Max Shi

I pledge my honor that I have abided by the Stevens Honor System
Max Shi

MA 232 Homework 2

2.3

1\. a. $\begin{bmatrix}1&0&0\\-5&1&0\\0&0&1\end{bmatrix}$ b. $\begin{bmatrix}1&0&0\\0&1&0\\0&-7&1\end{bmatrix}$ c. $\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$

2\. $\begin{bmatrix}0\\0\\1\end{bmatrix}$, $\begin{bmatrix}0\\1\\0\end{bmatrix}$, 3, 1

3\. $E_{21} = \begin{bmatrix}1&0&0\\-4&1&0\\0&0&1\end{bmatrix}$ $E_{31} = \begin{bmatrix}1&0&0\\0&1&0\\2&0&1\end{bmatrix}$ $E_{32} = \begin{bmatrix}1&0&0\\0&1&0\\0&-2&1\end{bmatrix}$

$E_{32}E_{31}E_{21}A = U$

$E_{32}E_{31} = \begin{bmatrix}1&0&0\\0&1&0\\2&-2&1\end{bmatrix}$. $E_{21} = \begin{bmatrix}1&0&0\\-4&1&0\\10&-2&1\end{bmatrix}$

2.4

6\. $A+B = \begin{bmatrix}2&2\\3&0\end{bmatrix}$ $(A+B)^2 = \begin{bmatrix}2&2\\3&0\end{bmatrix}\begin{bmatrix}2&2\\3&0\end{bmatrix} = \begin{bmatrix}10&4\\6&6\end{bmatrix}$

$A^2+2AB+B^2 = \begin{bmatrix}1&2\\0&0\end{bmatrix} + 2\begin{bmatrix}7&0\\0&0\end{bmatrix} + \begin{bmatrix}1&0\\3&0\end{bmatrix} = \begin{bmatrix}16&2\\3&0\end{bmatrix}$

$(A+B)^2 - A^2 - B^2 = \begin{bmatrix}8&2\\3&6\end{bmatrix}$ $BA = \begin{bmatrix}1&2\\3&6\end{bmatrix}$

$AB = \begin{bmatrix}1&2\\3&6\end{bmatrix} = BA$ $(A+B)^2 = A^2+AB+BA+B^2$.

14\. $A^2 - AB - BA + B^2$

$A^2 - AB - BA + B^2$

$A(A-B) - B(A-B)$, and $A^2 - AB - BA + B^2$

15\. a. True b. False c. True. d. False

16\. a) $m \times n$ multiplications b) $m \times n \times p$ c) n^3 multiplications

17\. a) $\begin{bmatrix}0\\0\end{bmatrix}$ b) $\begin{bmatrix}1&0&0\end{bmatrix}$ c) $\begin{bmatrix}0&1\end{bmatrix}$ d) $\begin{bmatrix}3&-2\end{bmatrix}$

26\. $\begin{bmatrix}0\\4\\1\end{bmatrix}\begin{bmatrix}1&2&1\end{bmatrix} = \begin{bmatrix}3&3&0\\10&14&4\\7&8&1\end{bmatrix}$

27. row 3 of A · col 1 of B = 0   27

row 3 of A · col 2 of B = 0, thus $AB_{21,31,32}$ will be 0 and it will be lower triangular

$\begin{bmatrix} x \\ x \\ 0 \end{bmatrix} [0\ x\ x] = \begin{bmatrix} 0 & x & x \\ 0 & x & x \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} x \\ x \\ x \end{bmatrix} [0\ 0\ x] = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}$, both are upper triangular.

32 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.5

1. $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \frac{1}{ad-bc}$ $A^{-1} = \begin{bmatrix} 0 & -3 \\ -4 & 0 \end{bmatrix} \cdot \frac{1}{-12} = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix} \cdot \frac{1}{4+0} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & \frac{1}{2} \end{bmatrix}$

$C^{-1} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix} \cdot \frac{1}{21-20} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

9. They are invertible. You would have to exchange columns 1 and 2 of B^{-1} to reach A^{-1}

10. $\left[\begin{array}{cccc|cccc} 0&0&0&2&1&0&0&0 \\ 0&0&3&0&0&1&0&0 \\ 0&4&0&0&0&0&1&0 \\ 5&0&0&0&0&0&0&1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 5&0&0&0&0&0&0&1 \\ 0&4&0&0&0&0&1&0 \\ 0&0&3&0&0&1&0&0 \\ 0&0&0&2&1&0&0&0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1&0&0&0&0&0&0&\frac{1}{5} \\ 0&1&0&0&0&0&\frac{1}{4}&0 \\ 0&0&1&0&0&\frac{1}{3}&0&0 \\ 0&0&0&1&\frac{1}{2}&0&0&0 \end{array}\right]$ $A^{-1} = \begin{bmatrix} 0&0&0&\frac{1}{5} \\ 0&0&\frac{1}{4}&0 \\ 0&\frac{1}{3}&0&0 \\ \frac{1}{2}&0&0&0 \end{bmatrix}$

$\left[\begin{array}{cccc|cccc} 3&2&0&0&1&0&0&0 \\ 4&3&0&0&0&1&0&0 \\ 0&0&6&5&0&0&1&0 \\ 0&0&7&6&0&0&0&1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 4&3&0&0&0&1&0&0 \\ 3&2&0&0&1&0&0&0 \\ 0&0&7&6&0&0&0&1 \\ 0&0&6&5&0&0&1&0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1&1&0&0&-1&1&0&0 \\ 3&2&0&0&1&0&0&0 \\ 0&0&1&1&0&0&-1&1 \\ 0&0&6&5&0&0&1&0 \end{array}\right]$

$\left[\begin{array}{cccc|cccc} 1&1&0&0&-1&1&0&0 \\ -3&2&0&0&-1&0&0&0 \\ 0&0&1&1&0&0&-1&1 \\ 0&0&-6&-5&0&0&-1&0 \end{array}\right] \Rightarrow$ ~~$\begin{bmatrix} 1&1&0&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 0&0&0&1 \end{bmatrix}$~~ invert by each block

$\begin{bmatrix} 3&2 \\ 4&3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3&-2 \\ -4&3 \end{bmatrix} \frac{1}{1}$ $\begin{bmatrix} 6&5 \\ 7&6 \end{bmatrix} \Rightarrow \begin{bmatrix} 6&-5 \\ -7&6 \end{bmatrix} \frac{1}{1}$

$\Rightarrow \begin{bmatrix} 3&-2&0&0 \\ -4&3&0&0 \\ 0&0&6&-5 \\ 0&0&-7&6 \end{bmatrix}$

18. $A^2B = I$, $A(AB) = I$, thus AB is an inverse.

23. $[A\ I] \Rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & -3 & -2 & 1 & -2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 4 & 1 & -2 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1/4 & -2/4 & 3/4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 3/4 & -1/2 & 1/4 \\ 0 & 1 & 0 & \vdots & -1/2 & 1 & -1/2 \\ 0 & 0 & 1 & \vdots & 1/4 & -1/2 & 3/4 \end{bmatrix}$

30. $\begin{bmatrix} a & b & b & 1 & 0 & 0 \\ a & a & b & 0 & 1 & 0 \\ a & a & a & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & b & 1 & 0 & 0 \\ 0 & a-b & 0 & -a & 1 & 0 \\ 0 & a-b & a-b & -a & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & b & 1 & 0 & 0 \\ 0 & a-b & 0 & -a & 1 & 0 \\ 0 & 0 & a-b & -a & -a+b & 1 \end{bmatrix}$

pivots must not be $0 \Rightarrow a \neq 0$, $a \neq b$

$\begin{bmatrix} 2 & c & c & 1 & 0 & 0 \\ c & c & c & 0 & 1 & 0 \\ 8 & 7 & c & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & c & c \\ 8 & 7 & c \\ 2 & c & c \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & c & c \\ 0 & 7-4c & -3c \\ 2 & c & c \end{bmatrix} \Rightarrow$

$c \neq 0$ $\quad c \neq 7$ $\quad c \neq 2$

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5. $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ $\quad B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

8. $E_{21} \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -b & 0 & 1 \end{bmatrix}$ $\quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -c & 1 \end{bmatrix}$

$E_{32}E_{31}E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -b & -c & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$

$E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$

18. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$a \neq 0$, $a \neq b$, $b \neq c$, $c \neq d$.

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2.4
#38 $A = \begin{bmatrix} - a_1^T - \\ - a_2^T - \\ - a_m^T - \end{bmatrix}$ $A^T = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \dots & a_m \\ | & | & & | \end{bmatrix}$

$A^TA =$ (by column row multiplication method...)

$= \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix}\begin{bmatrix} - a_1^T - \end{bmatrix} + \dots + \begin{bmatrix} | \\ a_m \\ | \end{bmatrix}\begin{bmatrix} - a_m^T - \end{bmatrix}$

$= a_1a_1^T + a_2a_2^T + \dots + a_ma_m^T$

$A^TCA = \begin{bmatrix} | & | \\ a_1 & a_2 \\ | & | \end{bmatrix}\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{bmatrix} - a_1^T - \\ - a_2^T - \end{bmatrix}$

$\left(\begin{bmatrix} | \\ a_1 \\ | \end{bmatrix}\begin{bmatrix} c_1 & 0 \end{bmatrix} + \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix}\begin{bmatrix} 0 & c_2 \end{bmatrix} \right)$

$c_1\begin{bmatrix} | & 0 \\ a_1 & \\ | & 0 \end{bmatrix} + c_2\begin{bmatrix} 0 & | \\ & a_2 \\ 0 & | \end{bmatrix}$

$\begin{bmatrix} | & | \\ c_1a_1 & c_2a_2 \\ | & | \end{bmatrix}\begin{bmatrix} - a_1^T - \\ - a_2^T - \end{bmatrix}$

$c_1\begin{bmatrix} | \\ a_1 \\ | \end{bmatrix}\begin{bmatrix} - a_1^T - \end{bmatrix} + c_2\begin{bmatrix} | \\ a_2 \\ | \end{bmatrix}\begin{bmatrix} - a_2^T - \end{bmatrix}$

$= c_1a_1a_1^T + c_2a_2a_2^T$

general formula = $c_1a_1a_1^T + c_2a_2a_2^T + \dots + c_na_na_n^T$

2.5
35. $\left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & & & \\ 0 & 1 & -b & 0 & & 1 & & \\ 0 & 0 & 1 & -c & & & 1 & \\ 0 & 0 & 0 & 1 & & & & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -b & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & b & bc \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & a & ab & abc \\ 0 & 1 & 0 & 0 & 0 & 1 & b & bc \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$ $A^{-1} = \begin{bmatrix} 1 & a & ab & abc \\ 0 & 1 & b & bc \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$

41. $T = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ pivots.

T's inverse $= \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$UL = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \Rightarrow (UL)^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

23 $\begin{bmatrix} A & B \\ 0 & S \end{bmatrix} \begin{bmatrix} I & -A^{-1}B \\ 0 & I \end{bmatrix} = \begin{bmatrix} A & \overset{-B+B}{-AA^{-1}B + B} \\ 0 & S \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & S \end{bmatrix}$

$S = D - CA^{-1}B$

$D = I_2$, $C = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, $\Rightarrow A^{-1} = [-\frac{1}{2}]$

$A = [-2]$ $B = [3\ 3]$

$CA^{-1}B = \begin{bmatrix} 4 \\ 4 \end{bmatrix}[-\frac{1}{2}][3\ 3]$

$\begin{bmatrix} 2 \\ 2 \end{bmatrix}[3\ 3]$

$\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix} = \begin{bmatrix} -5 & -6 \\ -6 & -5 \end{bmatrix} = S.$

24. A_k factors into L_k U_k...

Because the last value in the row for L_k is 0 and the last value in the column for U_k is 0, this means when doing multiplication, the values in the stars will not factor into the multiplication. Thus, this combined with L_k and U_k always being lower and upper triangular, respectively, $A_k = L_k U_k$, and thus, A_k factors into $L_k U_k$.