Q1) Prove that $E[x^2] \geq (E[x])^2$ when do we have equality?

$$Var(x) = E[x^2] - (E[x])^2$$

we know that $Var(x) \geq 0$

$\Rightarrow E[x^2] - (E[x])^2 \geq 0 \Rightarrow E[x^2] \geq (E[x])^2$

We would have equality when $Var(x) = 0$

Q) A coin is biased such that a head is three times as likely to occur as a tail. Find the expected no. of tails when this coin is tossed twice.

$X =$ no. of tails

$$E[X] = \sum_{0}^{2} x \cdot p_X(x) = 2 \times \left(\tfrac{1}{4} \times \tfrac{3}{4}\right) + 2 \times \left(\tfrac{1}{4} \times \tfrac{1}{4}\right)$$

$$= \frac{6}{16} + \frac{2}{16} = \frac{8}{16} = \frac{1}{2}$$

Q) Suppose an antique jewellery dealer is interested in purchasing a gold necklace for which the probabilities are 0.22, 0.36, 0.28 & 0.14 respectively, that she will be able to sell it for profit of \$250, sell it for profit of \$150, sell it to break even, or sell it for a loss of \$150 what is her expected profit?

$X =$ profit

$$X \in [-150, 150, 0, 250]$$

$$E[X] = \sum_{-150}^{250} x\, p_X(x) = -150 \times 0.14 + 150 \times 0.36 + 250 \times 0.22$$

$$= -(1.4 \times 15) + 36 \times 15 + 2.2 \times 25$$

$$-21 + 54 + 55 = \underline{\underline{88}}$$

Q A lot containing 7 components is sampled by a quality inspector, the lot contains 4 good components & 3 defective. A sample of 3 is taken by the inspector. Find the expected value of the no. of good components in this sample.

$X =$ no. of good components

$$p\{X=1\} = \frac{{}^4C_1 \cdot {}^3C_2}{{}^7C_3} = \frac{4 \times 3 \times \cancel{1 \times 2}}{7 \times \cancel{6} \times 5} = \frac{12}{35}$$

$$p\{X=2\} = \frac{{}^4C_2 \cdot {}^3C_1}{{}^7C_3} = \frac{\frac{\cancel{4}^{2} \times 3}{\cancel{2}} \times 3 \times \cancel{2 \times 2}}{7 \times \cancel{6} \times 5} = \frac{18}{35}$$

$$p\{X=3\} = \frac{{}^4C_3}{{}^7C_3} = \frac{4}{7 \times 5} = \frac{4}{35}$$

$$p\{X=0\} = \frac{{}^3C_3}{{}^7C_3} = \frac{1}{35}$$

$$E[X] = 1 \times \frac{12}{35} + 2 \times \frac{18}{35} + 3 \times \frac{4}{35}$$

$$= \frac{12}{35} + \frac{36}{35} + \frac{12}{35} = \frac{24+36}{35} = \frac{\cancel{60}^{12}}{\cancel{35}_{7}} = \frac{12}{7}$$

$$E[X] = \frac{12}{7}$$

Q An industrial process manufactures items that can be classified as either defective or not defective. The probability that the item is defective is 0.1. An experiment is conducted in which 5 items are drawn randomly from the process. Let X be the no. of defectives in the sample of 5.

What is the PMF of X?

$X =$ no. of defective items

$$P\{X=1\} = 0.1 \times (0.9)^4 \; {}^5C_1$$

$$p\{X=2\} = (0.1)^2 \times (0.9)^3 \; {}^5C_2$$

$$p\{X=3\} = (0.1)^3 \times (0.9)^2 \; {}^5C_3$$

$$p\{X=4\} = (0.1)^4 \times 0.9 \; {}^5C_4$$

$$p\{X=5\} = (0.1)^5 \; {}^5C_5$$

$$\text{\& } p_X(x) = (0.1)^x (0.9)^{5-x} \cdot {}^5C_x$$