I pledge my honor that I have abided by the Stevens Honor System [signature]

MA 346 HW2

1.3 2. c. $\sum_{n=0}^{10} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{10!}$

$= \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} + \frac{1}{0.4030\times10^5} + \frac{1}{0.3628\times10^6} + \frac{1}{0.3628\times10^7}$

$= 1 + 1 + 0.5 + 0.1666 + 0.4166\times10^{-1} + 0.8333\times10^{-2} + 0.1388\times10^{-2} + 0.1984\times10^{-3} + 0.2481\times10^{-4} + 0.2755\times10^{-5} + 0.2755\times10^{-6}$

$= 2.2666\times10^1 + 0.4166\times10^{-1} + 0.8333\times10^{-2} + 0.1388\times10^{-2} + 0.1984\times10^{-3} + 0.2481\times10^{-4} + 0.2755\times10^{-5} + 0.2755\times10^{-6}$

$= 2.716\times10^1 + 0.1984\times10^{-3} + 0.2481\times10^{-4} + 0.2756\times10^{-5} + 0.2756\times10^{-6}$

$= 0.2716\times10^1 = \boxed{2.716}$

absolute error $= |e - 0.2716\times10^1| = \boxed{2.281\times10^{-3}}$

relative error $= \frac{|2.281\times10^{-3}|}{e} = \boxed{8.39\times10^{-4}}$

d. $\sum_{j=0}^{10} \frac{1}{(10-j)!} = \frac{1}{10!} + \frac{1}{9!} + \frac{1}{8!} + \frac{1}{7!} + \frac{1}{6!} + \frac{1}{5!} + \frac{1}{4!} + \frac{1}{3!} + \frac{1}{2!} + \frac{1}{1!} + \frac{1}{0!}$

$= \frac{1}{0.3628\times10^7} + \frac{1}{0.3628\times10^6} + \frac{1}{0.4030\times10^5} + \frac{1}{5040} + \frac{1}{720} + \frac{1}{120} + \frac{1}{24} + \frac{1}{6} + \frac{1}{2} + \frac{1}{1} + \frac{1}{1}$

$= 0.2756\times10^{-6} + 0.2756\times10^{-5} + 0.2481\times10^{-4} + 0.1984\times10^{-3} + 0.1388\times10^{-2} + 0.8333\times10^{-2} + 0.4166\times10^{-1} + 0.1666 + 0.5 + 1 + 1$

$= 0.3031\times10^{-5} + 0.2481\times10^{-4} + \dots$

$= 0.2784\times10^{-4} + 0.1984\times10^{-3} + \dots$

$= 0.2262\times10^{-3} + 0.1388\times10^{-2} + \dots$

$= 0.1614\times10^{-2} + 0.8333\times10^{-2} + \dots$

$= 0.9947\times10^{-2} + 0.4166\times10^{-1} + \dots$

$= 0.5160\times10^{-1} + 0.1666 + \dots$

$= 0.2182\times10^{-1} + 0.5 + \dots$

$= 0.7182\times10^{-1} + 2 = \boxed{2.718}$

absolute error $= |e - 2.718| = \boxed{0.2818\times10^{-3}}$

relative error $= \frac{0.2818}{e} = \boxed{0.1037\times10^{-3}}$

Taylor with h notation questions @ back.

6.1 2a. Graphing $2x+y=-1$, $x+y=2$, $x-3y=5$, there is no solution to this system of equations, as all three lines never pass through the same point.

2d. The solution is a line, where $x_1 = \frac{-5x_3+5}{6}$, $\cancel{x_2} = \frac{2x_3-2}{3}$. Because there are three variables and only two equations, the most precise a solution can be is a line.

5d.
$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 2 & 1 & -1 & 1 & 1 \\ 4 & -1 & -2 & 2 & 0 \\ 3 & -1 & -1 & 2 & -3 \end{bmatrix} \xRightarrow{i=1=p} \begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & -1 & -3 \\ 0 & -5 & -2 & -2 & -8 \\ 0 & -4 & -1 & -1 & -10 \end{bmatrix} \xRightarrow{i=2=p}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & -1 & -3 \\ 0 & 0 & 3 & 3 & 7 \\ 0 & 0 & 3 & 3 & 2 \end{bmatrix} \xRightarrow{i=3=p} \begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & -1 & -3 \\ 0 & 0 & 3 & 3 & 7 \\ 0 & 0 & 0 & 0 & -19 \end{bmatrix}$$

No unique solution

6d.
$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 2 & 1 & -1 & 1 & 1 \\ -1 & 2 & 3 & -1 & 4 \\ 3 & -1 & -1 & 2 & -3 \end{bmatrix} \xRightarrow{i=1=p} \begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & -1 & -3 \\ 0 & 3 & 0 & 0 & 6 \\ 0 & -4 & 0 & -1 & -9 \end{bmatrix} \xRightarrow{i=2=p} \begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & -1 & -3 \\ 0 & 0 & -3 & -3 & -3 \\ 0 & 0 & 4 & 3 & 3 \end{bmatrix} \xRightarrow{i=3=p}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & -1 & -3 \\ 0 & 0 & -3 & -3 & -3 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$-x_4 = -1 \Rightarrow x_4 = 1$

$\Rightarrow -3x_3 - 3x_4 = -3 = -3x_3 - 3 = -3 \Rightarrow x_3 = 0$

$-x_2 - x_3 - x_4 = -3 \Rightarrow -x_2 - 0 - 1 = -3 \Rightarrow -x_2 = -2 \Rightarrow x_2 = 2$

$x_1 + x_2 + x_4 = 2 \Rightarrow x_1 + 2 + 1 = 2 \Rightarrow x_1 = -1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

10)
$$\left[\begin{array}{cccc} 1 & -1 & \alpha & -2 \\ -1 & 2 & -\alpha & 3 \\ \alpha & 1 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{cccc} 1 & -1 & \alpha & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 1+\alpha & 1-\alpha^2 & 2+2\alpha \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{cccc} 1 & -1 & \alpha & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1-\alpha^2 & 2+2\alpha-1-\alpha \end{array}\right]$$

a. No solutions when $1-\alpha^2=0$ and $2+2\alpha-(1+\alpha)\neq 0$

$\alpha^2=1$ $\quad \alpha=1 \Rightarrow 2+2(1)-(1+1)=3$

$\alpha=\pm 1$ $\quad \alpha=-1 \Rightarrow 2-2-(1-1)=0$.

No solutions when $\alpha=1$

b. Infinite solutions when $1-\alpha^2=0$ and $2+2\alpha-(1+\alpha)=0$.

@ $\alpha=-1$.

Infinite solutions when $\alpha=-1$.

c. $x_2=1$

$(1-\alpha^2)x_3=2+2\alpha-(1+\alpha)$ $\qquad x_1-1+\alpha x_3=-2$

$x_3(1-\alpha^2)=1+\alpha$ $\qquad x_1+\alpha x_3=-1$

$x_3=\frac{1+\alpha}{1-\alpha^2}$ $\qquad x_1=-1-\alpha\left(\frac{1+\alpha}{1-\alpha^2}\right)$

$$\text{Solution}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} -1-\alpha\left(\frac{1+\alpha}{1-\alpha^2}\right) \\ 1 \\ \frac{1+\alpha}{1-\alpha^2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 5 \\ -1 & 3 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 2 & 0 \\ 1 & 5 & 2 \end{bmatrix}$$

1. $(AB)'$

$$(AB)' = \begin{bmatrix} 2+6+3 & 4+4+15 & 1+0+6 \\ 8+6+5 & 16+4+25 & 4+0+10 \\ -2+9+0 & -4+6+0 & -1+0+0 \end{bmatrix}'$$

$$= \begin{bmatrix} 11 & 23 & 7 \\ 19 & 45 & 14 \\ 7 & 2 & -1 \end{bmatrix}' = \begin{bmatrix} 11 & 19 & 7 \\ 23 & 45 & 2 \\ 7 & 14 & -1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 1 & 0 & 2 \end{bmatrix} \qquad A' = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 2 & 3 \\ 3 & 5 & 0 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 2+6+3 & 8+6+5 & -2+9+0 \\ 4+4+15 & 16+4+25 & -4+6+0 \\ 1+0+6 & 4+0+10 & -1+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 19 & 7 \\ 23 & 45 & 2 \\ 7 & 14 & -1 \end{bmatrix} = (AB)' \quad \checkmark$$

2. $AB = \begin{bmatrix} 11 & 23 & 7 \\ 19 & 45 & 14 \\ 7 & 2 & -1 \end{bmatrix}$

minors: $\begin{bmatrix} -45-28 & -19-98 & 38-315 \\ -23-14 & -11-49 & 22-161 \\ 322-315 & 154-133 & 495-437 \end{bmatrix}$

$$\times \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} -73 & 117 & -277 \\ 37 & 60 & 139 \\ 7 & -21 & 58 \end{bmatrix}$$

$\det AB = 11 \cdot -73 + 23 \cdot 117 + 7 \cdot -277$

$= -51$

$AB^{-1} = \frac{1}{\det} (\text{minors})'$

$$= \frac{1}{-51} \begin{bmatrix} -73 & 37 & 7 \\ 117 & 60 & -21 \\ -277 & 139 & 58 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 5 \\ -1 & 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 2 & 0 \\ 1 & 5 & 2 \end{bmatrix}$$

$A^{-1} = \frac{1}{\det} \cdot (\text{minors})'$ $\qquad$ $B^{-1} = \frac{1}{\det} \cdot (\text{minors})'$

$$\text{minors} = \begin{bmatrix} -15 & 5 & 12+2 \\ -9 & -3 & 3+2 \\ 10-6 & 5-12 & 2-8 \end{bmatrix} \qquad \text{minors} = \begin{bmatrix} 4 & 6 & 15-2 \\ 8-5 & 4-1 & 10-4 \\ -2 & -3 & 4-12 \end{bmatrix}$$

$$\cdot \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} -15 & -5 & 14 \\ 9 & -3 & -5 \\ 4 & 7 & -6 \end{bmatrix} \qquad \cdot \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} 4 & -6 & 13 \\ -3 & 3 & -6 \\ -2 & 3 & -8 \end{bmatrix}$$

$\det = 1 \cdot -15 + 2 \cdot -5 + 3 \cdot 14 = 17$ $\qquad$ $\det = 8 + (-6 \cdot 4) + 13 \cdot 1 = -3$

$$A^{-1} = \frac{1}{17} \begin{bmatrix} -15 & 9 & 4 \\ -5 & -3 & 7 \\ 14 & -5 & -6 \end{bmatrix} \qquad B^{-1} = \frac{1}{-3} \begin{bmatrix} 4 & -3 & -2 \\ -6 & 3 & 3 \\ 13 & -6 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-15}{17} & \frac{9}{17} & \frac{4}{17} \\ \frac{-5}{17} & \frac{-3}{17} & \frac{7}{17} \\ \frac{14}{17} & \frac{-5}{17} & \frac{-6}{17} \end{bmatrix} \qquad = \begin{bmatrix} \frac{4}{-3} & 1 & \frac{2}{3} \\ 2 & -1 & -1 \\ -\frac{13}{3} & 2 & \frac{8}{3} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} \frac{4}{-3} \cdot \frac{-15}{17} + -\frac{5}{17} + \frac{2}{3} \cdot \frac{14}{17} & \frac{9}{17} \cdot \frac{-4}{3} + \frac{-3}{17} + \frac{2}{3} \cdot \frac{-5}{17} & \frac{4}{17} \cdot \frac{-4}{3} + \frac{7}{17} + \frac{2}{3} \cdot \frac{-6}{17} \\ 2 \cdot \frac{-15}{17} + \frac{5}{17} - \frac{14}{17} & 2 \cdot \frac{9}{17} + \frac{3}{17} - \frac{7}{17} & 2 \cdot \frac{4}{17} - \frac{7}{17} + \frac{6}{17} \\ -\frac{15}{17} \cdot \frac{-13}{17} + 2 \cdot \frac{-5}{17} + \frac{8}{3} \cdot \frac{14}{17} & -\frac{13}{3} \cdot \frac{9}{17} + 2 \cdot \frac{-3}{17} + \frac{8}{3} \cdot \frac{-6}{17} & -\frac{13}{3} \cdot \frac{4}{17} + 2 \cdot \frac{7}{17} + \frac{8}{3} \cdot \frac{-6}{17} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{73}{51} & \frac{-37}{51} & \frac{-7}{51} \\ \frac{-117}{51} & \frac{-60}{51} & \frac{21}{51} \\ \frac{227}{51} & \frac{-139}{51} & \frac{-58}{51} \end{bmatrix} = (AB)^{-1}$$

3. $\det(AB) = -51$ $\qquad$ $\det A = 17$ $\quad$ $\det B = -3$

(from part 2) $\qquad$ $\det A \cdot \det B = -51$

4. $$A(p,:) = \begin{bmatrix} -1 & 3 & 0 \\ 1 & 2 & 3 \\ 4 & 2 & 5 \end{bmatrix} \qquad A(:,p) = \begin{bmatrix} 3 & 1 & 2 \\ 5 & 4 & 2 \\ 0 & -1 & 3 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & 1 \end{bmatrix}$$

$$M * A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 5 \\ 40-1 & 20+3 & 50+0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 5 \\ 39 & 23 & 50 \end{bmatrix}$$

Taylor with h notation.

1) $f(x_0+h) = f(x_0) + f'(x_0)h + \frac{f''(x_0)}{2!}h^2$

$f(x) = \ln(x)$ $f'(x) = \frac{1}{x}$ $f''(x) = \frac{-1}{x^2}$

$f(3-2h) = \ln(3) + \frac{1}{3}\cdot(-2h) + \frac{-1}{\xi(3)^2}(-2h)^2$

$= \underbrace{\ln(3) + (-\frac{2}{3}h)}_{P_2(h)} + \underbrace{\frac{-1}{\xi(3)^2}(-2h)^2}_{R_n(h)}$

2) $f(x_0+h) = f(x_0) + f'(x_0)h + \frac{f''(x_0)}{2!}h^2$

$f(x) = (x)^m$ $f'(x) = mx^{m-1}$ $f''(x) = m\cdot(m-1)(x)^{m-2}$

$f(x_0-h) = x^m + mx^{m-1}(-h) + \frac{m\cdot(m-1)\cdot x^{m-2}(-h)^2}{2!}$

$= x^m - mx^{m-1}h + \frac{m\cdot(m-1)\cdot x^{m-2}}{2}\cdot(h^2)$