If X is an exponential Random Variable:

$$P(X \geq a) = \int_a^\infty \lambda e^{-\lambda x} dx = e^{-\lambda a}$$

$$P(X \geq a) = e^{-\lambda a}$$

Mean & Variance of X:

$$E[X] = \int_0^\infty \underset{\mathrm{I}}{x}\, \underset{\mathrm{II}}{\lambda e^{-\lambda x}} dx = \left[-xe^{-\lambda x} + \int e^{-\lambda x} dx\right]_0^\infty$$

$$= \left[-xe^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda}\right]_0^\infty = \left[xe^{-\lambda x} + \frac{e^{-\lambda x}}{\lambda}\right]_\infty^0$$

$$\frac{1}{\lambda} - \lim_{x\to\infty} \frac{x}{e^{\lambda x}} = \frac{1}{\lambda} - \lim_{x\to\infty} \frac{1}{\lambda e^{\lambda x}} = \frac{1}{\lambda}$$

mean/Expectation Value $= \frac{1}{\lambda}$

$$E[x^2] = \int_0^\infty x^2 \lambda e^{-\lambda x} dx$$

$$\Rightarrow -x^2 e^{-\lambda x} + 2\int x e^{-\lambda x} dx$$

$$\Rightarrow -x^2 e^{-\lambda x} + 2\left[-\frac{x}{\lambda} e^{-\lambda x} + \int \frac{e^{-\lambda x}}{\lambda}\right]$$

$$\Rightarrow \left[-x^2 e^{-\lambda x} - \frac{2x e^{-\lambda x}}{\lambda} - \frac{2e^{-\lambda x}}{\lambda^2}\right]_0^\infty$$

$$\Rightarrow \left[x^2 e^{-\lambda x} + \frac{2x}{\lambda} e^{-\lambda x} + \frac{2e^{-\lambda x}}{\lambda^2}\right]_\infty^0$$

$$\Rightarrow \left[0 + 0 + \frac{2}{\lambda^2} - 0 + 0 + 0\right] = \frac{2}{\lambda^2}$$

$$Var(X) = E[x^2] - (E[X])^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

$$\boxed{E[x] = \frac{1}{\lambda}, \quad Var(X) = \frac{1}{\lambda^2}}$$

Eg: The time until a small meteorite first lands anywhere in Sahara desert is modeled as an exponential random variable with mean of 10 days. The time is currently midnight. What is the probability that a meteorite first lands some time b/w 6 AM to 6 PM of the first day?

$$E[x] = \frac{1}{\lambda} = 10 \Rightarrow \lambda = \frac{1}{10}$$

6 hrs $= \frac{1}{4}$ day.

$$P\left(\frac{1}{4} \leq X \leq \frac{3}{4}\right) = \int_{1/4}^{3/4} \lambda e^{-\lambda x} = -\left[e^{-\lambda x}\right]_{1/4}^{3/4}$$

$$= \left[e^{-\frac{x}{10}}\right]_{3/4}^{1/4} = e^{-1/40} - e^{-3/40}$$

or $P\left(X \geq \frac{1}{4}\right) - P\left(X > \frac{3}{4}\right) = e^{-1/40} - e^{-3/40}$

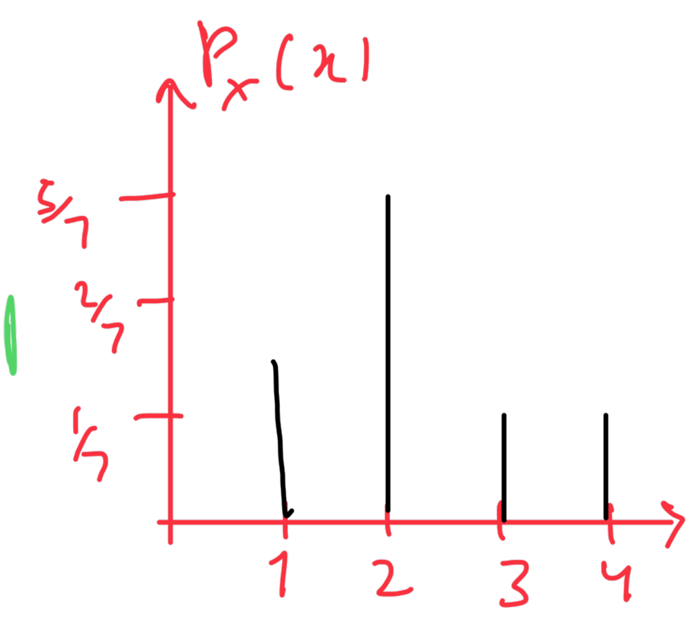
Cumulative Distribution Function:

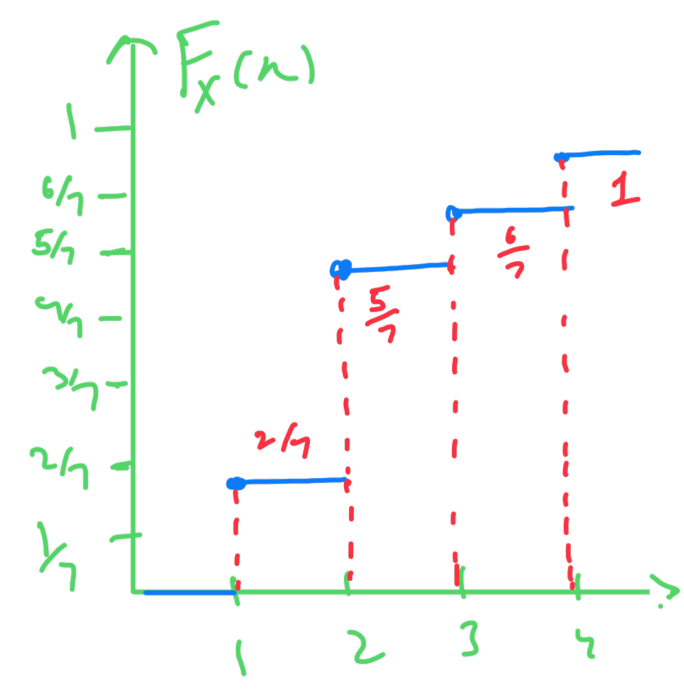
The CDF of a random variable is denoted by F_X & provides the probability $P(X \leq x)$. For every x, we have:

$$F_X(x) = P(X \leq x) = \begin{cases} \sum_{k \leq x} p_X(k) & \text{If } X \text{ is discrete} \\ \int_{-\infty}^{x} f_X(t)\, dt & \text{If } X \text{ is continuous} \end{cases}$$

Let X be a discrete random Variable:

$$p_X(x) = \begin{cases} 2/7 & x = 1 \\ 3/7 & x = 2 \\ 1/7 & x = 3 \\ 1/7 & x = 4 \end{cases}$$

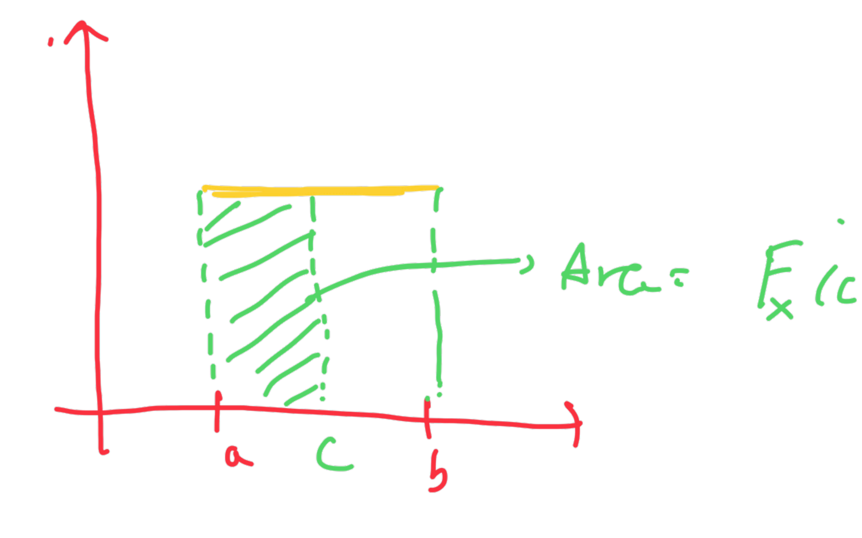


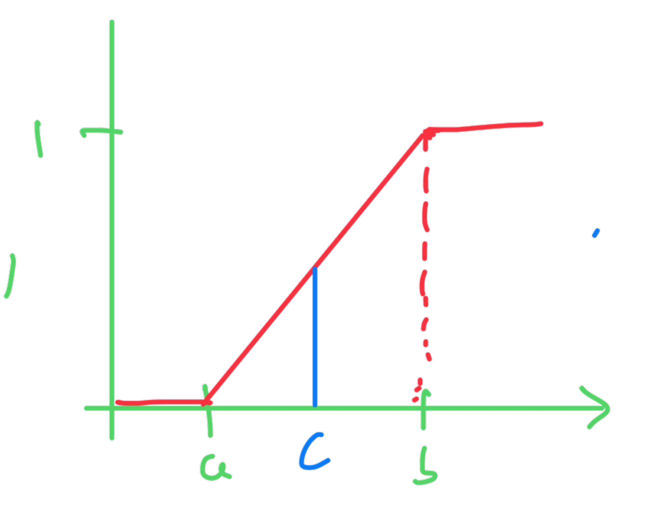


Let X be a Uniform Random Variable

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$P(X \leq x) = \int_{-\infty}^{x} \frac{1}{b-a} dx = \frac{x-a}{b-a}$$





Properties of CDF:

1) If $x \leq y$ $F_X(x) \leq F_X(y)$

2) $F_X(x) \to 0$ as $x \to -\infty$ and $F_X(x) \to 1$ as $x \to \infty$

3) If X is discrete, the $F_X(x)$ is a piecewise constant function

4) If X is continuous then, $F_X(x)$ is a continuous function

5) If X is continuous then,

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt \quad \& \quad \boxed{f_X(x) = \frac{d F_X(x)}{dx}}$$