Mao Li   I pledge my honor that I have abided by the Stevens Honor System. Mao Li

1. $\mathcal{L}^{-1}\left\{\frac{s^2+s+20}{s(s^2+2s+5)}\right\}$

$\frac{s^2+s+20}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$

$s^2+s+20 = A(s^2+2s+5) + (Bs+C)(s)$

$s^2+s+20 = As^2 + 2As + 5A + Bs^2 + Cs$

$As^2 + Bs^2 = s^2 \Rightarrow A+B=1 \Rightarrow B=-3$

$2As + Cs = s \Rightarrow 2A+C=1 \Rightarrow C=-7$

$5A = 20 \Rightarrow A=4$

$$\mathcal{L}^{-1}\left\{\frac{4}{s} + \frac{(-3s-7)}{s^2+2s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{4}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{-3s-7}{s^2+2s+5}\right\}$$

$$= 4 + \mathcal{L}^{-1}\left\{\frac{-3s-7}{s^2+2s+5}\right\}$$

$$= 4 + \mathcal{L}^{-1}\left\{\frac{-3s-7}{s^2+2s+1+4}\right\}$$

$$+ \mathcal{L}^{-1}\left\{\frac{-3s-7}{(s+1)^2+4}\right\}$$

$$+ \mathcal{L}^{-1}\left\{\frac{-3s-3}{(s+1)^2+4}\right\} + \mathcal{L}^{-1}\left\{\frac{-4}{(s+1)^2+4}\right\}$$

$$+ -3\,\mathcal{L}\left\{\frac{s+1}{(s+1)^2+4}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+4}\right\}$$

$$= 4 - 3e^{-t}\cos 2t - 2e^{-t}\sin 2t$$

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Max Shi

2. $y'+2y=te^{-2t}$ $\quad y(0)=-3$

$\mathcal{L}\{y'+2y\}=\mathcal{L}\{te^{-2t}\}$

$sY-y(0)+2Y=\frac{1}{(s+2)^2}$

$sY+3+2Y=\frac{1}{(s+2)^2}$

$sY+2Y=\frac{1}{(s+2)^2}-3$

$Y(s+2)=\frac{1}{(s+2)^2}-3$

$Y=\frac{1}{(s+2)^3}-\frac{3}{s+2}$

$\mathcal{L}^{-1}\{Y\}=\mathcal{L}^{-1}\{\frac{1}{(s+2)^3}-\frac{3}{s+2}\}$

$y=\mathcal{L}^{-1}\{\frac{1}{(s+2)^3}\}-\mathcal{L}^{-1}\{\frac{3}{s+2}\}$

$a=-2, n=2, n!=2$

$y=\frac{1}{2}\mathcal{L}^{-1}\{\frac{2}{(s+2)^3}\}-3\mathcal{L}^{-1}\{\frac{1}{s+2}\}$

$y=\frac{1}{2}t^2e^{-2t}-3e^{-2t}$

3. $f(t)=y'+y$, $y(0)=0$, $f(t)=\begin{cases}0, & 0\le t\le 1\\ 5, & 1\le t<\infty\end{cases}$

$f(t)=5u(t-1)$

$y'+y=5u(t-1)$

$\mathcal{L}\{y'+y\}=\mathcal{L}\{5u(t-1)\}$

$sY-y(0)+Y=5\cdot\frac{e^{-t}}{s}$

$sY-Y=5\cdot\frac{e^{-t}}{s}$

$Y(s-1)=\frac{5e^{-t}}{s}$

$Y=e^{-t}\left(\frac{5}{s(s+1)}\right)$

$Y=e^{-t}\left(\frac{5}{s}-\frac{5}{s+1}\right)$

$\mathcal{L}^{-1}\{Y\}=\mathcal{L}^{-1}\{e^{-t}\left(\frac{5}{s}-\frac{5}{s+1}\right)\}$

$y=5u(t-1)-\left(5e^{-(t-1)}\right)\left(u(t-1)\right)$

$y=5(u(t-1))(1-e^{1-t})$

$\frac{5}{s(s+1)}=\frac{A}{s}+\frac{B}{s+1}$

$5=A(s+1)+Bs$

$5=As+Bs+A$

$A=5, B=-5$