I pledge on my honor that I have abided by the Stevens Honor System

MA 346 HW 7

4.1.2b

x	$f(x)$	$f'(x)$
1.0	1.00	1.3125
1.2	1.2625	1.985
1.4	1.6595	1.985

$$f'(1.0) \approx \frac{f(1+h)-f(1)}{h} \approx \frac{f(1.2)-f(1.0)}{0.2} = 1.3125$$

$$f'(1.2) \approx \frac{f(1.2+h)-f(1.2)}{h} \approx \frac{f(1.4)-f(1.2)}{0.2} = 1.985$$

$$f'(1.4) \approx \frac{f(1.4+h)-f(1.4)}{h} \approx \frac{f(1.2)-f(1.4)}{-0.2} = 1.985$$

4b. $f(x) = x^2 \ln x + 1 \Rightarrow f'(x) = 2x\ln x + x \quad f''(x) = 2\ln x + 2 + 1$
$= 2\ln x + 3$

x	$f'(x)$	actual $f'(x)$	error
1.0	1.3125	1	0.3125
1.2	1.985	1.6376	0.3474
1.4	1.985	2.3421	0.3571

$$|\text{error}| < \left|\frac{h}{2} f''(\xi(x))\right| < \frac{h}{2} f''(1.4) \le \frac{0.2}{2}(2\ln(1.4)+3)$$
$$= 0.3673$$

6b

x	$f(x)$	$f'(x)$
7.4	-68.3193	-16.69325
7.6	-71.6982	-17.09575
7.8	-75.1576	-17.498
8.0	-78.6974	-17.9

$h = 0.2$

Midpoints

$$f'(7.6) = \frac{1}{0.2(2)}[f(7.8) - f(7.4)]$$
$$= \frac{1}{0.4}[-75.1576 - (-68.3193)]$$
$$= -17.09575$$

$$f'(7.8) = \frac{1}{(0.2)(2)}[f(8.0) - f(7.6)]$$
$$= \frac{1}{0.4}[-78.6974 - (-71.6982)]$$
$$= -17.498$$

Endpoints:

$$f'(7.4) = \frac{1}{2(0.2)}[-3f(7.4) + 4f(7.6) - f(7.8)]$$
$$= \frac{1}{0.4}[-3(-68.3193) + 4(-71.6982) - (-75.1576)] = -16.69325$$

$$f'(8.0) = \frac{1}{2(-0.2)}[-3f(8.0) + 4(f(7.8)) - f(7.6)]$$
$$= \frac{1}{-0.4}[-3(-78.6974) + 4(-75.1576) - (-71.6982)] = -17.9$$

4.26 $N_1(h) = 2.356194$ $N_1(\frac{h}{2}) = -0.4879837$
$N_1(\frac{h}{4}) = -0.8815732$ $N_1(\frac{h}{8}) = -0.9709157$

$O(h^2)$	$O(h^4)$	$O(h^6)$
$N_1(h) = 2.356194$		
$N_1(h/2) = -0.4879837$	$N_2(h) = -3.3321614$	
$N_1(h/4) = -0.8815732$	$N_2(h/2) = -1.2751627$	$N_3(h) = 0.787582$
$N_1(h/8) = -0.9709157$	$N_2(h/4) = -1.0602582$	$N_3(\frac{h}{2}) = -0.8453537$

$N_2(h) = N_1(\frac{h}{2}) + [N_1(\frac{h}{2}) - N_1(h)]$
$= -3.3321614$
$N_2(\frac{h}{2}) = N_1(\frac{h}{4}) + N_1(\frac{h}{4}) - N_1(\frac{h}{2}) = -1.2751627$
$N_2(h/4) = N_1(\frac{h}{8}) + N_1(\frac{h}{8}) - N_1(\frac{h}{4}) = -1.0602582$
$N_3(h) = N_2(\frac{h}{2}) + N_2(\frac{h}{2}) - N_2(h) = 0.787582$
$N_3(\frac{h}{2}) = N_2(\frac{h}{4}) + N_2(\frac{h}{4}) - N_2(\frac{h}{2}) = -0.8453537$

$N_4(h) = N_3(\frac{h}{2}) + N_3(\frac{h}{2}) - N_3(h) = -2.4782894$

4.2.9. $M = N_1(h) + k_1(h) + k_2 h^2 \ldots$ Eq 1
$M = N_1(\frac{h}{3}) + k_1(\frac{h}{3}) + k_2(\frac{h}{3})^2 \ldots$ Eq 2
3 Eq 2 − Eq 1 $\Rightarrow$ $2M = 2N_1(\frac{h}{3}) - N_1(h) + N_1(\frac{h}{3}) + k_2 O(h^2)$
$\Rightarrow M = N_1(\frac{h}{3}) + \left(\frac{N_1(h/3) - N_1(h)}{2}\right) + k_2 O(h^2)$ ⟨3⟩
$N_2(h) = N_1(\frac{h}{3}) + \left(\frac{N_1(\frac{h}{3}) - N_1(h)}{2}\right)$
$M = N_2(\frac{h}{9}) + k_2 O(h^2)$ ⟨4⟩
3 ⟨4⟩ − ⟨3⟩ $\Rightarrow$ $2M = 2N_2(\frac{h}{9}) + N_2(\frac{h}{9}) - N_2(h/3) + k_3(O(h^3))$
$M = N_2(\frac{h}{9}) + \left(\frac{N_2(\frac{h}{9}) - N_2(h/3)}{2}\right) + k_3 O(h^3)$

$N_3(\frac{h}{3}) = N_2(\frac{h}{9}) + \left(\frac{N_2(\frac{h}{9}) - N_2(h/3)}{2}\right)$
Because of the $k_3 O(h^3)$,
$N_3(h) = N_2(\frac{h}{3}) + \left(\frac{N_2(\frac{h}{3}) - N_2(h)}{2}\right)$ is an $O(h^3)$ approximation.

$$\int_{-0.5}^{0} x\ln(x+1)\,dx$$

4.3.8. 2b. $\int_a^b f(x)\,dx = \frac{h}{2}[f(x_0)+f(x_1)] - \frac{h^3}{12}f''(\xi)$

$h = b-a,\ x_0 = a, x_1 = b$

$h = 0.5$

$\frac{0.5}{2}[f(0)+f(-0.5)] = 0.25[0 + 0.5\ln 0.5]$

$= 0.173286$

6b. $\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}[f(x_0)+4f(x_1)+f(x_2)] - \frac{h^5}{90}f^{(4)}(\xi)$

$h = \frac{x_2 - x_0}{2}$

$\int_{-0.5}^{0} x\ln(x+1)\,dx \approx \frac{0.25}{3}[f(0)+4f(-0.25)+f(-0.5)]$

$h = 0.25$ $\approx \frac{1}{12}[0 + 4(-0.25\ln 0.75) + (-0.5\ln 0.5)]$

$\approx \frac{1}{12}[0.28768 + 0.3465] = 0.0528596$

8b. bound $= \frac{h^5}{90} f^{(4)}(\xi)$

$h = 0.25,\ \xi \in [-0.5, 0]$

$f'(x) = \ln(x+1) + 1 - \frac{1}{x+1}$

$f''(x) = \frac{1}{x+1} + \frac{1}{(x+1)^2}$ $f'''(x) = -\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}$ $f''''(x) = \frac{2}{(x+1)^3} + \frac{6}{(x+1)^4}$

$f''''(x)$ has no zeros on $[-0.5, 0]$, so check endpoints.

$f''''(-0.5) = \frac{2}{(0.5)^3} + \frac{6}{(0.5)^4} = 112$ $f''''(0) = \frac{2}{1^3} + \frac{6}{1^4} = 8.$

$\max f''''(x) = 112.$

$|\text{error}| \le \frac{0.25^5}{90} \cdot 112 = 0.001215278$

actual value $= \int_{-0.5}^{0} x\ln(x+1)\,dx$ $f = \ln(x+1)$ $g' = x$

$= \frac{x^2\ln(x+1)}{2} - \int \frac{x^2}{2(x+1)}\,dx$ $f' = \frac{1}{x+1}$ $g = \frac{x^2}{2}$

$u = x+1$ $du = 1$

$= \frac{x^2\ln(x+1)}{2} - \frac{1}{2}\left(\int \frac{(u-1)^2}{u}\,du\right)$

$= \frac{x^2\ln(x+1)}{2} - \frac{1}{2}\left(\frac{u^2}{2} - 2u + \ln u\right)$

$= \frac{x^2\ln(x+1)}{2} - \frac{1}{2}\left(\frac{(x+1)^2}{2} - 2(x+1) + \ln(x+1)\right)\Big|_{-0.5}^{0}$

$= 0.0525698\ 0129$

actual error $= |0.0525698 - 0.0528596|$

$= 0.0002848$

14. Trapezoidal = $\int_0^2 f(x)\,dx = \frac{2}{2}(f(2)+f(0)) = 5$
$f(2)+f(0) = 5$

Midpoint = $\int_0^2 f(x)\,dx = 2(1)f(1) = 4$
$f(1) = 2.$

Simpsons = $\int_0^2 f(x)\,dx = \frac{\frac{2}{2}}{3}(f(0)+4(f(1))+f(2))$
$= \frac{1}{3}(5+4(2)) = \frac{13}{3}$

17.

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46675

$\int_{1.8}^{2.6} f(x)\,dx$

Closed Newton Cotes:

n=1: Trapezoidal rule,
$\int_{1.8}^{2.6} f(x)\,dx = \frac{2.6-1.8}{\frac{1}{2}}[f(1.8)+f(2.6)] = 5.434756$

n=2: Simpson's Rule
$\int_{1.8}^{2.6} f(x)\,dx = \frac{2.6-1.8}{\frac{2}{3}}[f(1.8)+4f(2.2)+f(2.6)] = 5.034204$

n=4:
$\int_{1.8}^{2.6} f(x)\,dx = \frac{2}{45}\left(\frac{2.6-1.8}{4}\right)[7f(1.8)+32f(2.0)+12f(2.2)+$
$32(f(2.4)) + 7f(2.6)]$
$= 5.032921$

Open Newton Cotes:

n=0: Midpoint rule:
$\int_{1.8}^{2.6} f(x)\,dx = 2\left(\frac{2.6-1.8}{2}\right)f(2.2) = 4.833928$

n=2:
$\int_{1.8}^{2.6} f(x)\,dx = \frac{4}{3}\left(\frac{2.6-1.8}{4}\right)[2f(1.8) - f(2.2) + 2f(2.6)]$
$= 5.635032$

20. $\int_a^b f(x)\,dx = \frac{9}{4}h f(x_1) + \frac{3}{4}h f(x_2)$

$f=1 \Rightarrow \int_a^b 1\,dx = \frac{9}{4}h(1) + \frac{3}{4}h(1)$

$b-a = \frac{9}{4}\left(\frac{b-a}{3}\right) + \frac{3}{4}\left(\frac{b-a}{3}\right)$

$b-a = \frac{12}{4}\left(\frac{b-a}{3}\right) = b-a$ ✓

$f=x \Rightarrow \int_a^b x\,dx = \frac{9}{4}\left(\frac{b-a}{3}\right)x_1 + \frac{3}{4}\left(\frac{b-a}{3}\right)x_2$

$\frac{b^2-a^2}{2} = \frac{9}{4}\left(\frac{b-a}{3}\right)\left(a+\frac{b-a}{3}\right) + \frac{3}{4}\left(\frac{b-a}{3}\right)(b)$

$= \frac{9}{4}(h)(a+h) + \frac{3}{4}bh$

$= \frac{3}{4}h\,(3(a+h)+b)$

$= \frac{3}{4}\left(\frac{b-a}{3}\right)(3a+b-a+b)$

$= \frac{b-a}{4}(2a+2b) = \frac{2b^2-2a^2}{4} = \frac{b^2-a^2}{2}$

$f=x^2 \Rightarrow \int_a^b x^2\,dx = \frac{9}{4}h(a+h)^2 + \frac{3}{4}h(b)^2$

$\frac{b^3}{3}-\frac{a^3}{3} = \frac{9}{4}h(a^2+2ah+h^2) + \frac{3}{4}hb^2$

$= \frac{3}{4}h\left(3a^2+6ah+3h^2+b^2\right)$

$= \frac{b-a}{4}\left(3a^2+2ba-2a^2+\frac{3}{9}(b^2-2ab+a^2)+b^2\right)$

$= \frac{b-a}{4}\left(\frac{4}{3}a^2+\frac{4}{3}b^2+\frac{4}{3}ab\right)$

$= \frac{b-a}{3}\cdot(a^2+b^2+ab) = \frac{b^3-a^3}{3}$

$f=x^3 \Rightarrow \int_a^b x^3\,dx = \frac{9}{4}h(a+h)^3 + \frac{3}{4}h(b)^3$

$\frac{b^4}{4}-\frac{a^4}{4} = \frac{9}{4}\cdot\frac{(b-a)}{3}\left(a+\frac{b-a}{3}\right)^3 + \frac{3}{4}\frac{(b-a)}{3}b^3$

$= \frac{9}{4}\left(\frac{b-a}{3}\right)\left(\frac{b+2a}{3}\right)^3 + \frac{b^3(b-a)}{4}$

$= \frac{(b-a)(b+2a)^3}{4\cdot 9} + \frac{9b^3(b-a)}{4\cdot 9}$

$= \frac{(b-a)+(b+2a)^3+9b^3}{36}$

$= \frac{(b-a)(b^3+6b^2a+12a^2b+8a^3+9b^3)}{36}$

does not cancel.

degree of precision $= 2$

22. $\int_0^2 f(x)\,dx = Af(0)+Bf(1)+Cf(2)$

$f=1 \Rightarrow \int_0^2 1\,dx = A(1)+B(1)+C(1)$

$2 = A+B+C$

$f=x \Rightarrow \int_0^2 x\,dx = A(0)+B(1)+C(2)$

$\frac{x^2}{2}\Big|_0^2 = B+2C$

$4 = B+2C \Rightarrow 4-2C=B$

$f=x^2 \Rightarrow \int_0^2 x^2\,dx = A(0)^2+B(1)^2+C(2)^2$

$\frac{x^3}{3}\Big|_0^2 = B+4C$

$\frac{8}{3} = B+4C \Rightarrow \frac{8}{3} = 4-2C+4C \Rightarrow 2C = -\frac{4}{3} \Rightarrow C = -\frac{2}{3}$

$4-2C=B \Rightarrow 4-2(-\frac{2}{3}) = B \Rightarrow B = \frac{16}{3}$

$2=A+B+C \Rightarrow A = 2-\frac{16}{3}-(-\frac{2}{3}) \Rightarrow A = -\frac{8}{3}$

$C_0 = -\frac{8}{3},\ C_1 = \frac{16}{3},\ C_2 = -\frac{2}{3}$