MA 346 HW I

1.1 36. $2x\cos(2x) - (x+1)^2 = 0$

try $x = 0$: $f(0) = 0(\cos(0)) - (0+1)^2$
$f(0) = -1$

try $x = -\frac{\pi}{2}$: $f(-\frac{\pi}{2}) = -2 \cdot \frac{\pi}{2}\cos(2 \cdot \frac{\pi}{2}) - (-\frac{\pi}{2}+1)^2$
$= -\pi \cdot \cos\pi - (1 - \frac{\pi}{2})^2$
$= \pi - (1 - \frac{\pi}{2})^2$
$f(-\frac{\pi}{2}) \approx 2.816$

try $x = -\pi$: $f(-\pi) = -2 \cdot \pi \cos(-2\pi) - (1-\pi)^2$
$-2\pi(1) - (1-\pi)^2$
$f(-\pi) < 0$

Because f is continuous on $[-\pi, 0]$, by the Intermediate Value Theorem, f has solutions on $(-\pi, -\frac{\pi}{2})$ and $(-\frac{\pi}{2}, 0)$.

5c. $f(x) = 2x\cos(2x) - (x-2)^2$, $[2,4]$
First, $f(2)$ and $f(4)$.
$f(2) = 2 \cdot 2\cos(4) - (2-2)^2 = 4\cos(4) - 0 = 4\cos 4 = -2.614$
$f(4) = 2 \cdot 4\cos 8 - (4-2)^2 = 8\cos 8 - 4 = -5.164$
$f'(x) = 2\cos(2x) - 4x\sin(2x) - 2(x-2)$
$f'(x) = 0$ @ 3.131 (using graphing utility).
$f(3.131) = 6.262\cos(6.262) - (4.262)^2 = 4.981$
$\max_{2 \le x \le 4} |f(x)| = 5.164$ @ $x = 4$.

7a. $f(x) = x\sin\pi x - (x-2)\ln x$
$f(1) = \sin\pi - (1-2)\ln 1$
$= 0 - 0 = 0$.
$f(2) = 2\sin 2\pi - (2-2)\ln 2$
$= 0 - 0 = 0$.

Because $f(1) = f(2) = 0$, by Rolle's Theorem, there must be a point x, $1 \le x \le 2$, where $f'(x) = 0$.

11. $f(x) = e^x \cos x$

$P_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2$

$x_0 = 0$.

$f(0) = e^0 \cos 0 = 1$

$f'(x) = e^x \cos x - e^x \sin x$, $f'(0) = e^0 \cos 0 - e^0 \sin 0 = 1$

$f''(x) = e^x \cos x - e^x \sin x - e^x \sin x - e^x \cos x = -2e^x \sin x$

$f''(0) = 0.$

$P_2(x) = 1 + x$

a. $P_2(0.5) = 1 + 0.5 = 1.5$.

$f^3(x) = -2e^x \sin x - 2e^x \cos x$

$R_3(x) = \frac{-2e^{\xi(x)}(\sin \xi(x) + \cos \xi(x))}{6} x^3$

$e^x \cos x = 1.5 + \frac{0.5^3}{6}(-2e^{\xi(x)}(\sin \xi(x) + \cos \xi(x)))$

$0 \leq \xi(x) \leq 0.5$, so $\max(-2e^{\xi(x)}) = -2e^{0.5}$

$\max(\sin \xi(x) + \cos \xi(x)) = \sqrt{2}$ on any interval.

Therefore, $e^{0.5} \cos 0.5 - 1.5 = \frac{0.5^3}{6}(-2e^{\xi(x)}(\sin \xi(x) + \cos \xi(x)))$

$\leq \frac{0.5^3}{6} \cdot -2e^{0.5} \cdot -\sqrt{2}$

$\frac{0.5^3}{6} \cdot -2e^{0.5} \cdot -\sqrt{2} = 0.09715$ (error bound)

actual error $= |1.5 - e^{0.5} \cos 0.5| = |1.5 - 1.446| = 0.05311$

as expected, actual error < error bound.

b. $\max(-2e^x)$ on $[0, 1]$ is at $x = 1$, so set $\xi(x) = 1$.

$\max(\sin \xi(x) + \cos \xi(x)) = \sqrt{2}$ or $-\sqrt{2}$.

bound $= -2e^1 \cdot -\sqrt{2} \cdot \frac{1^3}{6} = 1.2814$

c. $\int_0^1 P(x)\,dx = \int_0^1 1 + x\,dx = [x + \frac{1}{2}x^2]_0^1 = 1 + \frac{1}{2} - (0 + 0) =$ 1.5

d. $\int_0^1 \frac{x^3}{6} - 2e^{\xi(x)}(\sin \xi(x) + \cos \xi(x))\,dx$

$\leq \int_0^1 \frac{x^3}{6} - 2e^1(-\sqrt{2})\,dx = \frac{2e\sqrt{2}}{6} \int_0^1 x^3\,dx = 0.320$ (error bound)

$|\int_0^1 e^x \cos x\,dx - \int_0^1 P_2(x)\,dx| = |1.378 - 1.5| = 0.122$ (actual bound)

18. $f(x) = (1-x)^{-1}$

$f'(x) = (1-x)^{-2}$ $f''(x) = 2(1-x)^{-3}$ $f'''(x) = 6(1-x)^{-4}$

$P_n(x) = 1 + 1x + \frac{2}{2}x^2 + \frac{6}{6}x^3 + \dots$

$P_n(x) = \sum_{i=0}^{n} x^i$

$R_n(x) := x^{n+1}$

$(1-x)^{-1} = P_n(x) + R_n(x)$

$|(1-x)^{-1} - P_n(x)| = R_n(x)$

$R_n(x) \le 10^{-6}$ $0 \le x \le 0.5$

$x^{n+1} \le 10^{-6}$ $\max(x) = 0.5$

$0.5^{n+1} \le 10^{-6}$

$n+1 \ge \log_{0.5} 10^{-6}$

$n \ge \log_{0.5} 10^{-6} - 1$

$n \ge 18.93$ $n = 19$

1.2. 4(c). $|\sqrt{2} - p^*| \le 10^{-4}$

$-10^{-4} \le \sqrt{2} - p^* \le 10^{-4}$

$-10^{-4} - \sqrt{2} \le -p^* \le 10^{-4} - \sqrt{2}$

$\sqrt{2} - 10^{-4} \le p^* \le \sqrt{2} + 10^{-4}$

5(d) i. $\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20}$

$\left(\frac{11}{33} + \frac{9}{33}\right) - \frac{3}{20}$

$\frac{20}{33} - \frac{3}{20}$

$\frac{400}{660} - \frac{99}{660} = \frac{301}{660}$

$= 0.456066$

ii. $\frac{1}{3} = 0.333$

$\frac{3}{11} = 0.272$

$\frac{3}{20} = 0.150$

$\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20} =$

$0.333 + 0.272 - 0.150 =$

0.455

iii. $\frac{1}{3} = 0.333$

$\frac{3}{11} = 0.273$

$\frac{3}{20} = 0.150$

$\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20} =$

$0.333 + 0.273 - 0.150 =$

0.456

iv. $\dfrac{\frac{301}{660} - 0.455}{\frac{301}{660}} = 2.32 \times 10^{-3}$ $\dfrac{\frac{301}{660} - 0.456}{\frac{301}{660}} = 1.33 \times 10^{-4}$

6 c. $(121 - 0.327) - 119$

$(0.121 \times 10^3 - 0.327 \times 10^0) - 0.119 \times 10^3$

$0.121 \times 10^3 - 0.119 \times 10^3$

0.200×10^1

$|0.200 \times 10^1 - 1.673| = 0.32700 \times 10^0$ (absolute error)

$\frac{0.327}{1.673} = 0.19546$ (relative error)

d. $(121 - 119) - 0.327$

$(0.121 \times 10^3 - 0.119 \times 10^3) - 0.327 \times 10^0$

$0.200 \times 10^1 - 0.327 \times 10^0$

0.167×10^1

$|0.167 \times 10^1 - 1.673| = 0.0030000$ (absolute error)

$\frac{0.003}{1.673} = 0.0017932$ (relative error)

13. a. $\lim_{x \to 0} \frac{x \cos x - \sin x}{x - \sin x} = \lim_{x \to 0} \frac{\cos x - x \sin x - \cos x}{1 - \cos x}$

$= \lim_{x \to 0} \frac{-x \sin x}{1 - \cos x} = \lim_{x \to 0} \frac{-\sin x - x \cos x}{\sin x} = \lim_{x \to 0} \frac{-\cos x - \cos x + x \sin x}{\cos x}$

$= \frac{-2}{1}$ (L'Hopital's rule)

b. $\cos 0.1 = 0.9950 \times 10^0$

$\sin 0.1 = 0.9983 \times 10^{-1}$

$f(0.1) = \frac{0.1000 \times 10^0 \cdot 0.9950 \times 10^0 - 0.9983 \times 10^{-1}}{0.1000 \times 10^0 - 0.9983 \times 10^{-1}}$

$= \frac{0.9950 \times 10^{-1} - 0.9983 \times 10^{-1}}{0.17 \times 10^{-3}} = \frac{-0.33 \times 10^{-3}}{0.1700 \times 10^{-3}} = -1.941$

c. $M_3(\cos(x)) = 1 - \frac{1}{2}x^2$

$M_3(\sin(x)) = x - \frac{1}{6}x^3$

$f(x) = \frac{x(1 - \frac{1}{2}x^2) - x + \frac{1}{6}x^3}{x - x + \frac{1}{6}x^3} = \frac{x - \frac{1}{2}x^3 - x + \frac{1}{6}x^3}{\frac{1}{6}x^3}$

$x^3 = 0.1 \times 10^{-2}$ $\quad \frac{-\frac{1}{2}x^3 - \frac{1}{6}x^3}{\frac{1}{6}x^3} = \frac{-\frac{1}{2}(0.1000 \times 10^{-2}) - \frac{1}{6}(0.1000 \times 10^{-2})}{\frac{1}{6}(0.1000 \times 10^{-2})}$

$= \frac{-0.5000 \times 10^{-3} - 0.1667 \times 10^{-3}}{0.1667 \times 10^{-3}} = \frac{-0.3333 \times 10^{-3}}{0.1667 \times 10^{-3}} = -2$

d. (i) $\frac{\left| -1.99899998 + 1.941 \right|}{\left| -1.9989998 \right|} = 2.9015 \times 10^{-2}$

(ii) $\frac{\left| -1.9989998 + (2) \right|}{\left| -1.9989998 \right|} = 5.0026 \times 10^{-4}$