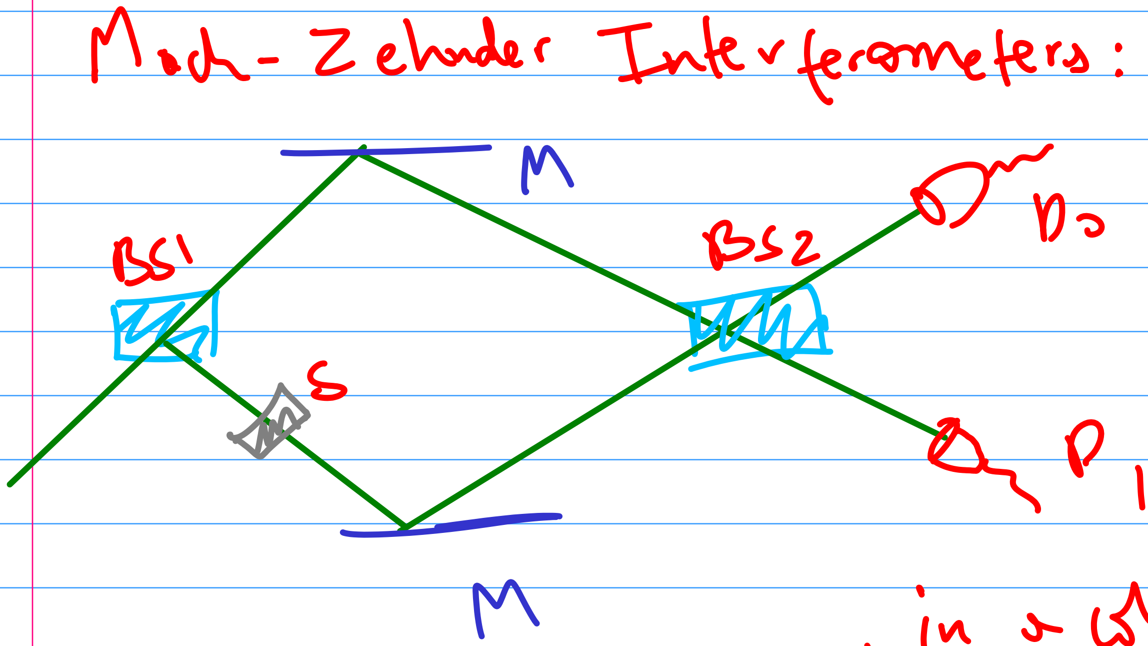
# Mach-Zehnder Interferometers:




We will use two complex no. to give the probability amplitudes in a column vector

P(upper) ~
$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$
P(bottom) ~

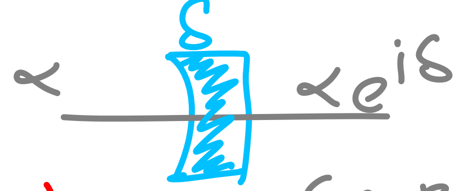
$\alpha, \beta \in \mathbb{C}$

$$|\alpha|^2 + |\beta|^2 = 1$$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a photon in the upper beam

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ . - - - - - - . . lower beam

Using superposition



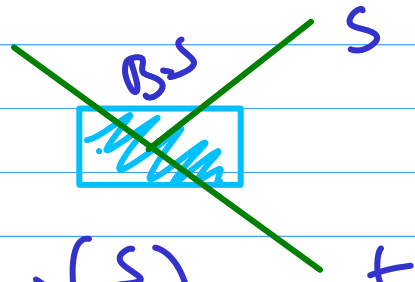

$\delta \in \mathbb{R}$

beam shifter of face $\delta$.

$$|\alpha| = |\alpha e^{i\delta}|$$

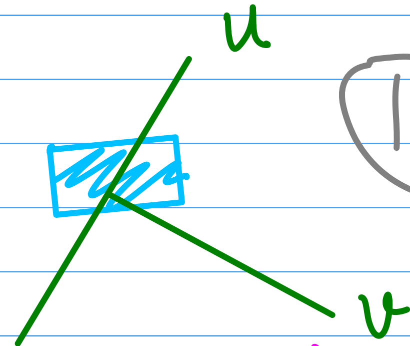
$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1\\ 0\end{pmatrix} + \beta\begin{pmatrix}0\\ 1\end{pmatrix}$$

$\begin{pmatrix}1\\ 0\end{pmatrix}$




$$\begin{pmatrix}1\\ 0\end{pmatrix} \rightarrow \begin{pmatrix}s\\ t\end{pmatrix}$$

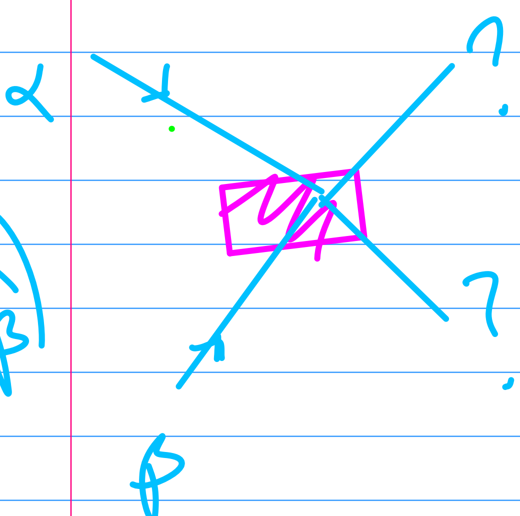
$$|s|^2 + |t|^2 = 1$$




$\begin{pmatrix}0\\ 1\end{pmatrix}$

$$\begin{pmatrix}0\\ 1\end{pmatrix} \rightarrow \begin{pmatrix}u\\ v\end{pmatrix};$$

$$|u|^2 + |v|^2 = 1$$




$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1\\ 0\end{pmatrix} + \beta\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$\downarrow$$

$$\alpha\begin{pmatrix}s\\ t\end{pmatrix} + \beta\begin{pmatrix}u\\ v\end{pmatrix}$$

$$= \begin{pmatrix}\alpha s + \beta u\\ \alpha t + \beta v\end{pmatrix}$$

$$= \underbrace{\boxed{\begin{pmatrix}s & u\\ t & v\end{pmatrix}}}_{BS}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

Balanced beam splitter

$$\boxed{|s|^2 = |t|^2 = |u|^2 = |v|^2 = 1/2}$$

$$BS = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \rightarrow \text{is this right?}$$

Let's try acting on this state:

$$\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$1^2 + 1^2 = 2 \neq 1$$

So, this cannot be a beam splitter.

$$BS = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha+\beta \\ \alpha-\beta \end{pmatrix}$$

$$\downarrow$$

$$\frac{1}{2}|\alpha+\beta|^2 + \frac{1}{2}|\alpha-\beta|^2 = 1$$

$$\Rightarrow \frac{1}{2}\left[(\alpha+\beta)(\bar{\alpha}+\bar{\beta}) + (\alpha-\beta)(\bar{\alpha}-\bar{\beta})\right] = 1$$

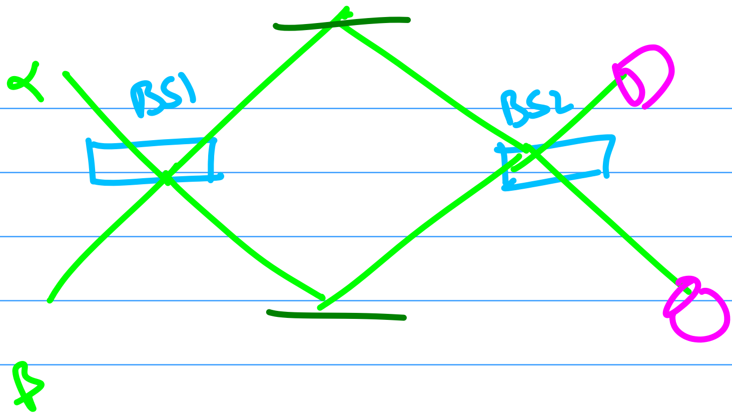
$$\Rightarrow \underline{|\alpha|^2 + |\beta|^2 = 1}$$

---

$$BS1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \qquad BS2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

<u>Both work</u>

$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$

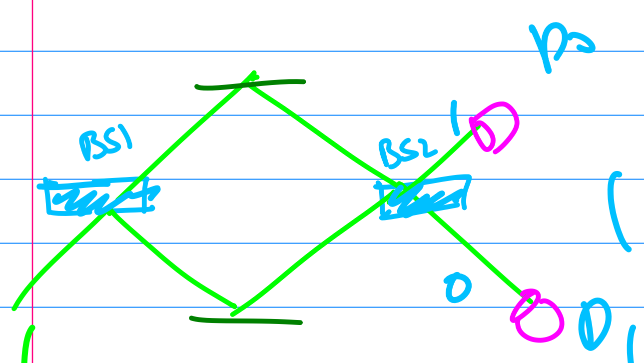



$$\text{output} = (BS2)(BS1)\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

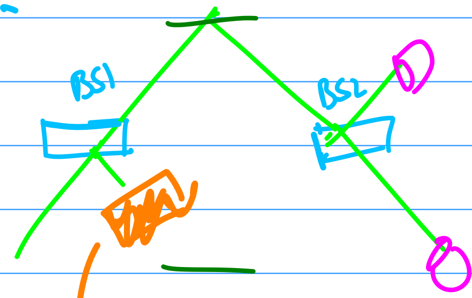
$$= \frac{1}{2}\begin{pmatrix}0 & 2\\ -2 & 0\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\beta\\ -\alpha\end{pmatrix}$$

$\begin{pmatrix}0\\ 1\end{pmatrix}$




$\begin{pmatrix}1\\ 0\end{pmatrix}$

---

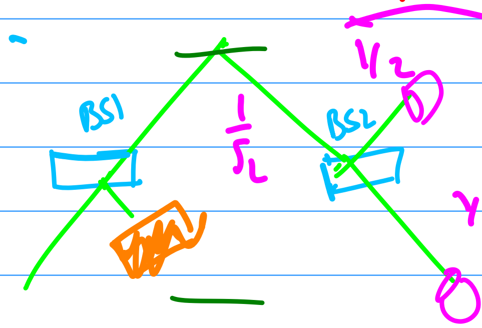



The previous equation assumes that nothing is blocking the beam so we can't use it for BS2

Brick
↳ to prevent photon from getting there

we can use it for BS1:

$$(BS_1)\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$$




$$= \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$$

so

$$(BS_2)\begin{pmatrix}1/\sqrt{2}\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\0\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \begin{pmatrix}1/2\\1/2\end{pmatrix}$$

| Outcome (Blocked: lower branch) | P |
|---|---|
| photon at the block | $1/2$ |
| " " " at D0 | $1/4$ |
| " " " at D1 | $1/4$ |