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I prefer my browser that
there are no ads in
the browser
system

Murphy

CS 135 Problem Set 2

1. a(1) $A \rightarrow B \equiv \neg A \vee B$ $\neg B \vee C$ negation

(2) $\neg C \rightarrow A \equiv C \vee A$

$\therefore \neg(B \rightarrow C) \equiv \neg(\neg B \vee C)$

(1) $\neg A$ B $\neg A$ B (1)

(2) $\neg A$

Invalid, counterexample: $A=F, B=F, C=T$

1b(1) $U \rightarrow W \equiv \neg U \vee W$

(2) $A \rightarrow W \equiv \neg A \vee W$

(3) $S \rightarrow U \equiv \neg S \vee U$

(4) $A \rightarrow S \equiv \neg A \vee S$

$\therefore \neg U \rightarrow \neg W$

$\neg U \rightarrow \neg W$ conclusion

$\neg \neg U \vee \neg W$ cond. identity

$U \vee \neg W$ double negation

$\neg(U \vee \neg W)$ negation conclusion

$\neg U \wedge \neg \neg W$ de Morgan's laws

$\neg U \wedge W$ double negation

$\neg U$

W

(1) $\neg U$ W

(2) $\neg A$ W $\neg A$ W

(3) $\neg S$ U

(4) $\neg A$ S

Invalid

counterexample:

$U=F$

$W=T$

$A=F$

$S=F$

$\neg U \rightarrow \neg W$

2a The domain x is all people in the world, and the domain d is all days spanning time.

b $\forall d \exists x \text{ Loves}(x, \text{Juliet}, d)$

c $\forall d \neg \text{Loves}(\text{Romeo}, \text{Juliet}, d)$

d $\exists x \forall d \exists y (\text{Loves}(x, y, d) \vee \neg \text{Loves}(x, y, d))$

e $\forall x \forall d_1 \forall y \forall d_2 (\text{Loves}(x, y, d_1) \wedge \text{Loves}(y, z, d_2)) \rightarrow \forall d_3 (\text{Loves}(x, y, d_3) \wedge \text{Future}(d_1, d_3))$

f $\forall x \forall y \forall d (\neg \text{Loves}(y, x, d) \rightarrow \neg \text{Loves}(x, z, d))$

g $\forall x \forall y \forall z \forall d_1 \forall d_2 (\text{Loves}(x, y, d_1) \wedge \text{Loves}(x, z, d_2) \wedge \text{Future}(d_1, d_2)) \rightarrow (\forall d_3 \text{ Future}(d_2, d_3) \wedge \neg \text{Loves}(y, x, d_3))$

1c. A: able to prevent evil
W: willing to prevent evil.
P: prevent evil
IM: impotent
M: malevolent
E: evil exists.
S: Superman exists.

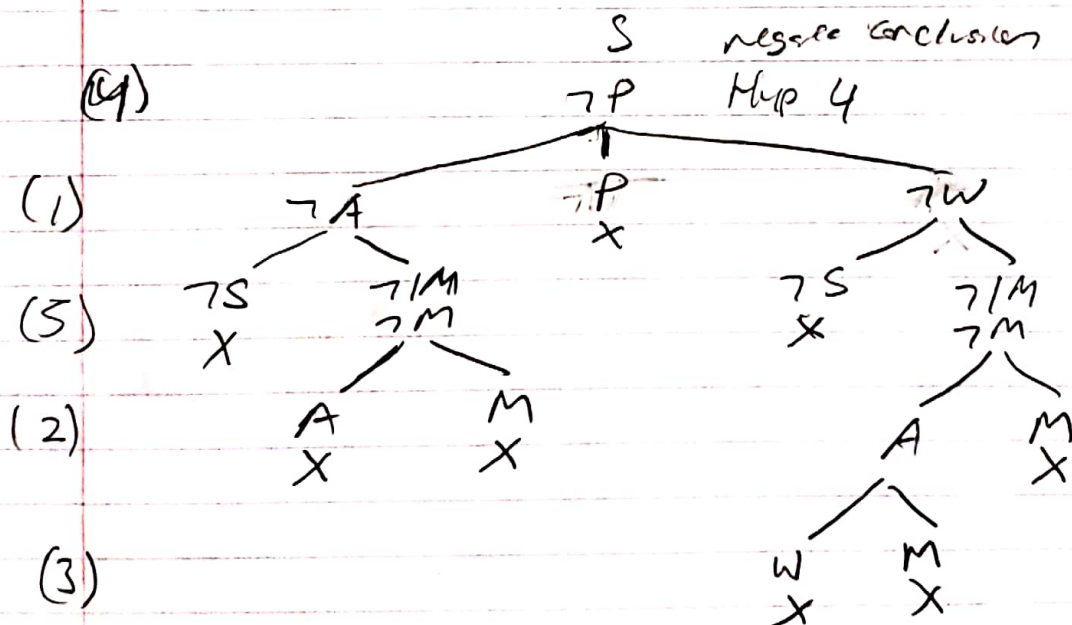
- (1) $(A \wedge W) \rightarrow P$
(2) $\neg A \rightarrow IM$
(3) $\neg W \rightarrow M$
(4) $\neg P$
(5) $S \rightarrow (\neg IM \wedge \neg M)$
 $\therefore \neg S$

$(A \wedge W) \rightarrow P$ Hyp 1
 $\equiv \neg(A \wedge W) \vee P$ cond. identity
 $\equiv \neg A \vee \neg W \vee P$ De Morgan's law

$\neg W \rightarrow M$ Hyp 3
 $W \vee M$ cond. identity

$\neg A \rightarrow IM$ Hyp 2
 $\equiv A \vee M$ cond. identity

$S \rightarrow (\neg IM \wedge \neg M)$ Hyp 5
 $\neg S \vee (\neg IM \wedge \neg M)$ cond. identity



✓ Argument Valid