Mato Shi   I pledge my word that I have not used help for the scanner (some illegible)

4. Given $f(t) = \mathcal{L}^{-1}\left\{\frac{2s-1}{s(s+1)^2}\right\} = -1 + e^{-t} + 3te^{-t}$,
find $g(t) = \mathcal{L}^{-1}\left\{\frac{2s-1}{s(s+1)^2}(1-e^{-2s})\right\}$.

$$g(t) = \mathcal{L}^{-1}\left\{\frac{2s-1}{s(s+1)^2}(1-e^{-2s})\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{2s-1}{s(s+1)^2} - \left(\frac{2s-1}{s(s+1)^2}\right)e^{-2s}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2s-1}{s(s+1)^2}\right\} - \mathcal{L}^{-1}\left\{\frac{2s-1}{s(s+1)^2}e^{-2s}\right\}$$

$$= -1 + e^{-t} + 3te^{-t} - \left(-1 + e^{-(t-2)} + 3(t-2)e^{t-2}\right) \cdot \mathcal{U}(t-2)$$

$$g(t) = -1 + e^{-t} + 3te^{-t} - \left(-1 + e^{2-t} + 3(t-2)e^{t-2}\right)\mathcal{U}(t-2)$$

5. $f(t) = \mathcal{L}^{-1}\left\{\frac{3s-15}{2s^2-4s+10}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{2} \cdot \frac{3s-15}{s^2-2s+5}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{3s-15}{s^2-2s+5}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{3s-15}{s^2-2s+1+4}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{3s-15}{(s-1)^2+2^2}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{3s-3-12}{(s-1)^2+2^2}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{3(s-1)}{(s-1)^2+2^2} - \frac{6 \cdot 2}{(s-1)^2+2^2}\right\}$$

$$= \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+2^2}\right\} - 3\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^2+2^2}\right\}$$

$$= \frac{3}{2}e^{t}\cos 2t - 3e^{t}\sin 2t$$

7. $f(t) = 4 - 4(\mathcal{U}(t-2))$

$y' + \frac{1}{2}y = 4 - 4\mathcal{U}(t-2)$, $y(0) = 0$

$\mathcal{L}\{y' + \frac{1}{2}y\} = \mathcal{L}\{4 - 4(\mathcal{U}(t-2))\}$

$sY - y(0) + \frac{1}{2}Y = \frac{4}{s} - \frac{e^{-2s}}{s}$

$sY + \frac{1}{2}Y = \frac{4}{s} - \frac{e^{-2s}}{s}$

$Y(s+\frac{1}{2}) = \frac{4}{s} - \frac{e^{-2s}}{s}$

$Y = \frac{4}{s(s+\frac{1}{2})} - \frac{e^{-2s}}{s(s+\frac{1}{2})}$

$Y = \frac{8}{s} - \frac{8}{s+\frac{1}{2}} - e^{-2s}\left(\frac{2}{s} - \frac{2}{s+1/2}\right)$

$\mathcal{L}^{-1}\{Y\} = \mathcal{L}^{-1}\left\{\frac{8}{s} - \frac{8}{s+1/2} - e^{-2s}\left(\frac{2}{s} - \frac{2}{s+1/2}\right)\right\}$

$y = 8 - e^{-1/2 t} - \left(2 - e^{-1/2(t-2)}\right)\mathcal{U}(t-2)$

$y = 8 - e^{-1/2 t} - \left(2 - e^{1 - 1/2 t}\right)\mathcal{U}(t-2)$

$\frac{A}{s} + \frac{B}{s+\frac{1}{2}} = \frac{C}{s(s+\frac{1}{2})}$

$A(s+\frac{1}{2}) + B(s) = C$

$As + \frac{1}{2}A + Bs = C$

$\frac{1}{2}A = C \Rightarrow A = 2C$

$As + Bs = 0 \Rightarrow As = -Bs \Rightarrow A = -B$

$\Rightarrow B = -2C$

for $C = 4$,
$A = 8$
$B = -8$

for $C = 1$,
$A = 2$
$B = -2$.

Scanned with CamScanner