I pledge my honor that I have abided by the Stevens Honor System

MA 346 HW 5

2.3 13. Using the matlab code written last HW:

a. False position: $[-1, 0]$: $p_{17} = -0.04065850$.
$[0, 1]$: $p_9 = 0.96239838$

b. Secant method: $[-1, 0]$: $p_5 = -0.0406592883 15$
$[0, 1]$: $p_{12} = -0.0406...$
(fails to find fixed point in $[0, 1]$)

c. Newton's method: $[-1, 0]$: $p_4 = -0.04065 92883$
$[0, 1]$: $p_4 = -0.040659...$
(fails to find fixed point in $[0, 1]$ with $p_0 = 0.5$)

2.4 2. Using the Matlab code...

a. $p_{15} = 0.7390787$. $[0, 1]$ $p_0 = 0.5$

b. $p_9 = -1.334345$. $[-3, 2]$, $p_0 = -2.5$
There's no solution in the interval.

c. $p_5 = 3.14156793$ $[3, 4]$ $p_0 = 3.5$

d. $p_{24} = 3.73310284$ $[3, 5]$, $p_0 = 4$.

4. a. $p_4 = 0.7390851$ $[0, 1]$ $p_0 = 0.5$

b. $p_{80} = -1.3343459$ $[-3, 2]$, $p_0 = -2.5$
(no solution on initial interval)

c. $p_5 = 3.1415679$ $[3, 4]$ $p_0 = 3.5$

d. $p_4 = 3.7330677$ $[3, 5]$ $p_0 = 4$

6a. $p_n = \frac{1}{n}, n \geq 1$

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^{\alpha}} = \lambda$$

$$\lim_{n\to\infty} \frac{\frac{1}{n+1} - p}{|\frac{1}{n} - p|^{\alpha}} = \lambda$$

If $\alpha = 1$, $p = 0$

$$\lim_{n\to\infty} \frac{\frac{1}{n+1} - 0}{\frac{1}{n} - 0} =$$

$$\lim_{n\to\infty} \frac{n}{n+1} \approx 1$$

(but less than 1).

Therefore, the sequence converges to 0 linearly.

b. $p_n = \frac{1}{n^2}, n \geq 1$

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^{\alpha}} = \lambda$$

Let $\alpha = 1$, $p = 0$

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)^2} - 0}{\frac{1}{n^2}}$$

$$\lim_{n\to\infty} \frac{n^2}{n^2+2n+1}$$

≈ 1 (but < 1).

Thus, with $\lambda < 1$ and $\alpha = 1$, this sequence converges linearly to 0.

2.5.4. $g(x) = 1 + (\sin x)^2$, $p_0^{(0)} = 1$

$p_1^{(0)} = g(p_0^0) = 1 + \sin(1)^2 = 1.7081$

$p_2^{(0)} = g(p_1^0) = 1 + \sin(1.7081)^2 = 1.9813$

$p_0^1 = \{\Delta^2\}(p_0^0) = p_0^0 - (p_1^0 - p_0^0)^2 / (p_2^0 - 2p_1^0 + p_0^0)$

$= 1 - (1.7081 - 1)^2 / (1.9813 - 2(1.7081) + 1)$

$= 2.1529$.

$p_1^1 = g(p_0^1) = 1 + (\sin(2.1529))^2 = 1.6977$

$p_2^1 = g(p_1^1) = 1 + \sin((1.6977))^2 = 1.9839$

$p_0^2 = \{\Delta^2\}(p_0^1) = p_0^1 - (p_1^1 - p_0^1)^2 / (p_2^1 - 2p_1^1 + p_0^1)$

$= 2.1529 - (1.6977 - 2.1529)^2 / (1.9839 - 2 \cdot 1.6977 + 2.1529)$

$= 1.87346$

3.1.6a.

x	0	0.25	0.5	0.75
$f(x)$	1	1.64872	2.71828	4.48169

$f(0.43)$.

1st degree: $f(0.25) = 1.64872$, $f(0.5) = 2.71828$.

$L_0(x) = \frac{x-0.5}{0.25-0.5}$ $\quad L_1(x) = \frac{x-0.25}{0.5-0.25}$

$= -4(x-0.5)$ $\quad = 4(x-0.25)$

$P(x) = L_0(x) \cdot f(x_0) + L_1(x) f(x_1)$

$= -4(x-0.5)(1.64872) + 4(x-0.25)(2.71828)$

$P(0.43) = -4(1.64872)(0.43-0.5) + 4(2.71828)(0.43-0.25)$

≈ 2.4188032.

2nd degree: $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

$L_0(x) = \frac{(x-0.5)(x-0.75)}{(0.25-0.5)(0.25-0.75)}$ $\quad L_1(x) = \frac{(x-0.25)(x-0.75)}{(0.5-0.25)(0.5-0.75)}$

$= 8(x-0.5)(x-0.75)$ $\quad = -16(x-0.25)(x-0.75)$

$L_2(x) = \frac{(x-0.25)(x-0.5)}{(0.75-0.25)(0.75-0.5)} = 8(x-0.25)(x-0.5)$

$P(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2)$

$= 13.18976(x-0.75)(x-0.5) + (-43.49248)(x-0.25)(x-0.75)$

$+ 35.85352(x-0.25)(x-0.5)$

$P(0.43) = 0.295 + 2.505 + -0.45175$

$= 2.34826$

3rd degree:

$L_0(x) = \frac{(x-0.25)(x-0.5)(x-0.75)}{(0-0.25)(0-0.5)(0-0.75)}$ $\quad L_1(x) = \frac{x(x-0.5)(x-0.75)}{(0.25-0)(0.25-0.5)(0.25-0.75)}$

$= -\frac{32}{3}(x-0.25)(x-0.5)(x-0.75)$ $\quad = 32(x)(x-0.5)(x-0.75)$

$L_2(x) = \frac{(x-0)(x-0.25)(x-0.75)}{(0.5-0)(0.5-0.25)(0.5-0.75)}$ $\quad L_3(x) = \frac{(x-0)(x-0.25)(x-0.5)}{(0.75)(0.75-0.25)(0.75-0.5)}$

$= -32(x)(x-0.25)(x-0.75)$ $\quad = \frac{32}{3}(x)(x-0.25)(x-0.5)$

$$P(x) = \sum_{i=0}^{4} L_i(x) f(x_i)$$

$$= 1 \cdot -\frac{32}{3}(x-0.25)(x-0.5)(x-0.75) +$$

$$1.64872 \cdot 32(x)(x-0.5)(x-0.75) +$$

$$2.71828 \cdot -32(x)(x-0.25)(x-0.75) +$$

$$4.48169 \cdot \frac{32}{3}(x)(x-0.25)(x-0.5)$$

$$P(0.43) = -0.043008 + 0.508178 + 2.154443 + -0.25900$$

$$= 2.360604734$$

8a $n=1 \Rightarrow f(x) = e^{2x}$ $[0.25, 0.5]$

$$\text{term} = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)\dots(x-x_n)$$

$f'(x) = 2e^{2x}$

$f''(x) = 4e^{2x}$

$f'''(x) = 8e^{2x}$

$$= \frac{f^2(\xi(x))}{2!}(x-x_0)(x-x_1)$$

$$\left(f^2(\xi(x)) = 4e^{2(\xi(x))} \quad (\max @ x = 0.5)\right.$$

$$= 4e^{2(0.5)} = 4e$$

$$= \frac{4e}{2}(x-0.5)(x-0.25)$$

$$\frac{4e}{2}(x^2 - 0.75x + 0.125)$$

min @ $x = 0.375$

$$|\text{error}| \leq \left|\frac{4e}{2}(0.375-0.25)(0.375-0.5)\right|$$

$$|\text{error}| \leq 0.0849$$

$$\text{error} = |e^{2 \cdot 0.43} - 2.4186032| = 0.05564$$

$0.05564 < 0.0849$ ✓

$n=2 \Rightarrow f(x) = e^{2x}$ $[0.25, 0.75]$

$$\frac{f^3(\xi(x))}{3!}(x-0.25)(x-0.5)(x-0.75)$$

$$\left(f^3(\xi(x)) = 8e^{2x} \quad (\max @ x = 0.75)\right)$$

$$= 8e^{2(0.75)} = 8e^{1.5}$$

$$\frac{8e^{1.5}}{6}(x-0.25)(x-0.5)(x-0.75)$$

max @ 0.3557 (desmos)

$$|\text{error}| \leq \frac{8e^{1.5}}{6}(0.3557-0.25)(0.3557-0.5)(0.3557-0.75)$$

$|\text{error}| \le 0.03593$

$\text{error} = |e^{2 \cdot 0.43} - 2.34886| = 0.014300$

$0.014300 \le 0.03593$ ✓

9. $P_3(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2) + L_3(x)f(x_3)$

$L_0(x) = \dfrac{(x-0.5)(x-1)(x-2)}{(-0.5)(-1)(-2)}$ $\quad L_1(x) = \dfrac{(x)(x-1)(x-2)}{(0.5-0)(0.5-1)(0.5-2)}$

$= -1(x-0.5)(x-1)(x-2)$ $\quad = \frac{8}{3}x(x-1)(x-2)$

$L_2(x) = \dfrac{(x)(x-0.5)(x-2)}{(1-0)(1-0.5)(1-2)}$ $\quad L_3(x) = \dfrac{x(x-0.5)(x-1)}{(2-0)(2-0.5)(2-1)}$

$= -2x(x-0.5)(x-2)$ $\quad = \frac{1}{3}(x)(x-0.5)(x-1)$

$-1x^3(0) + \frac{8}{3}(x^3)(y) + -2(3)x^3 + 2(\frac{1}{3})x^3 = 6x^3$

$\frac{8y}{3} + (-6) + \frac{2}{3} = 6$

$y = 4.25$

13a.

x	0	0.3	0.6
$f(x)$	1	1.1326	−0.7543

$L_0(x) = \dfrac{(x-0.3)(x-0.6)}{(-0.3)(-0.6)}$ $\quad L_1(x) = \dfrac{(x)(x-0.6)}{(0.3-0)(0.3-0.6)}$ $\quad L_2(x) = \dfrac{x(x-0.3)}{(0.6-0)(0.6-0.3)}$

$= \frac{50}{9}(x-0.3)(x-0.6)$ $\quad -\frac{100}{9}(x)(x-0.6)$ $\quad = \frac{50}{9}(x)(x-0.3)$

$P_2(x) = \frac{50}{9}(1)(x-0.3)(x-0.6) - \frac{100}{9}(1.1326)(x)(x-0.6) + \frac{50}{9}(-0.7543)(x)(x-0.3)$

$= \frac{50}{9}(x-0.3)(x-0.6) - 12.584469(x)(x-0.6) + -4.19076(x)(x-0.3)$

$\text{error term} = \dfrac{f^3(\xi(x))}{3!}(x-0)(x-0.3)(x-0.6)$

$f(x) = e^{2x}\cos 3x$ $\quad f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$

$f''(x) = 4e^{2x}\cos 3x - 6e^{2x}\sin 3x - 6e^{2x}\sin 3x - 9e^{2x}\cos 3x$

$= -5e^{2x}\cos 3x - 12e^{2x}\sin 3x$

$f'''(x) = -10e^{2x}\cos 3x + 15e^{2x}\sin 3x - 24e^{2x}\sin 3x - 36e^{2x}\cos 3x$

$= -46e^{2x}\cos 3x - 9e^{2x}\sin 3x$

$$E(x) = \frac{-46e^{2x}\cos(3x) - 9e^{2x}\sin(3x)}{3!}(x)(x-0.3)(x-0.6)$$

Graphing this equation, max found @ $x = 0.1403$

$E(x) = 0.104$

$|error| \leq 0.104$