{Avg} = \frac{\text{Sum of observations}}{\text{total no of observations}}$

Variance:

Variance of a random variable X is defined as:

$$\boxed{Var(X) = E[X - E[X]]^2}$$

Eg: Let X be a random variable with

$$P_X(x) = \begin{cases} 1/9 & x \in \{-4, -3, \ldots, 3, 4\} \\ 0 & \text{otherwise} \end{cases}$$

$E[X] = 0$

$Var(X) = E[X^2]$

So, we need PMF of X^2. Let $Z = X^2$

$$P_Z(z) = \begin{cases} 1/9 & z = 0 \\ 2/9 & z \in \{1, 4, 9, 16\} \\ 0 & \text{otherwise} \end{cases}$$

$E[Z] = \sum_{0}^{16} z P_Z(z)$

$= \frac{2}{9}(1 + 4 + 9 + 16) = \frac{2}{\cancel{9}\,3} \times \cancel{30}^{10} = \frac{20}{3}$

$= 6.66$