

Mat Shi

I pledge my honor that  
I have abided by the  
Stevens Honor System  
My Signature

# CS 334 Problem Set 4

CS 534 Problem Set 4

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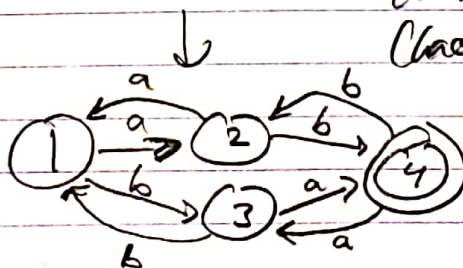
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2			✓	✓	✓	✓	✓		✓
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→ No new pairs marked.  
1=5, 1=7, 1=9, 2=8, 3=6,  
-, 5=7, 5=9, 7=9.

Class 1 = 1, 5, 7, 9  
Class 2 = 2, 8,  
Class 3 = 3, 6  
Class 4 = 4.



2. Let  $A$  be a regular language, and let  $D$  be a DFA that accepts  $A$ . Therefore, for every string not in  $A$ ,  $D$  ends up in a reject state. In other words, for every string in  $A$ 's complement,  $\bar{A}$ ,  $D$  ends in a reject state. Thus, if we flip the accept and reject states in  $D$  to produce  $\bar{D}$ , this DFA will end in an accept state for all strings in  $\bar{A}$ . Thus, a DFA accepts  $\bar{A}$ , proving  $A$  being regular implies  $\bar{A}$  is also regular.



26. Let  $A$  and  $B$  be regular languages. Let  $\bar{B}$  be the complement of  $B$ . By the proof in the previous problem,  $\bar{B}$  is regular.  $A - B$  is equivalent to the expression  $A \cap \bar{B}$ . Intersection of two sets,  $C$  and  $D$ ,  $C \cap D$ , is equivalent to  $(\bar{C} \cup \bar{D})$ . Thus,  $A \cap \bar{B} = (\bar{A} \cup B)$ , where  $\bar{A}$  is the complement of  $A$ .  $\bar{A}$  is also regular. Thus, because regular languages are closed under union,  $\bar{A} \cup B$  is regular. Furthermore, because regular languages are also closed under complement,  $(\bar{A} \cup B)$  is also regular. Thus,  $A - B$  is regular if  $A$  and  $B$  are regular languages.

27. Let us partition the input string into  $L$  into three parts  $u, v, w$ , where  $u$  and  $v$  are part of the alphabet, and  $w$  is the rest of the input string. There are then three cases for strings accepted by  $L$ . Let pumping length  $p = 2$ .

Case 1:  $uv = a^n$ .

In this case,  $w = a$  string with length  $\geq 0$  that has strictly any number of  $a$ 's, then  $b$ 's, then  $c$ 's. In this case, we assign  $x, y, z$  from the pumping lemma as such:  $x = u = a$ ,  $y = v = a$ ,  $z = w$ . This satisfies all three conditions, as  $xy^iz \in A \forall i \geq 0$ , as  $xy^i$  matches  $a^i$  and  $z$  matches  $b^k$  in the reg. ex.,  $|y| = 1 > 0$ , and  $|xy| = 2 \leq p$ , as  $p = 2$ .

Case 2:  $u \neq a$ , so  $u = b$  or  $u = c$ .

In this case, let  $uvw = s$ . For string  $s$  to be accepted by  $L$ , there are three combinations for  $u, v$ , and  $w$ . If  $u = c$ ,  $v$  must be  $c$ , and  $w$  is a string of length  $\geq 0$ , of only  $c$ 's. If  $u = b$ ,  $v$  can be either  $b$  or  $c$ . If it is  $c$ ,  $w$  is the same as if  $u = c$ . If  $u = b$ , then  $w =$  string of length  $\geq 0$  of strictly  $b$ 's then  $c$ 's. Let these be case A, B, and C respectively. All three properties of the pumping lemma are still fulfilled.  $|y| = 1 > 0$ ,  $|xy| = 2 \leq p$ , as  $p = 2$ .  $xy^iz \in A \forall i \geq 0$ , as  $i = 0$  in the  $a^i$  of the reg. ex. and!

Case A:

$x = u = c, y = v = c \forall i \geq 0, z = w = c^j, j \geq 0,$

→ this all matches  $c^k$ , and thus is  $\in L$ .



Case B:

$x = u = b$ ,  $y = v = c^i$ ,  $i \geq 0$ ,  $z = w = c^j$ ,  $j \geq 0$ , and since  $x$  matches  $b^j$ , and  $y'z$  matches  $c^k$ , case  $B \in L$ .

Case C:

$x = u = b$ ,  $y' = v' = b^i$ ,  $i \geq 0$ ,  $z = w = b^j c^k$ ,  $j, k \geq 0$ , and since  $xy'$  matches  $b^j$  from the regex, and  $z$  matches  $b^j c^k$  from the regex, Case  $C \in L$ .

Thus, case 2 fulfills all properties.

Case 3:  $u = a$ ,  $v \neq a$ , thus  $v = b$  or  $v = c$ .

$u = a$ , thus fulfilling  $a^i$ . However, in this case,  $i = 1$ , which means  $j = k$  for the regex  $b^j c^k$ . Thus, in order to satisfy the regex,  $v \neq c$ , as  $j$  can never equal  $k$  if  $v = c$ . Thus,  $v = b$ . However, in this case, set  $x = \epsilon$ ,  $y = u = a$ , and  $z = v$  plus a string with  $k$  number of  $b$ 's and  $k+1$  number of  $c$ 's, for  $k \geq 0$ . Thus,  $xy'z \in L$ ,  $\forall i \geq 0$ , as  $xy' = a^i$  which matches  $a^i$  in the regex, and  $z = vw = b^j c^k$  which matches the  $b^j c^k$ ,  $j = k$  in the regex. Furthermore,  $|y| = 1 > 0$ , and  $|xy| = 1 \leq p$ , as  $p = 2$ .

Therefore, through all three cases, all conditions on the pumping lemma are satisfied.

20. Assume that  $L$  is regular. Because regular languages are closed under union,  $\{a^i b^j c^i : i \geq 0\}$  must be regular. Because regular languages are closed under concatenation,  $\{b^j c^k : j, k \geq 0\}$  must also be regular, and therefore, must satisfy the pumping lemma. Let  $\{b^j c^k : j, k \geq 0\}$  be the language  $L_R$ . Because  $L_R$  is regular, it must be recognized by a DFA with  $p$  states for some number  $p \geq 1$ . Considering the string  $b^p c^p$ ,  $b^p c^p$  is recognized by  $L_R$ . Then, let  $x = \epsilon$ ,  $y = b^p$ , and  $z = c^p$ , for  $p \geq 1$ . This satisfies  $|p| > 1$  and  $|xy| \leq p$ , as  $|xy| = p$ . However, as  $y$  is pumped,  $xyy'z$  is not accepted by  $L_R$ , as  $xyy'z = b^{p+p} c^p$ , and  $p+p \neq p$ . Thus, there is a contradiction, which means  $L_R$  is not regular and  $L$  is not regular.

e) This has to do with the rules surrounding  $p \rightarrow q$ . If we let  $p = "L \text{ is a regular language}"$  and  $q = "L \text{ satisfies all conditions of the pumping lemma}"$ , we can show that we do not contradict the pumping lemma. In this case,  $p$  was false, which automatically makes the statement  $p \rightarrow q$  true, as  $q$  can now be any result. Thus, (c) and (d) do not contradict the pumping lemma.