Journée des doctorants

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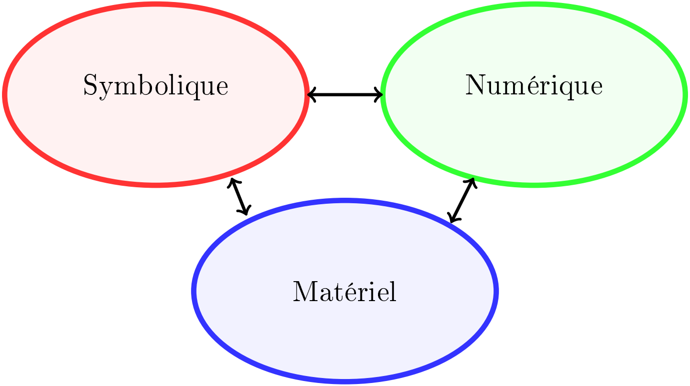
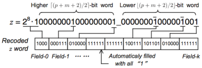
AriC: Arithmetic and Computing

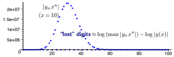
Améliorer le calcul, en termes de performance, d'efficacité et de fiabilité.



AriC: Arithmetic and Computing

- algorithmes arithmétiques & implantation:
  - virgule flottante;
  - multi-précision;
  - bornes d'erreur fines
- méthodes d'approximation:
  - uniforme/ponctuelle;
  - polynomiale/rationnelle
- calcul certifié et calcul formel:
  - algèbre linéaire à coefficients polynomiaux;
  - récurrences linéaires à coefficients polynomiaux;





$$\mathrm{Ai}(x) = A - Bx + \frac{A}{6}x^3 - \frac{B}{12}x^4 + \frac{A}{180}x^6 - \frac{B}{504}x^7 + \frac{A}{12960}x^9 - \cdots$$

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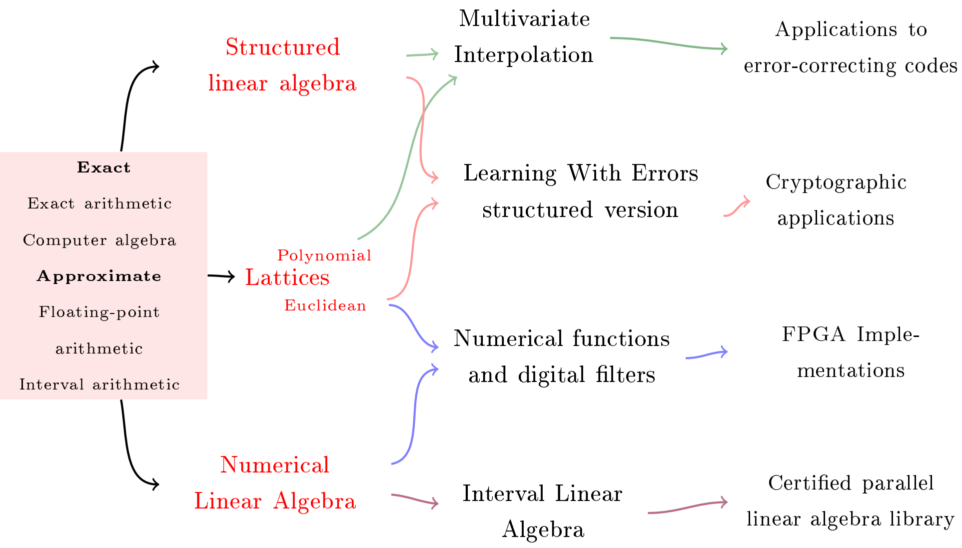
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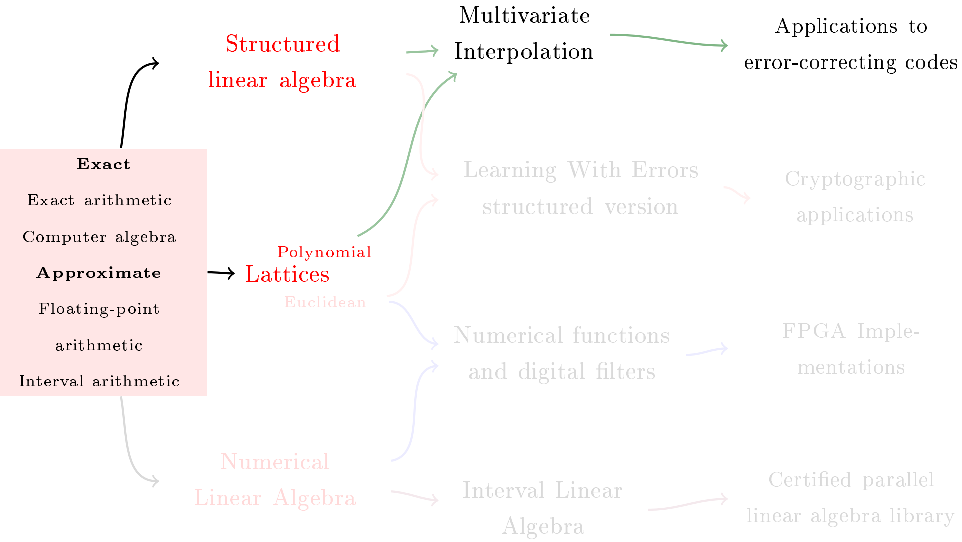
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AriC — Multivariate interpolation

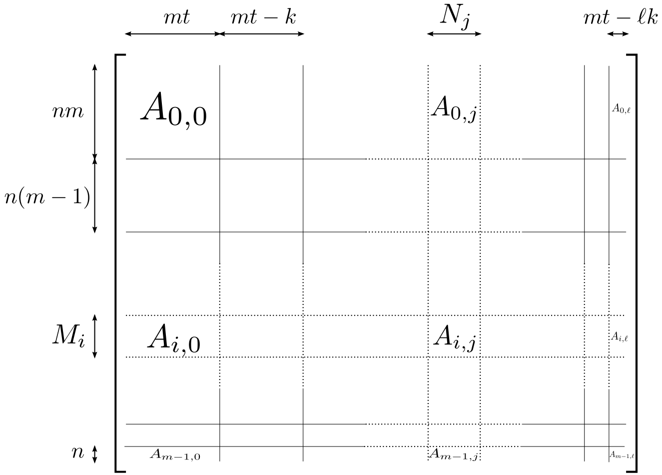
Problem: MULTIVARIATE POLYNOMIAL INTERPOLATION

Applications to:

- Error-correcting codes
  Enable reliable delivery of data over unreliable communication channels
  (add redundancy to the message)
- Crypto: private information retrieval
  look up information in an online database without letting the database servers know (or learn) the query terms or responses.

AriC — Tool: structured matrices

Find a solution of a (scalar) structured linear system,

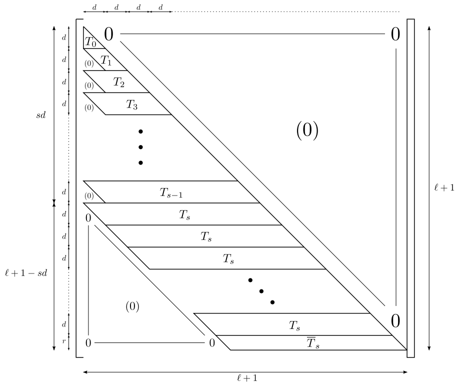


where $A_{i,j}$ is a Toeplitz / Hankel / Vandermonde / ... matrix

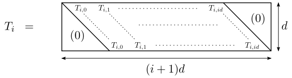
AriC — Tool: polynomial matrices

Find a short vector in the row space of a polynomial matrix
= lattice basis reduction (LLL), over the polynomials

Example matrix (for list-decoding Reed-Solomon codes):



where T_i has a **Toeplitz** structure:



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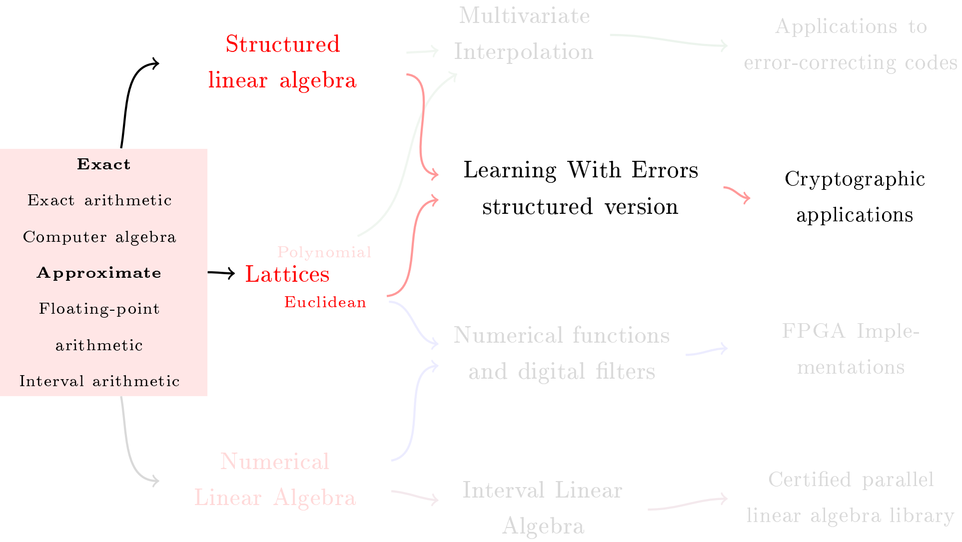
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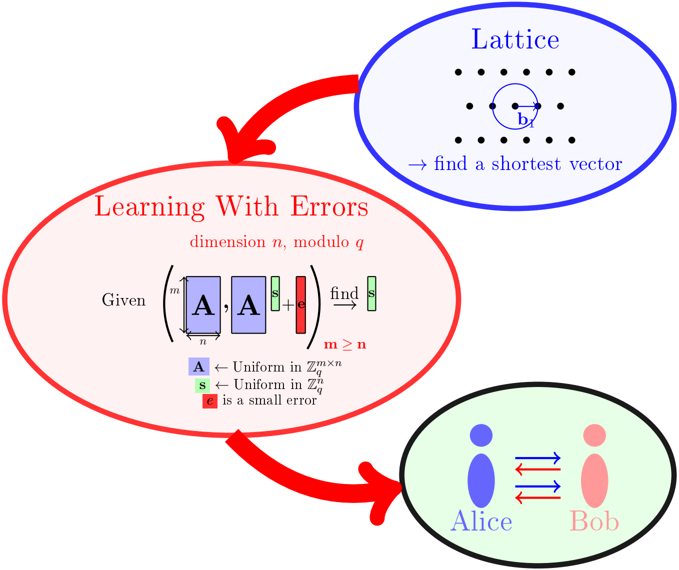
AriC – Lattice-Based Cryptography



AriC – Lattice-Based Cryptography

- Public Key Encryption
- Identity Based Encryption
- Fully Homomorphic Encryption

- Signature
- Group Signature
- Hash Function

- Cryptographic Multilinear Maps

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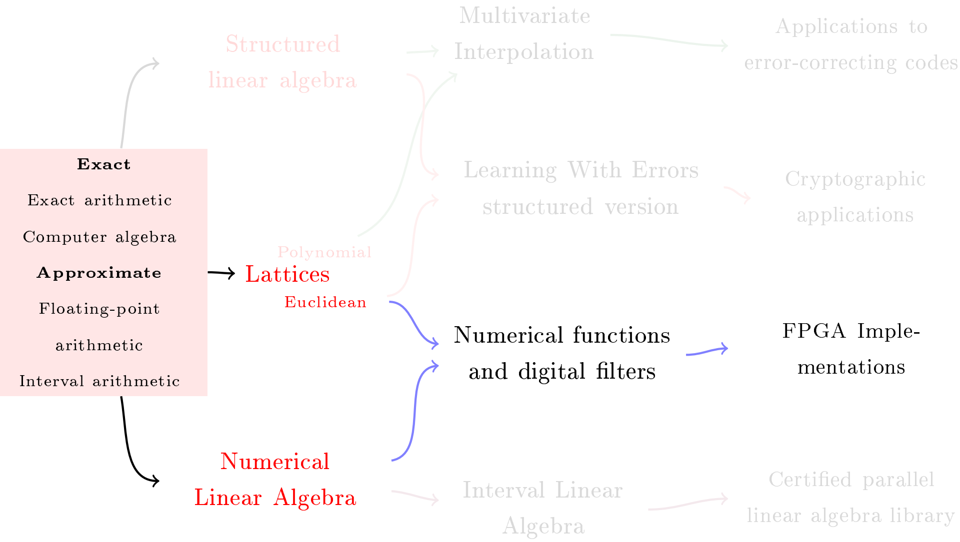

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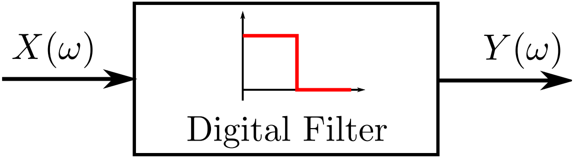
AriC – Digital Filter Design



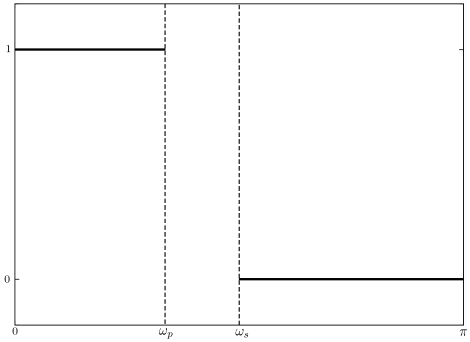
$$Y(\omega) = H_d(\omega) X(\omega), \omega \in [0, \pi]$$

Two types of filters:

- finite impulse response (**FIR**) $\Rightarrow H_d(\omega)$ **polynomial**
- infinite impulse response (**IIR**) $\Rightarrow H_d(\omega)$ **rational function**

AriC – Digital Filter Design

FIR case: $H_d(\omega) = \sum_{k=0}^{L} a_k \cos(\omega k)$



Steps:

1. Optimal filter computation:

$$H_d(\omega) = \sum_{k=0}^{L} a_k \cos(\omega k)$$

Naive rounding:

$$\overline{H}_d(\omega) = \sum_{k=0}^{L} \overline{a}_k \cos(\omega k)$$

2. Coefficient quantization:

$$H_d^*(\omega) = \sum_{k=0}^{L} a_k^* \cos(\omega k)$$

Goal: filter synthesis toolchain for embedded and FPGA targets

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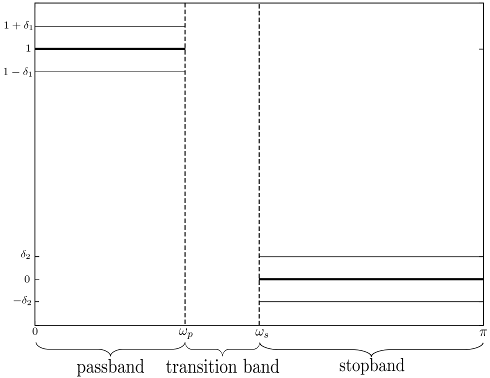

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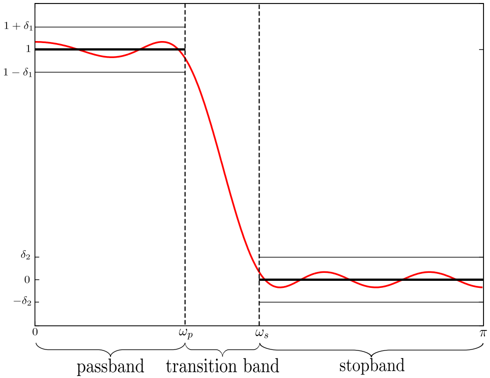

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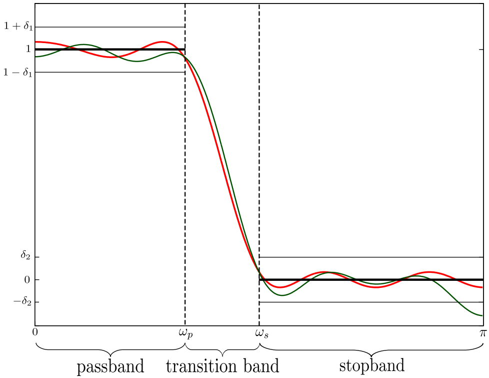

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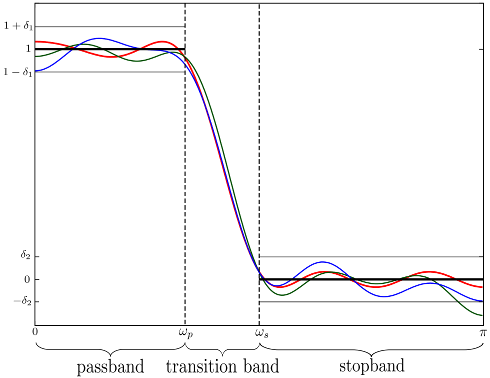

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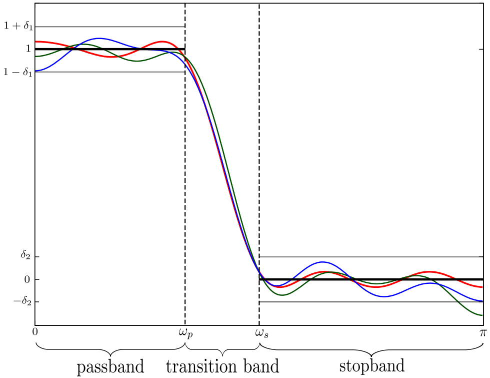

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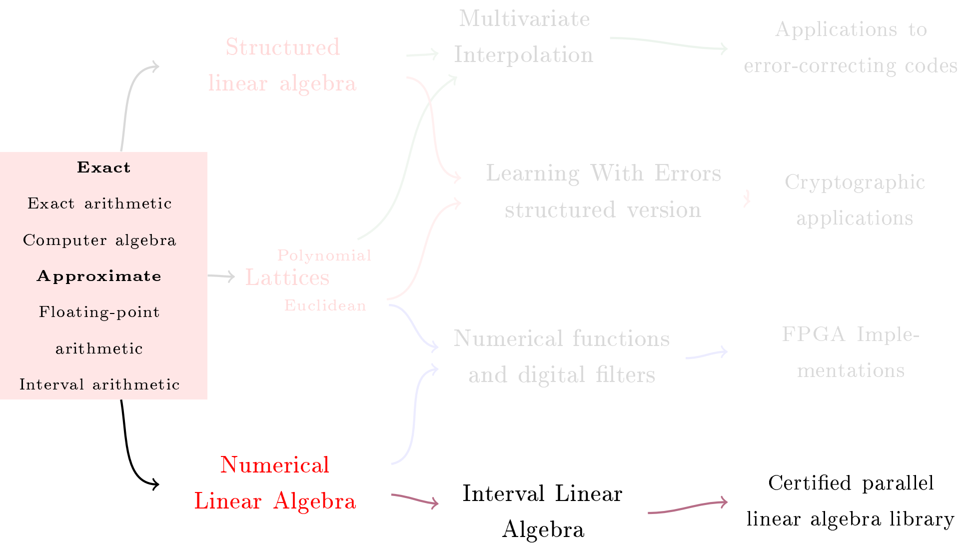

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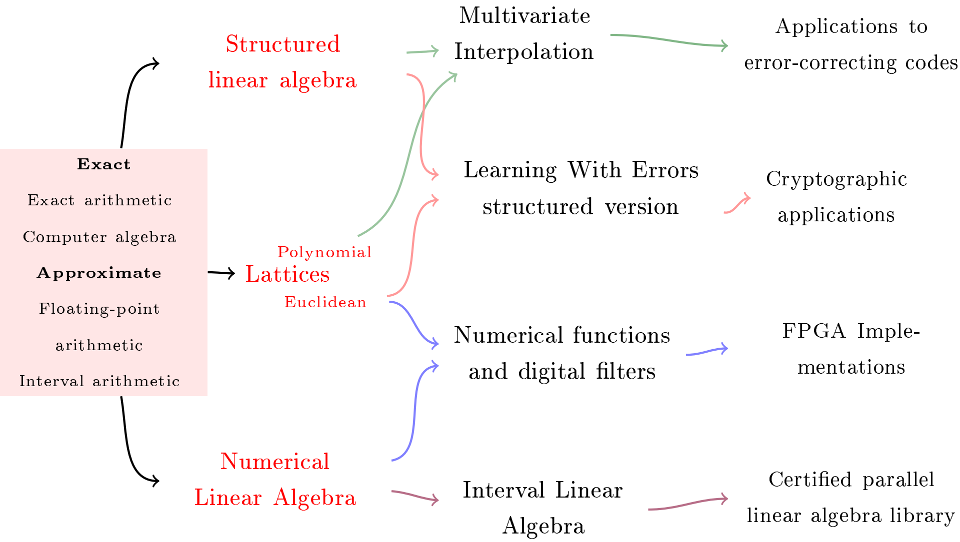
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