Max Shi MA 227 Test 2

I pledge my honor that I have abided by the Stevens Honor System.
[illegible]

1) $\vec{r}(t) = \langle \cos t, \sin t, t \rangle \quad 0 \le t \le 2\pi$

$\vec{r}\,'(t) = \langle -\sin t, \cos t, 1 \rangle$

$f(\vec{r}(t)) = \cos^2 t + \sin^2 t + t^2 = 1 + t^2$

$|\vec{r}\,'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{1+1} = \sqrt{2}$

$\int_C f\,ds = \int_0^{2\pi} (1+t^2)(\sqrt{2})\,dt = \sqrt{2}\int_0^{2\pi} 1+t^2\,dt$

$\text{Mass} = \sqrt{2}\left[t + \frac{t^3}{3}\right]_0^{2\pi}$

$\boxed{\text{Mass} = \sqrt{2}\left[2\pi + \frac{8\pi^3}{3}\right]}$

2. $C_1: \vec{r}(t) = \langle 0, t^2, t \rangle, \quad -1 \le t \le 1$

$\vec{r}\,'(t) = \langle 0, 2t, 1 \rangle, \quad f = \langle y, z^2, yz \rangle$

$\int_{C_1} y\,dx + z^2\,dy + yz\,dz = \int_{-1}^{1} t^2(0) + t^2(2t) + (t^2)(t)(1)\,dt$

$= \int_{-1}^{1} 2t^3 + t^3\,dt$

$= \int_{-1}^{1} 3t^3 = \left[\frac{3t^4}{4}\right]_{-1}^{1} = \frac{6}{4} = \frac{3}{2}$

$C_2: \vec{r}(t) = \langle 2t, 1, 3t+1 \rangle, \quad 0 \le t \le 1$

$\vec{r}\,'(t) = \langle 2, 0, 3 \rangle, \quad f = \langle y, z^2, yz \rangle$

$\int_{C_2} y\,dx + z^2\,dy + yz\,dz = \int_0^1 1(2) + (3t+1)^2(0) + (1)(3t+1)(3)\,dt$

$= \int_0^1 3 + 9t + 3\,dt$

$= \int_0^1 6 + 9t\,dt$

$= \left[6t + \frac{9t^2}{2}\right]_0^1 = 6 + \frac{9}{2}$

$\int_C f\,dr = \int_{C_1} f\,dr + \int_{C_2} f\,dr$

$= \frac{3}{2} + 6 + \frac{9}{2} = \boxed{12}$

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3. $P=1 \quad Q=2y\cos(y^2)+2y$

a) $P_y=0 \quad Q_x=0$

$P_y = Q_x \Rightarrow$ conservative.

$\int P\,dx = \int 1\,dx = x + h(y)$

$h'(y) = 2y\cos(y^2)+2y - \frac{d}{dy}(x)$

$h'(y) = 2y\cos y^2 + 2y - 0$

$h(y) = \int 2y\cos y^2 + 2y\,dy$

$= \int 2y\cos y^2 dy + \int 2y\,dy$

$\int \cos u\,du + y^2 + C$

$\sin u + y^2 + C$

$\sin y^2 + y^2 + C$

$\boxed{f(x,y) = x + \sin y^2 + y^2 + C}$

b) $\int_C \vec{F}\cdot d\vec{r} = f(b) - f(a)$

$\int_C \vec{F}\cdot d\vec{r} = f(0,\sqrt{\tfrac{\pi}{2}}) - f(1,0)$

$= [0 + \sin\tfrac{\pi}{2} + \tfrac{\pi}{2}] - [1 + \sin 0 + 0]$

$1 + \tfrac{\pi}{2} - 1 = \boxed{\tfrac{\pi}{2}}$

4a) $\oint_{C2} \vec{F}\cdot d\vec{r} = \iint_R Q_x - P_y\,dA$ ← unit circle, $\quad Q_x = 1,\ P_y = -1 \Rightarrow Q_x - P_y = 2$

$= \iint_R 2\,dA$

$= \int_0^{2\pi}\int_0^1 2r\,dr\,d\theta = \int_0^{2\pi}[r^2]_0^1\,d\theta = \int_0^{2\pi} d\theta = \boxed{2\pi}$

b) $Q_x - P_y = 2 \Rightarrow \iint_D 2\,dA = 2\iint_D dA = 24 = \iint_D (Q_x - P_y)\,dA$

$\oint_{C_1} F\cdot d\vec{r} - \oint_{C_2} \vec{F}\cdot d\vec{r} = \iint_D (Q_x - P_y)\,dA$

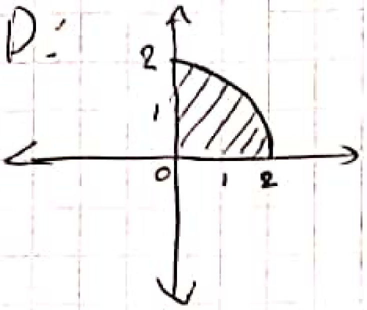
(negative because $\oint_{C_2}\vec{F}\cdot d\vec{r}$ is not positively oriented for D).

$\oint_{C_1} \vec{F}\cdot d\vec{r} - 2\pi = 24$

$\boxed{\oint_{C_1} \vec{F}\cdot d\vec{r} = 24 + 2\pi}$

5.

D:



$g(x,y) = xy \qquad g_x = y,\ g_y = x$

$\vec{r}(x,y) = \langle x, y, xy \rangle$

$|\vec{r}_x \times \vec{r}_y| = \sqrt{1 + g_x^2 + g_y^2}$

$= \sqrt{1 + y^2 + x^2}$

$f(\vec{r}(x,y)) = 1$

$\iint_S dS = \iint_D 1\left(\sqrt{1+y^2+x^2}\right) dA$

$= \int_0^2 \int_0^{\frac{\pi}{2}} \sqrt{1 + r^2\sin^2\theta + r^2\cos^2\theta}\,(r)\,d\theta\,dr \qquad r = 2 \Rightarrow u = 5,\ r = 0 \Rightarrow u = 1$

$= \int_0^2 \int_0^{\frac{\pi}{2}} \sqrt{1+r^2}\,(r)\,d\theta\,dr \qquad u = 1 + r^2,\ du = 2r \Rightarrow \frac{du}{2} = r\,dr$

$= \int_0^{\frac{\pi}{2}} \int_0^2 \sqrt{1+r^2}\, r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \int_1^5 \frac{\sqrt{u}}{2}\,du\,d\theta$

$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{3}u^{3/2}\right]_1^5 d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{3}\left[\sqrt{125} - 1\right] d\theta$

$= \frac{\pi}{2} \cdot \frac{1}{3}\left(\sqrt{125} - 1\right)$

$\iint_S dS = \boxed{\frac{\pi}{6}\left(\sqrt{125} - 1\right)}$

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