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MA 331

Professor Li

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I pledge my honor that I have abided by the Stevens Honor System.

1. Find the moment estimator and maximum likelihood estimator for θ of the uniform distribution on (0, θ).

Moment estimator: as X = U(0,θ), thus f(x) = 1/θ

$$\frac{1}{n}\sum_{i=1}^{n} X_i = E_\theta[X]$$
$$E_\theta[X] = \int_0^\theta x f(x) dx$$
$$E_\theta[X] = \int_0^\theta \frac{x}{\theta} dx$$
$$E_\theta[X] = \frac{x^2}{2\theta}|_0^\theta$$
$$E_\theta[X] = \frac{\theta^2}{2\theta} - \frac{0}{2\theta}$$
$$E_\theta[X] = \frac{\theta}{2}$$
$$\frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$$
$$\frac{\theta}{2} = \bar{X}$$
$$\hat{\theta} = 2\bar{X}$$

Maximum Likelihood Estimator:

$$f(x) = \frac{1}{\theta}$$
$$L(\hat{\theta}, x) = \prod_{i=1}^{n} \frac{1}{\theta}$$
$$L(\hat{\theta}, x) = \frac{1}{\theta^n}$$
$$\log L(\hat{\theta}, x) = \log \frac{1}{\theta^n}$$
$$\log L(\hat{\theta}, x) = \log 1 - \log(\theta^n)$$
$$\log L(\hat{\theta}, x) = 0 - \text{nlog}(\theta)$$
$$\frac{\partial \log L(\hat{\theta}, x)}{\partial \theta} = -\frac{n}{\theta}$$

This function is always decreasing and therefore will be maximized at the largest value of $x_n <= \theta$ in the sample. Thus, given that the random samples are ordered from least to greatest, and $0 <= x_n <= \theta$ for all n, the maximum likelihood estimator $= x_n$, the largest value x in the sample.

Textbook Problems

6.17.

Margin of error $= \frac{z^*\sigma}{\sqrt{n}}$

$$1.96\frac{6.5}{\sqrt{31}} = 2.288 \approx 2.29U/l$$

95% Confidence Interval = mean ± margin of error = (13.2-2.29, 13.2+2.29) = (10.91, 15.49)

6.27.

(a) Margin of error = $1.96 * \frac{8.3}{\sqrt{1200}} = 0.47$

95% Confidence Interval = = mean ± margin of error = (11.5-0.47, 11.5+0.47) = (11.03, 11.97)

(b) No because 17% of the students say they did not listen to radio, which means 17% of the sample population is definitely outside of the 95% confidence interval, which does not include 0.

(c) This should nevertheless be a good approximation because the sample size is large, and still represents the average listening hours of total population instead of a population that, for example, regularly listens to radio.

6.28.

(a) mean = 11.5*60 = 690, standard deviation = 8.3 * 60 = 498

(b) Margin of error = $1.96 * \frac{498}{\sqrt{1200}} = 28.18$,

95% confidence = (690 – 28.18, 690 + 28.18) = (661.82, 718.18)

(c) Multiplying the interval by 60 would have netted me the same answer.
(11.03*60, 11.97*60) = (661.8, 718.2)

6.58.

(a) P (z>1.77) = 0.0384

(b) P (z<1.77) = 0.9616

(c) P (z>1.77 or z<-1.77) = 0.0768

6.59.

(a) P (z > -1.69) = 0.9545

(b) P (z<-1.69) = 0.0455

(c) P (z>1.69 or z<-1.69) = 0.0910

6.71.

(a)

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$$z = (127.8 - 115) * \frac{\sqrt{25}}{30} = 2.13$$

$$P(z > 2.13) = 1 - 0.9834 = 0.0166$$

Based on the P value, there is a 1.66% chance the mean has not changed, supporting H_a, that the attitude toward school among older students is better.

(b)

We assumed that the tests were a simple random sample, and that the tests are normally distributed. The fact that it is an SRS is more important, as there were no known outliers or known skew to the sample.

6.73.

Sample mean = sum of all samples/number of samples = 2.73

$H_0 : \mu_0 = 0$
$H_a : \mu_0 \neq 0$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$$z = 2.73 * \frac{\sqrt{20}}{3} = 4.06$$

$$P(z > 4.06 \ or \ P < -4.06)$$

P is therefore very small, thus supporting H_a, that the computer's calculations and the driver's calculations differ significantly.

6.99.

(a)

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$$z = (2453.7 - 2403.7) * \frac{\sqrt{100}}{880} = 0.568$$

$$P(z > 0.568) = 1 - 0.7157 = 0.2843$$

(b)

$$z = 50 * \frac{\sqrt{500}}{880} = 1.27$$

$$P(z > 1.27) = 1 - 0.8980 = 0.1020$$

(c)

$$z = 50 * \frac{\sqrt{2500}}{880} = 2.84$$
$$P(z > 2.84) = 1 - .9977 = 0.0023$$

6.120.

(a)

$$P(X \le 2 \; and \; X \le 2 \; chosen \; from \; P_0) = \frac{(0.1 + 0.1 + 0.2)}{2} * \frac{0.1 + 0.1 + 0.2 + 0.2 + 0.2 + 0.2}{2}$$
$$0.2 * 0.5 = 0.1$$

There is a 10% chance of a Type I error.

(b)

$$P(X \ge 2 \; and \; X \ge 2 \; chosen \; from \; P_1) = \frac{0.1 * 7 + 0.3}{2} * \frac{0.1 * 4}{2} = 0.5 * 0.2 = 0.1$$

There is a 10% chance of a Type II error.

7.22. n = 16, t=2.15

(a) 7 degrees of freedom

(b) 1.895 < t < 2.365

(c) 0.05>P>0.025

(d) It is significant at the 5% level, but not significant at the 1% level.

(e) P = 0.0343

7.23. n = 27, t = 2.01

(a) 26 degrees of freedom

(b) 1.706 < t < 2.056

(c) 0.10 > P > 0.05

(d) It is not significant at either level.

(e) P = 0.549