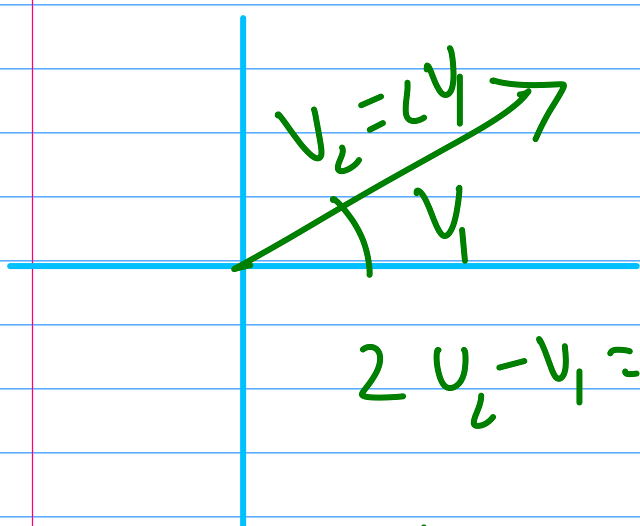
Suppose A is a $m \times n$ Matrix with $m < n$. Then there are non-zero sol^{n}s to $Ax = 0$

(more unknown than eq^{n}s)

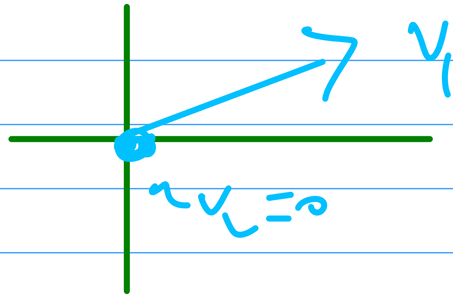
Reason: There will be free variables!!

Independence: Vectors $x_1, x_2 \ldots x_n$ are independent if no combination gives zero vector (except the zero comb)

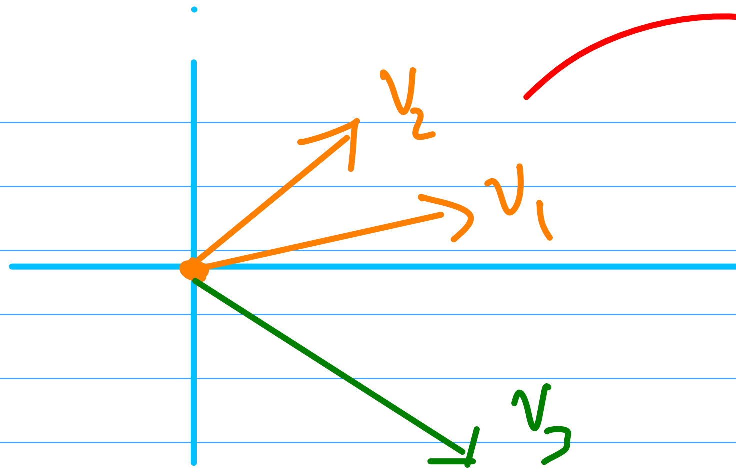
$c_1 x_1 + \ldots c_n x_n \neq 0$



$2v_2 - v_1 = 0$



$0 \cdot v_1 + n \cdot v_2 = 0$



$$A = \begin{array}{ccc} v_1 & v_2 & v_3 \\ \end{array}$$
$$A = \begin{pmatrix} 2 & 1 & 5/2 \\ 1 & 2 & -1 \end{pmatrix}$$

$$A = \begin{array}{ccc} v_1 & v_2 & v_3 \end{array}$$
$$\begin{pmatrix} 2 & 1 & 5/2 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Repeat when $v_1, \dots v_n$ are columns of A

They are independent if nullspace of A is $\{$zero vector$\}$

$N(A) = 0$ $\boxed{\text{rank} = n}$

They are dependent if

$Ac = 0$ for some

$\boxed{\text{rank} < n}$ non-zero c.

Vectors $v_1, \dots v_\ell$ span a space means: The space consists of all combs. of those vectors

<u>Basis</u> for a space is a sequence of vectors: $v_1, v_2, \dots v_d$

with 2 properties: 1. They are independent

2.) They span the space.

Example:

Space is $\mathbb{R}^3$

One basis is $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$

Another basis: $\left\{\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}2\\2\\5\end{pmatrix}, \begin{pmatrix}3\\3\\8\end{pmatrix}\right.$

The two rows are same in all of them so these cannot be basis.

For $\mathbb{R}^n$ n vectors give basis if the $n \times n$ matrix with those columns is **invertible**

Given a space: cols:

Every basis for the space has the same number of vectors

This no. is the dimension of the space.

Eg: Space $C(A)$: $\rightarrow \dim(A) = r$

$$\begin{array}{cccc} \left[\begin{matrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{matrix}\right] \\ \uparrow\ \uparrow\ \times\ \times \end{array}$$

$N(A) \rightarrow \dim(N(A)) = n-r$

$$\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

rank(A) = # of pivot columns = dimension of $C(A)$
$= 2$

$\rightarrow$ These 2 can be the basis or col 1, col 3 or col 2 & col 3

$\dim(N(A)) = n - r = \#$ of free variables.