Ma 227 - Multivariable Calculus **Hw-2** **Due: February 12, 2020 by 9:00 am**

Name (Printed):

Pledge and Sign:

A high quality scan of the solutions in pdf format is to be uploaded to Canvas before the deadline. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. The probability that two radio active materials with decay rates of $\frac{1}{10}$ per year (in the number of years s) and $\frac{1}{20}$ per year (in the number of years t) decay independently of each other is modeled by the joint probability density function:

$$f(s,t) = \begin{cases} ke^{-10s}e^{-20t} & \text{for } s \geq 0, t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) [7 pts.] Find k. [Hint: you need $\lim_{s\to\infty} e^{-s} = 0$.]

(b) [3 pts.] Express the probability that the material with decay rate of $\frac{1}{20}$ per year decays sooner than the one with decay rate of $\frac{1}{10}$ per year, but do **not** evaluate it.

2. Let E be the tetrahedron formed by coordinate planes, $x = 0$, $y = 0$,and $z = 0$, and the plane $2x + 3y + 4z = 12$. Set up the triple integral $\iiint_E x\, dV$ as an iterated integral in the orders:

(a) [3 pts.] $dzdydx$,

(b) [3 pts.] $dxdydz$.

(c) [4 pts.] Evaluate the triple integral using one of the iterated integrals above.

3. (a) [7 pts.] Find the total mass of the solid, which occupies the region E, bounded by the semi-cylinder $x^2 + y^2 = 4$, $y \geq 0$, and the planes $y = 0$, $z = 0$, and $y + z = 5$, where mass density is $\rho(x, y, z) = y$.

(b) [3 pts.] What is the x-coordinate, $\bar{x}$, of the center of mass? Why?

I pledge my honor that I have abided by the Stevens honor System

Max Shi MA-227 HW #2

1.(a) $\int_0^\infty \int_0^\infty k e^{-10s} e^{-20t}\, ds\, dt = 1$

$\int_0^\infty \left[-10ke^{-10s}\right]_0^\infty e^{-20t}\, dt = 1$

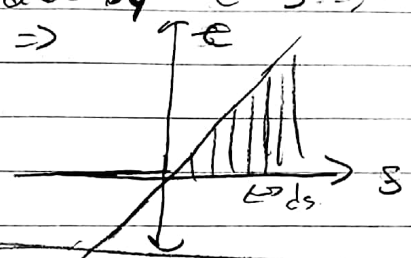
$\int_0^\infty 10ke^{-20t}\, dt = 1$

$\left[-200ke^{-20t}\right]_0^\infty = 1$

$200k = 1$

$\boxed{k = \frac{1}{200}}$

(b) modeled by $t < s$ =>



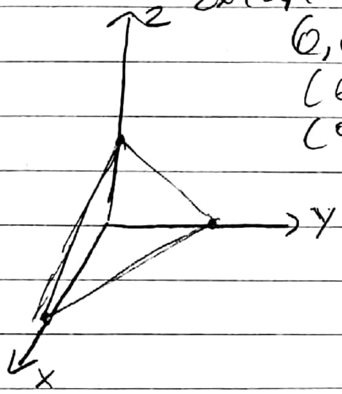
$\boxed{\int_0^\infty \int_0^s \frac{1}{200} e^{-10s} e^{-20t}\, dt\, ds}$

2. $2x + 3y + 4z = 12$

$(0,0,3)$

$(6,0,0)$

$(0,4,0)$



(a) $\int_0^6 \int_0^{4-2/3x} \int_0^{3-3/4y-1/2x} x\, dz\, dy\, dx$

$2x+3y=12 \quad 4z = 12-3y-2x$

$3y = 12-2x \quad z = 3 - \frac{3}{4}y - \frac{1}{2}x$

$y = 4 - \frac{2}{3}x$

(b) $\int_0^4 \int_0^{6-3/2y} \int_0^{3-3/4y-1/2x} x\, dz\, dx\, dy$

$2x+3y=12$

$2x = 12-3y$

$x = 6 - \frac{3}{2}y$

(c) Using part a,

$\int_0^6 \int_0^{4-2/3x} \int_0^{3-3/4y-1/2x} x\, dz\, dy\, dx$

$\int_0^6 \int_0^{4-2/3x} \left[xz\right]_0^{3-3/4y-1/2x} dy\, dx$

$\int_0^6 \int_0^{4-2/3x} (3x - 3/4\, yx - 1/2x^2)\, dy\, dx$

$\int_0^6 \left[3yx - \frac{3}{8}y^2x - 1/2\, yx^2\right]_0^{4-2/3x} dx$

$\int_0^6 3(4-2/3x)x - \frac{3}{8}(4-2/3x)^2x - \frac{1}{2}(4-2/3x)x^2\, dx$

$\int_0^6 12x - 2x^2 - \frac{3x}{8}(16 - 16/3x + \frac{4}{9}x^2) - 2x^2 + 1/3x^3\, dx$

$\int_0^6 12x - 2x^2 - 6x + 2x^2 - \frac{1}{6}x^3 - 2x^2 + 1/3x^3\, dx$

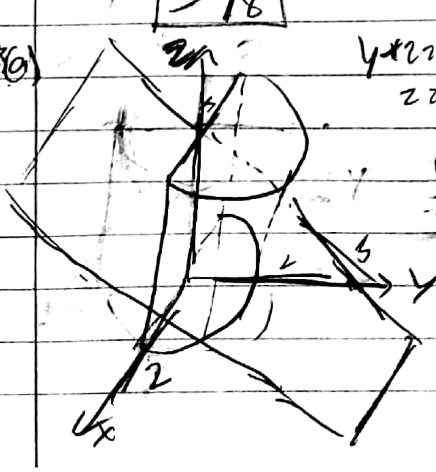
$\int_0^6 6x - 2x^2 + \frac{x^3}{6}\, dx$

$\left[3x^2 - \frac{2}{3}x^3 + \frac{x^4}{24}\right]_0^6$

$108 - 144 + 54$

$\boxed{= 18}$

3(a)



$y + z = 5$

$z = 5 - y$

$0 \le r \le 2$

$0 \le \theta \le \pi$

$0 \le z \le 5 - r\sin\theta$

$\int_0^\pi \int_0^2 \int_0^{5-r\sin\theta} r^2\sin\theta\, dz\, dr\, d\theta$

$\int_0^\pi \int_0^2 r^2\sin\theta \cdot (5 - r\sin\theta)\, dr\, d\theta$

$\int_0^\pi \int_0^2 5r^2\sin\theta - r^3\sin^2\theta\, dr\, d\theta$

$\int_0^\pi \left[\frac{5r^3}{3}\sin\theta - \frac{1}{4}r^4\sin^2\theta\right]_0^2 d\theta$

$\int_0^\pi \frac{40}{3}\sin\theta - 4\sin^2\theta\, d\theta$

$\int_0^\pi \frac{40}{3}\sin\theta - 2 + 2\cos 2\theta\, d\theta$

$\left[-\frac{40}{3}\cos\theta - 2\theta + \sin 2\theta\right]_0^\pi$

$= \frac{40}{3} - 2\pi + 0 + \frac{40}{3} - 0 - 0 = \boxed{\frac{80}{3} - 2\pi}$

$\boxed{\text{3b. } \bar{x} = 0 \text{ because the shape is symmetric about the yz-plane and the density function is also symmetric about the yz-plane.}}$