

Normal Random Variable

A continuous random variable X is said to be normal (or Gaussian) if it has a PDF of the form:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

Where μ & σ are two scalar parameters with $\sigma > 0$.

Q Verify Normalisation property for Gaussian random variable

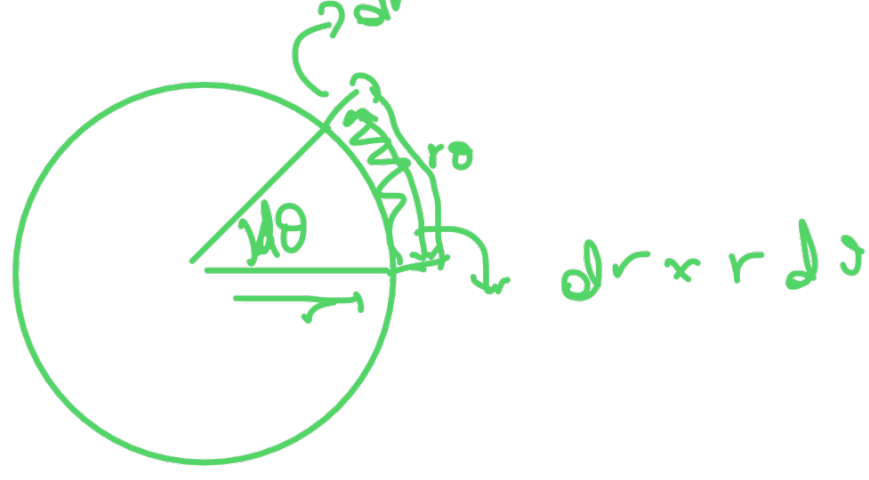
$$I = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$\frac{(x-\mu)}{\sigma} = t$
 $\Rightarrow dx = \sigma dt$

Gaussian integral

$$\Rightarrow I_1 = I = \frac{1}{\sigma} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$I_2 = I = \frac{1}{\sigma} \int_{-\infty}^{\infty} e^{-t_y^2} dt_y$$



$$I_1 \cdot I_2 = I^2 = \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(t_x^2 + t_y^2)} dt_x dt_y$$

Let $t_x^2 + t_y^2 = r^2$ $dA = dt_x dt_y = r \cdot dr \cdot d\theta$

and $t_x = r \cos \theta$

$t_y = r \sin \theta$

$$\Rightarrow I^2 = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\infty} e^{-r^2} r \cdot dr \cdot d\theta$$

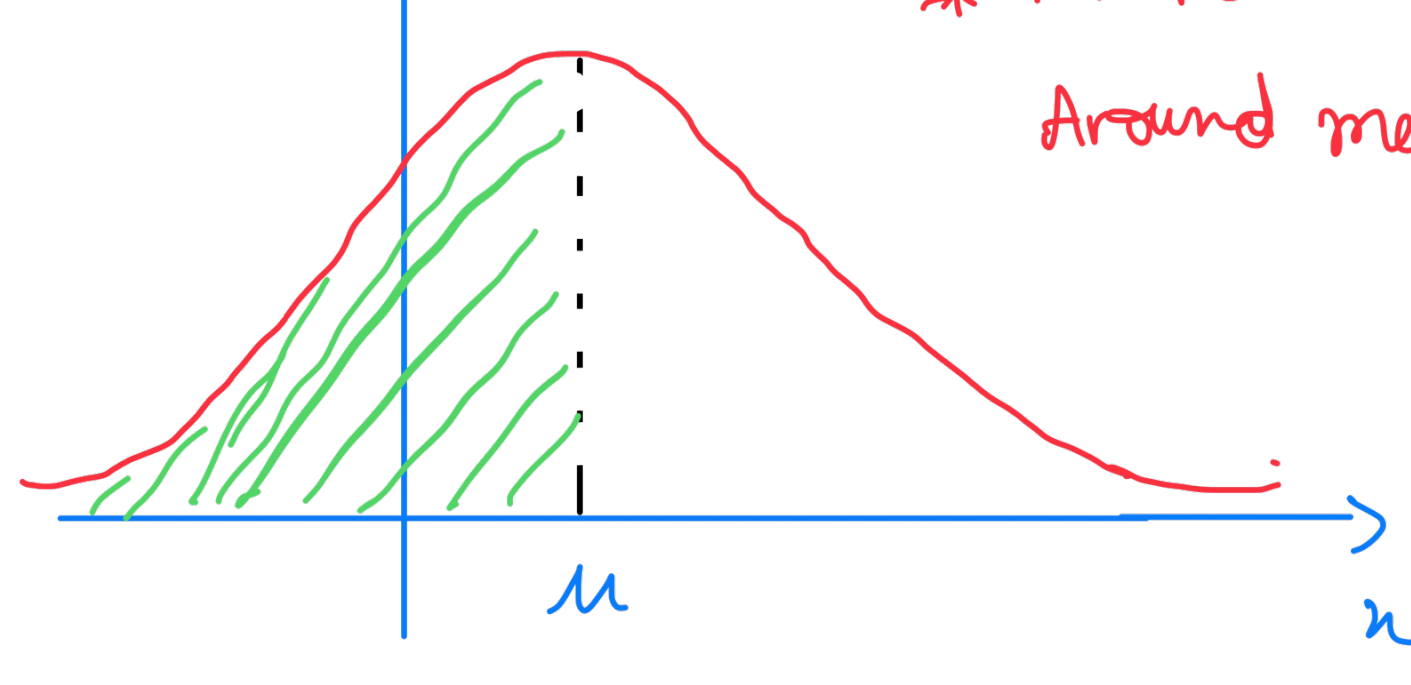
$$= \frac{1}{\pi} \times \frac{\pi}{2} \int_0^{\infty} \int_0^{\infty} e^{-r^2} r \cdot dr \cdot d\theta$$

$$= \frac{1}{2\pi} \cdot \int_0^{2\pi} \int_0^{\infty} e^{-r^2} dr \cdot d\theta = \int_0^{2\pi} \frac{1}{2\pi} d\theta$$

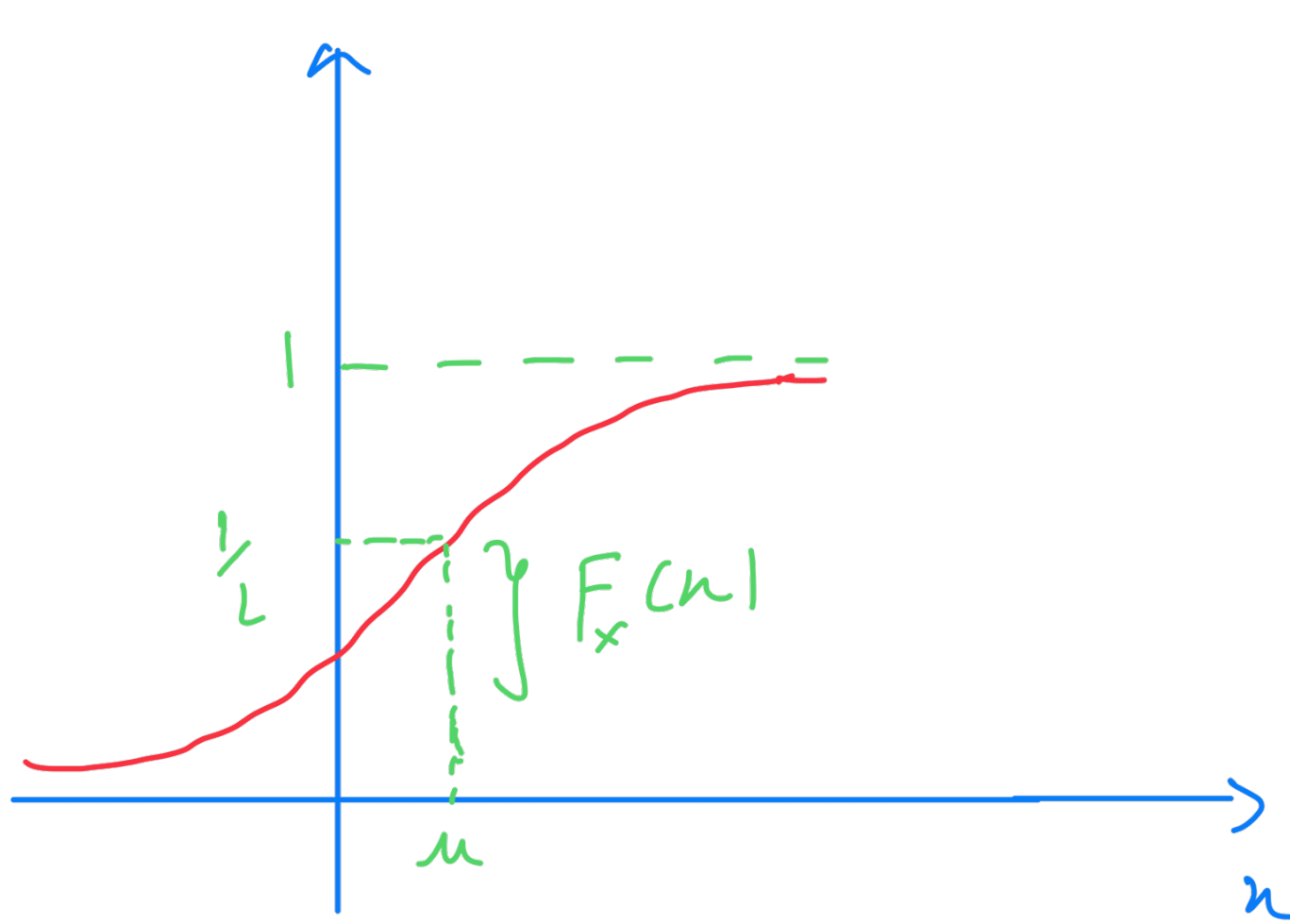
$$I^2 = \frac{1}{2\pi} \times 2\pi = 1$$

$$\Rightarrow \boxed{I = 1}$$

PDF $f_X(x)$



* The PDF is Symmetric Around mean (μ).



Mean and Variance

$$E[X] = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \underline{\underline{\mu}}$$

$$\text{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \underline{\underline{\sigma^2}}$$

Standard normal random Variable

A normal random variable with $\mu=0$ and $\sigma^2=1$. It is denoted by Y .

PDF: $f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$

Find CDF of standard normal random Variable:

$$P(Y \leq a) = \int_{-\infty}^a \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

This integral can be evaluated for all y .

people have already calculated the integral for various values of y & put it in table. This table is standard normal table.

Number in the table represents $P(Z \leq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0007	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0010	.0010	.0010
-2.9	.0016	.0016	.0016	.0015	.0015	.0014	.0014	.0014	.0014	.0014
-2.8	.0019	.0018	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0015
-2.7	.0023	.0022	.0022	.0021	.0021	.0020	.0020	.0019	.0019	.0019
-2.6	.0027	.0026	.0025	.0025	.0024	.0023	.0023	.0022	.0022	.0022
-2.5	.0031	.0030	.0029	.0028	.0028	.0027	.0027	.0026	.0026	.0026
-2.4	.0036	.0035	.0034	.0033	.0033	.0032	.0032	.0031	.0031	.0031
-2.3	.0041	.0040	.0039	.0038	.0038	.0037	.0037	.0036	.0036	.0036
-2.2	.0046	.0045	.0044	.0043	.0043	.0042	.0042	.0041	.0041	.0041
-2.1	.0051	.0050	.0049	.0048	.0048	.0047	.0047	.0046	.0046	.0046
-2.0	.0054	.0053	.0052	.0051	.0051	.0050	.0050	.0049	.0049	.0049
-1.9	.0059	.0058	.0057	.0056	.0056	.0055	.0055	.0054	.0054	.0054
-1.8	.0064	.0063	.0062	.0061	.0061	.0060	.0060	.0059	.0059	.0059
-1.7	.0069	.0068	.0067	.0066	.0066	.0065	.0065	.0064	.0064	.0064
-1.6	.0074	.0073	.0072	.0071	.0071	.0070	.0070	.0069	.0069	.0069
-1.5	.0078	.0077	.0076	.0075	.0075	.0074	.0074	.0073	.0073	.0073
-1.4	.0082	.0081	.0080	.0079	.0079	.0078	.0078	.0077	.0077	.0077
-1.3	.0086	.0085	.0084	.0083	.0083	.0082	.0082	.0081	.0081	.0081
-1.2	.0090	.0089	.0088	.0087	.0087	.0086	.0086	.0085	.0085	.0085
-1.1	.0094	.0093	.0092	.0091	.0091	.0090	.0090	.0089	.0089	.0089
-1.0	.0098	.0097	.0096	.0095	.0095	.0094	.0094	.0093	.0093	.0093
-0.9	.0101	.0100	.0099	.0098	.0098	.0097	.0097	.0096	.0096	.0096
-0.8	.0104	.0103	.0102	.0101	.0101	.0100	.0100	.0099	.0099	.0099
-0.7	.0107	.0106	.0105	.0104	.0104	.0103	.0103	.0102	.0102	.0102
-0.6	.0110	.0109	.0108	.0107	.0107	.0106	.0106	.0105	.0105	.0105
-0.5	.0113	.0112	.0111	.0110	.0110	.0109	.0109	.0108	.0108	.0108
-0.4	.0116	.0115	.0114	.0113	.0113	.0112	.0112	.0111	.0111	.0111
-0.3	.0119	.0118	.0117	.0116	.0116	.0115	.0115	.0114	.0114	.0114
-0.2	.0122	.0121	.0120	.0119	.0119	.0118	.0118	.0117	.0117	.0117
-0.1	.0125	.0124	.0123	.0122	.0122	.0121	.0121	.0120	.0120	.0120
0.0	.0128	.0127	.0126	.0125	.0125	.0124	.0124	.0123	.0123	.0123
0.1	.0131	.0130	.0129	.0128	.0128	.0127	.0127	.0126	.0126	.0126
0.2	.0134	.0133	.0132	.0131	.0131	.0130	.0130	.0129	.0129	.0129
0.3	.0137	.0136	.0135	.0134	.0134	.0133	.0133	.0132	.0132	.0132
0.4	.0140	.0139	.0138	.0137	.0137	.0136	.0136	.0135	.0135	.0135
0.5	.0143	.0142	.0141	.0140	.0140	.0139	.0139	.0138	.0138	.0138
0.6	.0146	.0145	.0144	.0143	.0143	.0142	.0142	.0141	.0141	.0141
0.7	.0149	.0148	.0147	.0146	.0146	.0145	.0145	.0144	.0144	.0144
0.8	.0152	.0151	.0150	.0149	.0149	.0148	.0148	.0147	.0147	.0147
0.9	.0155	.0154	.0153	.0152	.0152	.0151	.0151	.0150	.0150	.0150
1.0	.0158	.0157	.0156	.0155	.0155	.0154	.0154	.0153	.0153	.0153
1.1	.0161	.0160	.0159	.0158	.0158	.0157	.0157	.0156	.0156	.0156
1.2	.0164	.0163	.0162	.0161	.0161	.0160	.0160	.0159	.0159	.0159
1.3	.0167	.0166	.0165	.0164	.0164	.0163	.0163	.0162	.0162	.0162
1.4	.0170	.0169	.0168	.0167	.0167	.0166	.0166	.0165	.0165	.0165
1.5	.0173	.0172	.0171	.0170	.0170	.0169	.0169	.0168	.0168	.0168
1.6	.0176	.0175	.0174	.0173	.0173	.0172	.0172	.0171	.0171	.0171
1.7	.0179	.0178	.0177	.0176	.0176	.0175	.0175	.0174	.0174	.0174
1.8	.0182	.0181	.0180	.0179	.0179	.0178	.0178	.0177	.0177	.0177
1.9	.0185	.0184	.0183	.0182	.0182	.0181	.0181	.0180	.0180	.0180
2.0	.0188	.0187	.0186	.0185	.0185	.0184	.0184	.0183	.0183	.0183
2.1	.0191	.0190	.0189	.0188	.0188	.0187	.0187	.0186	.0186	.0186
2.2	.0194	.0193	.0192	.0191	.0191	.0190	.0190	.0189	.0189	.0189
2.3	.0197	.0196	.0195	.0194	.0194	.0193	.0193	.0192	.0192	.0192
2.4	.0200	.0199	.0198	.0197	.0197	.0196	.0196	.0195	.0195	.0195
2.5	.0202	.0201	.0200	.0199	.0199	.0198	.0198	.0197	.0197	.0197
2.6	.0205	.0204	.0203	.0202	.0202	.0201	.0201	.0200	.0200	.0200
2.7	.0207	.0206	.0205	.0204	.0204	.0203	.0203	.0202	.0202	.0202
2.8	.0209	.0208	.0207	.0206	.0206	.0205	.0205	.0204	.0204	.0204
2.9	.0211	.0210	.0209	.0208	.0208	.0207	.0207	.0206	.0206	.0206
3.0	.0213	.0212	.0211	.0210	.0210	.0209	.0209	.0208	.0208	.0208
3.1	.0215	.0214	.0213	.0212	.0212	.0211	.0211	.0210	.0210	.0210
3.2	.0217	.0216	.0215	.0214	.0214	.0213	.0213	.0212	.0212	.0212
3.3	.0219	.0218	.0217	.0216	.0216	.0215	.0215	.0214	.0214	.0214
3.4	.0221	.0220	.0219	.0218	.0218	.0217	.0217	.0216	.0216	.0216
3.5	.0223	.0222	.0221	.0220	.0220	.0219	.0219	.0218	.0218	.0218
3.6	.0225	.0224	.0223	.0222	.0222	.0221	.0221	.0220	.0220	.0220
3.7	.0227	.0226	.0225	.0224	.0224	.0223	.0223	.0222	.0222	.0222
3.8	.0229	.0228	.0227	.0226	.0226	.0225	.0225	.0224	.0224	.0224
3.9	.0231	.0230	.0229	.0228	.0228	.0227	.0227	.0226	.0226	.0226
4.0	.0232	.0231	.0230	.0229	.0229	.0228	.0228	.0227	.0227	.0227
4.1	.0234	.0233	.0232	.0231	.0231	.0230	.0230	.0229	.0229	.0229
4.2	.0235	.0234	.0233	.0232	.0232	.0231	.0231	.0230	.0230	.0230
4.3	.0236	.0235	.0234	.0233	.0233	.0232	.0232	.0231	.0231	.0231
4.4	.0237	.0236	.0235	.0234	.0234	.0233	.0233	.0232	.0232	.0232
4.5	.0238	.0237	.0236	.0235	.0235	.0234	.0234	.0233	.0233	.0233
4.6	.0239	.0238	.0237	.0236	.0236	.0235	.0235	.0234	.0234	.0234
4.7	.0240	.0239	.0238	.0237	.0237	.0236	.0236	.0235	.0235	.0235
4.8	.0241	.0240	.0239	.0238	.0238	.0237	.0237	.0236	.0236	.0236
4.9	.0242	.0241	.0240	.0239	.0239	.0238	.0238	.0237	.0237	.0237
5.0	.0243	.0242	.0241	.0240	.0240	.0239	.0239	.0238	.0238	.0238

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55568	.55963	.56358	.56753	.57147	.57542
+0.2	.57932	.58318	.58705	.59091	.59478	.59864	.60251	.60637	.61022	.61407
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65916	.66287	.66646	.67003	.67358	.67714	.68068	.68430	.68789
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72244
+0.6	.72572	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79114	.79412	.79708	.80003	.80296	.80587	.80877	.81165	.81451
+0.9	.81754	.81957	.82151	.82341	.82530	.82718	.82904	.83147	.83389	.83641
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86658	.86884	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88688	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93329	.93469	.93606	.93741	.93874	.94005	.94135	.94263	.94390	.94515
+1.6	.94642	.94769	.94894	.95018	.95140	.95261	.95381	.95499	.95616	.95732
+1.7	.95845	.95963	.95978	.96085	.96190	.96294	.96398	.96498	.96597	.96694
+1.8	.96790	.96885	.96962	.97038	.97112	.97184	.97256	.97326	.97395	.97462
+1.9	.97528	.97593	.97657	.97720	.97781	.97841	.97899	.97956	.97997	.98052
+2	.98127	.98178	.98231	.98282	.98332	.98380	.98426	.98471	.98514	.98561
+2.1	.98614	.98658	.98701	.98742	.98782	.98820	.98856	.98891	.98924	.98958
+2.2	.98971	.99014	.99056	.99097	.99137	.99176	.99213	.99248	.99282	.99315
+2.3	.99352	.99389	.99424	.99458	.99491	.99523	.99554	.99584	.99613	.99641
+2.4	.99670	.99698	.99724	.99749	.99773	.99796	.99818	.99839	.99859	.99878
+2.5	.99897	.99916	.99933	.99949	.99964	.99978	.99991	.99999	.99999	.99999
+2.6	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
+2.7	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
+2.8	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
+2.9	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
+3	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
+3.1	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999