I pledge my honor that I have abided by the Stevens Honor System - [illegible]

MA 346 HW 4

2.1. 2, $f(x) = 3(x+1)(x-\frac{1}{2})(x-1) = 0$

a. $[-2, 1.5]$

$\frac{-2+1.5}{2} = -0.25 = p_1$

$f(-0.25) = 3(0.75)(-0.75)(-1.25)$

$= 2.109375$

$[-2, -0.25]$

$\frac{-2+-0.25}{2} = -1.125 = p_2$

$f(-1.125) = 3(-0.125)(-1.625)(-2.125)$

$= -1.295$

$[-1.125, -0.25]$

$\boxed{p_3 = \frac{-1.125+-0.25}{2} = -0.6875}$

b. $[-1.25, 2.5]$

$p_1 = \frac{-1.25+2.5}{2} = 0.625$

$f(0.625) = 3(1.625)(0.125)(-0.375)$

$= -0.228$

$[0.625, 2.5]$

$p_2 = \frac{0.625+2.5}{2} = 1.5625$

$f(1.5625) = 3(2.5625)(1.1625)(0.5625)$

$= 5.0269$

$[0.625, 1.5625]$

$\boxed{p_3 = \frac{0.625+1.5625}{2} = 1.09375}$

3 a. $x - 2^{-x} = 0$ $[0, 1]$

$p_1 = 0.5$

$f(0.5) = 0.5 - 2^{-0.5} = -0.207$

$[0.5, 1] \Rightarrow p_2 = 0.75$

$f(p_2) = 0.75 - 2^{-0.75} = 0.1553$

$[0.5, 0.75] \Rightarrow p_3 = 0.625$

$f(p_3) = 0.625 - 2^{-0.625} = -0.023$

$[0.625, 0.75] \Rightarrow p_4 = 0.6875$

$f(p_4) = 0.6875 - 2^{-0.6875} = 0.067$

$[0.625, 0.6875] \Rightarrow p_5 = 0.65625$

$f(p_5) = 0.65625 - 2^{-0.65625} = 0.0222$

$[0.625, 0.65625] \Rightarrow p_6 = 0.640625$

$f(p_6) = -0.0008.1$

$[0.640625, 0.65625] \Rightarrow p_7 = 0.6486875$

$f(p_7) = 0.0108$

$[0.640625, 0.6486875] \Rightarrow p_8 = 0.64465625$

$f(p_8) = 0.005011$

$[0.640625, 0.64465625] \Rightarrow p_9 = 0.642640625$

$f(p_9) = 0.00210$

$[0.640625, 0.642640625] \Rightarrow p_{10} = 0.6416...$

$f(p_{10}) = 0.00064$

$[0.640625, p_{10}] \Rightarrow p_{11} = 0.641128...$

$f(p_{11}) = -0.00008$

$[p_{11}, p_{10}] \Rightarrow p_{12} = 0.6413...$

$f(p_{12}) = 0.000281$

$[p_{11}, p_{12}] \Rightarrow p_{13} = 0.64128...$

$f(p_{13}) = 0.000998$

$[p_{11}, p_{13}] \Rightarrow p_{14} = 0.64119...$

$f(p_{14}) = 0.000008885 < 10^{-5}$

$\boxed{p_{14} = 0.064119\,1896}$

b. $e^x - x^2 + 3x - 2 = 0$ $[0, 1]$

$p_1 = 0.5$

$f(p_1) = 0.8987$

$[0, p_1] \Rightarrow p_2 = 0.25$

$f(p_2) = -0.02847\ldots$

$[p_2, p_1] \Rightarrow p_3 = 0.375$

$f(p_3) = 0.4393\ldots$

$[p_2, p_3] \Rightarrow p_4 = 0.3125$

$f(p_4) = 0.20668\ldots$

$[p_2, p_4] \Rightarrow p_5 = 0.28125$

$f(p_5) = 0.089433\ldots$

$[p_2, p_5] \Rightarrow p_6 = 0.265625$

$f(p_6) = 0.030564\ldots$

$[p_2, p_6] \Rightarrow p_7 = 0.2578125$

$f(p_7) = 0.001066\ldots$

$[p_2, p_7] \Rightarrow p_8 = 0.2539$

$f(p_8) = -0.01369$

$[p_8, p_7] \Rightarrow p_9 = 0.255859\ldots$

$f(p_9) = -0.006314\ldots$

$[p_9, p_7] \Rightarrow p_{10} = 0.25683\ldots$

$f(p_{10}) = -0.002623\ldots$

$[p_{10}, p_7] \Rightarrow p_{11} = 0.25732\ldots$

$f(p_{11}) = -0.00077\ldots$

$[p_{11}, p_7] \Rightarrow p_{12} = 0.257568\ldots$

$f(p_{12}) = 0.00014386$

$[p_{11}, p_{12}] \Rightarrow p_{13} = 0.257446$

$f(p_{13}) = -0.000317$

$[p_{13}, p_{12}] \Rightarrow p_{14} = 0.25750732$

$p_{14} = -0.0000867$

$[p_{14}, p_{12}] \Rightarrow p_{15} = 0.25753784$

$f(p_{15}) = 0.00002855$

$f(p_{16}) = -0.0000029$

$f(p_{17}) = -2.759 \times 10^{-7}$

$\boxed{p_{17} = 0.257530212403}$

a. $2x\cos(2x) - (x+1)^2 = 0$, $[-3, -2]$

$f(p_1) = -3.66837$

$f(p_2) = -0.61391$

$f(p_3) = 0.630247$

$f(p_4) = 0.038675$

$f(p_5) = -0.28083$

$f(p_6) = -0.11955$

$f(p_7) = -0.040278$

$f(p_8) = -0.000985$

$f(p_9) = 0.018574$

$f(p_{10}) = 0.00880$

$f(p_{11}) = 0.0039104$

$f(p_{12}) = 0.0014629$

$f(p_{13}) = 0.0002389$

$f(p_{14}) = -0.00037307$

$f(p_{15}) = -0.0000670$

$f(p_{16}) = 0.0000859$

$f(p_{17}) = 0.00008946$

$\boxed{p_{17} = -2.19130706787}$

$f(-1) = +,\ f(0) = -$

c. $2x\cos(2x) - (x+1)^2 = 0$ $[-1, 0]$

$f(p_1) = -0.7903$
$f(p_2) = -0.1686$
$f(p_3) = 0.2963$
$f(p_4) = 0.0528815$
$f(p_5) = -0.0608144$
$f(p_6) = -0.004680$
$f(p_7) = 0.023925$
$f(p_8) = 0.0095780$
$f(p_9) = 0.002437$
$f(p_{10}) = -0.00112424$
$f(p_{11}) = 0.00065600$
$f(p_{12}) = -0.00023429$
$f(p_{13}) = 0.0002168$
$f(p_{14}) = -0.0000117$
$f(p_{15}) = 0.0000995$
$f(p_{16}) = 0.00004388$
$f(p_{17}) = 0.0000160$

$\boxed{p_{17} = -0.798164}$

d. $x\cos x - 2x^2 + 3x - 1 = 0$ $[0.2, 0.3]$

$f(p_1) = -0.13277$
$f(p_2) = -0.06158$
$f(p_3) = -0.02711$
$f(p_4) = -0.01016$
$f(p_5) = -0.00175$
$f(p_6) = 0.002428$
$f(p_7) = 0.0003372$
$f(p_8) = -0.0007089$
$f(p_9) = -0.00018563$
$f(p_{10}) = 0.00007596$
$f(p_{11}) = -0.00005482$
$f(p_{12}) = 0.000016570$
$f(p_{13}) = -0.00002212$
$f(p_{14}) = -0.00000577$

$\boxed{p_{14} = 0.2975280761722}$

$f(1.2) = +,\ f(1.3) = -$

d. $x\cos x - 2x^2 + 3x - 1 = 0$ $[1.2, 1.3]$

$f(p_1) = 0.0915$
$f(p_2) = -0.65458$
$f(p_3) = -0.01722$
$f(p_4) = 0.01086$
$f(p_5) = -0.008638$
$f(p_6) = -0.003468$
$f(p_7) = -0.0011886$
$f(p_8) = -0.0000084$
$f(p_9) = 0.0005183$
$f(p_{10}) = 0.00025401$
$f(p_{11}) = 0.0000918$
$f(p_{12}) = 0.00002071$
$f(p_{13}) = -0.0000148$
$f(p_{14}) = 0.00000293$

$\boxed{p_{14} = 1.2566223144 5}$

$7^{1/5} = 1.47577316159$

2.2.6. a. $p_1 = 343$
$p_2 = -2.2\times10^{25}$
diverges.

b. $p_1 = 7$
$p_2 = -335$
diverges, but slower than A.

c. $p_1 = 2.2$
$p_2 = 1.8197$
$p_3 = 1.58347$
$p_4 = 1.4894609$
$p_5 = 1.4760224$
$p_6 = 1.4757732$
$\boxed{p_7 = 1.47577316159}$

d. $p_1 = 1.5$
$p_2 = 1.4558$
$p_3 = 1.49874$
$p_4 = 1.45190$
$p_5 = 1.497577$
$p_6 = 1.45319227902$
$p_7 = 1.49642536371$
$p_8 = 1.45439611886$
$p_9 = 1.49543858728$
$p_{10} = 1.45552881001$

c converges fastest, then d, then b, then a.
$c > d > b > a$ (performance)

13a. $g'(x) = \frac{2x - e^x}{3}$. On the interval $[0,1]$, $g'(x)$ does not change sign, and $g'(0) = -\frac{1}{3}$ and $g'(1) = \frac{2-e}{3}$. Because $g''(x) = \frac{x - e^x}{3}$, with no zeros on $[0,1]$, $-\frac{1}{3}$ is a minimum, and $k = 1/3$.

Putting $p_0 = 0.5$, we get:

Error $\longrightarrow |p_n - p| \leq k^n \max\{p_0 - a, b - p_0\}$
$10^{-5} \leq 1/3^n \max\{0.5-0, 1-0.5\}$
$10^{-5} \leq 1/3^n (0.5)$
$\log_{1/3}\left(\frac{10^{-5}}{0.5}\right) \geq n \Rightarrow n \geq 9.848 \approx 9$ iterations

$p_0 = 0.5$ $p_1 = f(p_0) = 0.2004$ err $= 0.299$
$p_2 = f(p_1) = 0.2727$ err $= 0.0723$
$p_3 = f(p_2) = 0.253607$ err $= 0.0191$
$p_4 = f(p_3) = 0.25855$ err $= 0.004494$
$p_5 = f(p_4) = 0.257265$ err $= 0.001284$
$p_6 = f(p_5) = 0.25759$ err $= 0.0003333$
$p_7 = f(p_6) = 0.257512$ err $= 0.00008$
$p_8 = f(p_7) = 0.257534$ err $= 0.00022$
$p_9 = f(p_8) = 0.2575290$
err $= 0.00000582$

$\boxed{p_9 = 0.2575290}$

f. $g(x) = 0.5(\sin x + \cos x)$, $g'(x) = 0.5(\cos x - \sin x)$

range of $g(x)$ is $[-1,1]$, so fixed point can be on $x = [0,1]$. $g''(x) = 0.5(-\sin x - \cos x) \neq 0$ on interval $[0,1]$, So max/min are at endpoints $g'(0) = 0.5$, $g'(1) = 0.5(\cos 1 - \sin 1)$. Max on this interval is 0.5, so $k = 0.5$. Let $p_0 = 0.5$

$|p - p_n| \geq k^n \max\{p_0 - a, b - p_0\}$

$10^{-5} \geq 0.5^n \max\{0.5 - 0, 1 - 0.5\}$

$\log_{0.5}\left(\frac{10^{-5}}{0.5}\right) \geq n \Rightarrow n \leq 15.609$

$p_0 = 0.5$ $\quad p_1 = f(p_0) = 0.678$ err $= 0.17$

$p_2 = f(p_1) = 0.70307$ err $= 0.024$

$p_3 = f(p_2) = 0.704711$ err $= 0.0016410$

$p_4 = f(p_3) = 0.70480$ err $= 0.0000944$

$p_5 = f(p_4) = 0.7048167$ err $= 0.00000538$

$\boxed{p_5 = 0.7048167}$

14. $3x^2 - e^x = 0$ $\quad 1 = 6x - e^x \Rightarrow x = 0.42, 2.735$

$g'(x) = 6x - e^x$ $\quad -1 = 6x - e^x \Rightarrow x = 0, 2.918$

$g''(x) = 6 - e^x$ $\quad |g'(x)| \leq 1$ on intervals $[0, 0.42]$ $[2.735, 2.918]$

On these intervals $f(x) = [-1.101, -0.993]$, and $[7.034, 7.081]$, respectively.

Thus, there is no way to satisfy the Fixed Point Theorem, and the algorithm will not converge.

2.3. 2. $f(x) = -x^3 - \cos x$, $p_0 = -1$, $f'(x) = -3x^2 + \sin x$

$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = -1 - \frac{-(-1)^3 - \cos -1}{-3(-1)^2 + \sin -1} = -0.88$

$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} = -0.88 - \frac{-(-0.88)^3 - \cos -0.88}{-3(-0.88)^2 + \sin -0.88} = -0.86568$

We cannot use $p_0 = 0$ because $f'(p_0) = -3(0)^2 + \sin 0 = 0$.

4. $f(x) = -x^3 - \cos x \quad p_0 = -1, p_1 = 0$

a. $p_n = p_{n-1} - \dfrac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$ $\quad f(p_0) = f(-1) = 1 - \cos -1$

$f(p_1) = f(0) = -1$

$p_2 = p_1 - \dfrac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)}$

$p_2 = 0 - \dfrac{f(0)(0 - (-1))}{f(0) - f(-1)} = -\dfrac{-1(1)}{-1 - (1 - \cos(-1))} = -0.685$

$$\boxed{p_3 = p_2 - \dfrac{f(-0.685)(-0.685 - 0)}{f(-0.685) - f(0)} = -1.282}$$

b. p_2 is the same as secant method: $-0.685 = p_2$

$p_1 \cdot p_2 > 0$, so use p_2 and p_0. $\quad p_0 = -1$

$p_3 = p_2 - \dfrac{f(-0.685)(-0.685 - (-1))}{f(-0.685) - f(-1)}$

$$\boxed{p_3 = -0.841355}$$