Max Shi MA221 Exam 2

I pledge on my honor that I have not given or received any unauthorized assistance on this examination. -Max Shi

1(a) $\frac{d^2y}{dx^2} + 4y = 0$

$y'' + 4y = 0$

aux. eq. $m^2 + 4 = 0$

$m = \pm 2i$

$y = c_1 \cos 2x + c_2 \sin 2x$

(b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

$y'' - y' = 0$

aux. eq. $m^2 - m = 0$

$m(m-1) = 0$

$m = 0, 1$

$y = c_1 e^x + c_2$

(c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$y'' - 2y' + y = 0$

aux. eq. $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

$y = c_1 e^x + c_2 x e^x$

2 (a)(i) roots of y_c are $\cos 2x, \sin 2x$.

$4x^2 \Rightarrow Ax^2 + Bx + C$

$y_p = Ax^2 + Bx + C$.

(ii) $3\sin 2x \Rightarrow A\sin 2x + B\cos 2x$ (repeated roots)

$y_p = Ax\sin 2x + Bx\cos 2x$

(b) (i) roots are $e^x, 1$

$5x \Rightarrow Ax + B$ (repeated)

$y_p = Ax^2 + Bx$

(ii) $5xe^x \Rightarrow (Ax+B)e^x = Axe^x + Be^x$ (repeated)

$y_p = (Ax^2 + Bx)e^x$

(c) (i) roots are e^x, xe^x.

$5x \Rightarrow Ax + B$.

$y_p = Ax + B$.

(ii) $5xe^x \Rightarrow (Ax+B)e^x$ (repeated)

$y_p = (Ax^3 + Bx^2)e^x$

3. (a) $y'' - 5y' + 4y = 0$.

aux eq. $m^2 - 5m + 4 = 0$

$(m-4)(m-1) = 0$

$\Rightarrow m = 4, 1$

$y = c_1 e^{4x} + c_2 e^x$

(b) $y_p = Axe^{4x}$

$y_p' = 4Axe^{4x} + Ae^{4x}$

$y_p'' = 16Axe^{4x} + 4Ae^{4x} + 4Ae^{4x}$

$= 16Axe^{4x} + 8Ae^{4x}$

$L[y_p] = 16Axe^{4x} + 8Ae^{4x} - 5(4Axe^{4x} + Ae^{4x}) + 4(Axe^{4x}) = e^{4x}$

$= (16A - 20A + 4A)xe^{4x} + (8A - 5A)e^{4x} = e^{4x}$

$3Ae^{4x} = e^{4x}$

$3A = 1$

$A = \frac{1}{3}$.

$y_p = \frac{x}{3}e^{4x}$

(c) $y_p = Ax + B$, $y_p' = A$, $y_p'' = 0$.

$L[y_p] = 0 - 5(A) + 4(Ax + B) = x$

$= 4Ax + 4B - 5A$

$4A = 1 \Rightarrow A = \frac{1}{4}$

$4B - 5A = 0 \Rightarrow 4B = 5A$

$\Rightarrow B = \frac{5}{16}$

$y_p = \frac{1}{4}x + \frac{5}{16}$

(d) from b, set coefficient of e^{4x} to 12

$3Ae^{4x} = 12e^{4x}$

$3A = 12$

$A = 4.$

from c, set coefficient of x to 16.

$4Ax = 16x$, $4B = 5A$

$4A = 16$ $\quad 4B = 5(4)$

$A = 4$ $\quad B = 5.$

$y = c_1e^{4x} + c_2e^{x} + 4x + 5 + 4xe^{x}$

Add together. $y = y_c + y_{p_1} + y_{p_2}$

~~$y = c_1e^{4x} + c_2e^{x} + 4x + 5 + 4xe^{x}$~~

4. $W[y_1, y_2] = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix} = 2xe^{4x} + e^{4x} - 2xe^{4x}$

$f(x) = 4e^{2x}\ln x$

$W[y_1, y_2] = e^{4x}.$

$e^{4x} \neq 0$ on the interval $(0, \infty)$ therefore linearly independent on the interval $(0, \infty)$

$u_1' = \frac{-y_2 f}{W} = \frac{-xe^{2x}(4e^{2x}\ln x)}{e^{4x}}$

$u_1' = -4x\ln(x)$

$u_1 = -\int 4x\ln(x)\,dx$

$= -4\int x\ln x\,dx$

$= -4\left(\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right) \Rightarrow u_1y_1 = -2x^2\ln(x)e^{2x} + x^2e^{2x}$

$u_2' = \frac{y_1 f}{W} = \frac{e^{2x}(4e^{2x}\ln x)}{e^{4x}} = 4\ln x$

$\Rightarrow u_2 = \int 4\ln x\,dx$

$= 4(x\ln x - x) \Rightarrow u_2y_2 = 4x^2\ln(x)e^{2x} - 4x^2e^{2x}$

$y_p = -2x^2\ln(x)e^{2x} + x^2e^{2x} + 4x^2\ln(x)e^{2x} - 4x^2e^{2x}$

$2x^2\ln(x)e^{2x} - 3x^2e^{2x}$

$y = c_1e^{2x} + c_2xe^{2x} + 2x^2\ln(x)e^{2x} - 3x^2e^{2x}$

5. $x^2y'' - 4xy' + 6y = 0.$

auxeq. $a = 1,\ b = -4,\ c = 6$

$b - a = -5$

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2, 3. \Rightarrow y_c = c_1x^2 + c_2x^3$ $\Rightarrow y = c_1x^2 + c_2x^3$

$y_p = 0$ $\quad y' = 2c_1x + 3c_2x^2$

$8 = 2c_1(2) + 3c_2(2)^2$

$8 = 4c_1 + 12c_2$

$8 = 4(-2c_2) + 12c_2$

$8 = -8c_2 + 12c_2$

$8 = 4c_2$

$c_2 = 2.$

$0 = c_1(2)^2 + c_2(2)^3$

$4c_1 = -8c_2$

$c_1 = -2c_2$

$c_1 = -4.$

$y = -4x^2 + 2x^3$