Ma 227 - Multivariable Calculus **Hw-3** **Due: March 4, 2020**

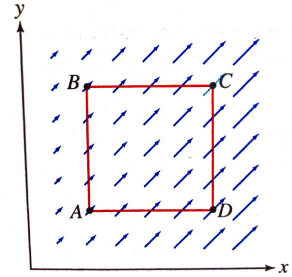
Name (Printed):

Pledge and Sign:

A high quality scan of the solutions in pdf format is to be uploaded to Canvas before the deadline. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. The figure below shows a force field. $\vec{\mathbf{F}}$



(a) [**3 pts.**] Over which of the two paths ADC, or ABC does $\vec{\mathbf{F}}$ perform less work? Explain!

(b) [**3 pts.**] If you have to work against $\vec{\mathbf{F}}$ to ove an object from C to A, which of the paths CBA or CDA, requires less work? Explain!

2. Evaluate each line integral.

(a) [**6 pts.**] Find the total mass of a tube in the shape of $\vec{\mathbf{r}}(t) = \langle \cos t, \sin t, t^2 \rangle$ (in centimeters) for $0 \leq t \leq 2\pi$ if the mass density is $\rho(x, y, z) = \sqrt{z}$ g/cm.

(b) [**6 pts.**] Evaluate $\displaystyle\int_C x^3\, dx + \; yzdy + ydz$, where C is the piecewise smooth path composed of C_1 the semicircle of radius 1 centered at the origin in the xy-plane, with $y \geq 0$, oriented counterclockwise when viewed from the positive z-axis, and C_2 the straight line segment from the point $(-1, 0, 0)$ to the point $(0, 0, 1)$.

3. (a) [**6 pts.**] Recall that gravitional force exerted on a mass m placed at (x, y, z) by a mass M placed on the origin is given by $\vec{\mathbf{F}} = \dfrac{-GMm}{|\vec{\mathbf{r}}|^3}\vec{\mathbf{r}}$, where $\vec{\mathbf{r}} = \langle x, y, z \rangle$ is the position vector of mass m. A potential for this force field is given by $f = \dfrac{GMm}{|\vec{\mathbf{r}}|}$, i.e. $\nabla f = \vec{\mathbf{F}}$. Assuming M is the earth and $GM \approx 4 \times 10^{14}$ m^{3}s^{-2}, use Fundamental Theorem of Line Integrals to compute the work W against earth's gravitional field to move a satellite of mass $m = 600$ kg along any path from an orbit of altitude 2000 km to an orbit of altitude 4000 km.

(b) **[6 pts.]** Find a potential function $f(x, y)$ for the conservative vector field $\vec{\mathbf{F}} = \langle 6x \sin y - \sin x, 3x^2 \cos y + 1 \rangle$.

Max Shi

I pledge my honor that I have abided by the Stevens honor System

Max Shi

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1. (a) $\vec{F}$ performs less work ~~over~~ over ABC. Paths BC in ABC and AD in ADC are identical, only AB and DC are different. The magnitude of the forces tangent to AB is less than the magnitude tangent to DC. Therefore, $\vec{F}$ does less work over ABC

(b) To do the least work against $\vec{F}$, we choose the path that F does less work on from A to C, as $\int_{ABC} \vec{F}\cdot d\vec{r} = -\int_{CBA} \vec{F}\cdot d\vec{r}$. As the work done on ABC was less than ADC, we choose CBA ~~to do~~ the least work against $\vec{F}$.

2(a) $\int_C \rho\, ds = \int_C \rho(x(t), y(t), z(t))\, |r'(t)|\, dt$

$\rho(x(t), y(t), z(t)) = \sqrt{t^2} = t$

$r'(t) = \langle -\sin t, \cos t, 2t \rangle$

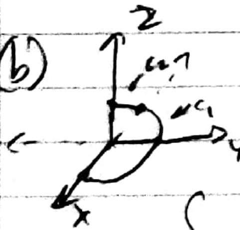
$|r'(t)| = \sqrt{\sin^2 t + \cos^2 t + 4t^2} = \sqrt{1+4t^2}$

$\int_C \rho\, ds = \int_0^{2\pi} t\sqrt{1+4t^2}\, dt$ $\quad u = 1+4t^2 \quad t=0 \Rightarrow u=1$

$\frac{1}{8}\int_1^{16\pi^2+1} \sqrt{u}\, du$ $\quad du = 8t\,dt \Rightarrow \frac{du}{8} = t\,dt \quad t=2\pi \Rightarrow u = 16\pi^2+1$

$\frac{1}{8}\left[\frac{2}{3}u^{3/2}\right]_1^{16\pi^2+1}$

$= \frac{1}{12}\left((16\pi^2+1)^{3/2} - 1\right)$

(b) 

$r_1(t) = \langle \cos t, \sin t, 0 \rangle \quad 0 \le t \le \pi \Rightarrow r_1'(t) = \langle -\sin t, \cos t, 0 \rangle$

$r_2(t) = \langle -1+t, 0, t \rangle \quad 0 \le t \le 1 \Rightarrow r_2'(t) = \langle 1, 0, 1 \rangle$

$\vec{F} = \langle x^3, yz, y \rangle$

$\int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_2} \vec{F}\cdot d\vec{r}$

$= \int_0^{\pi} -\cos^3 t \sin t + 0 + 0\, dt + \int_0^1 (-1+t)^3 \cdot 1 + 0 + 0\, dt$ $\quad t=\pi \Rightarrow u=-1$, $t=0 \Rightarrow u=1$

$= \int_0^{\pi} -\cos^3 t \sin t\, dt + \int_0^1 -1 + 3t - 3t^2 + t^3\, dt$ $\quad u = \cos t$, $du = -\sin t$

$= \int_1^{-1} u^3\, dt + \int_0^1 -1 + 3t - 3t^2 + t^3\, dt$

$= \frac{1}{4}\left[u^4\right]_1^{-1} + \left[-t + \frac{3}{2}t^2 - t^3 + \frac{1}{4}t^4\right]_0^1$

$= 0 + \left(-1 + \frac{3}{2} - 1 + \frac{1}{4}\right)$

$= -\frac{1}{4}$

3 (a) $\int \vec{F} \cdot d\vec{r} = f(B) - f(A)$

$\int_{2000}^{4000} \vec{F} \cdot d\vec{r} = f(4000) - f(2000)$, $GM = 4 \times 10^{14} m^3 s^{-2}$, $m = 600\,kg$

$4000\,km = 4 \times 10^6 m$ $\quad 2000\,km = 2 \times 10^6 m$

$f(4000) = \frac{4 \times 10^{14} \cdot 600}{4 \times 10^6}$ $\quad f(2000) = \frac{4 \times 10^{14} \cdot 600}{2 \times 10^6}$

$= 1 \times 10^8 \cdot 600$ $\quad = 2 \times 10^8 \cdot 600$

$= 6 \times 10^{10}$ $\quad = 1.2 \times 10^{11}$

$6 \times 10^{10} - 1.2 \times 10^{11} = 6.0 \times 10^{10}\,J$

6×10^{10} J against earth's gravitational field.

(b) $P(x,y) = 6x\sin y - \sin x$, $\quad Q(x,y) = 3x^2\cos y + 1$.

$P_y = 6x\cos y$, $\quad Q_x = 6x\cos y$

$P_y = Q_x$

$\int P\,dx = \int 6x\sin y - \sin x\,dx = 3x^2\sin y + \cos x + g(y)$

$g'(y) = Q - \frac{d}{dy}(\int P\,dx)$

$g'(y) = 3x^2\cos y + 1 - \frac{d}{dy}(3x^2\sin y + \cos x)$

$g'(y) = 3x^2\cos y + 1 - 3x^2\cos y$

$g'(y) = 1$

$g(y) = y$

$f(x,y) = 3x^2\sin y + \cos x + y$.