Ma 221 - Differential Equations **Homework 6** **Due: Mar 13, 2020**

***Name (Printed)*:** ***Lecture Instructor*:** ***Lecture Section*:**

***Collaborators*:**

I pledge my honor that I have abided by the Stevens Honor System. ***Sign*:**

General Instructions: *Write up solutions to the following set of questions and submit in class on the date indicated.* ***Please staple this cover sheet to your solution pages.***

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for disorganized work or insufficient explanations.

Collaboration with classmates is acceptable and encouraged but all students must write up the solutions on their own. Collaborators (up to groups of three) should be identified on the top of the front page. All submitted work must be pledged and signed.

1. **Laplace Transforms from the definition:** $F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$

   Sketch the graph of the function $f(t)$ and calculate its Laplace Transform.

$$f(t) = \begin{cases} 2t, & 0 \le t < 2 \\ 4, & 2 \le t < 5 \\ 0, & 5 \le t < \infty \end{cases}$$

2. Determine the inverse Laplace transforms.

   (a) $f(t) = \mathcal{L}^{-1}\left\{\dfrac{2}{s^4}\right\}$ (b) $f(t) = \mathcal{L}^{-1}\left\{\dfrac{2s-11}{2s^2+3s-2}\right\}$

3. Determine the inverse Laplace transforms.

   (a) $f(t) = \mathcal{L}^{-1}\left\{\dfrac{-2s+5}{s^2+9}\right\}$ (b) $f(t) = \mathcal{L}^{-1}\left\{\dfrac{-5s-20}{(s^2+4)(s+1)}\right\}$

4. Use the Laplace transform to solve the following initial value problem for $y(t)$.

$$y''(t) + 2y'(t) = 4t, \qquad y(0) = 1,\ y'(0) = -1$$

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No.

Date 3·12·20

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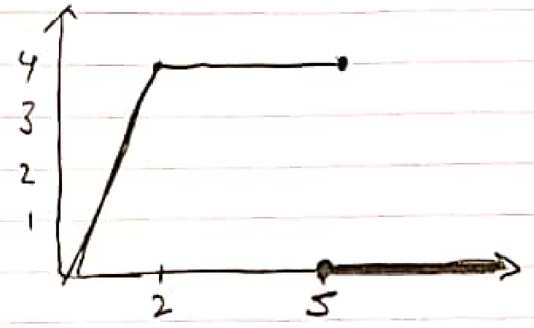
1.



$\mathcal{L}\{f(t)\} =$

$\int_0^2 e^{-st} \cdot 2t\,dt + \int_2^5 e^{-st}dt + \int_5^\infty 0\,dt.$

$u = 2t \quad v = \frac{1}{-s}e^{-st}$

$du = 2dt \quad dv = e^{-st}dt$

$\left[-\frac{2t}{s}e^{-st}\right]_0^2 - \int_0^2 -\frac{2}{s}e^{-st}dt + \left[\frac{-1}{s}e^{-st}\right]_2^5$

$-\frac{4}{s}e^{-2s} + 0 - \left[\frac{2}{s^2}e^{-st}\right]_0^2 + \left(-\frac{1}{s}e^{-5s} + \frac{1}{s}e^{2s}\right)$

$-4/s\,e^{-2s} - \frac{2}{s^2}e^{-2s} + \frac{2}{s^2} - 1/s\,e^{-5s} + \frac{1}{s}e^{-2s}$

$-\frac{3}{s}e^{-2s} - \frac{1}{s}e^{-5s} - \frac{2}{s^2}e^{-2s} + \frac{2}{s^2}$

2(a) $f(t) = \mathcal{L}^{-1}\left\{\frac{2}{s^4}\right\} \quad n = 3 \Rightarrow n! = 6.$

$\frac{2}{s^4} = \frac{2}{6}\cdot\frac{6}{s^4} \Rightarrow \mathcal{L}^{-1}\left\{\frac{2}{s^4}\right\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{6}{s^4}\right\} = \frac{1}{3}t^3$

(b) $f(t) = \mathcal{L}^{-1}\left\{\frac{2s-11}{2s^2+3s-2}\right\}$

$\frac{2s-11}{2s^2+3s-2} = \frac{2s-11}{(2s-1)(s+2)} \Rightarrow \frac{2s-11}{(2s-1)(s+2)} = \frac{A}{2s-1} + \frac{B}{s+2}$

$\Rightarrow 2s-11 = A(s+2) + B(2s-1)$

$\Rightarrow 2s-11 = As + 2A + 2Bs - B$

$\Rightarrow A + 2B = 2, \quad -11 = 2A - B.$

$\Rightarrow A = 2 - 2B \Rightarrow -11 = 2(2-2B) - B$

$\Rightarrow -11 = 4 - 4B - B$

$\Rightarrow -15 = -5B$

$\Rightarrow B = 3 \Rightarrow A = -4$

$\mathcal{L}^{-1}\left\{\frac{2s-11}{2s^2+3s-2}\right\} = \mathcal{L}^{-1}\left\{\frac{-4}{2s-1} + \frac{3}{s+2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{-4}{2s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{3}{s+2}\right\}$

$= -2\mathcal{L}^{-1}\left\{\frac{1}{s-1/2}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$

$= -2e^{1/2 t} + 3e^{-2t}$

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3(a) $f(t) = \mathcal{L}^{-1}\left\{\frac{-2s+5}{s^2+9}\right\}$

$$\frac{-2s+5}{s^2+9} = \frac{-2s}{s^2+9} + \frac{5}{s^2+9} = -2\frac{s}{s^2+9} + \frac{5}{3}\frac{3}{s^2+9}$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{-2s+5}{s^2+9}\right\} = \mathcal{L}^{-1}\left\{-2\frac{s}{s^2+9} + \frac{5}{3}\cdot\frac{3}{s^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{-2\frac{s}{s^2+9}\right\} + \mathcal{L}^{-1}\left\{\frac{5}{3}\cdot\frac{3}{s^2+9}\right\} = -2\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} + \frac{5}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$= -2\cos 3t + \frac{5}{3}\sin 3t$$

(b) $f(t) = \mathcal{L}^{-1}\left\{\frac{-5s-20}{(s^2+4)(s+1)}\right\}$

$$\frac{-5s-20}{(s^2+4)(s+1)} = \frac{As+B}{s^2+4} + \frac{C}{s+1}$$

$s=-1$ $\quad -5(-1)-20 = C(s^2+4)$

$$-5s-20 = (As+B)(s+1) + C(s^2+4)$$

$$-5s-20 = As^2+As+Bs+B+Cs^2+4C$$

$\Rightarrow A+C=0, \quad A+B=-5, \quad B+4C=-20$

$C=-A \quad B=-5-A$

$-5-A+4(-A)=-20$

$-5-5A=-20$

$-5A=-15$

$A=3, \; C=-3, \; B=-8$

$$\mathcal{L}^{-1}\left\{\frac{-5s-20}{(s^2+4)(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{3s-8}{s^2+4} + \frac{-3}{s+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3s}{s^2+4}\right\} - \mathcal{L}^{-1}\left\{\frac{8}{s^2+4}\right\} + \mathcal{L}^{-1}\left\{\frac{-3}{s+1}\right\}$$

$$= 3\cos 2t - 4\sin 2t - 3e^{-t}$$

4. $y''(t)+2y'(t) = 4t$, $\quad y(0)=1, \; y'(0)=-1$

$\mathcal{L}\{y''(t)+2y'(t)\} = \mathcal{L}\{4t\}$

$s^2Y - sy(0) - y'(0) + 2sY - 2y(0) = 4\frac{1}{s^2}$

$s^2Y - s + 1 + 2sY - 2 = 4\frac{1}{s^2}$

$s^2Y + 2sY = \frac{4}{s^2} + 1 + s$

$Y(s^2+2s) = \frac{4}{s^2} + 1 + s$

$Y = \frac{4+s^2+s^3}{(s^2)(s)(s+2)}$

$\mathcal{L}^{-1}\{Y\} = \mathcal{L}^{-1}\left\{\frac{4+s^2+s^3}{(s^3)(s+2)}\right\}$

$y = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s^2} + \frac{2}{s^3}\right\}$

$y = 1 - t + t^2$

$$\frac{4+s^2+s^3}{s^3(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+2}$$

$4+s^2+s^3 = As^2(s+2) + Bs(s+2) + C(s+2) + Ds^3$

for $s=0$,

$4 = 2C \Rightarrow C=2$.

for $s=-2$,

$4+4-8 = 8D \Rightarrow D=0$.

$4+s^2+s^3 = As^3+2As^2+Bs^2+2Bs + 2s + 4$

$s^3+s^2-2s = As^3 + (2A+B)s^2 + 2Bs$

$A=1, \; B=-1$,