Ma 227 - Multivariable Calculus **Hw-1** **Due: Feb. 5, 2020 by 9:00 am**

Name (Printed): ______________________________

Pledge and Sign: ______________________________

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A high quality scan of the solutions in pdf format is to be uploaded to Canvas before the deadline. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Given the iterated integral $\int_0^{1/2} \int_{2x}^1 e^{y^2}\, dydx$:

(a) [3 pts.] Draw the region of integration.

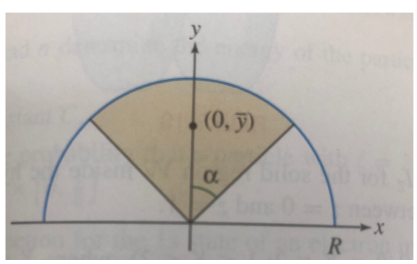
(b) [7 pts.] Evaluate the integral.

2. Consider the region D in the xy-plane bounded by the upper semi-circles of $x^2 + y^2 = 2x$ and $x^2 + y^2 = 4x$, and the piece of the x-axis between $x = 2$ and $x = 4$.

(a) [3 pts.] Sketch the region D.

(b) [7 pts.] Find the volume of the solid which lies below the plane $z = y$ and above the region D, described above.

3. [10 pts.] Consider the lamina L which occupies the shaded region in the figure below. The blue curve is the circle $x^2 + y^2 = R^2$. L has mass density $\rho(x, y) = \sqrt{x^2 + y^2}$ kg/m^2 at a point (x, y). Show that the y-coordinate, $\bar{y}$, of the center of mass of L is given by: $\bar{y} = \left(\frac{3R}{4}\right)\left(\frac{\sin\alpha}{\alpha}\right)$. Recall that $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$.



MA227 HW1

Max Shi

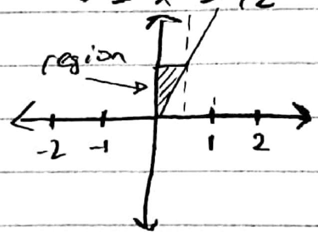
Pledge:
I pledge my honor that I have abided by the Stevens Honor System.

1. $\int_0^{1/2}\int_{2x}^{1} e^{y^2}\,dy\,dx$

$2x \le y \le 1$

$0 \le x \le 1/2$

(a)



Flipping the order of integration:

$\int_0^1\int_0^{y/2} e^{y^2}\,dx\,dy$

$\int_0^1 \left[x e^{y^2}\right]_0^{y/2} dy$

$\int_0^1 \frac{y}{2} e^{y^2}\,dy$ $\quad u = y^2 \quad y=1 \Rightarrow u=1$

$du = 2y\,dy \quad y=0 \Rightarrow u=0$

$\frac{1}{4}du = \frac{y}{2}dy$

$\frac{1}{4}\int_0^1 e^u\,du$

$\frac{1}{4}\left[e^u\right]_0^1$

$\frac{1}{4}e^1 - \frac{1}{4}e^0$

(b) $= \frac{1}{4}e - \frac{1}{4}$

2. $x^2 + y^2 = 2x$ (A) $\quad x^2+y^2 = 4x$ (B) $\quad x=2,\ x=4$

$x^2 - 2x + 1 + y^2 = 1 \quad x^2-4x+4+y^2 = 4$

$(x-1)^2 + y^2 = 1 \quad (x-2)^2 + y^2 = 4$

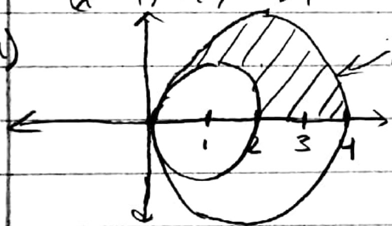
(A) $\Rightarrow r^2 = 2r\cos\theta$

$r = 2\cos\theta$

(B) $\Rightarrow r^2 = 4r\cos\theta$

$r = 4\cos\theta$

(a)



change to polar

$0 \le \theta \le \pi/2$

$2\cos\theta \le r \le 4\cos\theta$

$z = y \Rightarrow z = r\sin\theta$

$\int_0^{\pi/2}\int_{2\cos\theta}^{4\cos\theta} r\sin\theta\, r\,dr\,d\theta$

$\int_0^{\pi/2}\left[\frac{r^3}{3}\sin\theta\right]_{2\cos\theta}^{4\cos\theta} d\theta = \int_0^{\pi/2}\frac{(4\cos\theta)^3}{3}\sin\theta - \frac{(2\cos\theta)^3}{3}\sin\theta\,d\theta$

$\Rightarrow \int_0^{\pi/2}\left(\frac{64\cos^3\theta}{3} - \frac{8\cos^3\theta}{3}\right)\sin\theta\,d\theta = \frac{56}{3}\int_0^{\pi/2}\cos^3\theta\sin\theta\,d\theta$

$\Rightarrow \frac{56}{3}\int_0^{\pi/2}\cos^3\theta\sin\theta\,d\theta \quad u = \cos\theta \quad \theta = \pi/2 \Rightarrow u = 0$

$\frac{56}{3}\int_1^0 -u^3\,du \quad du = -\sin\theta\,d\theta \quad \theta = 0 \Rightarrow u = 1$

$\frac{56}{3}\int_0^1 u^3\,du$

$\Rightarrow \frac{14}{3}\left[u^4\right]_0^1$

(b) $= \frac{14}{3}$

3. $\int_{\pi/2-\alpha}^{\pi/2+\alpha}\int_0^R \rho(x,y)\, r\, dr\, d\theta$

$\rho(x,y) = \sqrt{x^2+y^2}$

$= \sqrt{r^2}$

$= r$

$M = \int_{\pi/2-\alpha}^{\pi/2+\alpha}\int_0^R r^2\, dr\, d\theta$

$\int_{\pi/2-\alpha}^{\pi/2+\alpha}\left[\frac{r^3}{3}\right]_0^R d\theta$

$\int_{\pi/2-\alpha}^{\pi/2+\alpha}\frac{R^3}{3}\, d\theta$

$\frac{R^3}{3}\left[\theta\right]_{\pi/2-\alpha}^{\pi/2+\alpha}$

$\frac{R^3}{3}\left(\frac{\pi}{2}+\alpha-\frac{\pi}{2}+\alpha\right)$

$= \frac{R^3}{3}(2\alpha)$

$M_{\cancel{x}} = \int_{\pi/2-\alpha}^{\pi/2+\alpha}\int_0^R r^2(r\sin\theta)\, dr\, d\theta$ (y)

$\int_{\pi/2-\alpha}^{\pi/2+\alpha}\int_0^R r^3\sin\theta\, dr\, d\theta$

$\int_{\pi/2-\alpha}^{\pi/2+\alpha}\left[\frac{r^4}{4}\right]_0^R \sin\theta\, d\theta$

$\int_{\pi/2-\alpha}^{\pi/2+\alpha}\frac{R^4}{4}\sin\theta\, d\theta$

$\frac{R^4}{4}\left[-\cos\theta\right]_{\pi/2-\alpha}^{\pi/2+\alpha}$

$\frac{R^4}{4}\left(-\cos(\pi/2+\alpha)+\cos\left(\frac{\pi}{2}-\alpha\right)\right)$

$\frac{R^4}{4}\left(-\cos\left(\frac{\pi}{2}-(-\alpha)\right)+\cos\left(\frac{\pi}{2}-\alpha\right)\right)$

Applying $\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

$\frac{R^4}{4}\left(-\sin(-\alpha)+\sin\alpha\right)$

$\frac{R^4}{4}(2\sin\alpha)$

$M_x = \frac{R^4}{2}(\sin\alpha)$

$\bar{y} = \frac{M_x}{M}$

$\bar{y} = \frac{R^4}{2}(\sin\alpha)\cdot\frac{3}{R^3 2\alpha}$

$\bar{y} = \frac{3R}{4}\cdot\frac{\sin\alpha}{\alpha}$

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