Max Sim Workshop 12 9/8/2020

I pledge my honor that I have abided by the Stevens Honor System (signature)

(a) $u(x,t) = \sum_{n=1}^{\infty} A_n \cos(2\sqrt{\lambda_n}\, t) + B_n \sin(2\sqrt{\lambda_n}\, t)) \cos(\sqrt{\lambda_n}\, x)$

let $\lambda_n = \alpha_n^2$

$X = c_1 \sin(\alpha_n x) + c_2 \cos(\alpha_n x) \xRightarrow{c_1 = 0} X = c_2 \cos(\alpha_n x)$

$X' = \alpha_n c_1 \cos(\alpha_n x) - \alpha_n c_2 \sin(\alpha_n x)$

$0 = \alpha_n c_1 \cos 0 - \alpha_n c_2 \sin 0$

$c_1 = 0$

$0 = c_2 \cos(2\pi \alpha_n)$

$2\pi \alpha_n = \pi n - \frac{\pi}{2}$

$\alpha_n = \frac{n}{2} - \frac{1}{4}$

$\lambda_n = \left(\frac{n}{2} - \frac{1}{4}\right)^2, n = 1, 2, 3, \ldots$

(b) $u(x,0) = \sum_{n=1}^{\infty} (A_n \cos 0 + B_n \sin 0) \cos\left(\left(\frac{n}{2} - \frac{1}{4}\right)x\right)$

$= \sum_{n=1}^{\infty} A_n \cos\left(\left(\frac{n}{2} - \frac{1}{4}\right)x\right) = 3\cos\left(\frac{3x}{4}\right) - 2\cos\left(\frac{5x}{4}\right)$

$n = 2$, $A_2 = 3$ $n = 3$, $A_3 = -2$, $A_n = 0, n \neq 2, 3$

$u_t(x,0) = \sum_{n=1}^{\infty} \left(-2\left(\frac{n}{2} - \frac{1}{4}\right) A_n \sin 0 + 2\left(\frac{n}{2} - \frac{1}{4}\right) B_n \cos 0\right) \cos\left(\left(\frac{n}{2} - \frac{1}{4}\right)x\right)$

$= \sum_{n=1}^{\infty} \left(n - \frac{1}{2}\right) B_n \cos\left(\left(\frac{n}{2} - \frac{1}{4}\right)x\right) = -5\cos\frac{x}{4} + 8\cos\frac{11x}{4}$

$n = 1$ $n = 6$

$\left(1 - \frac{1}{2}\right) B_1 = -5$ $\left(6 - \frac{1}{2}\right) B_6 = 8$

$\frac{1}{2} B_1 = -5$ $\frac{11}{2} B_6 = 8$

$B_1 = -10$. $B_6 = \frac{16}{11}$ $B_n = 0, n \neq 1, 6$.

$u(x,t) = -10\sin\left(\frac{1}{2}t\right)\cos\left(\frac{1}{4}x\right)$
$+ 3\cos\left(\frac{3}{2}t\right)\cos\left(\frac{3}{4}x\right)$
$- 2\cos\left(\frac{5}{2}t\right)\cos\left(\frac{5}{4}x\right)$
$+ \frac{16}{11}\sin\left(\frac{11}{2}t\right)\cos\left(\frac{11}{4}x\right)$

(c) $f(x) = \begin{cases} \frac{1}{\pi}x, & 0 < x < \pi \\ 2 - \frac{1}{\pi}x, & \pi < x < 2\pi \end{cases}$

$A_n = \frac{2}{2\pi}\int_0^{2\pi} f(x)\cos\left(\frac{nx}{2}\right)dx = \frac{1}{\pi}\left(\int_0^{\pi} \frac{x}{\pi}\cos\left(\frac{nx}{2}\right)dx + \int_{\pi}^{2\pi}\left(2 - \frac{x}{\pi}\right)\cos\left(\frac{nx}{2}\right)dx\right)$

$B_n = 0$

$n = 1, 2, 3, \ldots$