I pledge my honor that I have abided by the Stevens Honor System [illegible]

MA346 HW 9.

4.6.2b. $\int_1^{1.6} \frac{2x}{x^2-4}\,dx$ $\qquad h = \frac{b-a}{2} = \frac{1.6-1}{2} = 0.3$

$\mathcal{E} = \frac{1}{16}\left(\frac{h^5}{90}\right) f^4(\xi)$

Using desmos, max $|f^4(\xi)|$ $1 \le \xi \le 1.6 = 2343.7103$

$\mathcal{E} = \frac{1}{16}\,\frac{(0.3)^5}{90}(2343.7103) = 0.003955011$

$S(1,1.6) = \frac{0.3}{3}\left(f(1) + 4(f(1.3)) + f(1.6)\right)$

$= 0.1\left(-\frac{2}{3} + 4(-1.12554) + -2.22\overline{2}\right)$

$= -0.7391048$

$S(1,1.3) = \frac{0.15}{3}\left(f(1) + 4(f(1.15)) + f(1.3)\right)$

$= 0.05\left(-2/3 + 4(-0.8590103) + (-1.1255411)\right)$

$= -0.261412$

$S(1.3,1.6) = \frac{0.15}{3}\left(f(1.3) + 4(f(1.45)) + f(1.6)\right)$

$= 0.05\left(-1.1255411 + 4(-1.528326) + -2.222\overline{2}\right)$

$= -0.473053365$

$|S(1,1.6) - S(1,1.3) - S(1.3,1.6)| = 0.049638$

$15\mathcal{E} = 0.059325$

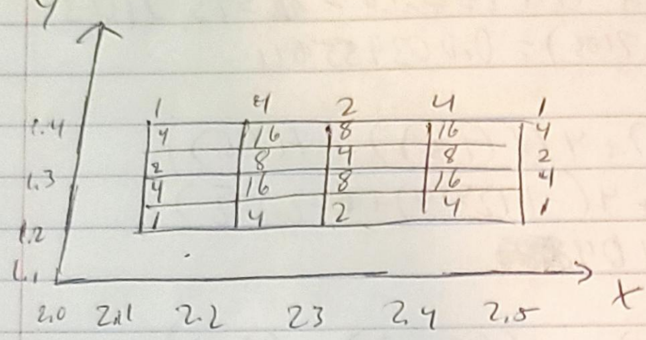
$|S(1,1.6) - S(1,1.3) - S(1.3,1.6)| \le 15\mathcal{E}$ ✓

$\left|\int_1^{1.6} f(x)\,dx - S(1,1.3) - S(1.3,1.6)\right| = |-0.73396917 + 0.261412 + 0.473053365|$

$= 0.000496195 < \mathcal{E}$

4.8.1a. $\int_{2.1}^{2.5}\int_{1.2}^{1.4} xy^2\,dy\,dx$

$h = \frac{2.5-2.1}{4} = 0.1 \qquad k = \frac{1.4-1.2}{4} = 0.05$



$$\int_{2.1}^{2.5}\int_{1.2}^{1.4} xy^2\,dy\,dx = \frac{(0.1)(0.05)}{9}\sum_{i=0}^{4}\sum_{j=0}^{4} w_{i,j}\,(x_i y_j^2)$$

$$= \frac{1}{1800}\left(1\cdot(2.1\cdot 1.2^2) + 4(2.2\cdot 1.2^2) + 2(2.3\cdot 1.2^2 \ldots\right.$$

1st row $= 1(2.1\cdot 1.2^2) + 4(2.2\cdot 1.2^2) + 2(2.3\times 1.2^2) + 4(2.4\times 1.2^2) + 1(2.5\times 1.2^2)$

$= 3.024 + 12.672 + 6.624 + 13.824 + 3.6$

$= 39.744$

2nd row $= 4(2.1\times 1.25^2) + 16(2.2\times 1.25^2) + 8(2.3\times 1.25^2) + 16(2.4\times 1.25^2) + 4(2.4\times 1.25^2)$

$= 13.125 + 55 + 28.75 + 60 + 15$

$= 172.5$

3rd row $= 2(2.1\times 1.3^2) + 8(2.2\times 1.3^2) + 4(2.3\times 1.3^2) + 8(2.4\times 1.3^2) + 2(2.5\times 1.3^2)$

$= 93.288$

4th row $= 201.204$ 5th row $= 54.096$.

$$\frac{1}{1800}(39.744 + 172.5 + 93.288 + 201.204 + 54.096) = \frac{560.832}{1800}$$

$$= 0.3115733$$

7ii (5h). $\int_{-\pi}^{3\pi/2}\int_0^{2\pi} y\sin x + x\cos y \, dy\, dx$.

$n=3, m=4$.

$u = \frac{1}{\frac{3\pi}{2}-(-\pi)}\left(2x - \frac{3\pi}{2} - (-\pi)\right)$ $v = \frac{1}{2\pi - 0}(2y - 2\pi - 0)$

$v = \frac{1}{2\pi}(2y - 2\pi)$

$= \frac{2}{5\pi}\left(2x - \frac{\pi}{2}\right)$ $y = \frac{2\pi v + 2\pi}{2} = \pi v + \pi$

$\frac{2\pi - 0}{2} = \pi$

$x = \frac{\frac{5\pi}{2}u + \frac{\pi}{2}}{2} = \frac{5\pi}{4}u + \frac{\pi}{4}$

$\frac{\frac{3\pi}{2}-(-\pi)}{2} = \frac{5\pi}{4}$

$\int_{-\pi}^{\frac{3\pi}{2}}\int_0^{2\pi} y\sin x + x\cos y \, dy\, dx =$

$\pi \cdot \frac{5\pi}{4} \cdot \int_{-1}^{1}\int_{-1}^{1} (\pi v + \pi)\sin\left(\frac{5\pi}{4}u + \frac{\pi}{4}\right) + \left(\frac{5\pi}{4}u + \frac{\pi}{4}\right)\cos(\pi v + \pi)\, dv\, du$

$n=3 \Rightarrow u_0 = -0.7745966692$ $m=4 \Rightarrow v_0 = -0.8611363116$

$u_1 = 0$ $v_1 = -0.3399810436$

$u_2 = -u_0 = 0.7745966692$ $v_2 = 0.3399810436$

$v_3 = 0.8611363116$

$c_{u,0} = 0.\overline{5}$

$c_{u,1} = 0.\overline{8}$ $c_{v,3} = c_{v,0} = 0.3478548451$

$c_{u,2} = 0.\overline{5}$ $c_{v,1} = c_{v,2} = 0.6521451549$.

$\frac{5\pi^2}{4}\sum_{i=0}^{2}\sum_{j=0}^{3} c_{u,i}\, c_{v,i}\, f(u_i, v_j)$

$= \Big(\underset{(0,0)}{-2.1103557616} + \underset{(0,1)}{0.5056943992261} + \underset{(0,2)}{-0.0933448848} +$

$\underset{(0,3)}{-2.91968698966} + \underset{(1,0)}{0.807221336304} + \underset{(1,1)}{0.47152031} +$

$\underset{(1,2)}{1.34713154681} + \underset{(1,3)}{1.99021469751} + \underset{(2,0)}{3.41539779} +$

$\underset{(2,1)}{-2.31964340067} + \underset{(2,2)}{-2.80967647005} + \underset{(2,3)}{2.2533392}\Big) \cdot \frac{5\pi^2}{4}$

$= -11.83624$

5.1.2d $y' = \frac{ty+y}{ty+t}$, $2 \le t \le 4$, $y(2)=4$

$$y' = \frac{ty}{ty+t} + \frac{y}{ty+t}$$

$$= \left(\frac{y}{y+1}\right) + \frac{1}{t}\left(\frac{y}{y+1}\right)$$

$$= \left(1+\frac{1}{t}\right)\left(\frac{y}{y+1}\right)$$

$$\Rightarrow y'\left(\frac{y+1}{y}\right) = \left(\frac{t+1}{t}\right) \Rightarrow y + \ln y = t + \ln t + c_1$$

$$y = W(te^{t+c_1})$$

$y(2)=4$ $\quad 4 + \ln 4 = 2 + \ln 2 + c_1$

$2 + \ln 4 - \ln 2 = c_1 = 2 + \ln 2.$

$$|f(t,y_1) - f(t,y_2)| = \left|\frac{ty_1+y_1}{ty_1+t} - \frac{ty_2+y_2}{ty_2+t}\right|$$

$$= \frac{\cancel{t^2y_1y_2 + ty_1y_2 + t^2y_1 + ty_1 - t^2y_1y_2 - ty_1y_2 - t^2y_2 - ty_2}}{(ty_1+t)(ty_2+t)}$$

$$= \frac{t^2y_1 + ty_1 - t^2y_2 - ty_2}{(ty_1+t)(ty_2+t)}$$

$$\frac{y_1(t^2+t) - y_2(t^2+t)}{t^2y_1y_2 + t^2y_1 + t^2y_2 + t^2}$$

$$\frac{(y_1-y_2)(t^2+t)}{t^2y_1y_2 + t^2y_1 + t^2y_2 + t^2}$$

does not satisfy Lipschitz condition, theorem is not applicable

The lambert W function gives different values, so there is no unique solution. Thus, it is not well-posed.

5.2.2d. $y' = t^{-2}(\sin 2t - 2ty)$, $1 \le t \le 2$, $y(1) = 2$, $h = 0.25$.

$w_0 = y(1) = 2$

$w_1 = w_0 + 0.25((1.25)^{-2}(\sin(2\cdot 1.25) - 2\cdot 1.25\cdot w_0)) = 1.2957555$

$w_2 = w_1 + 0.25(1.5^{-2}(\sin(2\cdot 1.5) - 2\cdot 1.5\cdot w_1)) = 0.87957703$

$w_3 = w_2 + 0.25((1.75)^{-2}(\sin(2\cdot 1.75) - 2\cdot 1.75\cdot w_2)) = 0.59959108419$

$w_4 = w_3 + 0.25((2)^{-2}(\sin(2\cdot 2) - 2\cdot 2\cdot w_3)) = 0.402899157186$

4d. $y(t) = \dfrac{4 + \cos 2 - \cos 2t}{2t^2}$ $f = y'(t) = t^{-2}(\sin 2t - 2ty)$

$|f(t,y_1) - f(t,y_2)| = |t^{-2}(\sin 2t - 2ty_1) - t^{-2}(\sin 2t - 2ty_2)|$

$= |-2ty_1 + 2ty_2| = |y_1 - y_2|\, 2t$

$L = 2t = 4$, $1 \le t \le 2$.

$y'(t) = \dfrac{-4 - \cos(2) + \cos(2t) + t\sin(2t)}{t^3}$

$y''(t) = \dfrac{(2t^2 - 3)\cos(2t) - 4t\sin(2t) + 3(4 + \cos 2)}{t^4}$

$y''(t)$ max @ $t = 1$, so $M = y''(1) = 7.531$

t_i:	1.25	1.5	1.75	2.0
Actual error	0.10744343	0.136893	0.13841869	0.12729394
error bound	0.404386	1.50362	4.491661	12.613989

$(i=1)$

$|y(t_i) - w_i| \le \dfrac{hM}{2L}\left[e^{L(t_i - a)} - 1\right] = \dfrac{0.25\cdot 7.531}{2(4)}\left[e^{4(0.25)} - 1\right]$

$= 0.404386.$

$(i=2)$ ... $\dfrac{(0.25)(7.531)}{2\cdot 4}\left(e^{4(0.5)} - 1\right) = 1.50362$