

Units of h : $[h] = \frac{[E]}{[\nu]}$

$$= \frac{M L^2 T^{-2}}{T^{-1}} = M L^2 T^{-1}$$

$\Rightarrow [h] = M L^2 T^{-1}$

$[h] = [L]$

↑
Angular
momentum

Spin $\frac{1}{2}$ particle $\equiv |\vec{S}| = \frac{1}{2} h$

$[h] = [r] [p]$

$\bullet \quad m \quad \frac{h}{p} \rightarrow \frac{h}{mc}$

we can associate a length with it.

\equiv Compton Wavelength of a particle

(\neq de Broglie Wavelength)

There exists a particle. What will be the wavelength of the photon that has the energy equal to the rest energy of the particle

$$mc^2 = E_r = h\nu = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{mc^2}$$

$$\Rightarrow \lambda = \frac{h}{mc}$$

Let us calculate the Compton wavelength of an e^- .

$$\lambda_c(e^-) = \frac{h}{mc} = \frac{2\pi\hbar c}{mc^2}$$

$$= \frac{2\pi (197.33 \text{ MeV}\cdot\text{fm})}{0.511 \text{ MeV}}$$

$$= 2426 \text{ fm} = 2.426 \text{ pm}$$

"photon"

Quanta for energy,
also has momentum

$$E^2 - p^2 c^2 = m^2 c^4$$

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$(E = \frac{1}{2} m v^2, p = m \vec{v} \Rightarrow E = \frac{p^2}{2m})$$

photons: $M_\gamma = 0$

$$E_\gamma = p c$$

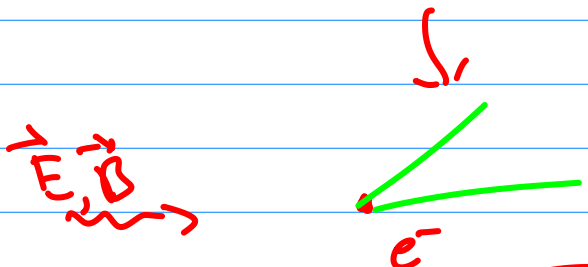
$$p_\gamma = \frac{E_\gamma}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$p_\gamma = \frac{h}{\lambda}$$

Compton Scattering

photon scattering on e^- that are "virtually" free.

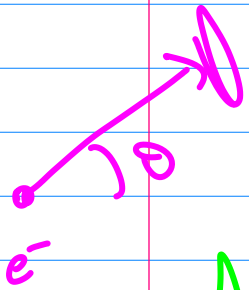
→ violation of classical Thompson Scattering



The diagram shows an electron (e^-) at a vertex. Two green lines representing electromagnetic waves originate from this vertex. One line is labeled with \vec{E} and \vec{B} (incident wave), and the other with \vec{E}' and \vec{B}' (scattered wave). A red arrow points from the text "units of Area" to the differential cross-section equation below.

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{2} (1 + \cos^2\theta)$$

units of Area

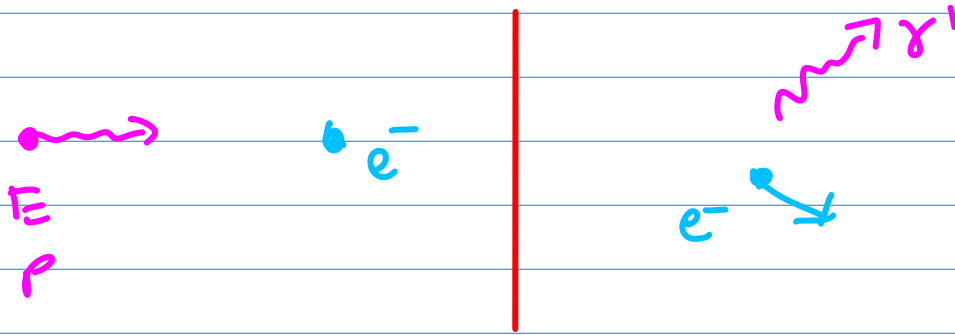


Area should be thought of as the area that captures (from the incoming beam) the energy that is being sent into this solid angle.

Outgoing wave has the same frequency as the original wave.

What did Compton find?

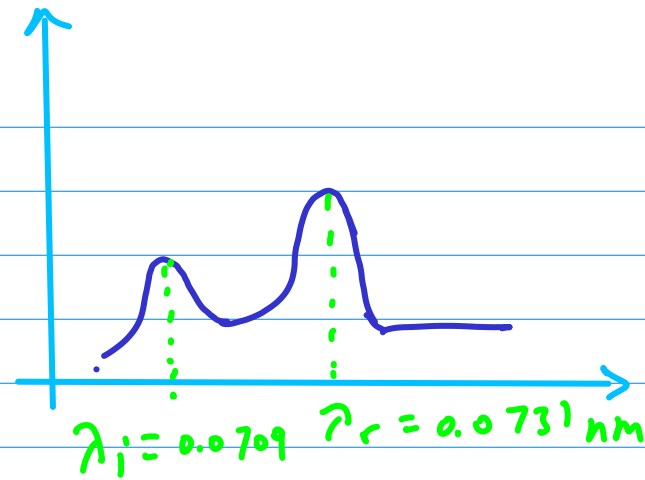
Treat the photon as a particle!



Here, photon loses energy: $\lambda_f > \lambda_i$

$$\lambda_f = \lambda_i + \underbrace{\frac{h}{m_e c}}_{\lambda_c(e^-)} (1 - \cos \theta)$$

Experiment



Molybdenum, $E_x = 17.49 \text{ keV}$

X-rays $\lambda = 0.00709 \text{ nm}$

$$\Delta \lambda = 0.0731 - 0.0709 = 0.0022 \text{ nm}$$

$$\Delta \lambda = 0.0022 \text{ nm} \approx \lambda_c(E) = 2.46 \text{ pm} = 0.00246 \text{ nm}$$

