Maxoshi

I pledge my honor that I have abided by the Stevens Honor System

Test 2

Theorem 2.8

1. $g_1(x) = x+1-xe^x$

$g'(x) = 1+0-e^x-xe^x$ $[0,1]$

$g''(x) = -e^x-e^x-xe^x$

$= -2e^x-xe^x \neq 0$ on $[0,1]$

$g'(0) = 1-1 = 0$

$g'(1) = 1-e-e = |1-2e| > 1$

Test with fixed point.

$p_0 = 0.5$

$p_1 = f(0.5) = 2.324$

$p_2 = f(p_1) = 27.079$

$p_3 = f(p_2) = 1.86\times10^{13}$

(diverges)

$g_2(x) = \frac{1+x}{1+e^x}$ $\quad g_2'(x) = \frac{1}{e^x+1} - \frac{(x+1)e^x}{(e^x+1)^2}$

$= \frac{-xe^x-1}{(e^x+1)^2}$ $[0,1]$

max $= g_2'(0) = \frac{-0-1}{(e^0+1)^2} = \frac{-1}{4}$ $\qquad g''(x) = \frac{e^x((x-1)e^x-x-3)}{(e^x+1)^3}$

min $= g'(1) = \frac{-1e-1}{(e+1)^2} \approx -0.269$

On desmos, $|g_2'(x)| \leq 1$ for all $x \in [0,1]$

Thus, converges linearly to fixed point on $[0,1]$

$g_3(x) = e^{-x}$ $\quad g_3'(x) = -e^{-x}$ $\quad g''_3(x) = e^{-x} \neq 0$

extrema at $x=0$, $x=1$.

$g_3'(0) = -e^{-0} = -1$, $g''(0) = e^{-0} = 1$ (positive)

$g_3'(1) = -e^{-1} = \frac{-1}{e} < 1$

Thus, by theorem 2.8, $|g_3'(x)| \leq k$ for all $x \in (a,b)$, so the theorem holds as $k<1$. Thus, converges linearly to fixed point.

(as $g'(x) = -1$ @ $x=0$, not on $(0,1)$.)

2.a) the interval is $\frac{b-a}{2^{n-1}}$, so in this case,
on the nth step, the width is $\frac{5-1}{2^{n-1}} = \frac{2^2}{2^{n-1}} =$
$\boxed{2^{3-n}}$

b) In each interval, the root must be in one of the halves. Therefore, as the midpoint, it is at most $\frac{b-a}{2^{n-1}} \cdot \frac{1}{2}$ distance away from both ends. This would be the worst case scenario, therefore, the maximum distance is $= \frac{b-a}{2^n}$ on the nth iteration.
In this example, it would be $\frac{2^2}{2^n} = \boxed{2^{2-n}}$

3a) $c\left(\frac{1}{x}\right) - a = 0 \Rightarrow c = a^2 \Rightarrow f(x) = y^2x - y \Rightarrow f'(x) = y^2$
Newton's method: $p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$
$f(x) = 2.89x - 1.7 \quad f'(x) = 2.89$
$p_0 = 1 \quad f(p_0) = 1.19 \quad f'(p_0) = 2.89 \quad p_1 = 1 - \frac{1.19}{2.89} = 0.58823$
$p_1 = 0.58823 \quad f(p_1) = -3.4\times10^{-10} \quad f'(p_1) = 2.89 \quad p_2 = p_1 - \frac{-3.4\times10^{-10}}{2.89} = 0.58823$
$p_2 = 0.58823 \quad f(p_2) = -3.4\times10^{-10} \quad f'(p_2) = 2.89 \quad p_3 = p_2 - \frac{-3.4\times10^{-10}}{2.89} = 0.58823$

Error is so small, more iterations will not change.
$p_2 = p_3 = p_4 = p_5 = 0.588228798$

4. $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 7 \\ -1 & 2 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 9 \\ 0 & 4 & 4 \end{bmatrix} \updownarrow \rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 4 \\ 0 & 0 & 9 \end{bmatrix}$

a) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ as r_2 and r_3 were swapped

b) $PA = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 2 & 5 \\ 2 & 4 & 7 \end{bmatrix} \begin{matrix} \\ +r_1 \\ -2r_1 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 4 \\ 0 & 0 & 9 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ $\quad \begin{matrix} r_1' = r_1 \\ r_2' - r_1 = r_2 \\ r_3' + 2r_1 = r_3 \end{matrix}$

$A = P^t L U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 4 \\ 0 & 0 & 9 \end{bmatrix}$