

Résoudre le système linéaire à coefficients complexes suivant :

$$\begin{cases} (1+i)x + (1-2i)y + (-1+3i)z = 2+i \\ x - 2y + z = 0 \\ ix + (2-i)y - 2z = 0 \end{cases}$$

$$\begin{aligned} & \begin{cases} (1+i)x + (1-2i)y + (-1+3i)z = 2+i \\ x - 2y + z = 0 \\ ix + (2-i)y - 2z = 0 \end{cases} \\ & \begin{matrix} \ell_1 \leftarrow \ell_2 \\ \ell_2 \leftarrow \ell_1 - (1+i)\ell_2 \\ \ell_3 \leftarrow \ell_3 - i\ell_2 \end{matrix} \begin{cases} x - 2y + z = 0 \\ ((1-2i) + (2+2i))y + ((-1+3i) - (1+i))z = 2+i \\ ((2-i) + 2i)y + (-2-i)z = 0 \end{cases} \\ & \Leftrightarrow \begin{cases} x - 2y + z = 0 \\ 3y + (-2+2i)z = 2+i \\ (2+i)y + (-2-i)z = 0 \end{cases} \\ & \Leftrightarrow \begin{cases} x - 2y + z = 0 \\ 3y + (-2+i)z = 2+i \\ (-5i)z = -3-4i \end{cases} \\ & \Rightarrow \begin{cases} x = \frac{4}{5} - \frac{3}{5}i \\ y = \frac{4}{5} - \frac{3}{5}i \\ z = \frac{4}{5} - \frac{3}{5}i \end{cases} \end{aligned}$$

$$\mathcal{S} = \left\{ \left(\frac{4}{5} - \frac{3}{5}i, \frac{4}{5} - \frac{3}{5}i, \frac{4}{5} - \frac{3}{5}i \right) \right\}$$