

I please my honor that I  
have added for the Stevens Honor  
Section - me my

HW 6 MA 346

32. 5.  $P_2 \approx f(0.5)$

$$f(0) = 1, f(1) = 2, f(0.5) = 8, f(x) = 2^{4x}$$

$$f(0.5) = 2^2 = 4$$

$$7. x_0 = 0 \quad P_0 = 1$$

$$x_1 = 1 \quad P_1 = 3 \quad P_{0,1}(2.5) = 6$$

$$x_2 = 2 \quad P_2 = 3 \quad P_{1,2} = 3 \quad P_{0,1,2} = 2.25$$

$$x_3 = 3 \quad P_3 = 1 \quad P_{2,3} = \frac{13}{3} \quad P_{1,2,3}(2.5) = 3$$

$$P_{0,1}(x) = 2x + 1$$

$$P_{0,2}(x) = x + 1$$

$$P_{0,1,2}(x) = \frac{(x-x_0)P_1 - (x-x_1)P_0}{x_1-x_0}$$

$$P_{0,2}(x) = \frac{(x-x_0)P_2 - (x-x_2)P_0}{x_2-x_0}$$

$$= \frac{(x-0)P_1 - (x-1)P_0}{1-0}$$

$$x+1 = \frac{P_2 x - P_0 x + 2P_0}{2}$$

$$2x+1 = P_1 x - P_0 x + P_0$$

$$2x+2 = P_2 x - x + 2$$

$$P_0 = 1$$

$$3x = P_2 x \Rightarrow P_2 = 3$$

$$P_1 = 3$$

$$P_{1,2} = \frac{(x-x_1)P_2 - (x-x_2)P_1}{x_2-x_1} = \frac{(2.5-1)3 - (2.5-2)3}{2-1} = \frac{4.5-1.5}{1} = 3$$

$$P_{0,1,2}(2.5) = \frac{(x-x_0)(P_{1,2}) - (x-x_2)(P_{0,1})}{x_2-x_0} = \frac{(2.5-0)(3) - (2.5-2)(6)}{2-0}$$

$$= \frac{7.5-3}{2} = 2.25$$

$$P_{0,1,2,3} = \frac{(x-x_0)P_{1,2,3} - (x-x_3)P_{0,1,2}}{x_3-x_0} = \frac{(2.5-0)(3) - (2.5-3)(1.25)}{3-0}$$

$$= 2.875$$



$$33.7a. -0.1 \quad 5.30$$

$$0.0 \quad 2.00 \quad -33$$

$$0.2 \quad 3.19 \quad 5.95 \quad 129.83$$

$$0.3 \quad 1.00 \quad -21.9 \quad -92.83 \quad -556.6$$

$$f(x_0, x_1) = \frac{f(0.0) - f(0.1)}{0.1} = \frac{2.00 - 5.3}{0.1} = -33$$

$$f(x_1, x_2) = \frac{f(0.2) - f(0.1)}{0.1} = \frac{3.19 - 2}{0.1} = 5.95$$

$$f(x_2, x_3) = \frac{f(0.3) - f(0.2)}{0.1} = \frac{1.00 - 3.19}{0.1} = -21.9$$

$$f(x_0, x_1, x_2) = \frac{f(x_2, x_1) - f(x_1, x_0)}{x_2 - x_0} = \frac{5.95 - (-33)}{0.2 - (-0.1)} = 129.83$$

$$f(x_1, x_2, x_3) = \frac{f(x_3, x_2) - f(x_2, x_1)}{x_3 - x_1} = \frac{-21.9 - 5.95}{0.3 - 0.1} = -92.83$$

$$f(x_0, \dots, x_3) = \frac{f(x_3, x_2, x_1) - f(x_2, x_1, x_0)}{x_3 - x_0} = \frac{-92.83 - 129.83}{0.3 - (-0.1)} = -556.6$$

$$p_3(x) = 5.3 - 33(x+0.1) + 129.83(x+0.1)x - 556.6(x+0.1)(x)$$

$$b. -0.1 \quad 5.30 \quad (x-0.2)$$

$$0.0 \quad 2.00 \quad -33$$

$$0.2 \quad 3.19 \quad 5.95 \quad 129.83$$

$$0.3 \quad 1.00 \quad -21.9 \quad -92.83 \quad -556.6$$

$$0.35 \quad 0.97260 \quad -0.548 \quad 142.346 \quad 671.942 \quad 2730.243$$

$$f(x_3, x_4) = \frac{0.97260 - 1.00}{0.35 - 0.3} = -0.548$$

$$f(x_2, x_3, x_4) = \frac{-0.548 - (-21.9)}{0.35 - 0.2} = 142.346$$

$$f(x_1, x_2, x_3, x_4) = 142.346 - (-92.83) = 671.942$$

$$f(x, \dots, x_4) = \frac{671.942 - (-556.6)}{0.35 - (-0.1)} = 2730.24$$



$$P_4(x) = 5.3 - 33(x+0.1) + 129.83(x+0.1)(x) - 556.6(x+0.1)(x)(x-0.2) + 2730.243(x+0.1)(x)(x-0.2)(x-0.3)$$

12.  $x \quad f(x) \quad \Delta \quad \Delta^2 \quad \Delta^3 \quad \Delta^4$

0	4				
1	9	5	1	-4	
2	15	6	-3	?	1
3	18	3	?		
		?			

$$P(x) = 4 + 5(x) + \frac{1}{2!}(x)(x-1) - \frac{4}{3!}(x)(x-1)(x-2) + \frac{1}{4!}(x)(x-1)(x-2)(x-3)$$

$$P(x) = 4 + 5x + \frac{1}{2}(x^2 - x) - \frac{2}{3}(x^3 - 3x^2 + 2x) + \frac{1}{24}(x^4 - 6x^3 + 11x^2 - 6x)$$

$$= \frac{1}{24}x^4 - \frac{1}{4}x^3 + \frac{5}{4}x^2 + \frac{1}{4}x + 4$$

The coefficient of  $x^3$  is  $-\frac{1}{4}$ .

3.4.1a.

$x$	$f(x)$	$f'(x)$
8.3	17.56492	3.116256
8.6	18.50515	3.151762

2.  $f(2) = 17.56492$

$f[2_0, 2_1] = 3.116256$

$2_0 = 8.3 \quad 17.56492$

$f[2_0, 2_1, 2_2] = 0.05948$

$f[2_1, 2_2] = 3.1341$

$f[2_0, 2_3] = 0.00202$

$2_2 = 8.6 \quad 18.50515$

$f[2_1, 2_2, 2_3] = 0.05887$

$f[2_2, 2_3] = 3.151762$

$2_3 = 8.6 \quad 18.50515$

$$H_3(x) = 17.56492 + 3.116256(x-8.3) + 0.05948(x-8.3)^2 - 0.00202(x-8.3)^3(8.6)$$



$$2a. H_3(x) = 17.86492 + 3.116256(x-8.3) + 0.05948(x-8.3)^2 - 0.002022(x-8.3)^2(x-8.6)$$

$$H_3(8.4) = 17.86492 + 0.3116256 + 0.0005948 + 0.000024044 = 17.8771444$$

$$\text{error} = |x \ln x - H_3(x)| \\ |17.87714633 - 17.87714444| = 0.000001889 = 1.889 \times 10^{-6}$$

$$35.1. f(0)=0, f(1)=1, f(2)=2.$$

$$S_0(x) = a_0 + b_0x + c_0x^2 + d_0x^3$$

$$S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3$$

$$S_0(0) = f(0) \Rightarrow a_0 = 0 \quad S_0(1) = f(1) \Rightarrow b_0 + c_0 + d_0 = 1$$

$$S_1(1) = f(1) \Rightarrow a_1 = 1 \quad S_1(2) = f(2) \Rightarrow 1 + b_1 + c_1 + d_1 = 2$$

$$S_0'(1) = S_1'(1) \Rightarrow b_0 + 2c_0 + 3d_0 = b_1$$

$$S_0''(1) = S_1''(1) \Rightarrow 2c_0 + 6d_0 = 2c_1$$

$$S''(0) = 0 \Rightarrow 2c_0 = 0 \Rightarrow c_0 = 0.$$

$$S''(2) = 0 \Rightarrow 2c_1 + 6d_1 = 0.$$

$$1) a_0 = 0 \quad 2) b_0 + c_0 + d_0 = 1 \quad 3) a_1 = 1 \quad 4) 1 + b_1 + c_1 + d_1 = 2$$

$$5) b_0 + 2c_0 + 3d_0 = b_1 \quad 6) 2c_0 + 6d_0 = 2c_1 \quad 7) c_0 = 0 \quad 8) 2c_1 + 6d_1 = 0$$

$$c_1 = -3d_1 \quad b_0 + 3d_0 = b_1 \quad 3d_0 = c_1 \quad b_0 + d_0 = 1$$

$$d_0 = -d_1 \quad b_0 = 1 - d_0$$

$$1 + b_0 + 3d_0 + 3d_0 + (-d_0) = 2$$

$$1 + b_0 + 5d_0 = 2$$

$$1 + (1 - d_0) + 5d_0 = 2 \quad d_0 = 0 \quad b_0 = 1 \quad c_0 = 0, a_0 = 0$$

$$2 + 4d_0 = 2 \quad b_1 = 1 \quad c_1 = 0 \quad d_1 = 0, a_1 = 1$$

$$S_0(x) = x$$

$$S_1(x) = x - 1 + 1 = x$$

$$S(x) = x, [0, 2]$$



2.  $f(0)=0, f(1)=1, f(2)=2$

Same as before, so

1)  $a_0=0$ , 2)  $b_0+c_0+d_0=1$ , 3)  $a_1=1$ , 4)  $1+b_1+c_1+d_1=2$   
 5)  $b_0+2c_0+3d_0=b_1$ , 6)  $2c_0+6d_0=2c_1$

$S'_0(0)=1 \Rightarrow b_0=1$        $S''(2)=1 \Rightarrow b_1+2c_1+3d_1=1$   
 $c_0+d_0=0$

$c_0=-d_0 \Rightarrow 4d_0=2c_1 \Rightarrow 2d_0=c_1$

$b_0+2(-d_0)+3d_0=b_1 \Rightarrow b_0+d_0=b_1$

$b_1+c_1+d_1=1 = b_1+2c_1+3d_1$

$c_1+2d_1=0 \Rightarrow c_1=-2d_1 \Rightarrow 2d_0=-2d_1 \Rightarrow d_0=-d_1$

$b_1+2(2d_0)-3d_0=1$

$b_1+d_0=1$

$b_1=1-d_0$

$1+(1-d_0)+2d_0-d_0=2$

$d_0=0, d_1=0, c_1=0$

$b_1=1, c_0=0, b_0=1$

$S_0(x)=x$

$S_1(x)=(x-1)+1=x$

$S(x)=x, [0,2]$

11.  $S_0(1)=1+(2)(1)-1^3=2=S_1(1)$

$S'_0(1)=2-3(1)^2=-1=S'_1(1)$

$S'_1(x)=b \Rightarrow b=-1$

$S''_1(2)=0 \Rightarrow 2c+6d=0 \Rightarrow c=-3d$

$S''_0(1)=S''_1(1) \Rightarrow -6=2c \Rightarrow c=-3 \Rightarrow d=1$

$b=-1, c=-3, d=1$

14.  $S_1(1)=1+B+2-2 \Rightarrow B=0$

$S'_1(1)=S'_0(1) \Rightarrow b=B+4-6 \Rightarrow b=-2$

$f'(0)=S'_0(0)=B=0$

$f'(2)=S'_2(2)=b-8(1)+2(1)^2=-2-8+2=-8$