

Résoudre le système linéaire à coefficients complexes suivant :

$$\begin{cases} (1+i)x + (1-2i)y + (-1+3i)z = 2+i \\ x - 2y + z = 0 \\ ix + (2-i)y - 2z = 0 \end{cases}$$

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$$\begin{array}{l} \ell_1 \leftarrow \ell_2 \\ \ell_2 \leftarrow \ell_1 - (1+i)\ell_2 \\ \ell_3 \leftarrow \ell_3 - i\ell_2 \end{array} \quad \begin{cases} x - 2y + z = 0 \\ ((1-2i)+(2+2i))y + ((-1+3i)-(1+i))z = 2+i \\ ((2-i)+2i)y + (-2-i)z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 2y + z = 0 \\ 3y + (-2+2i)z = 2+i \\ (2+i)y + (-2-i)z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 2y + z = 0 \\ 3y + (-2+i)z = 2+i \\ (-5i)z = -3 - 4i \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{4}{5} - \frac{3}{5}i \\ y = \frac{4}{5} - \frac{3}{5}i \\ z = \frac{4}{5} - \frac{3}{5}i \end{cases}$$

$$\mathcal{S} = \left\{ \left(\frac{4}{5} - \frac{3}{5}i, \frac{4}{5} - \frac{3}{5}i, \frac{4}{5} - \frac{3}{5}i \right) \right\}$$