Dialysis Unit Analysis

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Abstract

Hemodialysis using hollow-fiber membranes provides life-sustaining treatment for people suffering from kindey failure for over two million people worldwide [4]. We look at the transport dynamics behind dialysis machines. and found that there were many lenses through which to view the transport, with the simplest and most straight forward being mass transfer coefficients. We looked at how flow rates effect clearance rates, and considered if clearance could be improved if the permeability of the membrane was varied. Our results were inconclusive.

1 Problem Statement

1.1 Background

End Stage Renal Disease (ESRD) effects over 692,000 Americans, and imposes a disproportionately high financial burden compared to other chronic diseases with regard to cost [4]. There have been attempts to dialisize humans for many years without success. However, with the availability of cellophane and heparin in 1945, a Dutch physician named Willem Kolff was able to successfully prolong a patients life. Since then the dialysis machine has been commercially available for over 50 years. The general design of the most common machines is dialysis tubing forming small fibers ($200\mu m$) which separates blood and dialysis flow [9].

1.2 Problem Description

The efficiency of dialysis units can be controlled by changing flow conditions and analyzing different membrane designs. These two parameters can have a synergistic effect. We are interested in optimizing the design of these parameters.

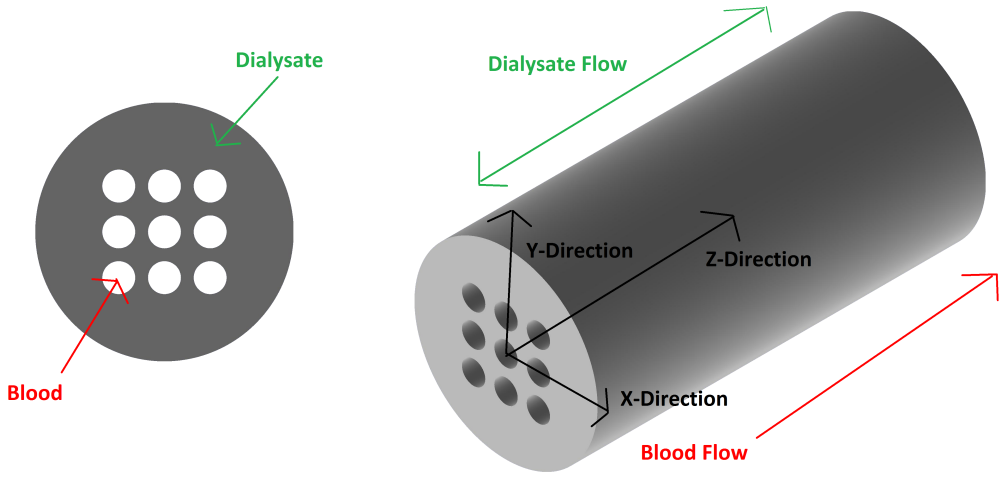

Figure 1: This figure gives an example of what a typical dialysis set-up might look like. Dialysate can be made to flow with, against, or perpendicular to the flow of blood. A typical machine has around 11,000 fibers (there are 9 shown), with each one approximately 200 μm in diameter.

2 Problem Formulation

2.1 Design and Governing Equations

As in most transport problems, it is reasonable to start with the conservation of mass equation for dilute and incompressible solutions that also follow Fick's law. This is given as:

$$\frac{\partial C_i}{\partial t} + \mathbf{v} \cdot \nabla C_i = D_{ij} \nabla^2 C_i + R_i \tag{1}$$

In our analysis of dialysis, we are interested in the problem when it is at equilibrium (i.e. Steady state), and where there is no internal reaction. Therefore, this leaves us with

$$0 = -D_{ij} \nabla^2 C_i + \mathbf{v} \cdot \nabla C_i \tag{2}$$

Relating this to dialysis, we have found 4 models which seem reasonable to apply. For transport across a membrane, our textbook offers the following approaches.

1. Transport through a porous medium
2. Mass transfer coefficients
3. The Kedem-Katchaslky Equation
4. The Patlak Equation

Each of these approaches uses different simplifications, and vary in how difficult their application is. After noting the simplicity of mass-transfer coefficients, we decided to take advantage of their use in analyzing ideal flow conditions. The mass transfer coefficient is the ratio between the flux of a solute across a membrane and the difference in concentration. This coefficient can be formally defined as:

$$k_f = -\frac{1}{S(C_s - C_0)} \int_S D_{ij} \left(\frac{\partial C_i}{\partial y} \right) \Big|_{y=0} dS$$

However, this coefficient is often simply measured in experimentation. It is very important to note, however, that the use of a mass transfer coefficient subsumes one is dealing with diffusion dominated transport. In the context of dialysis, this assumption is reasonable as very strong concentration gradients exist.

Regarding why we choose not to use the other approaches, choosing not to use the Patlak Equation, which can be thought of as a more precise Kedem-Katchaslky equation, was simple. Our textbook showed that for low Peclet number transport ($Pe < 1.2$), or equivalently, diffusion dominated transport, the assumptions made in the Kedem-Katchalsky equation are reasonable (error $< 10\%$). As a result, we did not see it necessary to verify the Patlak equation.

After performing a literature review, we did see widespread use of the Kedem-Katchalsky equations. The use of the equation involves considerably more inputs, and is more difficult to check. For this reason, we decided to check the "reasonableness" of our solutions using reported values derived from this equation.

Finally, with respect to simple transport through a porous medium, relations between the Kedem-Katchalksy equation and equations for transport through a porous medium can be related. Because of this, we thought it would be interesting to perform our analysis by using the two equations to see see what the "idea" membrane would be!

2.2 Problem Layout

The starting design is a set of parallel pipes (fibers) which represent porous membranes. Blood from the body of an individual with end stage renal disease will flow through the gap between these two pipes and a dialystate fluid will flow on the other side of each fiber. The dialystate will be a precisely controlled fluid, which will facilitate the mass transfer of noxious solutes from the blood.

We will consider the transport of urea and creatinine (and substances in general), which will be initially present in the blood and absent in the dialystate. Although both diffusion, due to a concentration difference, and convection due to velocity will contribute to transport, because of the use of mass transfer coefficients, we will first assume diffusive dominant transport. Next, the concentration profiles of the blood and dialystate will be considered separately from the mechanism of transport across the membrane, with governing equations

given in the Mass Transfer section. This will allow the flow rates, as well as cocurrent versus countercurrent flow, to be examined. Optimizing both the flow, as well as membrane properties, will then lead to a synergistic combination and maximal solute flux.

2.3 How We Will Analyze the Solutions

As has already been made clear, we are not implcicitly taking into account net solvent exchange (convective transport) with mass transfer coefficients. However, we can look at the concentration difference along the length of individual pores, or from the blood to the dialysate, and can consider the different overall transfer rates that appear in different approaches.

3 Solution

3.1 Overall Mass Transport

To determine the concentration profile in the z-direction, we need to look at general mass transfer across our membrane. We can begin with the simple equations:

$$d\dot{M}_i = -Q_B dC_{i_B} \tag{3}$$

$$d\dot{M}_i = Q_D dC_{i_D} \tag{4}$$

Here, $\dot{M}_i$ is molar flow rate, Q_B and Q_D are flow rates of blood and dialysate, respectively, and dC_{iB} is the differential concentration change of component i. A simple unit analysis verifies that

$$\frac{\text{moles}}{\text{sec}} = \frac{cm^3}{\text{sec}} \cdot \frac{\text{moles}}{cm^3}$$

We can now introduce our mass transfer coefficient

$$dM_i = k_0(C_{iB} - C_{iD})dA_m$$

where dA_m is a differential area, and combining gives us

$$dC_{iB} - dC_{iD} = -k_0 dA_m \left(\frac{1}{Q_B} + \frac{1}{Q_D}\right)(C_{iB} - C_{iD}) \tag{5}$$

This equation assumes co-current flow, but it can be easily modified for counter current flow. All that is required is negating the sign on equation (4). The result being

$$dC_{iB} - dC_{iD} = -k_0 dA_m \left(\frac{1}{Q_B} - \frac{1}{Q_D}\right)(C_{iB} - C_{iD}) \tag{6}$$

These equations can be integrated from the inlet to outlet to which gives the differences in concentration as a function of the flow rates. The only difference in the results is the sign

between the reciprocal flow rates. For co-current they are added (+), and for counter current they are subtracted (-). The equation for counter-current is:

$$ln\left(\frac{C_{iB}(0) - C_{iD}(0)}{C_{iB}(L) - C_{iD}(L)}\right) = k_0 A_m \left(\frac{1}{Q_B} - \frac{1}{Q_D}\right) \tag{7}$$

At first it was difficult to understand this equation, but a simple re-naming helps to shed light on the meaning.

$$\text{Let} \quad C_{diff}(z) = C_{iB}(z) - C_{iD}(z) \quad \text{and} k(z) = k_0 A_m(z)$$

then

$$ln\left(\frac{C_{diff}(0)}{C_{diff}(z)}\right) = k \cdot z$$

Which gives

$$C_{diff}(z) = C_{diff}(0)e^{-kz} \tag{8}$$

This form gives us insight into how co-current differs from counter-current. From here, we can define our functions for C_{iB} and C_{iD}. In the case of co-current flow

$$C_{iD}(z) = \int_0^z C_{diff}(b)\ db \quad \Longrightarrow \quad \frac{C_{diff}(0)(1 - e^{-kz})}{k}$$

In the case of counter-current flow, there are a couple more complications. Namely, we do not know $C_{diff}(0)$ or $C_{diff}(L)$. Therefore, it takes a little bit of ingenuity to solve the problem.

We know that at z=0, $C_{diff} + C_{iD} = C_{iB}$ which for simplicity sake, we will set this equal to one. Then

$$1 = C_{diff}(0) + \frac{C_{diff}(0)(-1 + e^{-kz})}{k} \quad \Longrightarrow \quad C_{diff}(0) = \frac{e^k k}{1 - e^k + e^k}$$

Note, there was a negative introduced to this integral (due to the opposite direction of flow). Substituting the above result back into the other boundary condition, we get a useful expression for our initial concentration difference.

$$C_{diff}(0) = \frac{1 - e^k}{e^k(k-1) + 1}$$

Plugging this result back to our equation for C_{iD} (counter current) allows us to define relations between co-current and counter-current. We can define a ratio between co and counter current final dialysate concentrations as a function of k values. Neglecting the difference in signs, we find this ratio to be

$$\frac{\text{Co-Current Concentration}}{\text{Counter-Current Concentration}} = \frac{k + e^{-k} - 1}{k}$$

Plotting this over a range of k values indicates what we expect: counter current is a more effective method of removing solute.

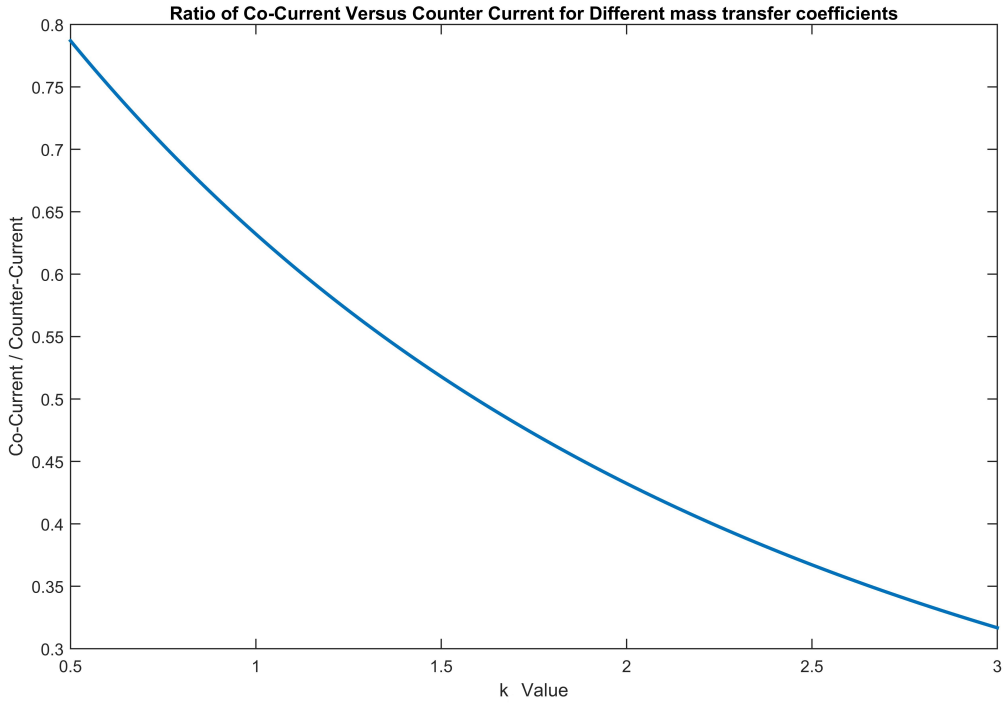

Figure 2: This figure shows how final dialysate concentrations vary based off of our "general" k value. This value roughly corresponds to overall resistance across the membrane. It is important to note, however, we did not control for the innate difference of meanings of the k-values (one is multiplied by the difference in flows, the other is multiplied by their sum). Therefore, these results are **more** pronounced than they really are.

A note on these k-values. It is important to recall that they do not mean the same thing in co-current and counter-current. For identical initial conditions, it is clear that the sum of reciprocal flow rates is larger than the difference between flow rates. Therefore, $k_{co-current} > k_{counter-current}$. Thus, the differences in effectiveness is not as severe as it appears from the graph alone.

In looking at the general concentration profiles in both the blood and the dialysate, we recognize that

$$C_{iB}(z) = C_{diff}(z) + C_{iD}(z)$$

Knowing this, along with all the above relations, we are able to plot the concentration profiles.

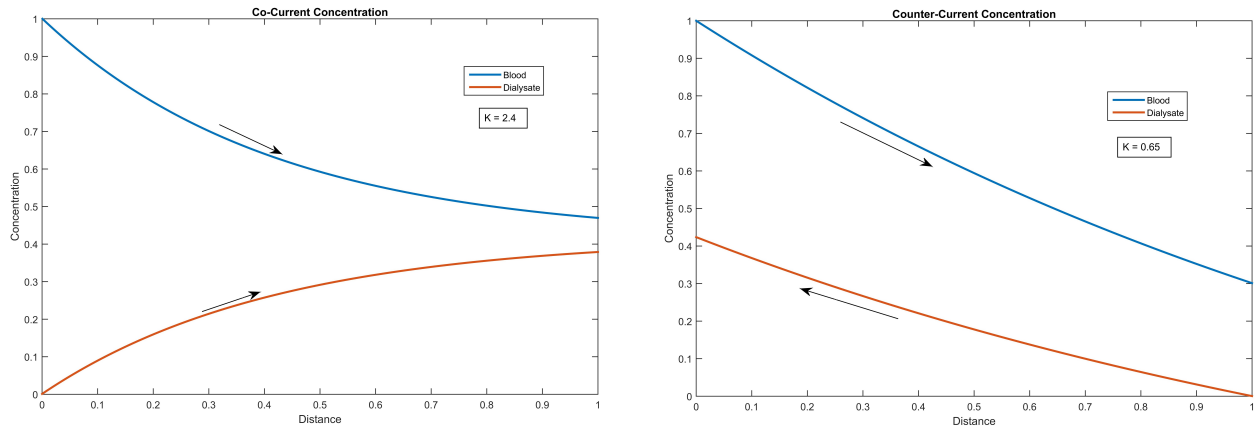

Figure 3: This figure shows how the concentration profiles differ for different dialysis set-ups. The values of unit and distance were made to be dimensionless, and the k-values were chosen to be representative of typical values.

3.2 Calculating the mass transfer coefficient

In order to calculate final concentration values in the dialysate, we need to calculate K_0 values for various solutes. The general equation governing our mass transfer coefficient (K_0) is

$$\frac{1}{k_0} = \frac{1}{k_D} + \frac{1}{P_m} + \frac{1}{k_B} \tag{9}$$

where k_B is the fluid phase conductivity of the blood, k_D is the fluid phase conductivity of the dialisate, and P_m is the permeability across the membrane. These can be thought of as resistances, and just as electrical resistances, they can be added in series. In principle, some values for these parameters can be derived from the Kedem-Katchalsky equation:

$$J_s = J_v(1 - \sigma_f)\overline{C}_s + PS\Delta C \tag{10}$$

where

$$J_v = L_p S(\Delta p - \sigma_s \Delta \pi)$$

and

$$\overline{C}_s = \frac{\Delta C}{ln(C_L/C_i)}$$

which can be approximated as

$$\overline{C}_s = \frac{C_L + C_i}{2}$$

In the above equations L_p is called the hydraulic conductivity, σ_s is called the osmotic reflection coefficient, σ_f is the filtration reflection coefficient (not necessarily equal), S is the surface area, and P is the micro-vascular permeability coefficient. Clearly, there is more that goes into mass transfer than just a difference in concentrations: the flow rate is important. The analysis in the book does not explicitly take this factor into account, and without estimating values, we are also not able to perform a more detailed analysis. Therefore, we will

use the approach taken in the book. As should be clear, the $J_v(1-\sigma_f)\overline{C}_s$ term is convective, and $PS\Delta C$ is diffusive.

Back to calculating the mass transfer coefficient, we will now look into finding values for each term in the K_0 equation separately. On the blood side, we begin by look at the Sherwood number:

$$Sh_B = \frac{\text{Convective mass transfer rate}}{\text{Diffusion rate}} = \frac{k_m H}{D_{eff}} = 1.62 Re^{1/3} Sc^{1/3} (d/L)^{1/3}$$

with

$$Sc = \mu/\rho D$$

In general, the Sherwood number approaches is asymptotic value in dialysis machines which is 4.30. All that is left is finding the D_{eff} value. The book gives a value for this which is:

$$D_{eff} = 0.53 D_{ij} + 5.292 \cdot 10^{-9} \cdot \dot{\gamma}$$

Where $\dot{\gamma}$ is the shear rate. Knowing this we now have an expression for k_m (which is what we are after).

As an aside, we can also look at out our Kedem-Katchalksy equation for an effective diffusion rate given by the coefficient in front of the concentration difference. To see if we obtain similar values in both approaches, we will take a small detour.

3.2.1 Sanity Check

Following the work of a paper published to measure membrane transport properties, we will simply look at varying values calculated by both methods. The published D_{eff} values in the below table were simply estimated from a chart.

D_{eff} Values	B-12	Urea
Published	0	2
From Book	3.7	10.91

Table 1: Table of Deff Values. Note, Each value should be multiplied by $10^{-6}(cm^2/sec)$

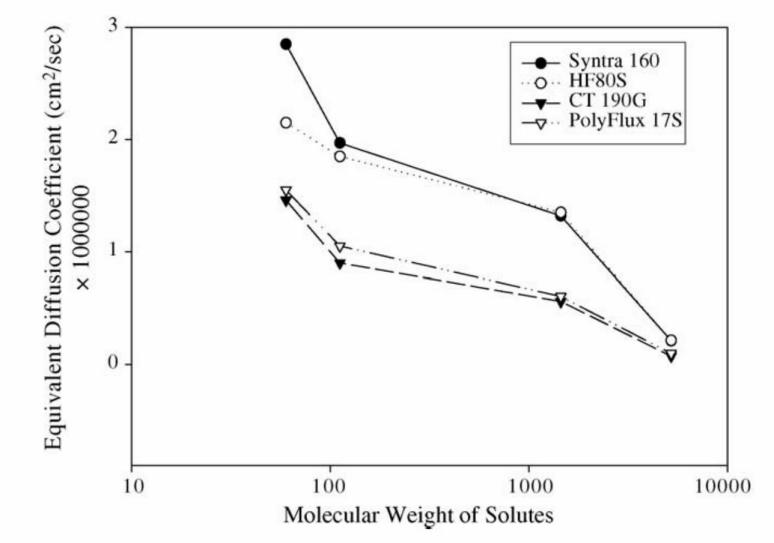

Figure 4: This chart was taken by a publication which looked at measuring hollow fiber membrane transport properties [6]

Just a quick glance shows notable differences. Although we are please to see at least a similar order of magnitude, we are not surprised the kedem-katchley equation gives an underestimate of the parameter as convective flux likely plays a role in the measured D_{eff}. However, determining the exact cause of these differences is out of the scope for this project, so we will simply continue on using the approach found in our textbook.

Back to mass transfer coefficients

Our general forms for the other conductivities, referring to our textbook, are:

$$\frac{1}{k_B} = \frac{\text{Thickness}}{4.30(0.53D_{ij} + 5.292 \cdot 10^{-9} \cdot \dot{\gamma})}$$

To find k_D, we consult the literature and find the following form:

$$\frac{1}{k_D} = \frac{1}{126.103(D_{ij})^{2/3}}$$

The membrane mass transfer coefficient is simply given as the permeability which is:

$$\frac{1}{k_m} = \frac{\text{Length of Membrane}}{D_{eff}}$$

Finally, we can calculate useful k_0 values. A plot of resistance values ($\frac{1}{k_0}$) over molecular weights is below. This information allows us to calculate the concentration profile along the z-axis in a co or counter current exchange. We are also able to look at the amount of solute removed.

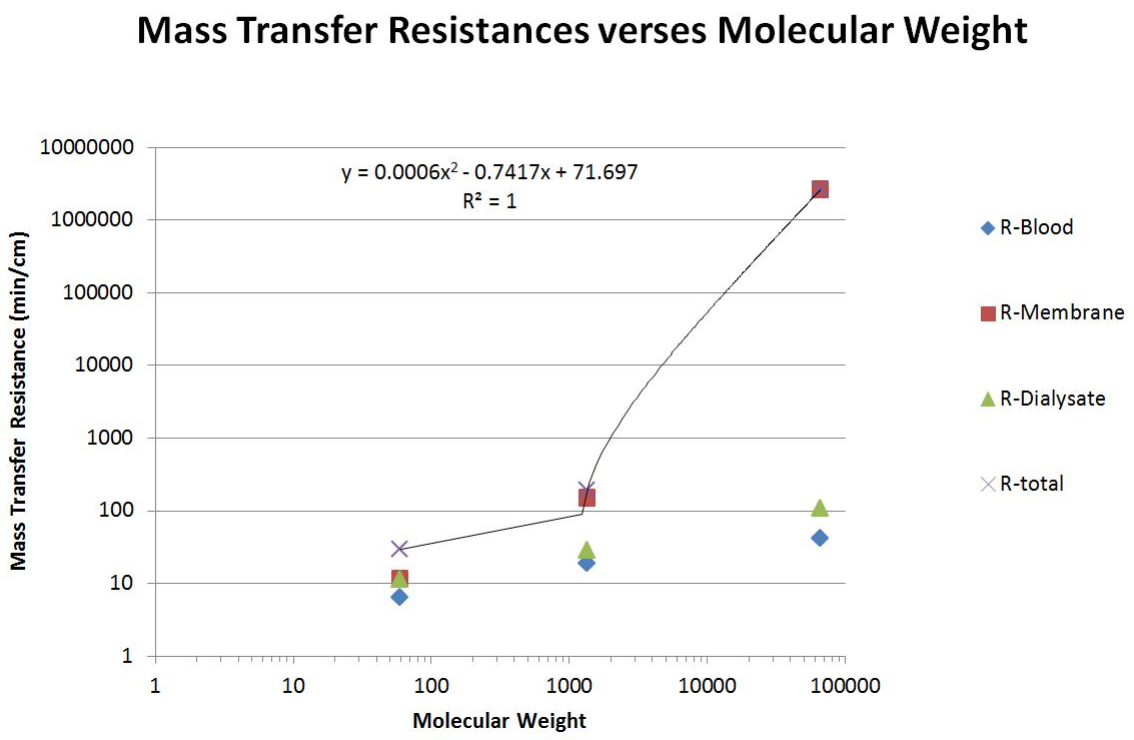

Figure 5: This plot shows different mass transfer resistances and was created from values found in our textbook. It is clear that for high MW substances the membrane is the main barrier. A fit was applied to show that resistance is roughly proportional to the square of MW.

3.3 Apparent Ideal Membrane

In just looking at how we calculate the mass transfer resistance values, it appears that we want membranes which have very partition coefficients (therefore permeabilities) for substances with low MW, and low partition coefficients for substances with high MW. A membrane with this property would help to create even larger differences in the concentration profiles between high MW substances and low MW substances. Intuitively, this should be a high density of very small pores.

4 Membrane Optimization Analysis

4.1 Governing Equation and Assumptions

To perform an analytic analysis of ideal pore size (and check our intuition), we return to governing equations. The average solute flux is given by the equation below, which incorporates both the diffusive mode of transport and the convective portion of transport.

$$\overline{N}_s = -HD_0\frac{dC}{dz} + WCv_m \tag{11}$$

This equation involves the solute-pore interactions, due to the introduction of the hydrodynamic resistance coefficients, H and W. H is dependent on K^-1, the enhanced friction coefficient and W is dependent on G, the lag coefficient. The equations for H and W in a cylindrical pore are shown below.

$$H = \frac{2}{R^2}\int_0^{R-\alpha} \frac{1}{K} r \, dr$$

$$W = \frac{4}{R^2}\int_0^{R-\alpha} G(1 - \frac{r^2}{R^2}) r \, dr$$

Solving these equations, by integrating over the radius of the pore available for the solute to be transported within, gives H and W in terms of $\emptyset$ and λ.

$$H(\lambda) = \emptyset(1 - 2.1044\lambda + 2.089\lambda^3 - 0.948\lambda^5)$$

$$W(\lambda) = \emptyset(2 - \emptyset)(1 - \frac{2}{3}\lambda^2 - 0.163\lambda^3)$$

Here, $\emptyset$ is also dependent on λ, which is the solute radius over the pore radius. Making H and W dependent on both the solute radius and pore radius, and allowing the pore geometry to be optimized in order to cause a maximal solute flux.

$$\emptyset = (1 - \lambda)^2$$

$$\lambda = \frac{\alpha}{R}$$

These equations for H and W are only valid for λ less than 0.4 though, as this assumption is necessary to solve approximately for K^-1 and G. Without this assumption the equations for H and W are much more complicated, making them impossible to derive here.

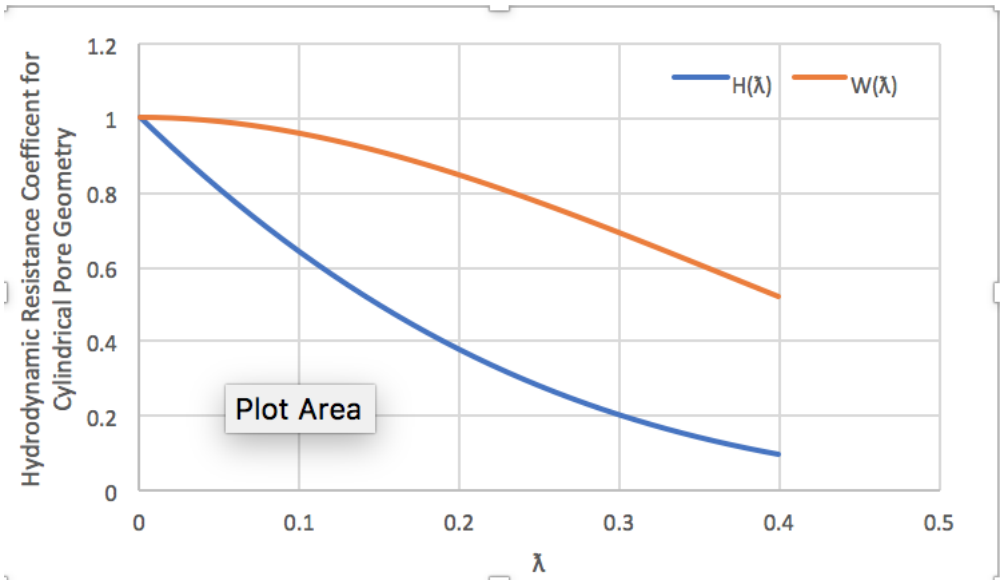

Figure 6: This figure shows how the hydrodynamic resistance coefficients are dependent on λ. A more linear relationship is shown for W, and a more exponential decay is shown for H.

Now the equations for H and W are also available for a slit pore, and are given in terms of $K^{-}1$ and G. But, $K^{-}1$ and G are different for this different geometry, causing a change in H and W from those seen in the cylindrical pore geometry.

$$H = \frac{1}{h}\int_0^{h-\alpha} \frac{1}{K} dy$$

$$W = \frac{3}{2h}\int_0^{h-\alpha} G(1 - \frac{y^2}{h^2}) dy$$

These equations can also be solved by integrating over the distance in the pore available for solute transport, and are then given as.

$$H(\lambda) = \emptyset(1 - 1.004\lambda + 0.418\lambda^3 + 0.21\lambda^4 - 0.169\lambda^5 + \lambda^6)$$

$$W(\lambda) = \frac{\emptyset}{2}(3 - \emptyset^2)(1 - \frac{1}{3}\lambda^2 + \lambda^3)$$

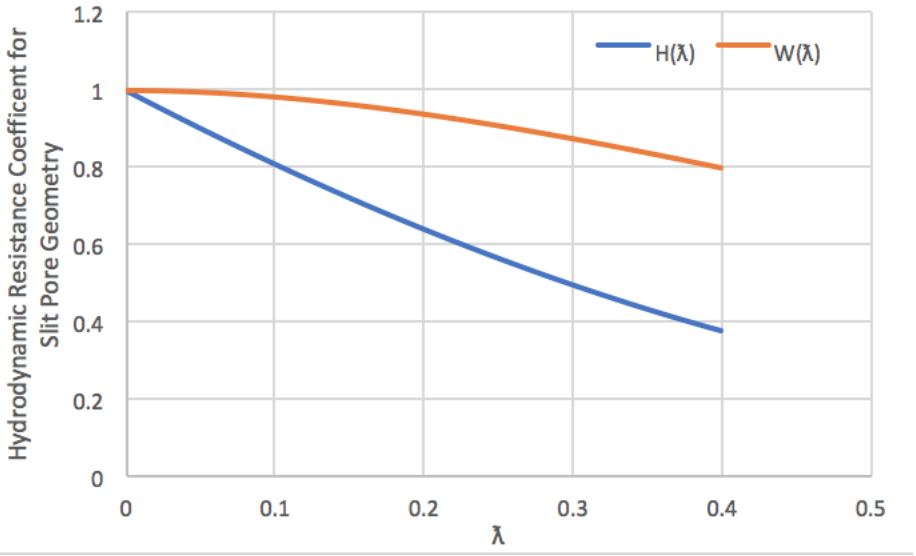

Figure 7: This figure gives the hydrodynamic resistance coefficients for a slit pore for λ between 0 and 0.4. The graph shows the same general trend as the one for the cylindrical pore, except at a lower slope. H decreases to around 0.1 for λ equals 0.4 in the cylindrical pore, while H equals around 0.38 for λ equals 0.4 in a slit pore. Thus an increase in the solute radius α causing an increase in λ has a lesser effect on the hydrodynamic resistance coefficients in the slit pore.

Assuming steady state conditions for the average flux, causes $\overline{N_s}$ to be constant and independent of the pore length or z. Then the $\overline{N_s}$ equation can be integrated to give.

$$\overline{N_s} = WC_0 v_m \frac{1 - (C_l \div C_0)e^{-Pe}}{1 - e^{-Pe}}$$

$$Pe = \frac{W v_m l}{H D_0}$$

This allows the effects of H and W to be quantified in the steady state situation, giving a measure of how they affect $\overline{N_s}$. This requires several conditions to be defined though, such as C_0 or the concentration of the solute in the blood, which is usually 1 $\frac{g}{L}$, for an individual with ESRD [7]. Next, the concentration of urea in the dialysate is given as around 0.5 $\frac{g}{L}$ at the end of the dialysis transfer [8], and the diffusion coefficient for urea in water, D_0 is 1.67*$10^9 \frac{m^2}{s}$ [3]. Finally the average velocity in a pore of length 4*10^{-6}m, width of about 10*10^{-9}m is 10*10^{-6} $\frac{m}{s}$ [1]. This then allows the average flux to be calculated for both the cylindrical pore and slit pore geometries, at different λ values.

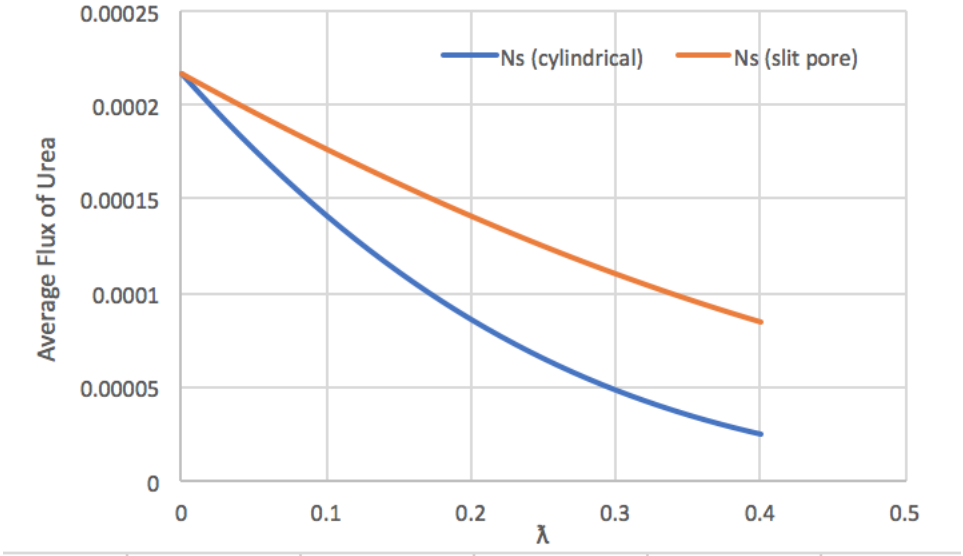

Figure 8: This figure shows two important trends, one concerning λ and the other the pore geometry itself. Firstly, as lambda increases from 0 to 0.4 the average flux for both pore geometries decreases. When lambda equals 0 though, the average flux is maximal for both pore geometries. This shows the optimal value of λ or the optimal pore design characteristics. The optimal pore will have an infinitely larger pore radius then solute radius, causing the limit of λ to approach 0. Furthermore, figure 3 shows that the slit pore geometry has a slower decrease in the average solute flux then the cylindrical pore geometry. This can be useful in controlling what solutes can pass through the pore, as if the pore size is constant, but the solute radii increases causing an increase in λ, then the increase in solute radii is proportional to a decrease in average flux. This is where a more severe decrease in average solute flux with an increase in λ could be useful, as it will allow more precise control of what solutes can pass the membrane and an overall larger pore size.

For some actual values, the solute radii of urea is 1.6 angstroms while the radius of hemoglobin is 32.5 angstroms [5]. Thus to achieve an average flux of 0.0001 in figure 8, a λ value of around 0.18 is needed for the cylindrical pores while a λ value of around 0.34 achieves the same average flux for slit pore geometry. This allows a pore radius of 180.5 angstroms for the cylindrical pore while a pore size of 95.6 angstroms is required for the slit

pore radius to reduce the average flux of hemoglobin to 0.0001. Then the transport of urea has a λ value of 0.00886 for the cylindrical pore and .0167 for the slit pore. This translates to an average flux of urea of at least 0.000208 for the cylindrical pore and 0.000208 for the slit pore. Thus, a larger pore size is possible in the cylindrical pore, to cause the same decrease in flux of the solute that transport is being prevented in. This larger pore then allows the flux of urea to be the same in the slit pore and cylindrical pore when hemoglobin controls the pore design. But as we desire to decrease the flux past the level possible using figure 3, and the solute radius of the molecule that needs to be prevented from transport is increased, then the flux in the cylindrical pore geometry may our surpass that of the slit pore geometry. But for now it can at least be said that the slit pore geometry is not better in considerations of dialysis membrane design, despite the fact that it offers a superior average flux for a all value of λ in the figure 8.

Next, as the Pe or the Peclet number is dependent on the length of the pore, this membrane characteristic can also be optimized. The same values of C_0, C_l, v_m, and D_0 that were used above were used again in the $\overline{N_s}$ equation, and W and H were set to equal 1 or λ was set to 0.

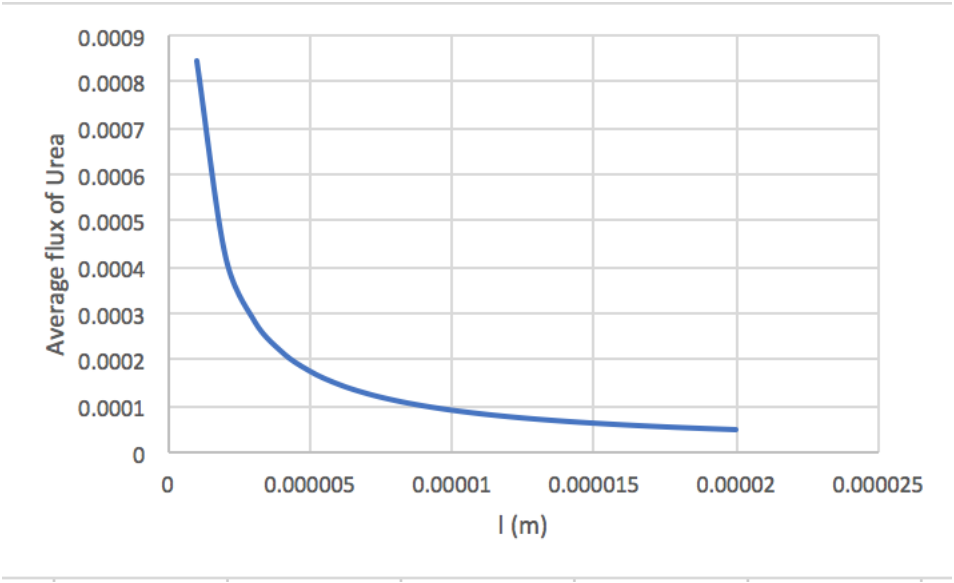

Figure 9: The above analysis of the average solute flux versus the pore length, shows that the smaller the pore length the larger the average flux. The figure also shows an exponential decay between the length of the pore and the average flux, which makes sense considering the peclet number is only found exponentiated by e. Thus, the optimal solute flux would be caused by an infinitesimally small pore length.

The solute flow rate J_s can then be related to the optimized average flux by the equation below.

$$J_s = \varepsilon S \overline{N_s} \tag{12}$$

This equation shows that the solute flow rate is proportional to the porosity, surface area

of the membrane and the average flux rate. Thus, increasing any of these values would result in an increase solute flow rate. Or in other words, the maximal flow rate is achieved by a membrane that has an infinite porosity, infinite surface area, and optimized average solute flux. This makes sense, as a large porosity means there are more pores for the solute to be transported through in a given area, while a large surface area means there is a larger area for the transport to occur, or a larger area of pores for the solute to flow through. Finally, an optimized flux means the largest possible rate of exchange of the solute from one side of the membrane to the other in each pore.

Overall, an ideal membrane for transport would have a λ value as close to zero as possible, a membrane thickness that is as small as possible, a porosity as large as possible, and a membrane surface area as large as possible. Furthermore, a cylindrical pore would allow better control of what solutes can and cannot pass through the membrane as it decreases more drastically with an increase in λ. This increase in λ is caused by an increase in solute radii, which occurs from the solutes that need to be transported to those that do not.

4.2 Flow Rate Analysis

In order to effectively analyze what "high" flow rates mean, we need to define some metrics.

4.3 Useful Metrics

To allow for more universial analysis, people often use common dimensionaless metrics. First, a clearance rate, K, is usually defined to characterize efficiency of a dialysis unit.

$$K = \frac{\dot{M}_i}{C_{iB}(\text{inlet}) - C_{iD}(\text{inlet})} \tag{13}$$

which can be written as

$$K = Q_B \frac{C_{iB}(0) - C_{iB}(L)}{C_{iB}(0) - C_{iD}(0)} = Q_D \frac{C_{iD}(L) - C_{iD}(0)}{C_{iB}(0) - C_{iD}(0)} \tag{14}$$

One can also define the extraction fraction, E, as K/Q_B. After some re-arranging, we can write E for counter-current exchange.

$$E = \frac{1 - exp(FR(1-Z))}{Z - exp(FR(1-Z))} \qquad \text{where} Z = \frac{Q_B}{Q_D} \quad \text{and} \quad FR = \frac{k_0 A_M}{Q_B} \tag{15}$$

Plotting this result shows us qualifies as a "high" flow ratio.

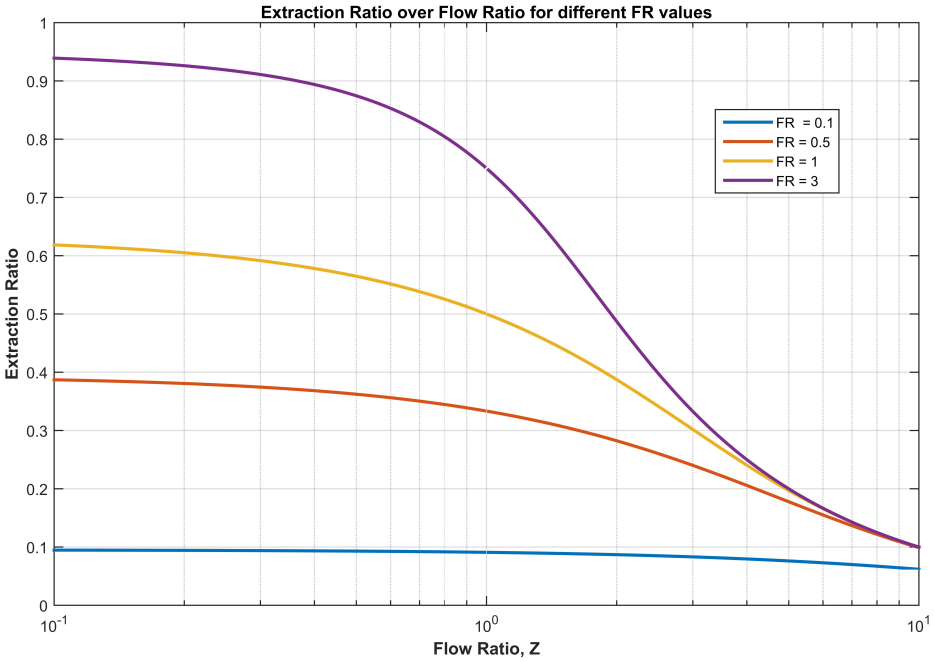

Figure 10: This plot shows how extraction fractions vary based off of flow ratios for various $k_0 * A_m/Q_B$ values. We see that for FR $= 3$, we are at

We can see that for an FR value of 3 (which is reasonable), a flow ratio of 1/2 already gives 93% the E value for a flow ratio of 1/10! This tells us, flow rates do reach maximum practical values.

5 Feasibility

Our end result is rather clear: the best dialysis uses high counter-current flow rates, and uses membranes that are very porous, and have small cylindrical pores. Our result is exactly what we would expect. One major short-coming of our approach, however is that only two flow set-ups were considered, with one ignored. We did not consider perpendicular flow to the dialysis fibers. Our reason for this was simple: one can not (easily) use our governing equation for concentration profiles with that set-up. However, intuitively, one would expect at least similar results to counter-current flow, which means we ignored a viable option.

We believe that our solution and analysis are not only valid, but widely employed. It was found time and time again through this project that simplifications can be a great time saver. Other considerations which were ignored in our analysis is any possible entrance effect into the pores, and turbulence layers on either side of the membrane. The fluid dynamics of boundary layers are very complicated, and we did not see utility in analyzing these effects.

Regarding our overall solution, we did not perform elaborate analysis on the ideal flow rate because of practical considerations: it is difficult to draw more than 200 ml/min of

blood from a patient (so Q_B is effectively fixed at its maximum value). Therefore, because dialysis is costly, and high flow-rates also risk membrane ruptures, it seems reasonable to cap the Q_D value at 500 ml/min. Therefore, we believe using these standard values without very in-depth analysis is reasonable. Overall, our analysis of dialysis gives insight as to how which parameters should change for increases in efficacy.

6 Acknowledgment

We would like to acknowledge all of the valuable analysis and values that our found in our textbook. We would have been lost in trying to understand the intricacies, nuances, and relations between the equations without it [2].

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