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Professor Borowski, CS 385

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I pledge my honor that I have abided by the Stevens Honor System.

Pg 67, #4

a. The algorithm computes the sum of all integers squared from 1 to n.
b. Its basic operation is multiplying and adding to the sum.
c. This basic operation is executed n times.
d. The efficiency class is θ(n)
e. The improvement to this algorithm would be to directly use the formula for this sum, which is n*(n+1)*(2n+1)/6, which would be in the constant efficiency class.

Pg 76, #1

a. $x(n) = x(n-1) + 5, n > 1, x(1) = 0$

$$x(n) = x(n-1) + 5$$
$$x(n-1) = x(n-2) + 5$$
$$x(n) = (x(n-2) + 5) + 5$$
$$x(n) = x(n-k) + 5k$$
$$n - k = 1$$
$$n - 1 = k$$
$$x(n) = x(n-(n-1)) + 5(n-1)$$
$$x(n) = x(1) + 5(n-1)$$
$$x(n) = 5n - 5$$

b. $x(n) = 3x(n-1), n > 1, x(1) = 4$

$$x(n) = 3x(n-1)$$
$$x(n-1) = 3x(n-2)$$
$$x(n) = 3\big(3(n-2)\big)$$
$$x(n) = 3^k x(n-k)$$
$$n - k = 1$$
$$n - 1 = k$$
$$x(n) = 3^{(n-1)} x(n-(n-1))$$
$$x(n) = 3^{n-1} x(1)$$
$$x(n) = 3^{n-1} * 4$$

c. $\mathrm{x(n) = x(n-1) + n, n > 0, x(0) = 0}$

$$x(n) = x(n-1) + n$$
$$x(n-1) = x(n-2) + n - 1$$
$$x(n) = x(n-2) + n - 1 + n$$

$$x(n) = x(n-k) + \sum_{i=0}^{k} n - i$$
$$n - k = 0$$
$$n = k$$
$$x(n) = x(n-n) + \sum_{i=0}^{n} n - i$$
$$x(n) = x(0) + \sum_{i=0}^{n} n - i$$
$$x(n) = \sum_{i=0}^{n} n - i$$
$$x(n) = \sum_{i=0}^{n} i$$
$$x(n) = \frac{n(n+1)}{2}$$

d. $x(n) = x\left(\frac{n}{2}\right) + n, n > 1, x(1) = 1$

$$x(n) = x\left(\frac{n}{2}\right) + n$$
$$x\left(\frac{n}{2}\right) = x\left(\frac{n}{4}\right) + \frac{n}{2}$$
$$x(n) = x\left(\frac{n}{4}\right) + \frac{n}{2} + n$$
$$x(n) = x\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} \frac{n}{2^i}$$
$$1 = \frac{n}{2^k}$$
$$2^k = n$$
$$k = \lg n$$
$$x(n) = x(1) + \sum_{i=0}^{\lg n - 1} \frac{n}{2^i}$$
$$x(n) = \sum_{i=0}^{\lg n} \frac{n}{2^i}$$

e. $x(n) = x\left(\frac{n}{3}\right) + 1, n > 1, x(1) = 1$

$$x(n) = x\left(\frac{n}{3}\right) + 1$$
$$x\left(\frac{n}{3}\right) = x\left(\frac{n}{9}\right) + 1$$
$$x(n) = x\left(\frac{n}{9}\right) + 1 + 1$$
$$x(n) = x\left(\frac{n}{3^k}\right) + k$$

$$1 = \frac{n}{3^k}$$
$$3^k = n$$
$$\mathrm{k} = \log_3 n$$
$$x(n) = x(1) + \log_3 n$$
$$x(n) = 1 + \log_3 n$$

Pg 76-77, #3

a. $x(n) = 2 + x(n-1), n > 1, n(1) = 0$

$$x(n-1) = 2 + x(n-2)$$
$$x(n) = 2 + 2 + x(n-2)$$
$$x(n) = 2k + x(n-k)$$
$$n - k = 1$$
$$n - 1 = k$$
$$x(n) = 2(n-1) + n(1)$$
$$x(n) = 2n - 2$$

b. The non-recursive, iterative method has the same number of executions as the recursive method, but it takes up less space in memory as a result of not having to utilize the stack to track recursive calls.