MA 346

1. $|f(x_0) - g(x_0)|$ $\quad g(x_0) = f(x_0+h)$

$|f(x_0) - f(x_0+h)|$

$$f(x_0+h) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!} h^k + \frac{f^{(n+1)}(\xi)}{(n+1)!} h^{n+1}$$

$= |f(x_0+h) - f(x_0)|$

$|f'(a)| = \left|\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}\right|$ $\quad h \cdot f'(x_0) = |f(x_0+h) - f(x_0)|$

If we let $h \neq 0$, then $\left|\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}\right| = \left|\frac{|f(x_0+h) - f(x_0)|}{h}\right|$, $h \neq 0$

Thus, $|f'(x_0)| = \left|\frac{f(x_0+h) - f(x_0)}{h}\right|$, $h \neq 0$.

$\Downarrow$

$$|f(x_0+h) - f(x_0)| = |h \cdot f'(x_0)|, \; h \neq 0.$$

1.2 error $\geq |h \cdot f'(x_0)|$ $\quad h = 0.01, x_0 = 1, f(x) = e^x, f'(x)$

error $\geq 0.01 \cdot e^1$

2. $f(x) = \ln(x)$

$f'(x) = x^{-1}$ $\quad f''(x) = -x^{-2}$ $\quad f'''(x) = 2x^{-3}$

$$f(x+h) = \ln x + \frac{x^{-1}}{1} h - \frac{x^{-2}}{2} h^2 + \frac{x^{-3}}{3} h^3$$

$$= \ln x + \sum_{i=1}^{n} -1^{(n+1)} \cdot \frac{x^{-n}}{n} h^n + -1^{(n+1)} \frac{\xi^{-n+1}}{n+1} h^{n+1}$$

$$f(x-h^2) = \ln x + \left(\sum_{i=1}^{n} -\frac{x^{-n}}{n} h^{2n}\right) - \frac{\xi^{-n+1}}{n+1} h^{2n+1}$$

$$f(x-h^2) = \ln x + \left(\sum_{i=1}^{n} \frac{-x^{-n}}{n} h^{2n}\right) - \frac{\xi^{-n+1}}{n+1} h^{2n+1}$$

3. $\begin{bmatrix} 1 & -2\alpha & 1 \\ 2\alpha & -1 & 1 \end{bmatrix}$

$-2\alpha r_1 = -2\alpha,\ 4\alpha^2 - 2\alpha$

$\begin{bmatrix} 1 & -2\alpha & 1 \\ 0 & -1+4\alpha^2 & -2\alpha \end{bmatrix}$

$(-1+4\alpha^2)x_2 = -2\alpha$

if $-1+4\alpha^2 = 0$ and $-2\alpha \neq 0$, then no solution.

$4\alpha^2 = 1$

$\alpha^2 = \frac{1}{4}$ $\quad$ ~~$\alpha = \frac{1}{2} \Rightarrow -2(\frac{1}{2}) \neq 0$~~

$\alpha = \pm\frac{1}{2}$ $\quad$ $\alpha = -\frac{1}{2} \Rightarrow -2(-\frac{1}{2}) \neq 0.$

If $\alpha = -\frac{1}{2}$, no solution.

Infinite solutions if $-1+4\alpha^2 = -2\alpha = 0$

$4\alpha^2 + 2\alpha - 1 = 0$

$\alpha = \frac{1}{2}$

If $\alpha = \frac{1}{2}$, there are infinite solutions.

Unique solution for all other α.

$(-1+4\alpha^2)x_2 = -2\alpha$

$x_2 = \dfrac{-2\alpha}{-1+4\alpha^2}$

$x_1 - 2\alpha x_2 = 1$

$x_1 - 2\alpha\left(\frac{-2\alpha}{-1+4\alpha^2}\right) = 1$

$x_1 + \frac{4\alpha^2}{-1+4\alpha^2} = 1$

$x_1 = 1 - \frac{4\alpha^2}{-1+4\alpha^2}$

In conclusion, if $\alpha = \frac{1}{2}$, then there are infinite solutions.
if $\alpha = -\frac{1}{2}$, the system has no solution.
Otherwise, the solution is $x_1 = 1 - \frac{4\alpha^2}{-1+4\alpha^2}$,
$x_2 = \frac{-2\alpha}{-1+4\alpha^2}$.