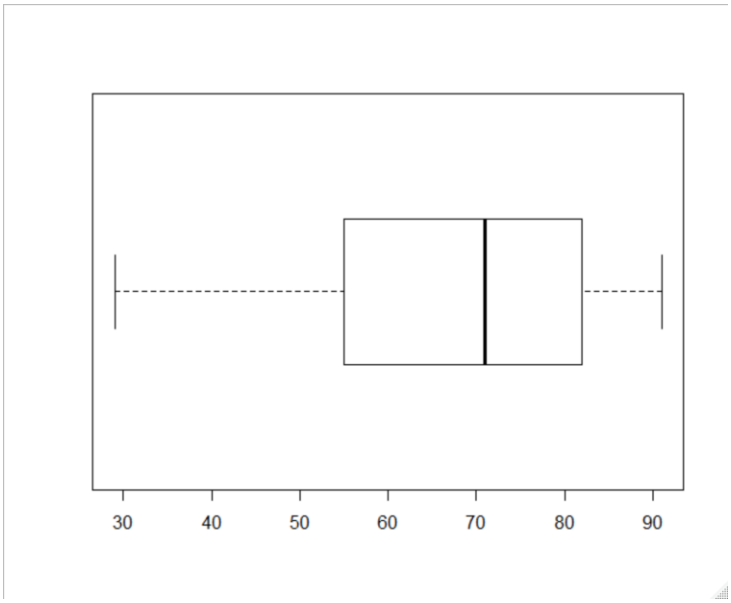
Max Shi
MA 331
Professor Li
November 29, 2019
I pledge my honor that I have abided by the Stevens Honor System.

Homework 7

10.32.

a.

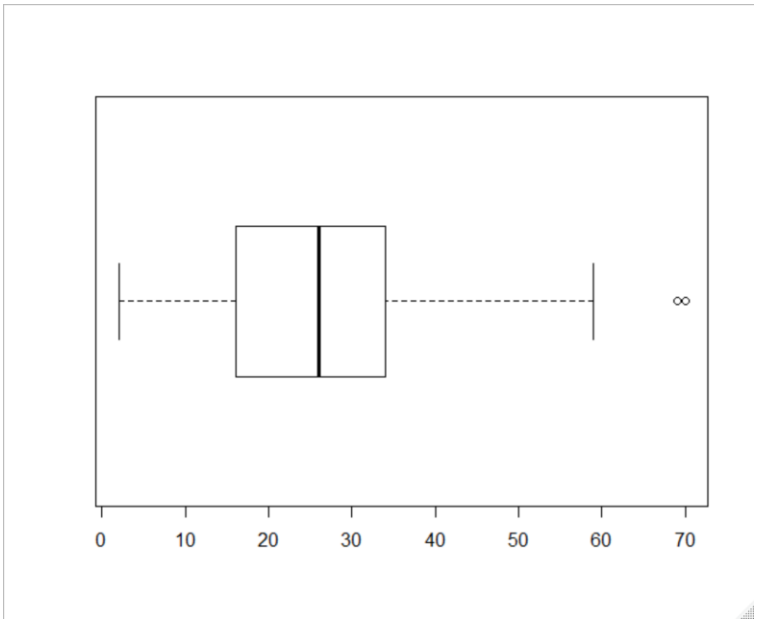
(ibi)



boxplot of ibi shows a left skew.

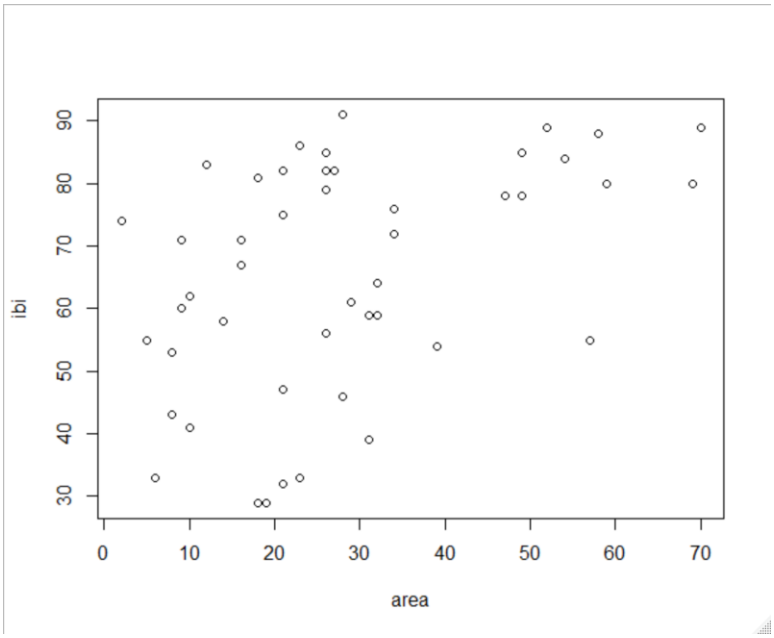
Mean of ibi = 65.94, standard deviation = 18.28.

(area)



The boxplot shows a slight right skew, as well as a few outliers in the data.
The mean is 28.29 and the standard deviation is 17.71.

b.



There seems to be a positive correlation, enforced by a calculated correlation of 0.445. There seems to be an outlier near the (58,55) point.

c.

In our plot, where IBI = y and area = x, the linear regression model is $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ for all i in our dataset, where x and y are independent, normally distributed variables.

d. $H_0 : \beta_1 = 0$

$H_a : \beta_1 \neq 0.$

e.

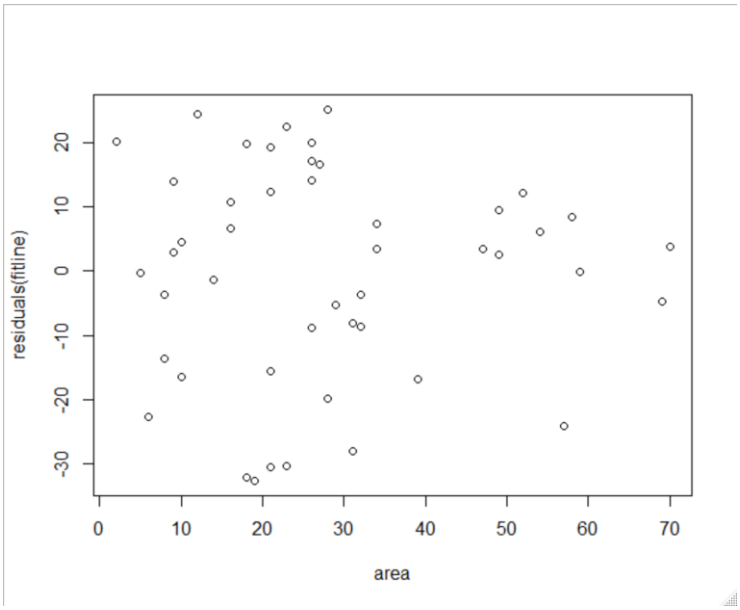
```
Residuals:
    Min      1Q  Median      3Q     Max
-32.666  -8.887   3.432  12.414  25.193

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  52.9230     4.4835  11.804 1.17e-15 ***
area          0.4602     0.1347   3.415  0.00132 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 16.53 on 47 degrees of freedom
Multiple R-squared:  0.1988,   Adjusted R-squared:  0.1818
F-statistic: 11.67 on 1 and 47 DF,  p-value: 0.001322
```

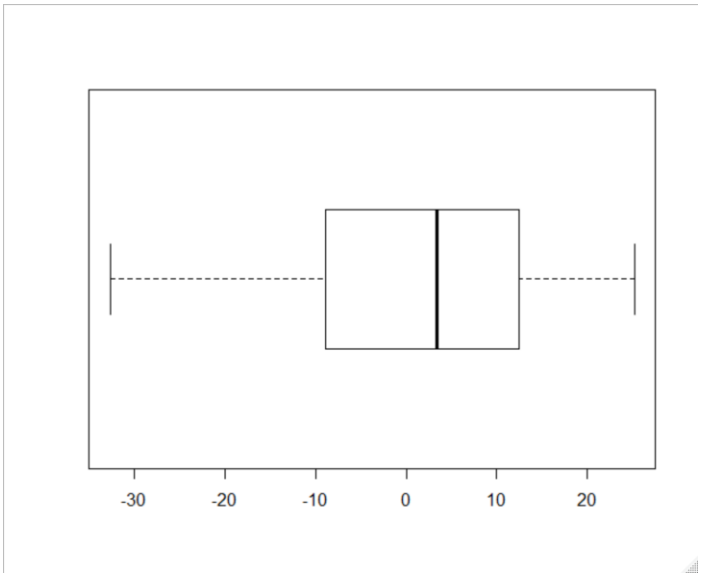
The linear regression gives $\beta_0 = 52.92$ and $\beta_1 = 0.460$. The standard error here is 16.53, and for the hypothesis tests, the t value is 3.415 and the p value is 0.00132.

f.



There does not seem to be anything too unusual in the plot.

g.



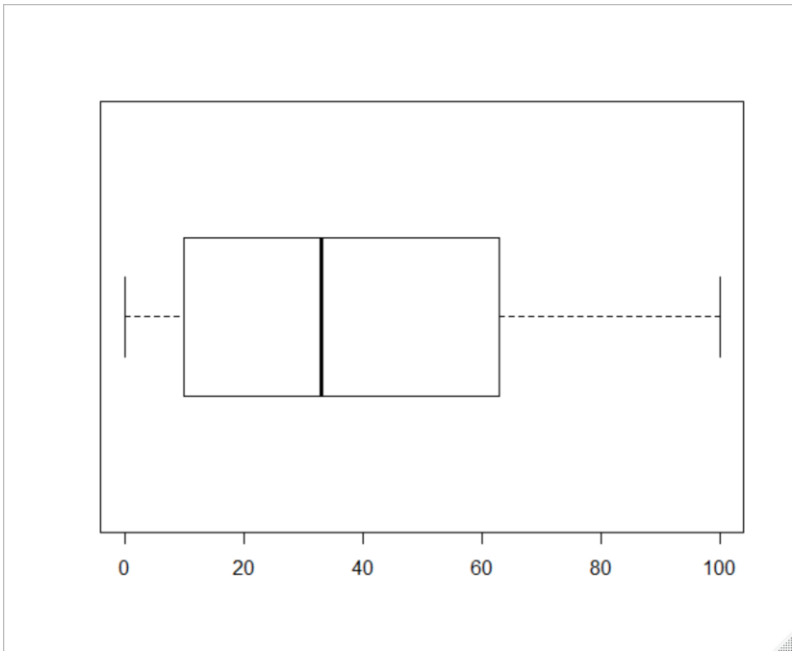
The residuals are left skewed.

g.

The assumptions here seem to be reasonable. There is nothing unusual about the plot of the area vs the residuals, and although the boxplots of the data and the residuals are skewed slightly, the large number of observations dismiss this concern.

10.33.

a. (percent forested)

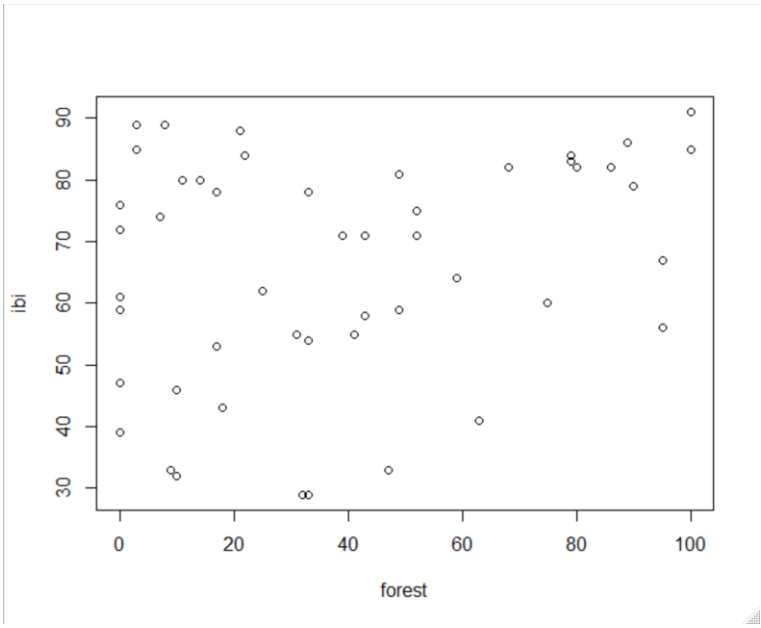


Boxplot shows a right skew.

The average is 39.39%, with a standard deviation of 32.02%.

(The observations of IBI can be taken from the previous exercise)

b.



From the plot of forest vs ibi, there seems to be a sort of positive correlation between the two variables. The calculated correlation in R is 0.269. However, among small values of x, the values of y are scattered.

c.

In our plot, where IBI = y and forest = x, the linear regression model is $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ for all i in our dataset, where x and y are independent, normally distributed variables.

d. $H_0 : \beta_1 = 0$
$H_a : \beta_1 \neq 0$.

e.

Linear Regression:

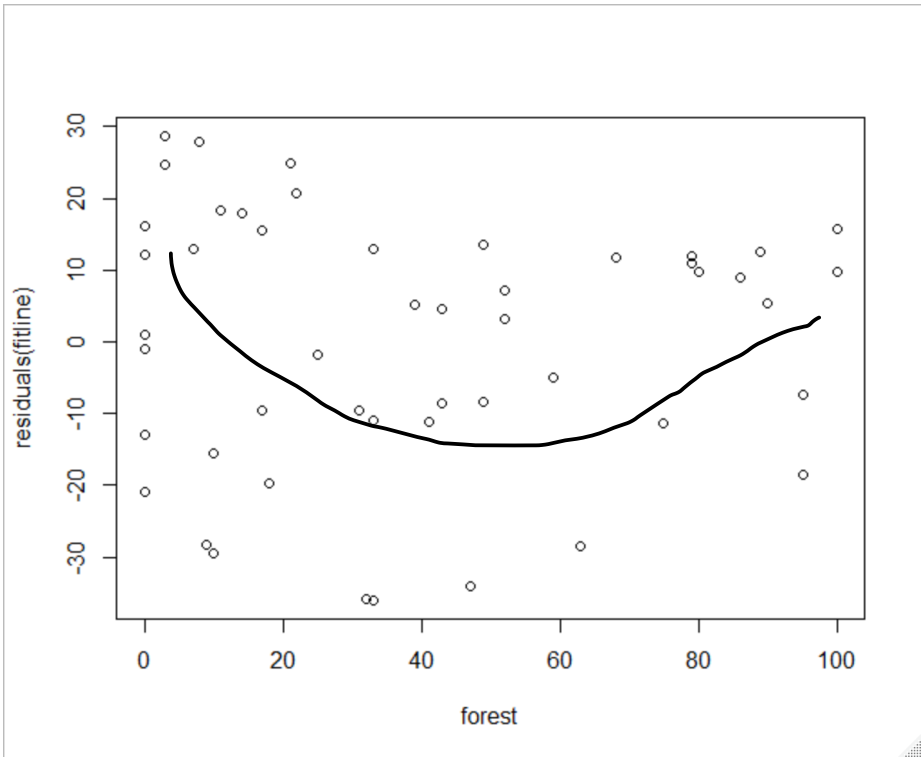
```
Residuals:
    Min      1Q  Median      3Q     Max
-35.961 -11.186   4.508  13.021  28.633

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 59.90725    4.03957  14.830   <2e-16 ***
forest       0.15313    0.07972   1.921   0.0608 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 17.79 on 47 degrees of freedom
Multiple R-squared:  0.07278,  Adjusted R-squared:  0.05305
F-statistic: 3.689 on 1 and 47 DF,  p-value: 0.06084
```

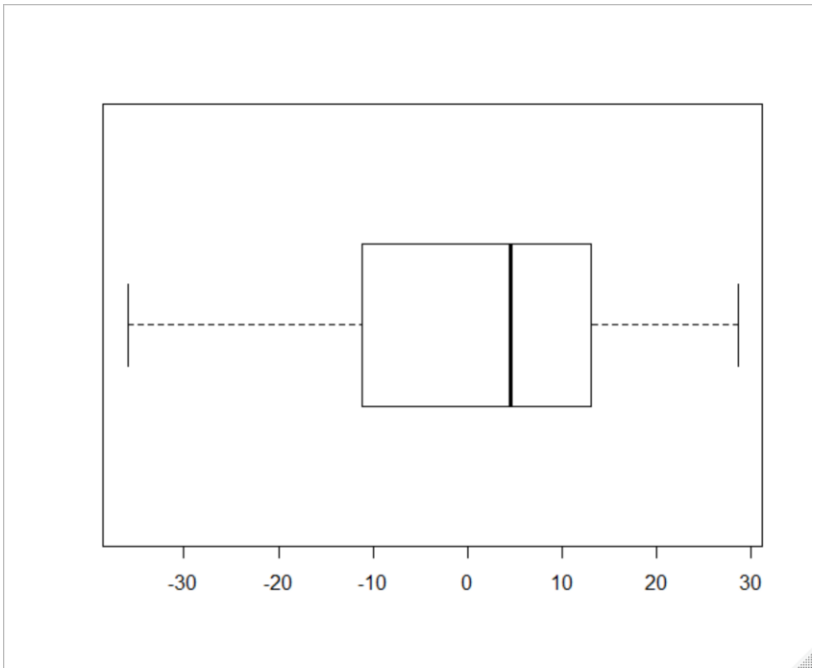
Here, $\beta_0 = 59.91$ and $\beta_1 = 0.153$ with error being 17.79. Hypothesis tests from d result in the t value being 1.921 and the P value being 0.0608.

f.



Residual plot shows a small curve, hopefully illustrated by the drawn line.

g.



The boxplot of the residuals shows a left skew, possibly suggesting a non-normal distribution.

h. They appear to be not very reasonable. The slight curve seen in the area vs residuals plot may suggest a use of a non-linear plot, and the residuals themselves do not seem to be of normal distribution either. Furthermore, although the data themselves are heavily left and right skewed, also suggesting use of a non-linear model.

10.34.

The two analyses see that using area instead of percent forest yields a much lower p value, as well as the lack of a curve in the x vs residual plot. It also shows a much stronger positive correlation. Thus, I would prefer to use area to predict IBI.

10.35.

With a (forest = 0, ibi = 0) observance added in, the regression becomes:

```
Residuals:
    Min      1Q  Median      3Q     Max
-56.969 -10.064   4.111  14.292  31.436

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 56.96922    4.32222   13.18   <2e-16 ***
forest       0.19821    0.08617    2.30   0.0258 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 19.52 on 48 degrees of freedom
Multiple R-squared:  0.09928,  Adjusted R-squared:  0.08052
F-statistic: 5.291 on 1 and 48 DF,  p-value: 0.02583
```

With a (forest = 100, ibi = 0) observance added in, the regression becomes:

```
Residuals:
    Min      1Q  Median      3Q     Max
-68.744 -10.065   5.901  15.655  26.991

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 61.80108    4.60916  13.408   <2e-16 ***
forest       0.06943    0.08844   0.785    0.436
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 20.43 on 48 degrees of freedom
Multiple R-squared:  0.01268,  Adjusted R-squared:  -0.007892
F-statistic: 0.6163 on 1 and 48 DF,  p-value: 0.4363
```

The statistical significance of the positive correlation goes up when the (0,0) value is added because it reinforces the idea of a positive correlation. The opposite happens when the (100,0) is added, because it does not reinforce the positive correlation that the regression suggests is happening.

10.36.

a. From R, calculating the 95% confidence interval yields

```
       fit      lwr      upr
1 71.32916 65.61416 77.04417
```
Thus, the confidence interval for a mean response corresponding to an area of 40 sq km = (65.614, 77.044).

b. From R, calculating the 95% prediction interval yields

```
       fit      lwr    upr
1 71.32916 37.57836 105.08
```
Thus, the prediction interval for a future response of 40 sq km = (37.578, 105.08)

c. The confidence interval predicts the average ibi of a piece of land with 40 sqkm of watershed. This is in comparison to the prediction interval, which gives the confidence interval of ibi with only one observation. More specifically, the confidence interval for a mean response gives the confidence interval for an average over multiple responses, while the prediction interval for a future response give the confidence interval for only one response. Thus, the intervals are centered on the same number, but the prediction interval is wider to account for error of prediction of only one observation.

d. I think these results could be applied to other streams in Arkansas, with minimal changes in other features such as climate, rainfall, etc. However, I would not apply this data to other streams in locations with many different changes in factors compared to this stream in Arkansas, for example, a stream in Texas with much less rainfall. It is likely there that percent forest or area of watershed is not the only factor playing into he IBI measurement.

10.37.

Using area, the prediction is:

```
39 63.50653 29.87487  97.13818
```
Using forest percentage, the prediction is:

```
39 73.53600 36.52237 110.54963
```
These two prediction intervals differ because of the criteria they use to make the prediction. Because both are linear regressions with a positive correlation, the lower the value, the smaller the prediction will be. This entry has a small area value, but a high forest value, meaning that the predicted ibi using the area as the factor will be lower than the prediction using forest percentage as the factor. However, the prediction intervals overlap, meaning that there is a good chance that an even better prediction could be combining these two factors into a multiple linear regression to examine the relationship there.