Max Shi MA 221 HW09 Brady Section H

I pledge my honor that I have abided by the Stevens Honor System

Max Shi

1. (a) $L = \pi$

$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$ $\qquad a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$ $\qquad = \frac{1}{\pi}\int_0^{\pi/2} dx$

$= \frac{1}{\pi}\int_0^{\pi/2}\cos(nx)\,dx$ $\qquad = \frac{1}{\pi}\left(\frac{\pi}{2}\right)$

$= \frac{1}{n\pi}\left[\sin(nx)\right]_0^{\frac{\pi}{2}}\,dx$ $\qquad = \frac{1}{2}$

$= \frac{1}{n\pi}\sin\left(\frac{\pi}{2}n\right), \quad n = 1, 2, \ldots$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$

$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\sin(nx)\,dx$

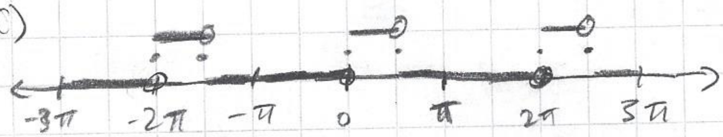
$= \frac{1}{n\pi}\left[-\cos(nx)\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{n\pi}\left(1 - \cos\left(\frac{\pi}{2}n\right)\right), \quad n = 1, 2, \ldots$

(b) $f(x) \sim \frac{1}{4} + \left(\frac{\cos x}{\pi} + 0 - \frac{\cos 3x}{3\pi} + 0 + \frac{\cos 5x}{5\pi}\right) + \left(\frac{\sin x}{\pi} + \frac{2\sin 2x}{2\pi} + \frac{\sin 3x}{3\pi} + 0 + \frac{\sin 5x}{5\pi}\right)$

$\sim \frac{1}{4} + \frac{\cos x}{\pi} - \frac{\cos 3x}{3\pi} + \frac{\cos 5x}{5\pi} + \frac{\sin x}{\pi} + \frac{\sin 2x}{\pi} + \frac{\sin 3x}{3\pi} + \frac{\sin 5x}{5\pi}$

(c)



The series converges to $\frac{1}{2}$ at all jump discontinuities

$x = -2\pi, -\frac{3\pi}{2}, 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}$

2. $L = 1$

$a_n = \frac{1}{1}\int_{-1}^{1} f(x)\cos\left(\frac{n\pi x}{1}\right)dx$ $\qquad a_0 = \int_{-1}^{1} f(x)\,dx$

$= \int_{-1}^{1} f(x)\cos(n\pi x)\,dx$ $\qquad a_0 = \int_{-1}^{1} 1 - x^2\,dx$

$= \int_{-1}^{1}(1-x^2)\cos(n\pi x)\,dx$ $\qquad = \left[x - \frac{x^3}{3}\right]_{-1}^{1}$

$= \int_{-1}^{1}\cos(n\pi x)\,dx - \int_{-1}^{1} x^2\cos(n\pi x)\,dx$ $\qquad = 1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right) = 4/3 = a_0$

Using $\int u^2\cos(bu)\,du = \frac{1}{b^3}\left(b^2u^2\sin(bu) + 2bu\cos(bu) - 2\sin(bu)\right)$

$= \frac{1}{n\pi}\left[\sin(n\pi x)\right]_{-1}^{1} - \frac{1}{(n\pi)^3}\left[(n\pi x)^2\sin(n\pi x) + 2n\pi x\cos(n\pi x) - 2\sin(n\pi x)\right]_{-1}^{1}$

$= 0 - \frac{1}{(n\pi)^3}\left(0 + 2n\pi\cos(n\pi) - 0 - (0 - 2n\pi\cos(-n\pi) - 0\right)$

$= \frac{-4}{n^2\pi^2}\cos(n\pi) = \frac{-4(-1)^n}{n^2\pi^2}$

$$b_n = \int_{-1}^{1} f(x)\sin(n\pi x)\,dx \qquad \int u^2\sin(bu)\,du = \frac{1}{b^3}\left(-b^2u^2\cos(bu) + 2bu\sin(bu) + 2\cos(bu)\right)$$

$$= \int_{-1}^{1}(1-x^2)\sin(n\pi x)\,dx$$

$$= \int_{-1}^{1}\sin(n\pi x)\,dx - \int_{-1}^{1}x^2\sin(n\pi x)\,dx$$

$$= -\frac{1}{n\pi}\left[\cos(n\pi x)\right]_{-1}^{1} - \frac{1}{(n\pi)^3}\left[-(n\pi x)^2\cos(n\pi x) + 2n\pi x\sin(n\pi x) + 2\cos(n\pi x)\right]_{-1}^{1}$$

$$= 0 - \frac{1}{(n\pi)^3}\left(-n\pi\cos(n\pi) + 2n\pi(\sin n\pi) + 2\cos(n\pi) - (-n\pi\cos(n\pi) - 2n\pi\sin(n\pi) + 2\cos(n\pi))\right)$$

$= -\frac{1}{(n\pi)^3}(0) = 0$. as $1-x^2$ is even!!

3. $y'' + 2y = 0$

$m^2 + 2 = 0$

$m = \pm\sqrt{2}\,i$

$y = C_1\cos(\sqrt{2}\,t) + C_2\sin(\sqrt{2}\,t)$

$k=0 \Rightarrow f_0(t) = \frac{1}{2}$

$y_p = A \quad 2A = \frac{1}{2}$

$y_p'' = 0 \quad A = \frac{1}{4}$

$y_p = 1/4$

$k=1 \Rightarrow f_1(t) = \frac{2}{\pi}\sin(t) \Rightarrow y_p = A\sin t + B\cos t \Rightarrow y_p'' = -A\sin t - B\cos t$

$-A\sin t - B\cos t + 2A\sin t + 2B\cos t = \frac{2}{\pi}\sin t$

$A\sin t + B\cos t = \frac{2}{\pi}\sin t$

$A = \frac{2}{\pi}, B = 0 \Rightarrow y_p = \frac{2}{\pi}\sin t$

$k=2 \Rightarrow f_2(t) = \frac{2}{3\pi}\sin 3t \Rightarrow y_p = A\sin 3t + B\cos 3t \Rightarrow y_p'' = -9A\sin 3t - 9B\cos 3t$

$-9A\sin 3t - 9A\cos 3t + 2A\sin 3t + 2B\cos 3t = \frac{2}{3\pi}\sin 3t$

$-7A\sin 3t - 7B\cos 3t = \frac{2}{3\pi}\sin 3t$

$-7A = \frac{2}{3\pi} \quad B = 0 \quad y_p = -\frac{2}{21\pi}\sin 3t$

$A = -\frac{2}{21\pi}$

$k \Rightarrow f_k(t) = \frac{2}{(2k-1)\pi}\sin((2k-1)t) \Rightarrow y_p = A\sin((2k-1)t) + B\cos((2k-1)t) \Rightarrow$

$y_p'' = -A(2k-1)^2\sin((2k-1)t) - B(2k-1)^2\cos((2k-1)t)$

$(-(2k-1)^2A + 2A)\sin((2k-1)t) + (-(2k-1)^2B + 2B)\cos((2k-1)t) = \frac{2}{(2k-1)\pi}$

$A(-(2k-1)^2 + 2) = \frac{2}{(2k-1)\pi} \Rightarrow A(-4k^2 + 4k - 1 + 2) = \frac{2}{(2k-1)\pi}$

$B = 0, \quad A = \frac{2}{(2k-1)(-4k^2+4k+1)\pi} \Rightarrow y_p = \frac{2\sin((2k-1)t)}{(2k-1)(-4k^2+4k+1)\pi}$

(a) $y_p(t) = 1/4 + \sum_{k=1}^{\infty}\frac{2\sin((2k-1)t)}{(2k-1)(-4k^2+4k+1)\pi}$

(b) $y_p(t) = \frac{1}{4} + \frac{2}{\pi}\sin(t) + \left(-\frac{2}{21\pi}\right)\sin(3t) + \left(\frac{-2}{115\pi}\right)\sin(5t)$

$k=3 \Rightarrow \frac{2\sin 5t}{(5)(-36+12+1)\pi} = \frac{-2}{115\pi}$