I pledge on my honor that I have abided by the Stevens Honor System
[illegible signature]

MA 346 Take Home Quiz

a. I agree. In $\sin(f(x))$, as $f(x)$ approaches 0, $\sin(f(x)) \approx f(x)$. In the function $\frac{1}{x}$, $\lim_{x\to\infty} \frac{1}{x} = 0$, thus, at large values, $\sin\frac{1}{x} = \frac{1}{x}$, and $\frac{1}{x} \in O(\frac{1}{x})$.

b. $f(x) = 2 + x - \arctan x$, $f'(x) = 1 - \frac{1}{x^2+1}$. Because $f'(x) < 1$, for all values of x, and $f(x)$ and $f'(x)$ is continuous for all x, $f(x)$ must have exactly one fixed point. This is an extension of theorem 2.3, that is applied to all values, as $a = -\infty$, and $b = \infty$. Therefore, I agree.

c. I agree. Bisection method is a guaranteed, but slow way to approach the fixed point. However, a few iterations of this method will help prevent Newton's method from diverging due to extreme values of $f(x)$. In some cases, however, Newton's method will always fail, no matter how many iterations of bisection is performed.

d. $p_n = \frac{1}{2^n}$. $\lim_{n\to\infty} \frac{\frac{1}{2^{n+1}}}{\left(\frac{1}{2^n}\right)^{\alpha}} \overset{?}{=} \lambda$ (Definition 2.7)

$p = 0$

$\alpha = 2$ (quadratic)

$$\lim_{n\to\infty} \frac{\frac{1}{2^{n+1}}}{\left(\frac{1}{2^n}\right)^2} = \lim_{n\to\infty} \frac{\frac{1}{2^{n+1}}}{\frac{1}{2^{2n}}} = \lim_{n\to\infty} \frac{2^{2n}}{2^{n+1}} = \infty$$

(not quadratically convergent)

I do not agree.

e. $f(x) = 6e^x - 2x^3 + 18x - 12$ (continuous on $[0,1]$)

$f(0) = 6 - 12 = -6$ (negative)

$f(1) = 6e - 2 + 18 - 12 = 6e + 4$ (positive)

$f'(x) = 6e^x - 6x^2 + 18$ (continuous on $[0,1]$)

By Intermediate Value Theorem, $f(x)$ must equal 0 on the interval $[0,1]$, therefore a root exists.

I agree.

2. actual values, $21^{1/3} = 2.758924176$

n	(a)	(b)
0	1	1
1	7.6666	1.95238
2	5.2302	2.12175
3	3.74269	2.242849
4	2.99485	2.334879
5	2.7770	2.407043
6	2.7590	2.468059288
7	2.758924	2.5122434
8	2.758924176	2.551057

The first equation converges faster to the true value than equation (b), therefore (a) is better.

3. (a) $P_3(x) = 2 - 1x + (x)(x-1) - 2(x)(x-1)(x-2)$

(b) We should get the same polynomial upon simplification. This is because the nodes themselves are the same and any interpolating polynomial to the nth degree with the same nodes is unique. Therefore the polynomial will be the same.

(c)

x_i	$f(x_i)$	1st DD	2nd DD	3rd DD	4th DD
-1	2				
		-1			
0	1		1	0.5	-2.5
		1	2	-2	
1	2				
		3	-12		
1	2				
		-9			
2	-7				

2nd DD =

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$$

$$= \frac{3-1}{1-0} = 2$$

$$f[x_2, x_3, x_4] = \frac{f[x_3, x_4] - f[x_2, x_2]}{x_4 - x_2}$$

$$= \frac{-9-3}{2-1} = -12$$

3rd DD =

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$$

$$= \frac{2-1}{1-(-1)} = \frac{1}{2}$$

$$f[x_1, x_2, x_3, x_4] = \frac{f[x_2, x_3, x_4] - f[x_1, x_2, x_3]}{x_4 - x_1}$$

$$= \frac{-12-2}{2-0} = -7$$

4th DD.

$$f[x_0, x_1, x_2, x_3, x_4] = \frac{f[x_1, x_2, x_3, x_4] - f[x_0, x_1, x_2, x_3]}{x_4 - x_0}$$

$$= \frac{-7-0.5}{2-(-1)} = -2.5$$

$$P_4(x) = 2 - x + (x)(x-1) + 0.5(x)(x-1)^2 - 2.5(x)(x-1)^2(x-2)$$

(d) The error is $\frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\dots(x-x_n)$

$n=3$, so $|\text{error}| \leq \frac{f^4(\xi(x))}{4!}(1.5-(-1))(1.5-0)(1.5-1)(1.5-2)$

$$|\text{error}| \leq \left|\frac{24}{4!}(2.5)(1.5)(0.5)(-0.5)\right|$$

$$\text{error} \leq 0.9375$$

4. $P_{1,2} = \dfrac{(x-x_1)P_1 - (x-x_2)P_2}{x_2 - x_1}$

$= \dfrac{(x-1.3)0.6200860 - (x-1.6)0.4554022}{1.6-1.3}$

$P_{1,2}(1.5) = \dfrac{(0.2)\,0.6200860 - (-0.1)\,0.4554022}{0.3}$

$= 0.5651914$

5. $h_0 = h_1 = 1 \quad n=2 \quad a_0 = f(-1) = 5 \quad a_1 = f(0) = 7, \; a_2 = f(1) = 9.$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ \frac{3}{1}(9-7) - \frac{3}{1}(7-5) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$

$c_0 = 0, \quad c_1 + 4c_2 + c_3 = 0, \quad c_7 = 0 \Rightarrow c_2 = 0.$

$b_0 = \frac{1}{h_0}(a_1 - a_0) - \frac{1}{3}(c_1 + 2c_0) = \frac{1}{1}(7-5) = 2$

$b_1 = \frac{1}{h_1}(a_2 - a_1) - \frac{1}{3}(c_2 - 2c_1) = \frac{1}{1}(9-5) = 2$

$d_0 = \frac{1}{3h_0}(c_1 - c_0) = 0$

$d_1 = \frac{1}{3h_0}(c_2 - c_1) = 0.$

$S(x) = \begin{cases} 5 + 2(x+1) + 0(x+1)^2 + 0(x+1)^3, & x \in [-1, 0] \\ 7 + 2x + 0x^2 + 0x^3 & , x \in [0, 1] \end{cases}$

$S(x) = \{ 7 + 2x, \; x [-1, 1].$