

Q1) When we throw a dart on a target, the probability of hitting the target is  $\frac{1}{4}$ . If dart is thrown three times, what is the probability of obtaining atleast one hit?

$P(s)$ : probability of hitting  
 $P(f)$ : . . . . . missing

$$P(X=1) = {}^3C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^2$$

$$P(X=2) = {}^3C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^1$$

$$P(X=3) = {}^3C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0$$

$$\text{Sum: } P(X=1) + P(X=2) + P(X=3) = 3 \times \frac{3}{4^3} + 3 \times \frac{3}{4^3} + \left(\frac{1}{4}\right)^3$$

$$= \frac{1}{4^3} (27 + 9 + 1) = \frac{37}{64}$$

$$P(s) = \frac{37}{64}$$

Q2) Manufacturer X produces PC at two different locations, Delhi & Bombay. 15% of PC produced at Delhi are defective & 5% of PC produced at Bombay are defective. If 10,00,000 PCs are produced at Delhi & 1,50,000 are produced at Bombay in a year, find the probability of purchasing a defective PC

$$P(\text{Defective}) = ( ) P(A)$$

$$\text{Total PC} = 11,50,000$$

$$P(D) = \frac{10,00,000}{11,50,000} = \frac{100}{115} = \frac{20}{23}$$

$$P(B) = \frac{1,50,000}{11,50,000} = \frac{3}{23}$$

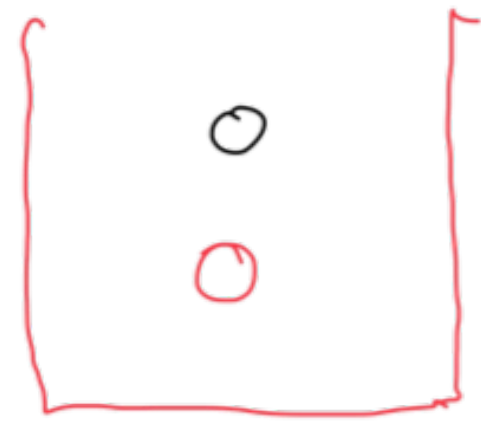
$$P(A) = P(D) \cdot P(A/D) + P(B) \cdot P(A/B)$$

$$= \frac{20}{23} \times \frac{3}{20} + \frac{1}{23} \times \frac{3}{23} = \frac{3}{23} \left(1 + \frac{1}{20}\right)$$

$$= \frac{3 \times 21}{23 \times 20} = \frac{63}{460}$$

$$P(A) = \frac{63}{460}$$

Q3) Consider two boxes:



A box is selected at random and a marble is drawn at random from that box.

What is the probability that marble is Black?

$$P(B) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{2}{3} = \frac{1}{4} + \frac{1}{3}$$

$$= 0.25 + 0.33 = 0.58$$

$$P(B) = 0.58$$

Suppose the marble drawn was red. What is the probability that 1<sup>st</sup> box was selected.

$$P(A_1/R) = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{2}{3}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{2}{9}}$$

$$= \frac{1}{1 + \frac{2}{3}} = \frac{3}{5}$$

$$P(A_1/R) = \frac{3}{5}$$

Independence: Let A & B be two events. The interesting case is, when the occurrence of B provides no information & does not alter the probability of occurrence of A

$$P(A/B) = P(A)$$

$$\Rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

(This eq<sup>n</sup> is adopted as the definition of two independent events because it can be used even if  $P(B)=0$ , in which case  $P(A/B)$  is undefined.)

Q Are disjoint events independent?

Definition of disjoint events,  $A \cap B = \emptyset$

$$\Rightarrow P(A \cap B) = 0, P(A) \geq P(B) > 0 \Rightarrow$$

$$P(A) \cdot P(B) > 0 \Rightarrow P(A) \cdot P(B) \neq P(A \cap B)$$

Q Consider two, four sided die.

$$\Omega = \left\{ \begin{matrix} (1,1) & \dots & (1,4) \\ \vdots & & \vdots \\ (4,1) & \dots & (4,4) \end{matrix} \right\}$$

Consider the following two events:

$$A = \{1^{\text{st}} \text{ roll is } 1\} \quad B = \{\text{Sum of two rolls is } 5\}$$

Are A & B independent?

$$P(A) = \frac{1}{4} \quad B = \{(1,4), (2,3), (3,2), (4,1)\}$$

$$P(B) = \frac{1}{4}$$

$$P(A \cap B) = \frac{1}{16}$$

$$P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = P(A \cap B)$$

$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$ , so the two events are independent.