

## PROBLEM SET 7: INSTRUMENTAL VARIABLES MGMT 737

1. **Compliers and Weak IV:** We explore Angrist and Evan (1998), who study the impact of children on parents' supply. This paper has two instruments: 1) whether the first two children of a mother are the same sex *Same sex*; 2) whether the second (successful) pregnancy is twins *Twins-2*. The 1980 Data for this paper is available in the repository in a csv `ang_ev_1980.csv`.

I was not able to perfectly match AE's sample but it is very close. As a result, the point estimates are not identical, but are close enough that you should be able to feel confident. You may use pre-built estimation packages unless otherwise stated.

*More than 2 children* is `morekids`, *Number of children* is `kidcount`, *worked for pay* is `mom.worked`, *Weeks worked* is `mom_weeks_worked` and the two dummy instruments are `samesex` and `twins_2`.

- (a) Replicate the coefficients from Table 5, Column 1, rows 1-4, using a linear regression.
  - (b) Replicate the coefficients from Table 5, Column 2, rows 3-4, using 2SLS. Convince yourself you could construct this estimate by hand using the result in the previous answer.
  - (c) Replicate the coefficients from Table 5, Column 7, rows 1-4, using a linear regression.
  - (d) For the endogeneous variable "More than 2 children", what is the complier share for each of the two instruments?
  - (e) For the endogeneous variable "More than 2 children" and each of the two instruments, what is the average share of the complier population with an education greater than high school (`moreths`)? What about different mother race shares?
  - (f) Using the Same sex instrument, construct the Weak IV robust Anderson-Rubin confidence intervals using the algorithm outlined in Chernozhukov and Hansen (2007) (see the slides)
2. **Kitagawa test** Now we implement the Kitagawa (2015) test for instrument validity. We will focus on the *worked for pay* outcome as our endogeneous variable  $Y$ , more than two children as our endogeneous variable, and our two instruments will be as before (same sex and twins). Implement the procedure below for both instruments.

- (a) Let  $P(y, d) = n_1^{-1} \sum_{i: Z_i=1} 1(Y_i = y, D_i = d)$ , where  $n_1$  is the number of observations where  $Z_i = 1$  in the sample, and  $Q(y, d) = n_0^{-1} \sum_{i: Z_i=0} 1(Y_i = y, D_i = d)$ , where  $n_1$  is the number of observations where  $Z_i = 0$  in the sample.  $n = n_0 + n_1$  Estimate  $P(y, d)$  and  $Q(y, d)$  and report them for the 4 possible pairs of  $(y, d)$ .
- (b) Next, we want to calculate the following statistic (Note that this is simplified relative to the general Kitagawa case because  $Y$  is binary):

$$T = \left( \frac{n_0 n_1}{n} \right)^{1/2} \max \left\{ \sup_{y \in \{0,1\}} \left\{ \frac{Q(y, 1) - P(y, 1)}{\xi \vee \sigma_{P,Q}(y, 1)} \right\}, \sup_{y \in \{0,1\}} \left\{ \frac{Q(y, 0) - P(y, 0)}{\xi \vee \sigma_{P,Q}(y, 0)} \right\} \right\}$$

where  $\xi$  is a trimming constant that you pick,  $\vee$  denotes the "max" between the two objects, and

$$\sigma_{P,Q}^2(y, d) = (1 - \hat{\lambda})P(y, d)(1 - P(y, d)) + \hat{\lambda}Q(y, d)(1 - Q(y, d)), \quad \hat{\lambda} = n_1/n.$$

Estimate  $\sigma$  for each of the four possible pairs of  $(y, d)$ , and then calculate the  $T$  based using a  $\xi = 0.1$ . Report this  $T$  statistic.

- (c) Now, we need to construct critical values for the statistic  $T$ . First, take the subsample of  $n_1$  observations where  $Z_i = 1$ . Sample a new set of  $n_1$  observations from this subsample, with replacement using weights  $H(y, d) = \hat{\lambda}P + (1 - \hat{\lambda})Q$ . (E.g. the weight of drawing each observation

is a function of this combined probability). Use this new data to construct a new empirical distribution  $P^*$ . Similarly construct a new sample of  $n_0$  observations based on the subsample of observations where  $Z_0 = 0$  using  $H(y, d)$ . Use this to construct  $Q^*$ . Then, calculate the  $T$  statistic above, but using  $P^*$  and  $Q^*$  in place of  $P$  and  $Q$ .

- (d) Repeat this last process 500 times and construct a distribution of the  $T^*$ . We reject the null of instrument validity (exclusion and monotonicity) at the 5 percent level if  $T$  is greater than the 95th percentile of the distribution of  $T^*$ . Identify what quantile  $T$  sits in the distribution of  $T^*$ .