Max Shi

MA 232 Homework 3

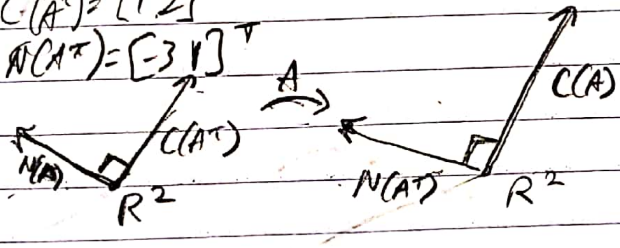
I pledge my honor that I have abided by the Stevens Honor System

Max Shi

1. 7. $x_1 - x_2 = 1 \quad *1 = x_1 - x_2 = 1$ add up
$x_2 - x_3 = 1 \quad *1 = x_2 - x_3 = 1 \Rightarrow x_1 - x_2 + x_2 - x_3 - x_1 + x_3 = 1+1-1$
$x_1 - x_3 = 1 \quad * -1 = -x_1 + x_3 = -1 \qquad \Rightarrow 0 = 1$

11. $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \to C(A) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad C(A^T) = [1\ 2]^T$ dim of all = 1
$N(A) = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad N(A^T) = [-3\ 1]^T$

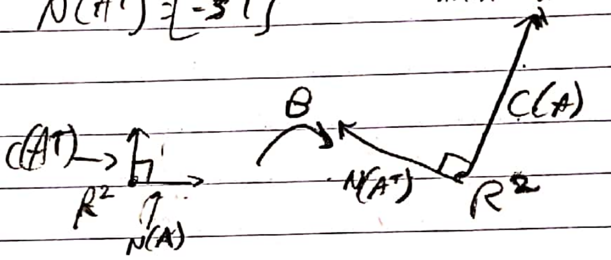
$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ $x_1 + 2x_2 = 0$
$x_1 = -2x_2$



$\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$ $x_1 = -3x_2$

$\begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} \to C(A) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad C(A^T) = [0\ 1]^T$
$N(A) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad N(A^T) = [-3\ 1]$ dim of all = 1

$\begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}$

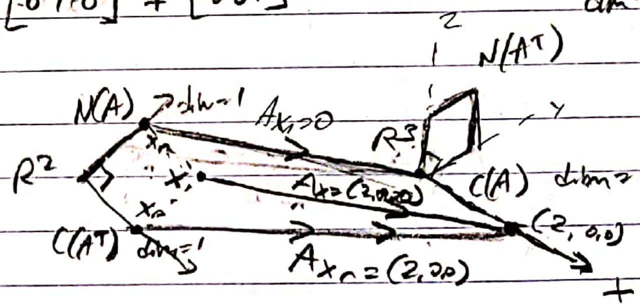


12. $\begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \to C(A) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C(A^T) = [1\ -1]$
$N(A) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad N(A^T) = [0\ 1\ 0]^T + [0\ 0\ 1]^T$ dim = 2

$\begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Span $[0\ 1\ 0]^T + [0\ 0\ 1]^T$

$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = N(A) + C(A^T)$
$= \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{x_n} + \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{x_r}$



17. $S = \{\langle 0,0,0\rangle\} \Rightarrow S^\perp = R^3$
$S = \{\text{span}(\langle 1,1,1\rangle)\} \Rightarrow S^\perp = \{\text{span}(\langle 1,0,-1\rangle, \langle 1,-1,0\rangle)\}$
$S = \{\text{span}(\langle 1,1,1\rangle, \langle 1,1,-1\rangle)\} \Rightarrow S^\perp = \{\text{span}(\langle 1,-1,0\rangle)\}$

$x + y + z$
$x + y - z$
$1\ -1\ 0$

$$e \cdot a = -2/5 \cdot 1 + 1/5 \cdot 1 + 1/5 \cdot 1 = 0$$

2. 1. $proj_A(B) = \frac{a^Tb}{a^Ta} a$

$$\frac{\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix}}{\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}}\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{1+2+2}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{5}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix}$$

$$e = b - p = \begin{bmatrix}1\\2\\2\end{bmatrix} - \begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix} = \begin{bmatrix}-2/5\\1/5\\1/5\end{bmatrix}$$

$proj_A(B) = \frac{a^Tb}{a^Ta}$

$$= \frac{\begin{bmatrix}-1 & -3 & -1\end{bmatrix}\begin{bmatrix}1\\3\\1\end{bmatrix}}{\begin{bmatrix}-1 & -3 & -1\end{bmatrix}\begin{bmatrix}-1\\-3\\-1\end{bmatrix}}\begin{bmatrix}-1\\-3\\-1\end{bmatrix} = \frac{-1-9-1}{1+9+1}\begin{bmatrix}-1\\-3\\-1\end{bmatrix} = \frac{-11}{11}\begin{bmatrix}-1\\-3\\-1\end{bmatrix} = \begin{bmatrix}1\\3\\1\end{bmatrix}$$

$$e = b - p = \begin{bmatrix}1\\3\\1\end{bmatrix} - \begin{bmatrix}1\\3\\1\end{bmatrix} = 0 \qquad e \cdot a = 0$$

3 a) $P = \frac{aa^T}{a^Ta} = \frac{\begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 1\end{bmatrix}}{3} = \frac{\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}}{3} = \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \cdot \frac{1}{3}$

$$P^2 = \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\frac{1}{3} \cdot \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\frac{1}{3} = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}^2 = \frac{1}{9}\begin{bmatrix}3&3&3\\3&3&3\\3&3&3\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$$Pb = \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\frac{1}{3} \cdot \begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}5\\5\\5\end{bmatrix} \cdot \frac{1}{3}$$

b) $P = \frac{\begin{bmatrix}-1\\-3\\-1\end{bmatrix}\begin{bmatrix}-1 & -3 & -1\end{bmatrix}}{11} = \frac{\begin{bmatrix}1&3&1\\3&9&3\\1&3&1\end{bmatrix}}{11}$

$$P^2 = \frac{1}{11^2}\begin{bmatrix}1&3&1\\3&9&3\\1&3&1\end{bmatrix}^2 = \frac{1}{121}\begin{bmatrix}11&33&11\\33&99&33\\11&33&11\end{bmatrix} = P$$

$$Pb = \begin{bmatrix}1&3&1\\3&9&3\\1&3&1\end{bmatrix}\frac{1}{11} \cdot \begin{bmatrix}-1\\-3\\-1\end{bmatrix} = \begin{bmatrix}-11\\-33\\-11\end{bmatrix} \cdot \frac{1}{11} = \frac{1}{11}\begin{bmatrix}1&3&1\\3&9&3\\1&3&1\end{bmatrix} = \begin{bmatrix}-1\\-3\\-1\end{bmatrix}$$

11. $A^TA\hat{x} = A^Tb$

$$\begin{bmatrix}1&0&0\\1&1&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}\hat{x} = \begin{bmatrix}1&0&0\\1&1&0\end{bmatrix}\begin{bmatrix}2\\3\\4\end{bmatrix}$$

$$\begin{bmatrix}-1&1\\2&-1\end{bmatrix}\begin{bmatrix}1&1\\2&1\end{bmatrix}\hat{x} = \begin{bmatrix}-1&1\\2&-1\end{bmatrix}\begin{bmatrix}2\\5\end{bmatrix}$$

$$\hat{x} = \begin{bmatrix}-1\\3\end{bmatrix}$$

$p = A\hat{x}$

$$p = \begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}2\\3\\0\end{bmatrix}$$

$$e = b - p \Rightarrow e = \begin{bmatrix}2\\3\\4\end{bmatrix} - \begin{bmatrix}2\\3\\0\end{bmatrix} = \begin{bmatrix}0\\0\\4\end{bmatrix}$$

$e \cdot A_1 = 0+0+0 = 0$

$e \cdot A_2 = 0+0+0 = 0$

b) $\begin{bmatrix}1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}1&1\\1&1\\0&1\end{bmatrix}\hat{x} = \begin{bmatrix}1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}4\\4\\6\end{bmatrix}$

$$\begin{bmatrix}-1&1\\3/2&-1\end{bmatrix}\begin{bmatrix}2&2\\3&2\end{bmatrix}\hat{x} = \begin{bmatrix}-1&1\\3/2&-1\end{bmatrix}\begin{bmatrix}8\\14\end{bmatrix}$$

$$\hat{x} = \begin{bmatrix}-2\\6\end{bmatrix}$$

$p = A\hat{x}$

$$p = \begin{bmatrix}1&1\\1&1\\0&1\end{bmatrix}\begin{bmatrix}-2\\6\end{bmatrix}$$

$$p = \begin{bmatrix}4\\4\\6\end{bmatrix}$$

$e = b - p = 0$

$e \cdot A_1 = 0$

$e \cdot A_2 = 0$

12. $P = A(A^TA)^{-1}A^T$

$$P_1 = \begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}\left(\begin{bmatrix}1&0&0\\1&1&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}\right)^{-1}\begin{bmatrix}1&0&0\\1&1&0\end{bmatrix}$$

$$\begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}\left(\begin{bmatrix}1&1\\2&1\end{bmatrix}\right)^{-1}\begin{bmatrix}1&0&0\\1&1&0\end{bmatrix}$$

$$\begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}\begin{bmatrix}-1&1\\2&-1\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&0\end{bmatrix}$$

$$\begin{bmatrix}0&1\\1&-1\\0&0\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&0\end{bmatrix}$$

$$P_1 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix} \qquad P_1b = \begin{bmatrix}2\\3\\0\end{bmatrix}$$

$$P_2 = \begin{bmatrix}1&1\\1&1\\0&1\end{bmatrix}\left(\begin{bmatrix}1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}1&1\\1&1\\0&1\end{bmatrix}\right)^{-1}\begin{bmatrix}1&1&0\\1&1&1\end{bmatrix}$$

$$\begin{bmatrix}1&1\\1&1\\0&1\end{bmatrix}\left(\begin{bmatrix}2&2\\3&2\end{bmatrix}\right)^{-1}\begin{bmatrix}1&1&0\\1&1&1\end{bmatrix}$$

$$\begin{bmatrix}1&1\\1&1\\0&1\end{bmatrix}\begin{bmatrix}-1&1\\3/2&-1\end{bmatrix}\begin{bmatrix}1&1&0\\1&1&1\end{bmatrix}$$

$$\begin{bmatrix}0&1/2\\0&1/2\\1&-1\end{bmatrix}\begin{bmatrix}1&1&0\\1&1&1\end{bmatrix}$$

$$P_2 = \begin{bmatrix}1/2&1/2&0\\1/2&1/2&0\\0&0&1\end{bmatrix}$$

$$P_2^2 = \begin{bmatrix}1/2&1/2&0\\1/2&1/2&0\\0&0&1\end{bmatrix}^2 = \begin{bmatrix}\frac14+\frac14&\frac14+\frac14&0\\\frac14+\frac14&\frac14+\frac14&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1/2&1/2&0\\1/2&1/2&0\\0&0&1\end{bmatrix}$$

$$P_2b = \begin{bmatrix}1/2&1/2&0\\1/2&1/2&0\\0&0&1\end{bmatrix}\begin{bmatrix}4\\4\\6\end{bmatrix} = \begin{bmatrix}4\\4\\6\end{bmatrix}$$

Scanned by CamScanner

13. $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}$ $P=A(A^TA)^{-1}A^T$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}\left(\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}\right)^{-1}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix} \quad Pb=\begin{bmatrix}1\\2\\3\\0\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}^{-1}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}=P$$

P is a square matrix and P

A^TA (AA)

17. $(I-P)^2=I^2-IP-PI+P^2$

$I-P-P+P^2$ $P^2=P$, so

$I-P-P+P$

$=I-P$ $I-P$ projects onto the left nullspace.

19. $\langle 1,1,0\rangle, \langle 2,0,1\rangle$

$\begin{bmatrix}1&2\\1&0\\0&1\end{bmatrix}$ $P=A(A^TA)^{-1}A^T=\begin{bmatrix}1&2\\1&0\\0&1\end{bmatrix}\left(\begin{bmatrix}1&1&0\\2&0&1\end{bmatrix}\begin{bmatrix}1&2\\1&0\\0&1\end{bmatrix}\right)^{-1}\begin{bmatrix}1&1&0\\2&0&1\end{bmatrix}$

$$\begin{bmatrix}2&2\\2&5\end{bmatrix}^{-1}=\begin{bmatrix}5/6&-1/3\\-1/3&1/3\end{bmatrix}$$

$$P=\begin{bmatrix}1&2\\1&0\\0&1\end{bmatrix}\begin{bmatrix}5/6&-1/3\\-1/3&1/3\end{bmatrix}=\begin{bmatrix}1/6&1/3\\5/6&-1/3\\-1/3&1/3\end{bmatrix}\begin{bmatrix}1&1&0\\2&0&1\end{bmatrix}=\begin{bmatrix}5/6&1/6&1/3\\1/6&5/6&-1/3\\1/3&-1/3&1/3\end{bmatrix}$$

20. $e=\langle 1,-1,-2\rangle$

$$Q=\frac{ee^T}{e^Te}=\frac{\begin{bmatrix}1\\-1\\-2\end{bmatrix}\begin{bmatrix}1&-1&-2\end{bmatrix}}{\begin{bmatrix}1&1&2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix}}=\frac{\begin{bmatrix}1&-1&-2\\-1&1&2\\-2&2&4\end{bmatrix}}{6}=\begin{bmatrix}1/6&-1/6&-1/3\\-1/6&1/6&1/3\\-1/3&1/3&2/3\end{bmatrix}$$

$$P=I-Q=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}1/6&-1/6&-1/3\\-1/6&1/6&1/3\\-1/3&1/3&2/3\end{bmatrix}=\begin{bmatrix}5/6&1/6&1/3\\1/6&5/6&-1/3\\1/3&-1/3&1/3\end{bmatrix}$$

3.1. $A=\begin{bmatrix}1&0\\1&1\\1&3\\1&4\end{bmatrix}$ $b=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}$

$$A^TA=\begin{bmatrix}1&1&1&1\\0&1&3&4\end{bmatrix}\begin{bmatrix}1&0\\1&1\\1&3\\1&4\end{bmatrix}=\begin{bmatrix}4&8\\8&26\end{bmatrix}^{-1}=\begin{bmatrix}13/20&-1/5\\-1/5&1/10\end{bmatrix}$$

$$A^Tb=\begin{bmatrix}1&1&1&1\\0&1&3&4\end{bmatrix}\begin{bmatrix}0\\8\\8\\20\end{bmatrix}=\begin{bmatrix}36\\112\end{bmatrix}$$

$$\hat{x}=\begin{bmatrix}13/20&-1/5\\-1/5&1/10\end{bmatrix}\begin{bmatrix}36\\112\end{bmatrix} \quad \hat{x}=\begin{bmatrix}1\\4\end{bmatrix}$$

$p=A\hat{x}$

$$=\begin{bmatrix}1&0\\1&1\\1&3\\1&4\end{bmatrix}\begin{bmatrix}1\\4\end{bmatrix}=\begin{bmatrix}1\\5\\13\\17\end{bmatrix}$$

$$e=b-p=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}-\begin{bmatrix}1\\5\\13\\17\end{bmatrix}=\begin{bmatrix}-1\\3\\-5\\3\end{bmatrix}$$

$$E=\|e\|^2=(1+9+25+9)$$
$$=44$$

9. $\begin{bmatrix}1&0&0\\1&1&1\\1&3&9\\1&4&16\end{bmatrix}\begin{bmatrix}C\\D\\E\end{bmatrix}=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}$ $\begin{bmatrix}1&1&1&1\\0&1&3&4\\0&1&9&16\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&1\\1&3&9\\1&4&16\end{bmatrix}\hat{x}=\begin{bmatrix}1&1&1&1\\0&1&3&4\\0&1&9&16\end{bmatrix}\begin{bmatrix}0\\8\\8\\20\end{bmatrix}$

4.9a is fitting to 4 points

4.9b. is projecting into R^4, the problem has not been changed.

10. $A=\begin{bmatrix}1&0&0&0\\1&1&1&1\\1&3&9&27\\1&4&16&64\end{bmatrix}$ $\hat{x}=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}$

$\left[\begin{array}{cccc|cccc}1&0&0&0&1&0&0&0\\1&1&1&1&0&1&0&0\\1&3&9&27&0&0&1&0\\1&4&16&64&0&0&0&1\end{array}\right]\xrightarrow{\substack{-r_1+r_2\\-r_1+r_3\\-r_1+r_4}}\left[\begin{array}{cccc|cccc}1&0&0&0&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&3&9&27&-1&0&1&0\\0&4&16&64&-1&0&0&1\end{array}\right]\xrightarrow{\substack{-3r_2+r_3\\-4r_2+r_4}}\left[\begin{array}{cccc|cccc}1&0&0&0&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&0&9&27&2&-3&1&0\\0&0&16&64&3&-4&0&1\end{array}\right]\xrightarrow{-\frac{16}{9}r_3+r_4}$

$\left[\begin{array}{cccc|cccc}1&0&0&0&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&0&9&27&2&-3&1&0\\0&0&0&16&-\frac{5}{9}&\frac{12}{9}&-\frac{16}{9}&1\end{array}\right]\xrightarrow{\substack{\frac{1}{9}r_3\\\frac{1}{16}r_4}}\left[\begin{array}{cccc|cccc}1&0&0&0&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&0&1&3&\frac{2}{9}&-\frac{3}{9}&\frac{1}{9}&0\\0&0&0&1&-\frac{5}{144}&\frac{1}{12}&-\frac{1}{9}&\frac{1}{16}\end{array}\right]\xrightarrow{-3r_4+r_3}\left[\begin{array}{cccc|cccc}1&0&0&0&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&0&1&0&\frac{47}{144}&-\frac{24}{12}&\frac{4}{9}&-\frac{3}{16}\\0&0&0&1&-\frac{5}{144}&\frac{1}{12}&-\frac{1}{9}&\frac{1}{16}\end{array}\right]$

$\rightarrow\left[\begin{array}{cccc|cccc}1&0&0&0&1&0&0&0\\0&1&0&0&-\frac{19}{12}&2&-\frac{2}{3}&\frac{1}{4}\\0&0&1&0&\frac{2}{3}&-\frac{7}{6}&\frac{5}{6}&-\frac{1}{3}\\0&0&0&1&-\frac{1}{12}&\frac{1}{6}&-\frac{1}{6}&\frac{1}{12}\end{array}\right]$ $\hat{x}=\begin{bmatrix}1&0&0&0\\-\frac{19}{12}&2&-\frac{2}{3}&\frac{1}{4}\\\frac{2}{3}&-\frac{7}{6}&\frac{5}{6}&-\frac{1}{3}\\-\frac{1}{12}&\frac{1}{6}&-\frac{1}{6}&\frac{1}{12}\end{bmatrix}\begin{bmatrix}0\\8\\8\\20\end{bmatrix}=\begin{bmatrix}0\\47\\-28\\5\end{bmatrix}\frac{1}{3}$

$p=A\hat{x}=\begin{bmatrix}1&0&0&0\\1&1&1&1\\1&3&9&27\\1&4&16&64\end{bmatrix}\begin{bmatrix}0\\47\\-28\\5\end{bmatrix}\frac{1}{3}=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}\Rightarrow p=b,\ e=0.$

12. $a^Ta\hat{x}=a^Tb$

$a^Ta=m\Rightarrow m\hat{x}=a^Tb$

$\sum_{i=1}^{m}1$ $\hat{x}=\frac{a^Tb}{m}$, $a^Tb=b_1+b_2\ldots b_m$ thus a^Tb is the sum of all b, and $\frac{a^Tb}{m}$ = mean.

(b) $e=b-a\hat{x}=b-\hat{x}=[b_1-\bar{x}\ \ b_2-\bar{x}\ \ldots\ b_m-\bar{x}]^T$,

$\|e\|^2=\sum_{i=1}^{m}(b_i-\bar{x})^2=$ variance $=\sigma^2$

$\|e\|=\sqrt{\|e\|^2}=$ standard deviation $=\sigma$

(c) $e=b-p$

$e=\begin{bmatrix}1\\2\\6\end{bmatrix}-\begin{bmatrix}3\\3\\3\end{bmatrix}=\begin{bmatrix}-2\\-1\\3\end{bmatrix}$ $\quad p\cdot e=3\cdot-2+3\cdot-1+3\cdot3=0.$

$b-p$

$P=A(A^TA)^{-1}A^T$

$=\begin{bmatrix}3\\3\\3\end{bmatrix}\left([3\,3\,3]\begin{bmatrix}3\\3\\3\end{bmatrix}\right)^{-1}[3\,3\,3]$

$[27]^{-1}=\left[\frac{1}{27}\right]$

$\begin{bmatrix}3\\3\\3\end{bmatrix}\left[\frac{1}{27}\right]=\frac{1}{9}\begin{bmatrix}1\\1\\1\end{bmatrix}[3\,3\,3]=\frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}=P$

13. $(A^TA)^{-1}A^Te = (A^TA)^{-1}A^T(b-Ax)$

$(A^TA)^{-1}A^Te = (A^TA)^{-1}A^Tb - (A^TA)^{-1}(A^TA)x$

$(A^TA)^{-1}A^T(e) = \hat{x} - Ix$

$Ax-b=e \Rightarrow Ax-b=0$ implies that $\hat{x}-x$ is also 0, and it's unbiased.

14. $(b-Ax)(b-Ax)^T$

$(A^TA)^{-1}A^T(b-Ax)(b-Ax)^T A(A^TA)^{-1}$

$(A^TA)^{-1}A^T \sigma^2 \cancel{I} (A(A^TA)^{-1})$

$(A^TA)^{-1}(A^TA)\sigma^2(A^TA)^{-1}$

$\Sigma_{\hat{x}} \Rightarrow = \sigma^2(A^TA)^{-1}$

15. $(A^TA) = [1\,1\,1\,1]\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = [4]$

$(A^TA)^{-1} = \frac{1}{4}$ $\qquad \sigma^2(A^TA)^{-1} = \frac{\sigma^2}{4}$

4. 1. Independent $\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$

2. orthogonal, independent $\begin{bmatrix}0.6\\0.8\end{bmatrix}\begin{bmatrix}0.8\\-0.6\end{bmatrix}$

3. $-\sin\theta\cos\theta + \sin\theta\cos\theta$ orthonormal, independent

3. $\begin{bmatrix}1&-1&\cdots\end{bmatrix}\begin{bmatrix}1&1&1&-1\\1&1&-1&1\\-1&1&1&1\\-1&-1&-1&1\end{bmatrix}$ $\begin{bmatrix}n^2&&&\\&n^2&&\\&&n^2&\\&&&n^2\end{bmatrix}$ $n=4 \Rightarrow \begin{bmatrix}16&0&0&0\\0&16&0&0\\0&0&16&0\\0&0&0&16\end{bmatrix} = 16I$

$\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix} = \begin{bmatrix}1&0&0\\0&4&0\\0&0&9\end{bmatrix}$ diagonal with 1, 4, 9.

5. $\begin{bmatrix}2\\-2\\0\end{bmatrix}$ $\begin{bmatrix}1\\1\\-1\end{bmatrix}$

$\begin{bmatrix}2\\-2\\0\end{bmatrix}\frac{1}{\sqrt{8}}$, $\begin{bmatrix}1\\1\\-1\end{bmatrix}\frac{1}{\sqrt{3}}$

11. $\langle 1,3,4,5,7\rangle$, $\langle -6,6,8,0,8\rangle$ (a2, b)

$B = b - \frac{A^Tb}{A^TA}A$

$B = b - \frac{[1\,3\,4\,5\,7]\begin{bmatrix}-6\\6\\8\\0\\8\end{bmatrix}}{[1\,3\,4\,5\,7]\begin{bmatrix}1\\3\\4\\5\\7\end{bmatrix}}\begin{bmatrix}1\\3\\4\\5\\7\end{bmatrix}$

$\frac{-6+18+32+0+56}{1+9+16+25+49}\begin{bmatrix}1\\3\\4\\5\\7\end{bmatrix}$

$\begin{bmatrix}-6\\6\\8\\0\\8\end{bmatrix} - 1\begin{bmatrix}1\\3\\4\\5\\7\end{bmatrix} = \begin{bmatrix}-7\\3\\4\\-5\\1\end{bmatrix}$

$Q = \frac{1}{10}\begin{bmatrix}1&-7\\3&3\\4&4\\5&-5\\7&1\end{bmatrix}$

$q_1 = \frac{A}{\|A\|} = \frac{1}{10}\begin{bmatrix}1\\3\\4\\5\\7\end{bmatrix}$ $\quad \frac{B}{\|B\|} = \frac{1}{10}\begin{bmatrix}-7\\3\\4\\-5\\1\end{bmatrix} = q_2$

$QQ^Tx = \begin{bmatrix}0.5&-0.18&-0.24&0.4&0\\-0.18&0.18&0.24&0&0.24\\-0.24&0.24&0.32&0&0.32\\0.4&0&0&0.5&0.3\\0&0.24&0.32&0.3&0.5\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$

$= [0.5\ -0.18\ -0.24\ 0.4\ 0]^T$

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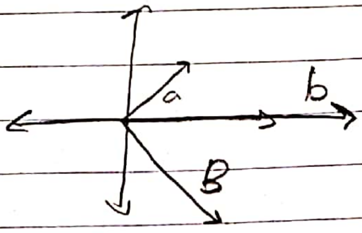
13. $B = b - \frac{a^T b}{a^T a} a$

$B = b - \frac{[1\ 1]\begin{bmatrix}4\\0\end{bmatrix}}{[1\ 1]\begin{bmatrix}1\\1\end{bmatrix}} \begin{bmatrix}1\\1\end{bmatrix}$

$B = b - \frac{4}{2} \begin{bmatrix}1\\1\end{bmatrix}$

$B = \begin{bmatrix}4\\0\end{bmatrix} - 2 \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$



14. $q_1 = \begin{bmatrix}1\\1\end{bmatrix}\frac{1}{\sqrt{2}}$ $b = \begin{bmatrix}4\\0\end{bmatrix}$ $B = b - \frac{a^T b}{a^T a} a$

$B = \begin{bmatrix}2\\-2\end{bmatrix}$

$q_2 = \begin{bmatrix}2\\-2\end{bmatrix}\frac{1}{2\sqrt{2}}$

$Q = \begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\end{bmatrix}$

$R = Q^T A$

$R = \begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\end{bmatrix}\begin{bmatrix}1 & 4\\ 1 & 0\end{bmatrix}$

$R = \begin{bmatrix}\sqrt{2} & 2\sqrt{2}\\ 0 & 2\sqrt{2}\end{bmatrix}$

18. $B = b - \frac{[1\ -1\ 0\ 0]\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}}{[1\ -1\ 0\ 0]\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}} \begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$

$\begin{bmatrix}0\\1\\-1\\0\end{bmatrix} - \frac{-1}{2}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\\ -1\\ 0\end{bmatrix}$

$C = c - \frac{a^T c}{a^T a} a - \frac{b^T c}{b^T b} b$

$c - \frac{[1\ -1\ 0\ 0]\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}}{2} a - \frac{[\frac{1}{2}\ \frac{1}{2}\ -1\ 0]\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}}{\frac{6}{4}} \begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\\ -1\\ 0\end{bmatrix}$

$C = \begin{bmatrix}0\\0\\1\\-1\end{bmatrix} - \frac{-2}{3}\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\\ -1\\ 0\end{bmatrix} = \begin{bmatrix}\frac{1}{3}\\ \frac{1}{3}\\ \frac{1}{3}\\ -1\end{bmatrix}$

$A = \langle 1, -1, 0, 0\rangle$

$B = \langle \frac{1}{2}, \frac{1}{2}, -1, 0\rangle$

$C = \langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1\rangle$

26. True $Q^T Q = I \Rightarrow Q^{-1} Q^{-1} = I$

True. $Qx = q_1 x_1 + q_2 x_2$

$\begin{bmatrix}q_1 & q_2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$ $\|Qx\|^2 = (q_1 x_1 + q_2 x_2)^2 = \|q_1\|^2 x_1^2 + q_1 x_1 q_2 x_2 + q_2 x_2 q_1 x_1 + \|q_2\|^2 x_2^2$

$= x_1^2 + x_2^2$.

$q_1 x_1 + q_2 x_2$

$q_1 x_1 + q_2$

30. The columns are independent and orthonormal.
Thus, $W^{-1} = W^T$

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5. 32. a. Let $r = C(A^T)$, $n = N(A)$, $c = C(A)$, and $l = N(A^T)$.

$r^T n = 0$, $c^T l = 0$.

r and n must be orthogonal, c and l must be orthogonal.

b. $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, $r = [1\ 2]^T$, $c = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $n = [-2\ 1]^T$, $l = [-3\ 1]^T$

6. 32. $P_1 = A(A^TA)^{-1}A^T$

$P_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\left([1\ 2\ 0]\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)^{-1}[1\ 2\ 0]$

$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1/5 \\ 2/5 \\ 0 \end{bmatrix}[1\ 2\ 0]$ $\begin{bmatrix} 1/5 & 2/5 & 0 \\ 2/5 & 4/5 & 0 \\ 0 & 0 & 0 \end{bmatrix} = P_1$

$P_2 P_1 = P_1$ because a projection

~~through~~ the first column will yield values in one dimension,

and further projection of one dimension into two dimensions will not create information

thus $P_2 P_1 = P_1$.

33. $\dfrac{b_1 + \ldots + b_{999}}{999} + \dfrac{1}{1000}\left(b_{1000} - \dfrac{b_1 + \ldots + b_{999}}{999}\right)$

$\dfrac{b_1 + \ldots + b_{999}}{999} - \dfrac{b_1 + \ldots + b_{999}}{999(1000)} + \dfrac{b_{1000}}{1000}$

$= b_1 + \ldots + b_{999}\left(\dfrac{1}{999} - \dfrac{1}{999(1000)}\right)$

$\left(\dfrac{1000}{999(1000)} - \dfrac{1}{999(1000)}\right)$

$\left(\dfrac{\cancel{999}}{\cancel{999}(1000)}\right)$

$= b_1 + \ldots + b_{999}\left(\dfrac{1}{1000}\right) + \dfrac{b_{1000}}{b_{1000}}$

$= \dfrac{b_1 + \ldots + b_{999} + b_{1000}}{1000}$

$= \dfrac{b_1 + \ldots + b_{1000}}{1000}$

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