

# 数独

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## 第 1 章

# 数独を量子アニーリングで解く

数独は  $M \times M$  のブロックを、行方向に  $M$  列分、列方向に  $M$  行分並べた、全  $M \times M = M^2$  ブロック、従って  $M^2 \times M^2$  個のセルからなる盤面で、どの行および列についても、また  $M \times M$  の各ブロックの中においても、同じ数値が 2 個以上現れてはならないという制約の下、各セルに  $1 \sim M^2$  までの数値を一つずつ入れて盤面を埋めていくクイズ。

### 1.1 問題の構成

決定変数  $q$  を各セル毎に  $M \times M = M^2$  個用意する。 $q_{i,j,n}$  は、 $i$  行  $j$  列目のセル内の  $M \times M = M^2$  個の決定変数。

ここで、 $i, j, n \in \{1, 2, \dots, M \times M = M^2\}$ 。

下の表は  $3 \times 3$  のブロック 1 個の例を表している。このブロックが横に 3 行、縦に 3 列、全部で 9 ブロックが並んでいる盤面がよく知られた数独問題になる。

	1 列目 ( $j = 1$ )				2 列目 ( $j = 2$ )				3 列目 ( $j = 3$ )			
1 行目 ( $i = 1$ )	セル ( $i = 1, j = 1$ )				セル ( $i = 1, j = 2$ )				セル ( $i = 1, j = 3$ )			
	1	2	...	9	1	2	...	9	1	2	...	9
	$q_{111}$	$q_{112}$	...	$q_{119}$	$q_{121}$	$q_{122}$	...	$q_{129}$	$q_{131}$	$q_{132}$	...	$q_{139}$
2 行目 ( $i = 2$ )	セル ( $i = 2, j = 1$ )				セル ( $i = 2, j = 2$ )				セル ( $i = 2, j = 3$ )			
	1	2	...	9	1	2	...	9	1	2	...	9
	$q_{211}$	$q_{212}$	...	$q_{219}$	$q_{221}$	$q_{222}$	...	$q_{229}$	$q_{231}$	$q_{232}$	...	$q_{239}$
3 行目 ( $i = 3$ )	セル ( $i = 3, j = 1$ )				セル ( $i = 3, j = 2$ )				セル ( $i = 3, j = 3$ )			
	1	2	...	9	1	2	...	9	1	2	...	9
	$q_{311}$	$q_{312}$	...	$q_{319}$	$q_{321}$	$q_{322}$	...	$q_{329}$	$q_{331}$	$q_{332}$	...	$q_{339}$

この決定変数は 0 か 1 かの 2 値変数で、数値  $1 \sim N$  をそのセルに置く (1) か置かない (0) かを表している。

制約条件は、次の様に考えることができる。 $(M \times M = M^2)$  を  $N$  と書く事にする)

1. 各セルの中では  $q_1 \sim q_N$  の内でどれか 1 つだけが 1 になる (セルに 2 つ以上の数値は入らない)

$$f_1 = \sum_i^N \sum_j^N \left( \sum_n^N q_{i,j,n} - 1 \right)^2$$

2. 同一の行 (列) にあるセルの数値と同じ数値は、同じ行 (列) の他のセルには入らない (第 1 項が行、第 2 項が列)

$$f_2 = \sum_i^N \left( \sum_n^N \sum_j^N q_{i,j,n} - 1 \right)^2 + \sum_j^N \left( \sum_n^N \sum_i^N q_{i,j,n} - 1 \right)^2$$

3. いずれのブロック  $(M \times M)$  においても、その中のセルの数値は重複しない

$$f_3 = \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_n^N \sum_x^M \sum_y^M q_{i_0+x-1, j_0+y-1, n} - 1 \right)^2$$

4. いずれの行 (列) 方向の数値の和も同じ値  $S (= 1 + 2 + \dots + N)$  になる (第 1 項が行、第 2 項が列)

$$f_4 = \sum_i^N \left( \sum_j^N \sum_n^N n \cdot q_{i,j,n} - S \right)^2 + \sum_j^N \left( \sum_i^N \sum_n^N n \cdot q_{i,j,n} - S \right)^2$$

5. いずれのブロック  $(M \times M)$  においても、その中のセルの数値の和は同じ値  $S (= 1 + 2 + \dots + N)$  になる

$$f_5 = \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_x^M \sum_y^M \sum_n^N n \cdot q_{i_0+x-1, j_0+y-1, n} - S \right)^2$$

6. 予め数値  $X \in \{1, \dots, N\}$  が決められている  $I$  行  $J$  列目のセルがある

$$f_6 = \sum_{\text{規定のセル } I, J} \left( \sum_n^N n \cdot q_{I, J, n} - X \right)^2$$

## 1.1.1 式の展開

$$\begin{aligned}
f_1 &= \sum_i^N \sum_j^N \left( \sum_n^N q_{i,j,n} - 1 \right)^2 \\
&= \sum_i^N \sum_j^N \left( \sum_{n_1}^N \sum_{n_2}^N q_{i,j,n_1} q_{i,j,n_2} - 2 \sum_n^N q_{i,j,n} \right) \\
f_2 &= \sum_i^N \left( \sum_n^N \sum_j^N q_{i,j,n} - 1 \right)^2 + \sum_j^N \left( \sum_n^N \sum_i^N q_{i,j,n} - 1 \right)^2 \\
&= \sum_i^N \left( \sum_{n_1}^N \sum_{j_1}^N q_{i,j_1,n_1} \sum_{n_2}^N \sum_{j_2}^N q_{i,j_2,n_2} - 2 \sum_n^N \sum_j^N q_{i,j,n} \right) \\
&\quad + \sum_j^N \left( \sum_{n_1}^N \sum_{i_1}^N q_{i_1,j,n_1} \sum_{n_2}^N \sum_{i_2}^N q_{i_2,j,n_2} - 2 \sum_n^N \sum_i^N q_{i,j,n} \right) \\
f_3 &= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_n^N \sum_x^M \sum_y^M q_{i_0+x-1, j_0+y-1, n} - 1 \right)^2 \\
&= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_{n_1}^N \sum_{x_1}^M \sum_{y_1}^M q_{i_0+x_1-1, j_0+y_1-1, n_1} \sum_{n_2}^N \sum_{x_2}^M \sum_{y_2}^M q_{i_0+x_2-1, j_0+y_2-1, n_2} \right. \\
&\quad \left. - 2 \sum_n^N \sum_x^M \sum_y^M q_{i_0+x-1, j_0+y-1, n} \right) \\
f_4 &= \sum_i^N \left( \sum_j^N \sum_n^N n \cdot q_{i,j,n} - S \right)^2 + \sum_j^N \left( \sum_i^N \sum_n^N n \cdot q_{i,j,n} - S \right)^2 \\
&= \sum_i^N \left( \sum_{j_1}^N \sum_{n_1}^N n_1 q_{i,j_1,n_1} \sum_{j_2}^N \sum_{n_2}^N n_2 q_{i,j_2,n_2} - 2S \sum_j^N \sum_n^N n q_{i,j,n} \right) \\
&\quad + \sum_j^N \left( \sum_{i_1}^N \sum_{n_1}^N n_1 q_{i_1,j,n_1} \sum_{i_2}^N \sum_{n_2}^N n_2 q_{i_2,j,n_2} - 2S \sum_i^N \sum_n^N n q_{i,j,n} \right) \\
f_5 &= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_x^M \sum_y^M \sum_n^N n \cdot q_{i_0+x-1, j_0+y-1, n} - S \right)^2 \\
&= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_{x_1}^M \sum_{y_1}^M \sum_{n_1}^N n_1 q_{i_0+x_1-1, j_0+y_1-1, n_1} \sum_{x_2}^M \sum_{y_2}^M \sum_{n_2}^N n_2 q_{i_0+x_2-1, j_0+y_2-1, n_2} \right. \\
&\quad \left. - 2S \sum_x^M \sum_y^M \sum_n^N n q_{i_0+x-1, j_0+y-1, n} \right) \\
f_6 &= \sum_{\text{規定のセル } I, J} \left( \sum_n^N n \cdot q_{I,J,n} - X \right)^2 \\
&= \sum_{\text{規定セルの } I, J} \left( \sum_{n_1}^N n_1 q_{I,J,n_1} \sum_{n_2}^N n_2 q_{I,J,n_2} - 2X \sum_n^N n q_{I,J,n} \right)
\end{aligned}$$

## 1.2 式の展開と実装

- 式を展開する上で留意する点は次の2点だけ
  - (1) 0 か 1 の何れかの値しかとらない二値変数の場合  $q^2 = q$  が成り立つ
  - (2) 定数は最小化に関係ないので無視できる
- 展開した制約式に現れる  $\sum$  を、そのまま for 文の繰り返しに移せば QUBO を生成できる
- QUBO ができたら、それを量子コンピュータのシミュレータである、SASampler() あるいは SQASampler() の第1引数に渡してあげると、計算結果の sampleset を受け取ることができる
- 数式上で  $N$  個の数値を  $\sum_{i=1}^N$  の様に扱っていても、プログラム上の始まりの値は 0 なので、全部で  $N$  個の数値を for 文で繰り返すとなると、終わりの値は  $N - 1$  になる
- また、魔方陣の盤面に置く数値は 0 ~ 9 の 10 個ではなくて、1 ~ 9 の 9 個である事にも注意してプログラムする必要がある

### 1.2.1 class

```
from openjij import SASampler, SQASampler
from collections import defaultdict, Counter
import numpy as np

class NumberPlace:
    def __init__(self, M=2):
        self.M = M
        self.N = M * M
        S = 0
        for i in range(1, self.N+1):
            S += i
        self.S = S
        with open('data.txt', 'r') as f:
            self.required = f.read().splitlines()
        self.idx = {}
        k = 0
        for i in range(self.N):
            for j in range(self.N):
                for n in range(self.N):
                    self.idx[(i,j,n)] = k
                    k += 1
        samplers = [SASampler(), SQASampler()]
        self.sampler = samplers[0]

    def get_param(self):
        return self.N, self.M, self.S, self.idx

    def block_ij(self):
        i0j0 = []
        for i in range(self.M):
            i0j0.append(self.M*i)
```

```
return i0j0
```

ブロック先頭セルの*i*<sub>0</sub>, *j*<sub>0</sub>は、

*M*=2なら [(0,0),(0,2)],  
[(2,0),(2,2)]

*M*=3なら [(0,0),(0,3),(0,6)],  
[(3,0),(3,3),(3,6)],  
[(6,0),(6,3),(6,6)]

*M*=2なら [(*M*\*0,*M*\*0),(*M*\*0,*M*\*1)],  
[(*M*\*1,*M*\*0),(*M*\*1,*M*\*1)]

*M*=3なら [(*M*\*0,*M*\*0),(*M*\*0,*M*\*1),(*M*\*0,*M*\*2)],  
[(*M*\*1,*M*\*0),(*M*\*1,*M*\*1),(*M*\*1,*M*\*2)],  
[(*M*\*2,*M*\*0),(*M*\*2,*M*\*1),(*M*\*2,*M*\*2)]

### 1.2.2 制約：*f*<sub>1</sub>

各セルの中では *q*<sub>1</sub> ~ *q*<sub>*N*</sub> の内でどれか 1 つだけが 1 になる（セルに 2 つ以上の数値は入らない）

$$f_1 = \sum_i^N \sum_j^N \left( \sum_n^N q_{i,j,n} - 1 \right)^2$$

$$= \sum_i^N \sum_j^N \left( \sum_{n_1}^N \sum_{n_2}^N q_{i,j,n_1} q_{i,j,n_2} - 2 \sum_n^N q_{i,j,n} \right)$$

```
def sub1(self, i, j, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        Q[(idx[(i, j, n1)], idx[(i, j, n1)])] -= 2.0 * L
    for n2 in range(N):
        Q[(idx[(i, j, n1)], idx[(i, j, n2)])] += 1.0 * L

def f1(self, L, Q):
    N, _, _, _ = self.get_param()
    for i in range(N):
        for j in range(N):
            self.sub1(i, j, L, Q)
    return Q
```

### 1.2.3 制約：*f*<sub>2</sub>

同一の行（列）にあるセルの数値と同じ数値は、同じ行（列）の他のセルには入らない（第 1 項が行、第 2 項が列）

$$\begin{aligned}
f_2 &= \sum_i^N \left( \sum_n^N \sum_j^N q_{i,j,n} - 1 \right)^2 + \sum_j^N \left( \sum_n^N \sum_i^N q_{i,j,n} - 1 \right)^2 \\
&= \sum_i^N \left( \sum_{n_1}^N \sum_{j_1}^N q_{i,j_1,n_1} \sum_{n_2}^N \sum_{j_2}^N q_{i,j_2,n_2} - 2 \sum_n^N \sum_j^N q_{i,j,n} \right) \\
&\quad + \sum_j^N \left( \sum_{n_1}^N \sum_{i_1}^N q_{i_1,j,n_1} \sum_{n_2}^N \sum_{i_2}^N q_{i_2,j,n_2} - 2 \sum_n^N \sum_i^N q_{i,j,n} \right)
\end{aligned}$$

```

def sub2R(self, i, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        for j1 in range(N):
            Q[(idx[(i, j1, n1)], idx[(i, j1, n1)])] -= 2.0 * L
            for n2 in range(N):
                for j2 in range(N):
                    Q[(idx[(i, j1, n1)], idx[(i, j2, n2)])] += 1.0 * L

def sub2C(self, j, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        for i1 in range(N):
            Q[(idx[(i1, j, n1)], idx[(i1, j, n1)])] -= 2.0 * L
            for n2 in range(N):
                for i2 in range(N):
                    Q[(idx[(i1, j, n1)], idx[(i2, j, n2)])] += 1.0 * L

def f2(self, L, Q):
    N, _, _, _ = self.get_param()
    for i in range(N):
        self.sub2R(i, L, Q)
    for j in range(N):
        self.sub2C(j, L, Q)
    return Q

```

### 1.2.4 制約： $f_3$

いずれのブロック ( $M \times M$ ) においても、その中のセルの数値は重複しない

$$\begin{aligned}
f_3 &= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_n^N \sum_x^M \sum_y^M q_{i_0+x-1, j_0+y-1, n} - 1 \right)^2 \\
&= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_{n_1}^N \sum_{x_1}^M \sum_{y_1}^M q_{i_0+x_1-1, j_0+y_1-1, n_1} \sum_{n_2}^N \sum_{x_2}^M \sum_{y_2}^M q_{i_0+x_2-1, j_0+y_2-1, n_2} \right. \\
&\quad \left. - 2 \sum_n^N \sum_x^M \sum_y^M q_{i_0+x-1, j_0+y-1, n} \right)
\end{aligned}$$

```

def sub3(self, i0, j0, L, Q):
    N, M, _, idx = self.get_param()
    for n1 in range(N):
        for x1 in range(M):
            for y1 in range(M):
                Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x1, j0+y1, n1)])] -= 2.0 * L
            for n2 in range(N):
                for x2 in range(M):
                    for y2 in range(M):
                        Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x2, j0+y2, n2)])]
+= L

def f3(self, L, Q):
    i0j0 = self.block_ij()
    for i0 in i0j0:
        for j0 in i0j0:
            self.sub3(i0, j0, L, Q)
    return Q

```

### 1.2.5 制約： $f_4$

いずれの行（列）方向の数値の和も同じ値  $S(= 1 + 2 + \dots + N)$  になる（第1項が行、第2項が列）

$$\begin{aligned}
 f_4 &= \sum_i \left( \sum_j \sum_n n \cdot q_{i,j,n} - S \right)^2 + \sum_j \left( \sum_i \sum_n n \cdot q_{i,j,n} - S \right)^2 \\
 &= \sum_i \left( \sum_{j_1} \sum_{n_1} n_1 q_{i,j_1,n_1} \sum_{j_2} \sum_{n_2} n_2 q_{i,j_2,n_2} - 2S \sum_j \sum_n n q_{i,j,n} \right) \\
 &+ \sum_j \left( \sum_{i_1} \sum_{n_1} n_1 q_{i_1,j,n_1} \sum_{i_2} \sum_{n_2} n_2 q_{i_2,j,n_2} - 2S \sum_i \sum_n n q_{i,j,n} \right)
 \end{aligned}$$

```

def sub4R(self, i, L, Q):
    N, _, S, idx = self.get_param()
    for j1 in range(N):
        for n1 in range(N):
            Q[(idx[(i, j1, n1)], idx[(i, j1, n1)])] -= 2.0 * (n1+1) * S * L
        for j2 in range(N):
            for n2 in range(N):
                Q[(idx[(i, j1, n1)], idx[(i, j2, n2)])] += (n1+1) * (n2+1) * L

def sub4C(self, j, L, Q):
    N, _, S, idx = self.get_param()
    for i1 in range(N):
        for n1 in range(N):
            Q[(idx[(i1, j, n1)], idx[(i1, j, n1)])] -= 2.0 * (n1+1) * S * L
        for i2 in range(N):
            for n2 in range(N):
                Q[(idx[(i1, j, n1)], idx[(i2, j, n2)])] += (n1+1) * (n2+1) * L

```



```

def f4(self, L, Q):
    N, _, _, _ = self.get_param()
    for i in range(N):
        self.sub4R(i, L, Q)
    for j in range(N):
        self.sub4C(j, L, Q)
    return Q

```

### 1.2.6 制約： $f_5$

いずれのブロック ( $M \times M$ ) においても、その中のセルの数値の和は同じ値  $S (= 1 + 2 + \dots + N)$  になる

$$\begin{aligned}
 f_5 &= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_x \sum_y \sum_n n \cdot q_{i_0+x-1, j_0+y-1, n} - S \right)^2 \\
 &= \sum_{\text{ブロック先頭セルの } i_0, j_0} \left( \sum_{x_1} \sum_{y_1} \sum_{n_1} n_1 q_{i_0+x_1-1, j_0+y_1-1, n_1} \sum_{x_2} \sum_{y_2} \sum_{n_2} n_2 q_{i_0+x_2-1, j_0+y_2-1, n_2} \right. \\
 &\quad \left. - 2S \sum_x \sum_y \sum_n n q_{i_0+x-1, j_0+y-1, n} \right)
 \end{aligned}$$

```

def sub5(self, i0, j0, L, Q):
    N, M, S, idx = self.get_param()
    for x1 in range(M):
        for y1 in range(M):
            for n1 in range(N):
                Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x1, j0+y1, n1)])] -= 2.0 * (n1
+1) * S * L
            for x2 in range(M):
                for y2 in range(M):
                    for n2 in range(N):
                        Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x2, j0+y2, n2)])]
-=(n1+1) * (n2+1) * L

def f5(self, L, Q):
    i0j0 = self.block_ij()
    for i0 in i0j0:
        for j0 in i0j0:
            self.sub5(i0, j0, L, Q)
    return Q

```

### 1.2.7 制約： $f_6$

予め数値  $X \in \{1, \dots, N\}$  が決められている  $I$  行  $J$  列目のセルがある

$$\begin{aligned}
f_6 &= \sum_{\text{規定のセル } I, J} \left( \sum_n^N n \cdot q_{I, J, n} - X \right)^2 \\
&= \sum_{\text{規定セルの } I, J} \left( \sum_{n_1}^N n_1 q_{I, J, n_1} \sum_{n_2}^N n_2 q_{I, J, n_2} - 2X \sum_n^N n q_{I, J, n} \right)
\end{aligned}$$

```

def f6(self, I, J, X, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        Q[(idx[(I, J, n1)], idx[(I, J, n1)])] -= 2 * (n1+1) * X * L
    for n2 in range(N):
        Q[(idx[(I, J, n1)], idx[(I, J, n2)])] += (n1+1) * (n2+1) * L
    return Q

```

### 1.2.8 評価関数： $f$

$$f = \lambda_1 \cdot f_1 + \lambda_2 \cdot (f_2 + f_3) + \lambda_3 \cdot (f_4 + f_5) + \lambda_4 \cdot \left( \sum_{\text{規定}} f_6 \right)$$

```

def f(self, lagrange1=1.0, lagrange2=1.0, lagrange3=1.0, lagrange4=1.0):
    Q = defaultdict(lambda: 0)
    _ = self.f1(lagrange1, Q)
    _ = self.f2(lagrange2, Q)
    _ = self.f3(lagrange2, Q)
    _ = self.f4(lagrange3, Q)
    _ = self.f5(lagrange3, Q)
    for a in self.required:
        IJX = a.split(',')
        _ = self.f6(int(IJX[0]), int(IJX[1]), int(IJX[2]), lagrange4, Q)
    return Q

def solv(self, Q, num_reads=1):
    sampleset = self.sampler.sample_qubo(Q, num_reads=num_reads)
    return sampleset

def result(self, sampleset):
    N, _, _, idx = self.get_param()
    result = [i for i in sampleset.first[0].values()]
    ans = [[None] * N for _ in range(N)]
    for i in range(N):
        for j in range(N):
            for n in range(N):
                if result[idx[(i, j, n)]] == 1:
                    ans[i][j] = n+1
    return ans

```

## 出力結果のチェック

出力された結果を、ふるいにかける仕掛け

```
def evaluate(self, sampleset, prn=True):
    # Extract sample solutions, energies, and sort them by frequency
    samples = sampleset.record['sample']
    energies = sampleset.record['energy']
    # Combine solutions and corresponding energies
    sample_data = [(tuple(sample), energy) for sample, energy in zip(samples,
    energies)]
    # Sort the results by appearance frequency and then energy
    sample_frequency = Counter(sample for sample, _ in sample_data)
    # Print sorted results by frequency and include energy
    if prn:
        print("\nSorted samples by frequency and energy:")
        for solution, freq in sample_frequency.most_common():
            energy = next(energy for sample, energy in sample_data if sample ==
            solution)
            print(f"Sample: {solution}, Frequency: {freq}, Energy: {energy:+.2f}")
    return sample_data, sample_frequency

def check1(self, a, debug=False):
    N, M, _, _ = self.get_param()
    b = np.array(a).reshape(N*N, N)
    # 各セルに数値は1つ?
    for i in range(N*N):
        s = 0
        for n in range(N):
            s += b[i][n]
        if s != 1:
            if debug:
                print(f'!: セルの中の数値が1つになっていない')
            return False
    # 各ブロックに重複する数値はない?
    i0j0 = self.block_ij()
    for i in i0j0:
        for j in i0j0:
            for n in range(N):
                s = 0
                for x in range(M):
                    for y in range(M):
                        bidx = (i+x)*N + j+y
                        s += b[bidx][n]
                if s != 1:
                    if debug:
                        print(f'!: ブロック内で数値が重複')
                    return False
    #
    for n in range(N):
        # 各行に重複する数値はない?
        for i in range(N):
            s = 0
            for j in range(N):
```

```

        bidx = i * N + j
        s += b[bidx][n]
    if s != 1:
        if debug:
            print(f'!: 行で数値が重複')
        return False
# 各列に重複する数値はない?
for j in range(N):
    s = 0
    for i in range(N):
        bidx = i * N + j
        s += b[bidx][n]
    if s != 1:
        if debug:
            print(f'!: 列で数値が重複')
        return False

#
return True

def check2(self, a, debug=False):
    N, M, S, _ = self.get_param()
    b = np.array(a).reshape(N, N)
    # 規定値は正しい?
    for a in self.required:
        IJX = a.split(',')
        if b[int(IJX[0])][int(IJX[1])] != int(IJX[2]):
            if debug:
                print('!: 規定値が違っている')
            return False
    # 各行の数値の和はS?
    for i in range(N):
        s = 0
        for j in range(N):
            s += b[i][j]
        if s != S:
            if debug:
                print(f'!: 行の総和={s}!={S}')
            return False
    # 各列の数値の和はS?
    for j in range(N):
        s = 0
        for i in range(N):
            s += b[i][j]
        if s != S:
            if debug:
                print(f'!: 列の総和={s}!={S}')
            return False
    # 各ブロックの数値の和はS?
    i0j0 = self.block_ij()
    for i in i0j0:
        for j in i0j0:
            s = 0
            for x in range(M):
                for y in range(M):

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        #print(i+x, j+y)
        s += b[i+x][j+y]
    if s != 1:
        if debug:
            print(f'!: ブロック内の総和={s}!={S}')
        return False
    #
    return True

def decode(self, a):
    N, M, _, _ = self.get_param()
    b = np.array(a).reshape(N**2, N)
    print(b)
    mat = []
    for v in b:
        num = 0
        for i, u in enumerate(v):
            if u==1:
                num = i+1
        mat.append(num)
    return mat

```

### 1.2.9 main

```

if __name__ == '__main__':
    M = 2
    N = M*M
    sudoku = NumberPlace(M)
    lagrange1 = 500    # 数値に重複なし
    lagrange2 = 26.3   # 行、列、ブロック、で重複なし
    lagrange3 = 0.001  # 和はS
    lagrange4 = 20.0   # 規定セル
    Q = sudoku.f(lagrange1, lagrange2, lagrange3, lagrange4)
    num_reads = 1000
    sampleset = sudoku.solve(Q, num_reads)
    ans = sudoku.result(sampleset)
    print(*ans, sep='\n')
    #
    for sample in sampleset.record['sample']:
        if sudoku.check1(sample, False):
            a = sudoku.decode(sample)
            if sudoku.check2(a, False):
                print(np.array(a).reshape(N, N))
                print()

```

## 1.3 実行結果

規定セルの値：data.txt

0, 1, 1  
1, 0, 2  
3, 0, 4  
3, 3, 1

期待したのは次の状態

[3, 1, 4, 2]  
[2, 4, 1, 3]  
[1, 2, 3, 4]  
[4, 3, 2, 1]

実行結果は次の通り。満足できる結果になっていない

[1, 1, 4, 4]  
[2, 4, 4, 3]  
[3, 2, 4, 1]  
[4, 4, 4, 1]

## 1.4 プログラムの全体

```
from openjij import SASampler, SQASampler
from collections import defaultdict, Counter
import numpy as np

class NumberPlace:
    def __init__(self, M=2):
        self.M = M
        self.N = M * M
        S = 0
        for i in range(1, self.N+1):
            S += i
        self.S = S
        with open('data.txt', 'r') as f:
            self.required = f.read().splitlines()
        self.idx = {}
        k = 0
        for i in range(self.N):
            for j in range(self.N):
                for n in range(self.N):
                    self.idx[(i,j,n)] = k
                    k += 1
        samplers = [SASampler(), SQASampler()]
        self.sampler = samplers[0]

    def get_param(self):
        return self.N, self.M, self.S, self.idx

    def block_ij(self):
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        i0j0 = []
        for i in range(self.M):
            i0j0.append(self.M*i)
        return i0j0

def sub1(self, i, j, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        Q[(idx[(i, j, n1)], idx[(i, j, n1)])] -= 2.0 * L
        for n2 in range(N):
            Q[(idx[(i, j, n1)], idx[(i, j, n2)])] += 1.0 * L

def f1(self, L, Q):
    N, _, _, _ = self.get_param()
    for i in range(N):
        for j in range(N):
            self.sub1(i, j, L, Q)
    return Q

def sub2R(self, i, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        for j1 in range(N):
            Q[(idx[(i, j1, n1)], idx[(i, j1, n1)])] -= 2.0 * L
            for n2 in range(N):
                for j2 in range(N):
                    Q[(idx[(i, j1, n1)], idx[(i, j2, n2)])] += 1.0 * L

def sub2C(self, j, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        for i1 in range(N):
            Q[(idx[(i1, j, n1)], idx[(i1, j, n1)])] -= 2.0 * L
            for n2 in range(N):
                for i2 in range(N):
                    Q[(idx[(i1, j, n1)], idx[(i2, j, n2)])] += 1.0 * L

def f2(self, L, Q):
    N, _, _, _ = self.get_param()
    for i in range(N):
        self.sub2R(i, L, Q)
    for j in range(N):
        self.sub2C(j, L, Q)
    return Q

def sub3(self, i0, j0, L, Q):
    N, M, _, idx = self.get_param()
    for n1 in range(N):
        for x1 in range(M):
            for y1 in range(M):
                Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x1, j0+y1, n1)])] -= 2.0 *
L
                for n2 in range(N):
                    for x2 in range(M):

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        for y2 in range(M):
            Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x2, j0+y2, n2
    ])] += L

def f3(self, L, Q):
    i0j0 = self.block_ij()
    for i0 in i0j0:
        for j0 in i0j0:
            self.sub3(i0, j0, L, Q)
    return Q

def sub4R(self, i, L, Q):
    N, _, S, idx = self.get_param()
    for j1 in range(N):
        for n1 in range(N):
            Q[(idx[(i, j1, n1)], idx[(i, j1, n1)])] -= 2.0 * (n1+1) * S * L
        for j2 in range(N):
            for n2 in range(N):
                Q[(idx[(i, j1, n1)], idx[(i, j2, n2)])] += (n1+1) * (n2+1)
    * L

def sub4C(self, j, L, Q):
    N, _, S, idx = self.get_param()
    for i1 in range(N):
        for n1 in range(N):
            Q[(idx[(i1, j, n1)], idx[(i1, j, n1)])] -= 2.0 * (n1+1) * S * L
        for i2 in range(N):
            for n2 in range(N):
                Q[(idx[(i1, j, n1)], idx[(i2, j, n2)])] += (n1+1) * (n2+1)
    * L

def f4(self, L, Q):
    N, _, _, _ = self.get_param()
    for i in range(N):
        self.sub4R(i, L, Q)
    for j in range(N):
        self.sub4C(j, L, Q)
    return Q

def sub5(self, i0, j0, L, Q):
    N, M, S, idx = self.get_param()
    for x1 in range(M):
        for y1 in range(M):
            for n1 in range(N):
                Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x1, j0+y1, n1)])] -= 2.0 *
    (n1+1) * S * L
            for x2 in range(M):
                for y2 in range(M):
                    for n2 in range(N):
                        Q[(idx[(i0+x1, j0+y1, n1)], idx[(i0+x2, j0+y2, n2
    ])] -= (n1+1) * (n2+1) * L

def f5(self, L, Q):
    i0j0 = self.block_ij()

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        for i0 in i0j0:
            for j0 in i0j0:
                self.sub5(i0, j0, L, Q)
        return Q

def f6(self, I, J, X, L, Q):
    N, _, _, idx = self.get_param()
    for n1 in range(N):
        Q[(idx[(I, J, n1)], idx[(I, J, n1)])] -= 2 * (n1+1) * X * L
        for n2 in range(N):
            Q[(idx[(I, J, n1)], idx[(I, J, n2)])] += (n1+1) * (n2+1) * L
    return Q

def f(self, lagrange1=1.0, lagrange2=1.0, lagrange3=1.0, lagrange4=1.0):
    Q = defaultdict(lambda: 0)
    _ = self.f1(lagrange1, Q)
    _ = self.f2(lagrange2, Q)
    _ = self.f3(lagrange2, Q)
    _ = self.f4(lagrange3, Q)
    _ = self.f5(lagrange3, Q)
    for a in self.required:
        IJX = a.split(',')
        _ = self.f6(int(IJX[0]), int(IJX[1]), int(IJX[2]), lagrange4, Q)
    return Q

def solv(self, Q, num_reads=1):
    sampleset = self.sampler.sample_qubo(Q, num_reads=num_reads)
    return sampleset

def result(self, sampleset):
    N, _, _, idx = self.get_param()
    result = [i for i in sampleset.first[0].values()]
    ans = [[None] * N for _ in range(N)]
    for i in range(N):
        for j in range(N):
            for n in range(N):
                if result[idx[(i,j,n)]] == 1:
                    ans[i][j] = n+1
    return ans

def evaluate(self, sampleset, prn=True):
    # Extract sample solutions, energies, and sort them by frequency
    samples = sampleset.record['sample']
    energies = sampleset.record['energy']
    # Combine solutions and corresponding energies
    sample_data = [(tuple(sample), energy) for sample, energy in zip(samples,
    energies)]
    # Sort the results by appearance frequency and then energy
    sample_frequency = Counter(sample for sample, _ in sample_data)
    # Print sorted results by frequency and include energy
    if prn:
        print("\nSorted samples by frequency and energy:")
        for solution, freq in sample_frequency.most_common():
            energy = next(energy for sample, energy in sample_data if sample ==

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        solution)
            print(f"Sample: {solution}, Frequency: {freq}, Energy: {energy:+.2f}
        })
        return sample_data, sample_frequency

def check1(self, a, debug=False):
    N, M, _, _ = self.get_param()
    b = np.array(a).reshape(N*N, N)
    # 各セルに数値は1つ？
    for i in range(N*N):
        s = 0
        for n in range(N):
            s += b[i][n]
        if s != 1:
            if debug:
                print(f'!: セルの中の数値が1つになっていない')
            return False
    # 各ブロックに重複する数値はない？
    i0j0 = self.block_ij()
    for i in i0j0:
        for j in i0j0:
            for n in range(N):
                s = 0
                for x in range(M):
                    for y in range(M):
                        bidx = (i+x)*N + j+y
                        s += b[bidx][n]
                if s != 1:
                    if debug:
                        print(f'!: ブロック内で数値が重複')
                    return False
    #
    for n in range(N):
        # 各行に重複する数値はない？
        for i in range(N):
            s = 0
            for j in range(N):
                bidx = i * N + j
                s += b[bidx][n]
            if s != 1:
                if debug:
                    print(f'!: 行で数値が重複')
                return False
        # 各列に重複する数値はない？
        for j in range(N):
            s = 0
            for i in range(N):
                bidx = i * N + j
                s += b[bidx][n]
            if s != 1:
                if debug:
                    print(f'!: 列で数値が重複')
                return False
    #

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```

        return True

def check2(self, a, debug=False):
    N, M, S, _ = self.get_param()
    b = np.array(a).reshape(N, N)
    # 規定値は正しい?
    for a in self.required:
        IJX = a.split(',')
        if b[int(IJX[0])][int(IJX[1])] != int(IJX[2]):
            if debug:
                print('!: 規定値が違っている')
            return False
    # 各行の数値の和はS?
    for i in range(N):
        s = 0
        for j in range(N):
            s += b[i][j]
        if s != S:
            if debug:
                print(f'!: 行の総和={s}!={S}')
            return False
    # 各列の数値の和はS?
    for j in range(N):
        s = 0
        for i in range(N):
            s += b[i][j]
        if s != S:
            if debug:
                print(f'!: 列の総和={s}!={S}')
            return False
    # 各ブロックの数値の和はS?
    i0j0 = self.block_ij()
    for i in i0j0:
        for j in i0j0:
            s = 0
            for x in range(M):
                for y in range(M):
                    #print(i+x, j+y)
                    s += b[i+x][j+y]
            if s != 1:
                if debug:
                    print(f'!: ブロック内の総和={s}!={S}')
                return False
    '''
    # 右下がりの対角要素の和はS?
    s = 0
    for i in range(N):
        for j in range(N):
            if i == j:
                s += b[i][j]
    if s != S:
        if debug:
            print(f'!: 右下がりの対角要素の総和={s}!={S}')
        return False

```

```

    # 右上がりの対角要素の和はS?
    s = 0
    for i in range(N):
        k = N - i - 1
        s += b[i][k]
    if s != S:
        if debug:
            print(f'!: 右上がりの対角要素の総和={s}!={S}')
        return False
    '''
    #
    return True

def decode(self, a):
    N, M, _, _ = self.get_param()
    b = np.array(a).reshape(N**2, N)
    print(b)
    mat = []
    for v in b:
        num = 0
        for i, u in enumerate(v):
            if u==1:
                num = i+1
        mat.append(num)
    return mat

if __name__ == '__main__':
    M = 2
    N = M*M
    sudoku = NumberPlace(M)
    lagrange1 = 500    # 数値に重複なし
    lagrange2 = 26.3   # 行、列、ブロック、で重複なし
    lagrange3 = 0.001  # 和はS
    lagrange4 = 20.0   # 規定セル
    Q = sudoku.f(lagrange1, lagrange2, lagrange3, lagrange4)
    num_reads = 1000
    sampleset = sudoku.solve(Q, num_reads)
    ans = sudoku.result(sampleset)
    print(*ans, sep='\n')
    #
    for sample in sampleset.record['sample']:
        if sudoku.check1(sample, False):
            a = sudoku.decode(sample)
            if sudoku.check2(a, False):
                print(np.array(a).reshape(N, N))
                print()

```

プログラム 1.1 数独

## 参考文献

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