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- [signature]

MA 346 HW 8

2b. $\int_{-0.5}^{0.5} x \ln(x+1)\,dx$, $n=6$

$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right]$

$h = \frac{b-a}{n} = \frac{0.5-(-0.5)}{6} = \frac{1}{6}$

$= \frac{1}{12}\left[f(-0.5) + 2(f(-1/3) + f(-1/6) + f(0) + f(1/6) + f(1/3)) + f(0.5)\right]$

$= \frac{1}{12}\left[0.34657 + 2(0.13576 + 0.03038 + 0 + 0.02569 + 0.09589) + 0.20273\right]$

$= 0.093728$

4b. $\int_{-0.5}^{0.5} x\ln(x+1)\,dx$

$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{n/2-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right]$

$h = \frac{b-a}{n} = \frac{1}{6}$

$\sum_{j=1}^{n/2-1} f(x_{2j}) = f(-1/6) + f(1/6) = 0.03038 + 0.02569 = 0.05607$

$\sum_{j=1}^{n/2} f(x_{2j-1}) = f(-1/3) + f(0) + f(1/3) = 0.13576 + 0 + 0.09589 = 0.23165$

$= \frac{1}{18}\left[0.34657 + 2(0.05607) + 4(0.23165) + 0.20273\right]$

$= 0.0882244$

6b. $\int_a^b f(x) = 2h\sum_{j=0}^{n/2} f(x_{2j})$ $h = \frac{b-a}{n+2} = \frac{0.5+0.5}{8} = \frac{1}{8}$

$= 2\left(\frac{1}{8}\right)\sum_{j=0}^{3} f(x_{2j}) = \frac{1}{4}(f(x_0) + f(x_2) + f(x_4) + f(x_6))$

$= \frac{1}{4}(f(-3/8) + f(-1/8) + f(1/8) + f(3/8))$

$= \cancel{\frac{1}{4}(0.33353 + 0.01669 + 0.01472 + 0.11942)}$

$= \cancel{0.12104}$

$= \frac{1}{4}(0.17625 + 0.01669 + 0.01472 + 0.11942)$

$= 0.08177$

4.5 16: $\int_0^1 x^2 e^{-x}\,dx \Rightarrow R_{3,3}$

$R_{1,1} = \frac{1}{2}\left[f(0)+f(1)\right] = \frac{1}{2}\left(0+e^{-1}\right) = 0.18394$

$R_{2,1} = \frac{1}{4}\left[f(0) + 2(f(0.5)) + f(1)\right] = \frac{1}{4}\left[0+2(0.15163)+e^{-1}\right]$
$= 0.167786$

$R_{3,1} = \frac{1}{8}\left[f(0)+2\left(f(0.25)+f(0.5)+f(0.75)\right)+f(1)\right]$
$= \frac{1}{8}\left[0+2(0.048675+0.151632+0.265706)+0.367879\right]$
$= 0.162488$

$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1}-R_{1,1}) = 0.1624013$

$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1}-R_{2,1}) = 0.160722$

$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2}-R_{2,2}) = 0.16061$

7.

x	1	2	3	4	5
$f(x)$	2.4142	2.6734	2.8974	3.0976	3.2804

$\int_1^5 f(x)\,dx$

$R_{1,1} = \frac{4}{2}\left[f(1)+f(5)\right] = 11.3892$

$R_{2,1} = \frac{4}{4}\left[f(1) + 2(f(3)) + f(5)\right] = 11.4894$

$R_{3,1} = \frac{4}{8}\left[f(1)+2\left(f(2)+f(3)+f(4)\right)+f(5)\right] = 11.5157$

$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1}-R_{1,1}) = \cancel{11.486}\ 11.5896$

$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1}-R_{2,1}) = 11.52446$

$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2}-R_{2,2}) = 11.52012$

9. $f(2.25) = A \quad f(2.5) = B \quad f(2.75) = C.$

$$R_{3,1} = \frac{1}{8}[f(2) + 2(A+B+C) + f(3)] = 0.43687$$
$$= \frac{1}{8}[0.51342 + 2A + 2B + 2C + 0.36788] = 0.43687$$
$$\frac{1}{4}(A+B+C) = 0.43687 - \frac{1}{8}(0.88130)$$
$$\frac{1}{4}(A+B+C) = 0.3267075$$

$$R_{1,1} = \frac{1}{2}(0.51342 + 0.36788) = 0.44065.$$

$$R_{2,1} = \frac{1}{4}(0.51342 + 2B + 0.36788) = \frac{1}{2}B + 0.220325.$$

$$R_{2,2} = R_{2,1} - \frac{1}{3}(R_{2,1} - R_{1,1})$$
$$= \frac{1}{2}B + 0.220325 - \frac{1}{3}(\frac{1}{2}B + 0.220325 - 0.44065)$$
$$= \frac{1}{2}B + 0.220325 - \frac{1}{6}B + 0.07344$$
$$= \frac{1}{3}B + 0.293766.$$

$$R_{3,2} = R_{3,1} - \frac{1}{3}(R_{3,1} - R_{2,1})$$
$$= 0.43687 - \frac{1}{3}(0.43687 - (\frac{1}{2}B + 0.220325))$$
$$= 0.43687 - \frac{1}{3}(0.216545 - \frac{1}{2}B)$$
$$= \frac{1}{6}B + 0.364688$$

$$R_{3,3} = R_{3,2} - \frac{1}{15}(R_{3,2} - R_{2,2})$$
$$0.43662 = \frac{1}{6}B + 0.364688 - \frac{1}{15}(\frac{1}{6}B + 0.364688 - (\frac{1}{3}B + 0.293766))$$
$$= \frac{1}{6}B + 0.364688 - \frac{1}{15}(-\frac{1}{6}B + 0.070922)$$
$$0.43662 = \frac{17}{90}B + 0.3599598$$
$$B = 0.405847$$

$n=2$

4.7.2b. $\int_1^{1.6} \frac{2x}{x^2-4}\,dx = \int_{-0.3}^{0.3} \frac{2(t+1.3)}{(t+1.3)^2-4}\,dt$

$= \int_{-1}^{1} \frac{2(\frac{10}{3}t+1.3)}{(\frac{10}{3}t+1.3)^2-4}\,dt$

$= \frac{2(\frac{10}{3}(0.5773502692)+1.3)}{(\frac{10}{3}(0.5773502692)+1.3)^2-4} + \frac{2(\frac{10}{3}(-0.5773502692)+1.3)}{(\frac{10}{3}(-0.5773502692)+1.3)^2-4}$

$= 1.008065106 + 0.47299$

$t = \frac{2x-1-1.6}{1.6-1} \Rightarrow t = \frac{2x-2.6}{0.6} \Rightarrow \frac{0.6t+2.6}{2} = x$

$\frac{3}{10}t+1.3 = x.$

$\int_1^{1.6} \frac{2x}{x^2-4}\,dx = \int_{-1}^{1} \frac{2(\frac{3}{10}t+1.3)}{(\frac{3}{10}t+1.3)^2-4}\,dt \cdot 0.3$

$\frac{3}{10}t+1.3,\ t=0.5773502692 \Rightarrow 1.473205081$

$\frac{3}{10}t+1.3,\ t=-0.5773502692 \Rightarrow 1.126794919$

$0.3\cdot\left(\frac{2\cdot1.47320581}{(1.473205081)^2-4} + \frac{2\cdot1.126794919}{1.126794919^2-4}\right) = 0.3(-2.43574\ldots)$

$= -0.730723086$

4(2b). $\frac{3}{10}t+1.3 \qquad t=0 \Rightarrow 1.3$

$t = 0.7745966692 \Rightarrow 1.532379001 \Rightarrow A$

$t = -0.7745966692 \Rightarrow 1.067620999 \Rightarrow B.$

$0.3\cdot\int_{-1}^{1} \frac{2(\frac{3}{10}t+1.3)}{(\frac{3}{10}t+1.3)^2-4}\,dt = 0.3\cdot\left(\frac{2\cdot A}{A^2-4}\cdot 0.5 + 0.\overline{8}\cdot\frac{2\cdot1.3}{1.3^2-4} + 0.5\cdot\frac{2\cdot B}{B^2-4}\right)$

$= 0.3(-2.445996741)$

$= -0.733799022$

11. $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \quad f'(x) = a_1 + 2a_2x + 3a_3x^2$

$$\int (a_0 + a_1x + a_2x^2 + a_3x^3)\,dx = af(-1) + b(f(1)) + cf'(-1) + df'(1)$$

$$= a(a_0 - a_1 + a_2 - a_3) + b(a_0 + a_1 + a_2 + a_3)$$
$$+ c(a_1 - 2a_2 + 3a_3) + d(a_1 + 2a_2 + 3a_3)$$

$a_0 \int 1\,dx +$ $\quad = a_0(a+b) + \cancel{a_1(\ldots) + a_1(\ldots)}$

$a_1 \int x\,dx +$ $\quad a_1(b+c+d-a) +$

$a_2 \int x^2\,dx +$ $\quad a_2(2d+a+b-2c) +$

$a_3 \int x^3\,dx$ $\quad a_3(3d+3c+b-a)$

$a_0 \int_{-1}^{1} dx = a_0(a+b) \qquad a_1 \int_{-1}^{1} x\,dx = a_1(b+c+d-a)$

$2 = a+b \qquad 0 = b+c+d-a$

$a_2 \int_{-1}^{1} x^2\,dx = a_2(2d+a+b-2c) \qquad a_3 \int_{-1}^{1} x^3\,dx = a_3(3d+3c+b-a)$

$2/3 = 2d+a+b-2c \qquad 0 = 3d+3c+b-a.$

$b = 2-a \qquad \cancel{0 = 2 + a + \ldots}$

$\cancel{2/3 = 2d + 2 - 2c}$

$\cancel{-\tfrac{4}{3} = 2d - 2c}$

$\cancel{-2/3 = d - c}$

$\cancel{c = d + 2/3}$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -2 & 2 & 2/3 \\ -1 & 1 & 3 & 3 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 1 & 2 \\ 0 & 0 & -2 & 2 & -4/3 \\ 0 & 2 & 3 & 3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 1 & 2 \\ 0 & 0 & -2 & 2 & -4/3 \\ 0 & 0 & 2 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 1 & 2 \\ 0 & 0 & -2 & 2 & -4/3 \\ 0 & 0 & 0 & 4 & -4/3 \end{bmatrix} \Rightarrow 4d = -4/3 \Rightarrow d = -1/3$$

$-2c + 2d = -4/3 \Rightarrow c = \frac{-4/3 - 2d}{-2} = \frac{-4/3 + 2/3}{-2} = 1/3$

$2b + c + d = 2 \Rightarrow 2b + 1/3 - 1/3 = 2 \Rightarrow b = 1$

$a + b = 2 \Rightarrow a + 1 = 2 \Rightarrow a = 1.$

$a = 1,\ b = 1,\ c = 1/3,\ d = -1/3$