I pledge my honor that I have abided by the Stevens Honor System, Max Shi

Max Shi Problem Set #5

1a. Basis: $n=0$ $2^0 = 2^{0+1} - 1$

$1 = 2^1 - 1$

$1 = 1$

Inductive hypothesis: $\exists k$ s.t. $2^0 + 2^1 + \dots + 2^k = 2^{k+1} - 1$, $k \geq 0$

Inductive step:

$$2^0 + 2^1 + \dots + 2^{k+1} = 2^{k+1+1} - 1$$
$$(2^0 + 2^1 \dots + 2^k) + 2^{k+1} = 2^{k+2} - 1$$
$$2^{k+1} - 1 + 2^{k+1} = 2^{k+2} - 1$$
$$2^{k+1} + 2^{k+1} - 1 = 2^{k+2} - 1$$
$$2(2^{k+1}) - 1 = 2^{k+2} - 1$$
$$2^{k+1+1} - 1 = 2^{k+2} - 1$$
$$2^{k+2} - 1 = 2^{k+2} - 1$$

This establishes the inductive step, and the claim follows from PI.

b basis: $n=0$ $a^0 = \frac{a^{0+1} - 1}{a-1}$

$1 = \frac{a-1}{a-1}$

$1 = 1$

Inductive hypothesis: $\exists k$ s.t. $a^0 + a^1 + \dots a^k = \frac{a^{k+1} - 1}{a-1}$, $k \geq 0$

Inductive step:

$$a^0 + a^1 + \dots a^{k+1} = \frac{a^{k+2} - 1}{a-1}$$
$$(a^0 + a^1 + \dots a^k) + a^{k+1} = \frac{a^{k+2} - 1}{a-1}$$
$$\frac{a^{k+1} - 1}{a-1} + a^{k+1} = \frac{a^{k+2} - 1}{a-1}$$
$$\frac{a^{k+1} - 1}{a-1} + \frac{(a-1)(a^{k+1})}{a-1} = \frac{a^{k+2} - 1}{a-1}$$
$$\frac{\cancel{a^{k+1}} - 1}{a-1} + \frac{a^{k+2} - \cancel{a^{k+1}}}{a-1} = \frac{a^{k+2} - 1}{a-1}$$
$$\frac{a^{k+2} - 1}{a-1} = \frac{a^{k+2} - 1}{a-1}$$

This establishes the inductive step, and the claim follows from PI

2. basis: $n=1$ $\quad 1\cdot2\cdot3 = \frac{1}{4}(1\cdot2\cdot3\cdot4)$

$1\cdot2\cdot3 = \frac{4}{4}(1\cdot2\cdot3)$

$1\cdot2\cdot3 = 1\cdot2\cdot3$

Inductive hypothesis:

$\exists k$ s.t. $(1\cdot2\cdot3)+(2\cdot3\cdot4)+\dots+(k(k+1)(k+2)) = \frac{1}{4}(k(k+1)(k+2)(k+3))$,

$k \geq 1$

Inductive step:

$(1\cdot2\cdot3)+(2\cdot3\cdot4)+\dots+(k+1)(k+2)(k+3) = \frac{1}{4}((k+1)(k+2)(k+3)(k+4))$

$(1\cdot2\cdot3)+(2\cdot3\cdot4)+\dots+(k(k+1)(k+2))+((k+1)(k+2)(k+3)) =$

$\frac{1}{4}(k+1)(k+2)(k+3)(k+4)$

$\frac{1}{4}(k(k+1)(k+2)(k+3)) + (k+1)(k+2)(k+3) = \frac{1}{4}(k+1)(k+2)(k+3)(k+4)$

$(k+1)(k+2)(k+3)\cdot(\frac{k}{4}+1)$

$(k+1)(k+2)(k+3)\frac{1}{4}(k+4) = \frac{1}{4}(k+1)(k+2)(k+3)(k+4)$

This establishes the inductive step, and the claim follows from PI.

3. Able: 4, 8, 11, 12, 15, 16, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34.

Claim: Can make n for all $n \geq 30$.

basis: $n=30 = 2\cdot11 + 2\cdot4$ $\quad n=32 = 4\cdot8$

$n=31 = 11 + 5\cdot4$ $\quad n=33 = 11\cdot3$

Inductive hypothesis: $\exists k$ s.t. we can make $k-3$ for all $k-3 \geq 30$.

Let $k-3 \geq 30$

Because we can make $k-3$ by inductive hypothesis, we can make $k+1$ because $k-3+4 = k+1$. Because we can make all n s.t. $30 \leq n \leq 33$, this cycle will repeat because $34 = 30+4$, and therefore by PI the claim will hold for all $n \geq 30$.

4. Claim: P(n) (power n) = 2^n $\forall n \geq 0$

Basis: n=0 power 0 = 2^0

n=0 so power 0 = 1

1 = 1

Inductive hypothesis: $\exists k$ power k = 2^k, $k \geq 0$

Inductive step:

power k+1 = 2^{k+1}

* 2 (power - (k+1) 1) = 2^{k+1}

2 * power (k+1-1) = 2^{k+1}

2 * power k = 2^{k+1}

2 * 2^k = 2^{k+1}

$2^{k+1} = 2^{k+1}$

This establishes the inductive step, and the claim follows from PI.

5. Claim: (tower m) is a tower of powers $2^{2^{2^{\cdot^{\cdot^{2}}}}}$ with height m.

Basis: m=0 : tower 0 = tower of powers of height 0.

tower 0 = 2^0

if (= 0 0) 1 = 2^0

1 = 1

Inductive hypothesis: $\exists k$ s.t. tower k = $2^{2^{\cdot^{\cdot^{2}}}}$ with height k.

(denote as $2^{2^{\cdot^{\cdot^{2}}}}_{h=k}$)

Inductive step: tower (k+1) = $2^{2^{\cdot^{\cdot^{2}}}}_{h=k+1}$

power (tower - (k+1) 1) = $2^{2^{\cdot^{\cdot^{2}}}}_{h=k+1}$

power (tower (k+1-1)) = $2^{2^{\cdot^{\cdot^{2}}}}_{h=k+1}$

(from prev. problem power n = 2^n) $\longrightarrow$ power (tower k) = $2^{2^{\cdot^{\cdot^{2}}}}_{h=k+1}$

power ($2^{2^{\cdot^{\cdot^{2}}}}_{h=k}$) = $2^{2^{\cdot^{\cdot^{2}}}}_{h=k+1}$

$2^{(2^{2^{\cdot^{\cdot^{2}}}}h=k)}$ = $2^{2^{\cdot^{\cdot^{2}}}}_{h=k+1}$

$2^{2^{2^{\cdot^{\cdot^{2}}}}}_{h=k+1} = 2^{2^{\cdot^{\cdot^{2}}}}_{h=k+1}$

This establishes the inductive step, and the claim follows from PI.