3. $4y'' + y = 4\sin(3x/2)$ $\qquad y(0) = 0$

$\qquad y'(\pi) = 0$

$$Y_p = A\sin\left(\frac{3x}{2}\right) + B\cos(3x/2)$$

$$Y_p' = \frac{3}{2}A\cos\left(\frac{3x}{2}\right) - \frac{3}{2}B\sin(3x/2)$$

$$Y_p'' = \frac{-9}{4}A\sin\left(\frac{3x}{2}\right) - \frac{9}{4}B\cos(3x/2)$$

$$4\sin\left(\frac{3x}{2}\right) = 4\left(\frac{-9}{4}A\sin\left(\frac{3x}{2}\right) - \frac{9}{4}B\cos\left(\frac{3x}{2}\right)\right) + A\sin\left(\frac{3x}{2}\right) + B\cos\left(\frac{3x}{2}\right)$$

$$4\sin\left(\frac{3x}{2}\right) = -9A\sin\left(\frac{3x}{2}\right) - 9B\cos\left(\frac{3x}{2}\right) + A\sin\left(\frac{3x}{2}\right) + B\cos\left(\frac{3x}{2}\right)$$

$$4\sin\left(\frac{3x}{2}\right) = -8A\sin\left(\frac{3x}{2}\right) - 8B\cos\left(\frac{3x}{2}\right)$$

$B = 0$

$A = -1/2 \qquad Y_p = \frac{-1}{2}\sin\left(\frac{3x}{2}\right)$

$$y = C_1\cos\left(\frac{x}{2}\right) + C_2\sin\left(\frac{x}{2}\right) - \frac{1}{2}\sin\left(\frac{3x}{2}\right)$$

$$y' = \frac{-C_1}{2}\sin\left(\frac{x}{2}\right) + \frac{C_2}{2}\cos\left(\frac{x}{2}\right) - \frac{3}{4}\cos\left(\frac{3x}{2}\right)$$

$$\begin{cases} \text{At } x = 0, \ 0 = y(0) = C_1 + C_2 \cdot 0 - \frac{1}{2}\cdot 0 \Rightarrow C_1 = 0 \\ \text{At } x = \pi, \ 0 = y'(\pi) = \frac{-C_1}{2} + C_2 \cdot 0 - 0 \Rightarrow C_1 = 0 \end{cases}$$

No constraints for C_2

$$\boxed{y = C_2\sin\left(\frac{x}{2}\right) - \frac{1}{2}\sin\left(\frac{3x}{2}\right)}$$

MA 221-Recitation Week 12

4. $y'' + 2y' + \lambda y = 0 \quad 0 < x < \pi \quad y(0) = 0,\ y(\pi) = 0$

$m^2 + 2m + \lambda m = 0 \quad m_{1,2} = -1 \pm \sqrt{1-\lambda} \rightarrow$ complex roots if $1-\lambda < 0,\ \lambda > 1$

$\alpha > 0 \quad \alpha^2 > 0 \quad 1 - \lambda < 0 \quad \alpha^2 = \lambda - 1 \quad m_{1,2} = -1 \pm \sqrt{-\alpha^2} = -1 \pm \alpha i$

$\underline{y = e^{-x}(C_1 \cos \alpha x + C_2 \sin \alpha x)} \quad y(0) = 0 \quad 0 = e^{-0}(C_1 \cdot 1 + C_2 \cdot 0),\ 0 = C_1$

$y(\pi) = 0 \quad 0 = e^{-\pi}(C_1 \cos(\alpha\pi) + C_2 \sin(\alpha\pi)) \quad 0 = e^{-\pi}(0 \cdot \cos(\alpha\pi) + C_2 \sin(\alpha\pi))$

$\alpha\pi = 0, \pi, 2\pi \ldots$

$\alpha = n$ from $(0, \infty)$

$\lambda = n^2 + 1,\ n = (0, \infty)$

$$\boxed{y_n = e^{-x}(C \sin(nx)) \qquad \lambda_n = n^2 + 1, \qquad n = 0, 1, 2, 3, \ldots, \infty}$$

(5) $\frac{du}{dt} = k\frac{d^2u}{dx^2} - au$ for $0 < x < \pi$ $t > 0$

Let $u(x,t) = X(x) \cdot T(t)$

$\frac{du}{dt} = XT'$ and $\frac{d^2u}{dt} = X''T$

So we get

$XT' = kX''T - aXT$

$XT' = (kX'' - aX)T$

$\frac{T'}{T} = \frac{kX'' - aX}{X} = \lambda$ ← set $= \lambda$

$T' = \lambda T$

$\frac{dT}{T} = \lambda dt$

$\ln(T) = \lambda t + \ln(C)$

$T = Ce^{\lambda t}$

$kX'' - aX = \lambda X$

$X'' = \left(\frac{a+\lambda}{k}\right)X.$

$m^2 = \frac{a+\lambda}{k}$

$m = \pm\sqrt{\frac{a+\lambda}{k}}$

Answers:

(1) If $\frac{a+\lambda}{k} = 0 \Rightarrow \lambda = -a$

$X(x) = C_1 + C_2 X$

$u(x,t) = (C_1 + C_2 X)e^{-at}$

(2) If $\frac{a+\lambda}{k} > 0$

Then $m = \pm\sqrt{\frac{a+\lambda}{k}}$

So, $X(x) = C_1 e^{\left(\sqrt{\frac{a+\lambda}{k}}\right)x} + C_2 e^{\left(-\sqrt{\frac{a+\lambda}{k}}\right)x}$

$u(x,t) = \left[C_1 e^{\left(\sqrt{\frac{a+\lambda}{k}}\right)x} + C_2 e^{\left(-\sqrt{\frac{a+\lambda}{k}}\right)x}\right]e^{\lambda t}$

(3) If $\frac{a+\lambda}{k} < 0$

Then $m = \pm i\sqrt{-\frac{a+\lambda}{k}}$

$X(x) = C_1\cos\left(\sqrt{-\frac{a+\lambda}{k}}\right)x + C_2\sin\left(\sqrt{-\frac{a+\lambda}{k}}\right)x$

$u(x,t) = \left[C_1\cos\left(\sqrt{-\frac{a+\lambda}{k}}\right)x + C_2\sin\left(\sqrt{-\frac{a+\lambda}{k}}\right)x\right]e^{\lambda t}$

MA 221 Ro Group 1 I pledge my honor that I have abided by the Stevens Honor System - [illegible]

6 $y'' + (\lambda - 1)y = 0,\ 0 < x < \pi,\ y(0) = 0,\ y'(\pi) = 0.$

$m^2 + (\lambda - 1) = 0$

$m = \sqrt{1-\lambda}$ For complex, let $1-\lambda < 0$

set $-\alpha^2 < 0$ s.t. $1-\lambda = -\alpha^2$

$m = \pm\sqrt{-\alpha^2}$

$= \pm \alpha i$

$y = C_1 \cos \alpha x + C_2 \sin \alpha x$

$y' = -\alpha C_1 \sin \alpha x + \alpha C_2 \cos \alpha x$

$y(0) = 0 \Rightarrow 0 = C_1 \cos 0 + C_2 \sin 0$

$0 = C_1$

$y'(\pi) = 0 \Rightarrow \alpha C_2 \cos(\alpha \pi) = 0$

$\cos \alpha \pi = 0$

$\alpha \pi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$\alpha = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$

$\alpha = n - \frac{1}{2},\ n = 1, 2, 3 \ldots$

$\alpha = n - \frac{1}{2} \Rightarrow -\alpha^2 = -(n-\frac{1}{2})^2$

$1 - \lambda = -(n - \frac{1}{2})^2$

$1 + (n - \frac{1}{2})^2 = \lambda$

$y_n = C \sin((n - \frac{1}{2})x)$

$\lambda_n = 1 + (n - \frac{1}{2})^2,\ n = 1, 2, 3 \ldots$