

Soit la fonction $V: \mathbb{R}^2 \rightarrow \mathbb{R}$ définie par

$$V(r, \theta) = \frac{\cos \theta}{r^2}$$

1. Donner l'ensemble de définition de V .



$$\mathcal{D}_V = \mathbb{R}^* \times \mathbb{R}$$

2. Calculer les dérivées partielles $\frac{\partial V}{\partial r}$ et $\frac{\partial V}{\partial \theta}$.



pour $(r, \theta) \in \mathcal{D}_V$, on a

$$\frac{\partial V}{\partial r} = -\frac{2}{r^3} \cos \theta, \quad \frac{\partial V}{\partial \theta} = -\frac{1}{r^2} \sin \theta$$

3. Calculer les dérivées partielles secondes $\frac{\partial^2 V}{\partial r^2}$, $\frac{\partial V}{\partial \theta}$ et $\frac{\partial^2 V}{\partial r \partial \theta}$.



pour $(r, \theta) \in \mathcal{D}_V$, on a

$$\begin{aligned} \frac{\partial^2 V}{\partial r^2} &= \frac{\partial}{\partial r} \left[-\frac{2}{r^3} \cos \theta \right] = \frac{6}{r^4} \cos \theta \\ \frac{\partial^2 V}{\partial \theta^2} &= \frac{\partial}{\partial \theta} \left[-\frac{1}{r^2} \sin \theta \right] = -\frac{1}{r^2} \cos \theta \\ \frac{\partial^2 V}{\partial r \partial \theta} &= \frac{\partial}{\partial r} \left[-\frac{1}{r^2} \sin \theta \right] = \frac{2}{r^3} \sin \theta \end{aligned}$$

4. En déduire que

$$\sin \theta \frac{\partial}{\partial r} \left[r^2 \frac{\partial V}{\partial r} \right] + \frac{\partial}{\partial \theta} \left[\sin \theta \frac{\partial V}{\partial \theta} \right] = 0.$$



On a

$$\begin{aligned} & \sin \theta \frac{\partial}{\partial r} \left[r^2 \frac{\partial V}{\partial r} \right] + \frac{\partial}{\partial \theta} \left[\sin \theta \frac{\partial V}{\partial \theta} \right] \\ &= \sin \theta \left[2r \frac{\partial V}{\partial r} + r^2 \frac{\partial^2 V}{\partial r^2} \right] + \left[\cos \theta \frac{\partial V}{\partial \theta} + \sin \theta \frac{\partial^2 V}{\partial \theta^2} \right] \\ &= \sin \theta \left[2r \left(-\frac{2}{r^3} \cos \theta \right) + r^2 \frac{6}{r^4} \cos \theta \right] + \left[\cos \theta \left(-\frac{1}{r^2} \sin \theta \right) + \sin \theta \left(-\frac{1}{r^2} \cos \theta \right) \right] \\ &= \frac{\cos \theta \sin \theta}{r^2} (-4 + 6 - 1 - 1) \\ &= 0 \end{aligned}$$