Max Shi   Prof Brandy   Lecture H

I pledge my honor that I have abided by the Stevens Honor System
Max Shi

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1. $\mathcal{L}^{-1}\left\{\frac{s^2+11s+20}{(s^2+4s+8)(s+1)^2}\right\}$

$$\frac{s^2+11s+20}{(s^2+4s+8)(s+1)^2} = \frac{As+B}{(s^2+4s+8)} + \frac{C}{(s+1)} + \frac{D}{(s+1)^2}$$

$$s^2+11s+20 = (As+B)(s+1)^2 + C(s+1)(s^2+4s+8) + D(s^2+4s+8)$$

$$s^2+11s+20 = (As+B)(s^2+2s+1) + C(s+1)(s^2+4s+8) + Ds^2+4Ds+8D$$

$$s^2+11s+20 = As^3+2As^2+As+Bs^2+2Bs+B+Cs^3+4Cs^2+8Cs+Cs^2+4Cs+8C+Ds^2+4Ds+8D$$

$$s^2+11s+20 = s^3(A+C)+s^2(2A+B+4C+C+D)+s(A+2B+8C+4C+4D)+B+8C+8D$$

$A+C=0,\ 2A+B+5C+D=1,\ A+2B+12C+4D=11,\ B+8C+8D=20$

$A=-C \Rightarrow B+3C+D=1,\ 2B+11C+4D=11,\ B+8C+8D=20.$

$B=1-3C-D \Rightarrow 2(1-3C-D)+11C+4D=11,\ 1-3C-D+8C+8D=20$

$5C+2D=9,\ 5C+7D=19$

$\Rightarrow 5D=10 \Rightarrow D=2$

$D=2 \Rightarrow C=1 \Rightarrow A=-1 \Rightarrow B=-4$

$$\mathcal{L}^{-1}\left\{\frac{s^2+11s+20}{(s^2+4s+8)(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{-s-4}{(s^2+4s+8)} + \frac{1}{s+1} + \frac{2}{(s+1)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-s-4}{s^2+4s+8}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2}\right\}$$

$n=1,\ n!=1,\ a=-1$

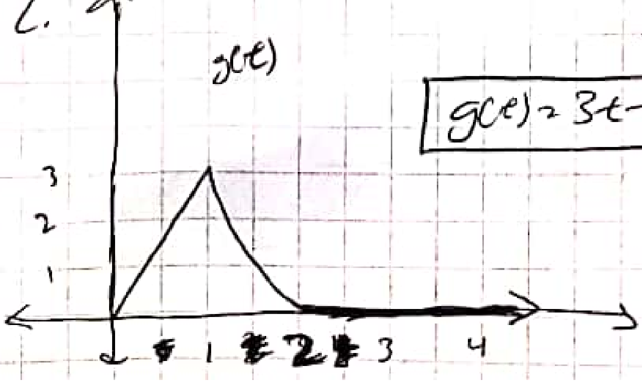
$$\mathcal{L}^{-1}\left\{\frac{-s-4}{(s+2)^2+2^2}\right\} + e^{-t} + 2te^{-t}$$

$$-\mathcal{L}^{-1}\left\{\frac{s+4}{(s+2)^2+2^2}\right\}$$

$$-\left(\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+2^2}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s+2)^2+2^2}\right\}\right)$$

$$\boxed{-\left(e^{-2t}\cos(2t) + e^{-2t}\sin(2t)\right) + e^{-t} + 2te^{-t}}$$

2.



$g(t) = 3t + (t^2-9t+8)u(t-1) - (t^2-6t+8)u(t-2)$

$$\boxed{g(t) = 3t + (t^2-9t+8)u(t-1) - (t^2-6t+8)u(t-2)}$$

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$$g(t) = 3t + (t^2-9t+8)u(t-1) - (t^2-6t+8)u(t-2)$$

$$= 3t + [(t^2-2t+1) + 2t - 1 - 9t + 8]u(t-1) - [(t^2-4t+4) + 4t - 4 - 6t + 8]u(t-2)$$

$$= 3t + [(t-1)^2 - 7t + 7]u(t-1) - [(t-2)^2 - 2t + 4]u(t-2)$$

$$= 3t + [(t-1)^2 - 7(t-1)]u(t-1) - [(t-2)^2 - 2(t-2)]u(t-2)$$

$$G(s) = \mathcal{L}\{3t + [(t-1)^2 - 7(t-1)]u(t-1) - [(t-2)^2 - 2(t-2)]u(t-2)\}$$

$$\boxed{G(s) = \frac{3}{s^2} + e^{-s}\left(\frac{2}{s^3} - \frac{7}{s^2}\right) - e^{-2s}\left(\frac{2}{s^3} - \frac{2}{s^2}\right)}$$

3. $g(t) = \mathcal{L}^{-1}\left\{-\frac{4}{s^3} + \frac{4}{s^2} + \left(\frac{4}{s^3} + \frac{4}{s^2}\right)e^{-2s}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{-4}{s^3}\right\} + \mathcal{L}^{-1}\left\{\frac{4}{s^2}\right\} + \mathcal{L}^{-1}\left\{e^{-2s}\left(\frac{4}{s^3} + \frac{4}{s^2}\right)\right\}$$

$n=2,\ n!=2 \qquad n=1,\ n!=1$

$$= -2\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} + 4\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \dots\dots$$

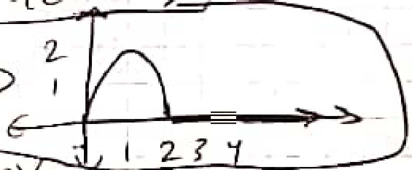
$$= -2t^2 + 4t + \mathcal{L}^{-1}\left\{\left[2\left(\frac{2}{s^3}\right) + 4\left(\frac{1}{s^2}\right)\right]e^{-2s}\right\}$$

$t-a = t-2$

$$= -2t^2 + 4t + [2(t-2)^2 + 4(t-2)]u(t-2)$$

$$\boxed{g(t) = -2t^2 + 4t + (2t^2 - 4t)u(t-2)}$$

$$\boxed{g(t) = \begin{cases} -2t^2 + 4t, & 0 \le t < 2 \\ 0, & 2 \le t \end{cases}} \Rightarrow$$



4. $f(t) = y' + 3y,\ y(0) = 1,\ f(t) = \begin{cases} (2-t)/2, & 0 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases}$

$$f(t) = \frac{2-t}{2} - \left(\frac{2-t}{2}\right)u(t-2) \qquad \mathcal{L}(f(t)) = \frac{1}{s} - \frac{1}{2s^2} + \frac{e^{-2s}}{2s^2}$$

$$= 1 - \frac{t}{2} + \left(\frac{t-2}{2}\right)u(t-2)$$

$$\frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$1 = A(s+3) + B(s)$$

$$A + B = 0,\ 3A = 1$$

$$A = \frac{1}{3},\ B = -\frac{1}{3}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{y' + 3y\}$$

$$\frac{1}{s} - \frac{1}{2s^2} + \frac{e^{-2s}}{2s^2} = \mathcal{L}\{y'\} + \mathcal{L}\{3y\}$$

$$\frac{1/2}{s^2(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3}$$

$$\frac{1}{s} - \frac{1}{2s^2} + \frac{e^{-2s}}{2s^2} = sY - y(0) + 3Y,\ y(0) = 1$$

$$\frac{1}{2} = A(s)(s+3) + B(s+3) + Cs^2$$

$$\frac{1}{s} - \frac{1}{2s^2} + \frac{e^{-2s}}{2s^2} = sY + 3Y - 1$$

$$\frac{1}{2} = As^2 + 3As + Bs + 3B + Cs^2$$

$$\frac{1}{s} - \frac{1}{2s^2} + \frac{e^{-2s}}{2s^2} + 1 = (s+3)Y$$

$$A + C = 0,\ 3A + B = 0,\ 3B = \frac{1}{2}$$

$$C = \frac{1}{18},\ A = -\frac{1}{18},\ B = \frac{1}{6}$$

$$\frac{1}{s(s+3)} - \frac{1}{2s^2(s+3)} + \frac{e^{-2s}}{2s^2(s+3)} + \frac{1}{s+3} = Y$$

$$\frac{1}{3s} - \frac{1}{3(s+3)} + \frac{1}{18s} - \frac{1}{6s^2} - \frac{1}{18(s+3)} + e^{-2s}\left(-\frac{1}{18s} + \frac{1}{6s^2} + \frac{1}{18(s+3)}\right) + \frac{1}{s+3} = Y$$

$$Y = \frac{6}{18s} - \frac{6}{18(s+3)} + \frac{1}{18s} - \frac{1}{6s^2} - \frac{1}{18(s+3)} + \frac{18}{18(s+3)} + e^{-2s}\left(-\frac{1}{18s} + \frac{1}{6s^2} + \frac{2}{18(s+3)}\right)$$

$$Y = \frac{7}{18s} + \frac{11}{18(s+3)} - \frac{1}{6s^2} + e^{-2s}\left(\frac{1}{6s^2} + \frac{1}{18(s+3)} - \frac{1}{18s}\right)$$

$$Y = \mathcal{L}^{-1}\left\{\frac{7}{18s} + \frac{11}{18(s+3)} - \frac{1}{6s^2} + e^{-2s}\left(\frac{1}{6s^2} + \frac{1}{18(s+3)} - \frac{1}{18s}\right)\right\}$$

$$Y = \frac{7}{18}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{11}{18}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \mathcal{L}^{-1}\left\{e^{-2s}\left(\frac{1}{6s^2} + \frac{1}{18(s+3)} - \frac{1}{18s}\right)\right\}$$

$$Y = \frac{7}{18} + \frac{11}{18}e^{-3t} - \frac{1}{6}t + u(t-2)\left(\frac{(t-2)}{6} + \frac{1}{18}e^{-3(t-2)} - \frac{1}{18}\right)$$

$$\boxed{Y = \frac{7}{18} + \frac{11}{18}e^{-3t} - \frac{1}{6}t + \left(\frac{t-2}{6} + \frac{1}{18}e^{6-3t} - \frac{1}{18}\right)u(t-2)}$$