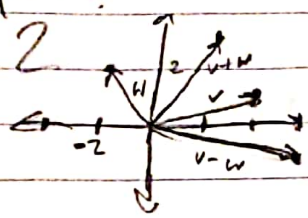
Max Shu

I pledge my honor that I have abided by the Stevens Honor System
Max Shu

MA232 HW1

Part I.

1.1. 2



5. $u+v+w = \begin{bmatrix} 1-3+2 \\ 2+1-3 \\ 3-2-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$2u+2v+w = \begin{bmatrix} 2(1)+2(-3)+2 \\ 2(2)+2(1)-3 \\ 2(3)+2(-2)-1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

1.2 1. $u \cdot v = -0.6(4)+0.8(3)$
$= -2.4+2.4$
$u \cdot v = 0$

$w = cu + dv \quad \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} c \\ 2c \\ 3c \end{bmatrix} + \begin{bmatrix} -3d \\ d \\ -2d \end{bmatrix}$

$\boxed{c=1,\ d=1 \text{ from } u+v+w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$

$u \cdot w = -0.6(1)+0.8(2)$
$= -0.6+1.6$
$u \cdot w = 1$

$u \cdot (v+w) = -0.6(4+1)+0.8(3+2)$
$-0.6(5)+0.8(5)$
$u \cdot (v+w) = 1$

$w \cdot v = 1(4)+3(2)$
$= 4+6$
$\boxed{w \cdot v = 10.}$

2. $||u|| = \sqrt{(-0.6)^2+(0.8)^2}$
$= \sqrt{0.36+0.64}$
$= 1$

$||v|| = 5 \quad ||w|| = \sqrt{5}$

$|u \cdot v| \le ||u||\,||v|| \quad 0 \le 1(5)$ ✓

$|v \cdot w| \le ||v||\,||w|| \quad 10 \le 5\sqrt{5}$
$2 \le \sqrt{5}$ ✓

5. $||v|| = \sqrt{1+9} = \sqrt{10}$ $\quad u_1 = \langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \rangle$ $\quad U_1 = \langle -\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \rangle$

$||w|| = \sqrt{9} = 3$ $\quad u_2 = \langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$ $\quad U_2 = \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$

29. $||v-w||$ - max = 8, min = 3
$v \cdot w$ - max = 15, min = -15

1.3 1. $b = 3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 12 \end{bmatrix}$ or $\langle 3, 7, 12 \rangle$

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} \text{row1} \cdot x \\ \text{row2} \cdot x \\ \text{row3} \cdot x \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 12 \end{bmatrix}$

2. $y_1 = 1$
$y_1+y_2 = 1$
$y_1+y_2+y_3 = 1$
$y_1 = 1, y_2 = 0, y_3 = 0$

$y_1 = 1$
$y_1+y_2 = 4$
$y_1+y_2+y_3 = 9$
$y_1 = 1, y_2 = 3, y_3 = 5$

3. $y_1 = c_1$
$y_1+y_2 = c_2 \Rightarrow c_1+y_2 = c_2 \Rightarrow y_2 = c_2-c_1$
$y_1+y_2+y_3 = c_3 \Rightarrow c_1+c_2-c_1+y_3 = c_3 \Rightarrow y_3 = c_3-c_2$

$\Rightarrow \overset{A}{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

independent, no way to neutralize all rows.

2.1. 3. they also satisfy the third equation.
$\langle -1,1,2\rangle$, $\langle -2,1,3\rangle$, $\langle -3,1,4\rangle$

14. $\begin{bmatrix} 2 & 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = 6$
the row

2.2 1. $l_{21} = 5 \Rightarrow \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & -6 & 6 \end{array}\right]$

3. Subtract eq 1 $\times -\frac{1}{2}$. $\begin{array}{l} 2x - 4y = 6 \\ 3y = 3 \end{array}$ $\left[\begin{array}{cc|c} 2 & -4 & 6 \\ 0 & 3 & 3 \end{array}\right] \Rightarrow \begin{array}{l} x = 5 \\ y = 1, \end{array}$

$\begin{array}{l} 2x - 4y = -6 \\ -x + 5y = 0 \end{array} \Rightarrow \begin{array}{l} 2x - 4y = -6 \\ 3y = -3 \end{array}$ $\begin{bmatrix} 2 & -4 & -6 \\ 0 & 3 & -3 \end{bmatrix} \Rightarrow \begin{array}{l} x = -5 \\ y = -1 \end{array}$

11. (a) any point on the line between the two points, e.g. the midpoint.
(b) They meet at the line between the two points.

12. $\begin{bmatrix} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ & -2 & 2 & 0 \end{bmatrix} \xrightarrow{-2q_1+q_2} \begin{bmatrix} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ & -2 & 2 & 0 \end{bmatrix} \xrightarrow{2q_2+q_3} \begin{bmatrix} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{bmatrix}$

$z = 1$, $y + 3(1) = 4 \Rightarrow y = 1$, $2x + 3(1) + (1)(1) = 8 \Rightarrow x = 2$ $\langle 2,1,1\rangle$

19. $\begin{bmatrix} 1 & 4 & -2 & 1 \\ 1 & 7 & -6 & 6 \\ & 3 & q & t \end{bmatrix} \xrightarrow{-q_1+q_2} \begin{bmatrix} 1 & 4 & -2 & 1 \\ 0 & 3 & -4 & 5 \\ 0 & 3 & q & t \end{bmatrix}$, $q = 4$ would make it singular.
$t = 5$ would give inf. many solutions.
$3y - 4(1) = 5 \Rightarrow y = 3$
$x + 4(3) - 2(1) = 1 \Rightarrow x = -9$

Part 2

1.1 28. $\begin{array}{l} v_1 + w_1 = 4 \\ w_2 + v_2 = 5 \\ v_3 + w_3 = 6 \end{array}$ $\begin{array}{l} 4 - w_1 = 2 \quad v_1 = w_1 + 2 \Rightarrow w_1 + 2 + w_1 = 4 \Rightarrow w_1 = 3 \Rightarrow v_1 = 1 \\ v_2 - w_2 = 5 \quad v_2 = 5 + w_2 \Rightarrow 5 + w_2 + w_2 = 5 \Rightarrow w_2 = 0 \Rightarrow v_2 = 5 \\ v_3 - w_3 = 8 \quad v_3 = 8 + w_3 \Rightarrow 8 + w_3 + w_3 = 6 \Rightarrow w_3 = -1 \Rightarrow v_3 = 7 \end{array}$
$v = \langle 1,5,7\rangle$ $w = (3,0,-1)$
6 unknown numbers.

29. $c_u + 2c_v + c_w = 0$
$3c_u + 7c_v + 5c_w = 1 \Rightarrow c_v + 2c_w = 1$
$c_v = -1$ $c_w = 1 \Rightarrow c_u = 1 \Rightarrow u - v + w = b$
$c_v = -3$ $c_w = 2 \Rightarrow c_u = 4 \Rightarrow 4u - 3v + 2w = b$.
There will not always be 2 combinations if u, v, and w have 0-combinations to produce b.

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31. $2c - d = 1 \Rightarrow 6e - 2e = 1 \Rightarrow 4e = 1,\ e = \frac{1}{4}$

$-c + 2d - e = 0 \Rightarrow -c + 3e = 0 \Rightarrow c = 3e$

$-d + 2e = 0 \Rightarrow d = 2e \qquad e = \frac{1}{4} \Rightarrow d = \frac{2}{4} \Rightarrow c = \frac{3}{4}$

1.2.33 $\langle \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle, \langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \rangle, \langle -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle, \langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \rangle$

$+ + - - \qquad - + - + \qquad - - + + \qquad + - - +$

1.3.14. let x be a number such that $x\langle a,b \rangle = \langle c,d \rangle$

therefore $c - ax = 0,\ d - bx = 0$

$\frac{c}{a} = x,\ \frac{d}{b} = x$

$\frac{c}{a} = \frac{d}{b} \Rightarrow \frac{b}{a} = \frac{d}{c}$

let $y = \frac{b}{a} = \frac{d}{c}$

separate to get $b = ay,\ d = cy$

therefore $y\langle a,c \rangle = \langle b,d \rangle$ and $\langle a,c \rangle$ is a multiple of $\langle b,d \rangle$

2.1.33 $w = cu + dv$

$w = \begin{bmatrix} c \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ d \end{bmatrix}$ if $w = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ then $w = 5u + 7v$.

Thus $Aw = 5Au + 7Av$.

10. 1, $i=1$, $-x_2 + 2x_1 - x_0 = 1 \Rightarrow 2x_1 - x_2 = 1$

2, $i=2$, $-x_3 + 2x_2 - x_1 = 2 \Rightarrow -x_1 + 2x_2 - x_3 = 2$

3, $i=3$, $-x_4 + 2x_3 - x_2 = 3 \Rightarrow -x_2 + 2x_3 - x_4 = 3$

4, $i=4$, $-x_5 + 2x_4 - x_3 = 4 \Rightarrow -x_3 + 2x_4 = 4$

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 1 \\ -1 & 2 & -1 & 0 & 2 \\ 0 & -1 & 2 & -1 & 3 \\ 0 & 0 & -1 & 2 & 4 \end{bmatrix} \xrightarrow{-a_2 \leftrightarrow a_1} \begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 2 & -1 & 0 & 0 & 1 \\ 0 & -1 & 2 & -1 & 3 \\ 0 & 0 & -1 & 2 & 4 \end{bmatrix} \xrightarrow{-2a_1 + a_2} \begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 0 & 3 & -2 & 0 & 5 \\ 0 & -1 & 2 & -1 & 3 \\ 0 & 0 & -1 & 2 & 4 \end{bmatrix} \xrightarrow{-a_3 \leftrightarrow a_2}$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 0 & 1 & -2 & 1 & -3 \\ 0 & 3 & -2 & 0 & 5 \\ 0 & 0 & -1 & 2 & 4 \end{bmatrix} \xrightarrow{-3a_2 + a_3} \begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 0 & 1 & -2 & 1 & -3 \\ 0 & 0 & 4 & -3 & 14 \\ 0 & 0 & -1 & 2 & 4 \end{bmatrix} \xrightarrow{-a_4 \leftrightarrow a_3} \begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 0 & 1 & -2 & 1 & -3 \\ 0 & 0 & 1 & -2 & -4 \\ 0 & 0 & 4 & -3 & 14 \end{bmatrix} \xrightarrow{-4a_3 + a_4}$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 0 & 1 & -2 & 1 & -3 \\ 0 & 0 & 1 & -2 & -4 \\ 0 & 0 & 0 & 5 & 30 \end{bmatrix} \xrightarrow{\frac{1}{5}a_4} \underbrace{\begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 0 & 1 & -2 & 1 & -3 \\ 0 & 0 & 1 & -2 & -4 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}}_{\text{REF}} \xrightarrow{-a_4 + a_2,\ 2a_4 + a_3} \begin{bmatrix} 1 & -2 & 1 & 0 & -2 \\ 0 & 1 & -2 & 0 & -9 \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \xrightarrow{-a_3 + a_1,\ 2a_3 + a_2}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 & -10 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \xrightarrow{2a_2 + a_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}}_{\text{RREF}} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 8 \\ 6 \end{bmatrix}$$