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MIDDLE MEN: THE VISIBLE MARKET-MAKERS

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This paper presents a framework in which middlemen emerge to intermediate between ex-ante homogeneous buyers and sellers in the presence of search frictions. Middlemen announce prices, and hold an inventory to provide more sure services. Middlemen can mitigate trade imbalances with price competition. Using this framework I illustrate how the frictionless limit can emerge and how middlemen can implement the short-side principle for the market price to be Walrasian. The recent progress in the literature on intermediation will also be discussed.
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1. Introduction

It has long been known that in a competitive market for a homogeneous good, the price adjusts itself to resolve trade imbalances where demand and supply meet. In most real-life markets, however, a gap exists between production and consumption. Consumers obtain a significant proportion of products at cost from intermediaries, who are prepared to buy and sell and set the price to match purchases to sales.[1] Although the role of middlemen in mitigating market frictions is familiar, the functioning of markets that middlemen provide has not been given the attention it deserves. The quest for a satisfactory description of "how markets operate" hinges on the role that middlemen play in the process of market exchanges.

The objective of this paper is to look inside the black box that justifies the very existence of middlemen. In their seminal work, Rubinstein and Wolinsky (1987) propose that intermediaries can be active under market frictions when they have a higher meeting rate than producers and consumers, while Spulber (1996) suggests that the middlemen's role of setting publicly observable prices is important. In the present paper, I offer a framework in which middlemen play both of these roles. In particular, I emphasize the importance of middlemen's inventory holdings under search frictions. Middlemen's inventories provide their customers with a high meeting rate. Here, middlemen, like supermarkets: (i) link producers and consumers; (ii) publicly announce prices; and (iii) hold inventories to smooth trade imbalances among producers and consumers.

In contrast to the existing models, where the middlemen's rates of meeting with consumers are exogenous, I present a simple model in which those rates are determined endogenously. Consider an economy that has two distinct markets for a homogeneous good. Producers are located in one market and each producer is able to serve only one consumer. The producers' market is characterized by random meetings with bilateral bargaining. In the other market middlemen are located and each middleman is able to serve $k_m \geq 1$ consumers. The middlemen's market is characterized by directed search with publicly announced prices. It features the middlemen's technology of spreading

[1] Burstein *et al.* (2003) document that distribution costs represent more than 40% of the retail price for the average consumption goods in the USA. Spulber (1999) estimates that market-based activities, fundamentally different from production activities, contribute at least 28% of US GDP.

price and capacity information efficiently. Consumers can choose which market to search under coordination frictions. In a market equilibrium, consumers are indifferent between searching in the producers' market and in the middlemen's market.

The model explicitly formulates the dependence of middlemen's ability to serve consumers on how many units they can have ready for sale. The producers have production technologies but are not able to sell multiple units per unit of time. In contrast, the middlemen do not have production technologies but are able to buy and sell multiple units. The middlemen can buy goods from different producers and transport them from the producers' market to the middlemen's market, and keep restocking the goods to operate the market all the time.[2]

Given a finite amount of the good, each middleman can stock more inventory when fewer middlemen are operating. As the proportion of middlemen becomes sufficiently small, each middleman stocks a sufficiently large amount of inventory. In the limit, the middlemen's market clears: at the individual level, each middleman guarantees consumers that trade will occur and there will be no stockouts, while at the aggregate level the middlemen's stocking in total generates the supply just equal to the total demand in the middlemen's market. If free entry of middlemen were allowed, this frictionless limit would emerge as the entry costs required to operate as a middleman become prohibitively high. Hence, middlemen can emerge to resolve trade imbalances that search frictions inject into the market.

Finally, using a version of the model where producers choose to operate only in the wholesale market, I show that the market price will become Walrasian in the same limit. When market frictions disappear in either side of the market, the short-side of the agents at the individual level coincides with the aggregate counterpart *and* the short-side of the agents take all the trade surplus. If there exists an aggregate excess supply, the stockout probability approaches zero at the individual level, and the proportion of trading surplus that any given consumer receives in each meeting approaches one. This case corresponds to the competitive limit. Conversely, if there exists an aggregate excess demand, the stockout probability approaches one at the individual level. Then, the proportion of trading surplus for consumers approaches zero. In this case, the probability of clearing out the goods approaches one at any given middleman. It might be interesting to imagine here that the market-clearing process manifests itself through merchant traders.

Before closing this introductory section, it is worth mentioning that my approach intends to offer a natural endogenous link between inventory holdings and a high meeting rate that middlemen provide to consumers, with middlemen's price posting and competition. This new approach provides a richer understanding of middlemen. Indeed, as Clower and Leijonhufvud (1975, p. 184) put, in the presence of frictions and in the absence of the Walrasian auctioneer, "natural forces of greed and competition might plausibly be invoked to provide a rationale for the gradual emergence of merchant traders and organized markets". They suggest an inventory-based competitive "supermarket" story, while my goal here is to explain why and how middlemen emerge in the first place.

[2] In the context of a modern specialized retailer like Wal-Mart, the ability of middlemen may also include the adaptation of new information technology, such as barcodes and computer tracking of inventories, that can complement its ability to stock goods frequently. In history, the acquisition of trading skills, which started from the second century CE among Jewish farmers and later provided them with the comparative advantage to become merchant tranders, included education and investment in religious literacy.

The rest of the paper is organized as follows. Section 2 presents the basic setup and establishes the steady-state equilibrium. Section 3 provides the main results on frictionless limits and of Walrasian price. Section 4 discusses the related literature and suggests some direction for future research. Section 5 concludes. All proofs are included in Appendix I.

2. Model

The model is a simplified version of Watanabe (2010).[3] Consider an economy inhabited by a continuum of homogeneous buyers, sellers and middlemen, represented by b, s and m, respectively. The population of buyers is normalized to one, and the population of sellers and middlemen are denoted by S and M, respectively. The population of agents is constant over time. All agents are risk-neutral and infinitely-lived. Time is discrete and each period is divided into two subperiods. During the first subperiod, retail markets are open for a storable, homogeneous and indivisible good to buyers. The retail markets are subject to search frictions as described in detail below. Each period, each buyer has unit demand while each seller can sell $k_s = 1$ unit, and each middleman can sell $k_m \geq 1$ units of the good. The selling capacity of each middleman k_m is determined by the demand–supply balancing in the wholesale market each period (see below). Sellers can produce any quantity of units they wish. The marginal production cost is constant and normalized to zero. The consumption value of the good is normalized to unity. If a buyer successfully purchases at a price p, then he or she obtains the per-period utility of one. Otherwise, the buyer receives zero utility. A seller or a middleman who sells z units at a price p obtains the revenue zp during the first subperiod. As in Watanabe (2010), I assume infinite discounting so that all agents have zero future payoffs at any time period. This assumption is to simplify the presentation, and is not crucial for our main results: see the discussion in Watanabe (2010, 2013).

There are two retail markets (see Figure 1). One market is operated by sellers and features random search and bilateral bargaining. The flow of contacts between sellers and buyers in this market is given by a matching technology $M = M(\tilde{B}, \tilde{S})$, where $\tilde{B}$ and $\tilde{S}$ denote the amount of buyers and sellers that actually participate in the sellers' market. The function M is continuous, concave and nonnegative, with $M(0, \tilde{S}) = M(\tilde{B}, 0) = 0$ for all $\tilde{B}, \tilde{S} \geq 0$. Without loss of generality, we assume that for $\tilde{B}, \tilde{S} > 0$ a buyer finds a seller with probability $\lambda^b = M(\tilde{B}, \tilde{S})/\tilde{B}$ and a seller finds a buyer with probability $\lambda^s = M(\tilde{B}, \tilde{S})/\tilde{S}$, satisfying $\tilde{B}\lambda^b = \tilde{S}\lambda^s$. $\lambda^b, \lambda^s \in (0, 1)$ is a constant. Matched partners follow an efficient bargaining process, which yields a linear sharing of the total surplus. To facilitate the presentation I assume that buyers take all the surplus so that the buyers' expected utility in the sellers' market is λ^b.

The other market is operated by middlemen and features directed search and price posting. It can be described as a simple two-stage game. In the first stage, middlemen simultaneously post a price which they are willing to sell at. Observing the prices, all buyers simultaneously decide which middleman to visit in the second stage. Each buyer can visit one middleman. If more buyers visit a middleman than its selling capacity, then the units are allocated randomly. Assuming buyers cannot coordinate their actions over which middleman to visit, a

[3] Watanabe (2010) sets up the producers' (i.e. the sellers' direct) market as a directed search market with price posing, whereas in the current formulation it is modelled as a random search market with bilateral bargaining. The latter approach is simpler and better fits with the current objective to clarify the middlemen's role of setting publicly observable prices. See also the discussion in Section 4.

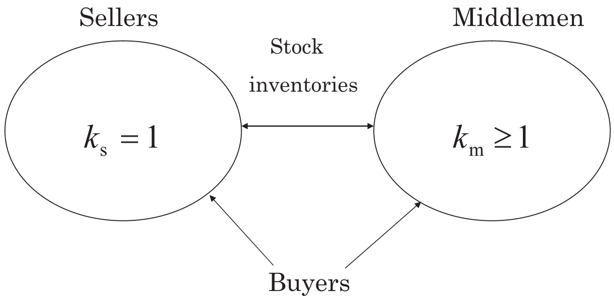

FIGURE 1. Middlemen economy

symmetric equilibrium is investigated where all buyers use the identical strategy for any configuration of the announced prices. I focus attention on a steady-state equilibrium where all middlemen post the identical price p_m and hold a capacity k_m every period.

In any given period, an equilibrium implies buyers are indifferent between visiting the sellers' market and the middlemen's market. Each individual seller or middleman is characterized by an expected queue of buyers. I construct a middlemen equilibrium where x_i is the expected queue of buyers at i. Each buyer visits some seller (and some middleman) with probability Sx_s (and Mx_m). They should satisfy the adding-up restriction,

$$Mx_m + Sx_s = 1, \tag{1}$$

requiring that the number of buyers visiting individual sellers and middlemen be summed up to the total population of buyers.

Once the retail market is closed, another market opens during the second subperiod. This is a wholesale market operated by sellers, and middlemen can restock their units for the future retail markets. There is no search friction in the wholesale market. In the steady-state equilibrium, each seller serves $\lambda^s = x_s\lambda^b$ buyers and each middleman serves $x_m\eta(x_m, k_m)$ buyers each period, where $\eta(x_m, k_m)$ is the probability of a buyer being served by a middleman. Hence, at the end of each period, there are $S(1 - x_s\lambda^b)$ units in total that sellers have failed to sell. The sellers do not have strict incentives to carry over the remaining unit. However, the steady-state equilibrium requires that each middleman restock $x_m\eta(x_m, k_m)$ units (to hold k_m units at the start of each period). The middlemen can restock the necessary units from the remaining sellers as long as it holds that the latter quantity does not exceed the former quantity. In particular, we are interested in the case where given the population measure of middlemen M the units of each middleman $k_m \geq 1$ should satisfy the condition:

$$Mx_m\eta(x_m, k_m) + Sx_s\lambda^b = S, \tag{2}$$

which states that the aggregate demand $Mx_m\eta(x_m, k_m)$ equals the aggregate supply $S(1 - x_s\lambda^b)$.[4]

[4] Under the infinite discounting of agents, the storage capability of sellers is irrelevant for middleman–seller trades and the restocking price must equal zero. That is, as the future payoffs are irrelevant middlemen do not buy the units at the end of any given period unless they can be obtained at zero price. A usual tie-breaking assumption guarantees that the sellers sell to a middleman.

Buyers' search. Assuming for the moment the existence of a symmetric equilibrium, I shall begin with the buyers' problem in the middlemen's market. The underlying matching environment is described as follows. First, given the coordinate frictions among a mass of agents, the number of buyers visiting a given middleman who has expected queue x is a random variable, denoted by n, which has the Poisson distribution, $\text{Prob}(n = k) = e^{-x}x^k/k!$. Let X denote a random variable representing the number of the *other* buyers appearing at a middleman. Then, the probability that a buyer is served when using a middleman denoted by η is given by

$$\begin{aligned}\eta &= \text{Prob}(X < k_m)\ \text{Prob}\ (\text{served}|X < k_m) + \text{Prob}(X \geq k_m)\ \text{Prob}(\text{served}|X \geq k_m) \\ &= \{\text{Prob}(X = 0) + \text{Prob}(X = 1) + \cdots\cdots + \text{Prob}(X = k_m - 1)\} \\ &\quad + \left\{\text{Prob}(X = k_m)\frac{k_m}{k_m + 1} + \text{Prob}(X = k_m + 1)\frac{k_m}{k_m + 2} + \cdots\right\}.\end{aligned}$$

In the above equation, the terms in the first bracket represent the service probability when there are $n_m = \{0, 1, 2, 3, \ldots, k_m - 1\}$ other buyers at the given middleman. In these cases, a given buyer will be served with probability one if he or she visits this middleman. The terms in the second bracket represent the service probability when there are $n_m = \{k_m, k_m + 1, k_m + 2, \ldots\}$ other buyers using the middleman. In these cases, stockouts occur and the given buyer is selected with probability $k_m/(n_m + 1)$. In sum, these two objects constitute the overall service probability for a given buyer. By applying the Poisson density to the buyers' arrival rate with an expected queue of buyers x_m, we obtain:[5]

$$\begin{aligned}\eta &= \sum_{j=0}^{k_m-1}\frac{x_m^j e^{-x_m}}{j!} + \sum_{j=k_m}^{\infty}\frac{x_m^j e^{-x_m}}{j!}\frac{k_m}{j+1} \\ &= \sum_{j=0}^{k_m-1}\frac{x_m^j e^{-x_m}}{j!} + k_m\sum_{j=0}^{\infty}\frac{x_m^j e^{-x_m}}{(j+1)!} - k_m\sum_{j=0}^{k_m-1}\frac{x_m^j e^{-x_m}}{(j+1)!} \\ &= \frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} + \frac{k_m}{x_m}(1 - e^{-x_m}) - \frac{k_m}{x_m}\left(\frac{\Gamma(k_m + 1, x_m)}{\Gamma(k_m + 1)} - e^{-x_m}\right) \\ &= \frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} + \frac{k_m}{x_m}\left(1 - \frac{\Gamma(k_m + 1, x_m)}{\Gamma(k_m + 1)}\right),\end{aligned} \tag{3}$$

where $\Gamma(k_m) = \int_0^\infty t^{k_m-1}e^{-t}dt = (k_m - 1)!$ and $\Gamma(k_m, x_m) = \int_{x_m}^\infty t^{k_m-1}e^{-t}dt$.

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[5] To reach the third equality, the following manipulations will be performed. The first term can be obtained by applying $\sum_{j=0}^{k-1} x^j e^{-x}/j! = \Gamma(k, x)/\Gamma(k)$ (the series definition of the cumulative gamma function). To obtain the closed-form expression in the second term, I set $h = j + 1$ and proceed as follows:

$$\sum_{j=0}^{\infty}\frac{x^j e^{-x}}{(j+1)!} = \sum_{h=1}^{\infty}\frac{x^{h-1}e^{-x}}{h!} = \frac{1}{x}\sum_{h=1}^{\infty}\frac{x^h e^{-x}}{h!} = \frac{1}{x}\left(\sum_{h=0}^{\infty}\frac{x^h e^{-x}}{h!} - e^{-x}\right) = \frac{1}{x}(1 - e^{-x}).$$

Finally, the third term can be obtained by combining the two manipulations described above.

One important property of the above probability is that it can be expressed in terms of the expected queue x_m and the available units $k_m \geq 1$; hence, we can write $\eta = \eta(x_m, k_m)$.[6] Furthermore,

- $\eta(x_m, k_m)$ is strictly decreasing in x_m and satisfies $\lim \eta(x_m, k_m) = 1$ as $x_m \to 0$ and $\lim \eta(x_m, k_m) = 0$ as $x_m \to \infty$, given $1 \leq k_m < \infty$.
- $\eta(x_m, k_m)$ is strictly increasing in k_m and satisfies $\eta(x_m, k_m) = (1 - e^{-x_m})/x_m$ when $k_m = 1$ and $\lim \eta(x_m, k_m) = 1$ as $k_m \to \infty$, given $0 < x_m < \infty$.

Clearly, the greater the expected number of buyers appearing at the given supplier, the smaller the probability that a given buyer is served if he or she visits the middleman. Hence, η is strictly decreasing in x_m given k_m. Similarly, a larger capacity implies a higher service rate; hence, η is strictly increasing in k_m given x_m (its mathematical proof is given in the proof of Theorem 1). The standard urn-ball matching function that is used in many search models corresponds to the case with $k_m = 1$ (see e.g. Butters, 1977).

Given η derived above, I now characterize the expected queue of buyers. In any equilibrium where V^b is a buyer's expected utility, should a middleman deviate by announcing a price p, the expected queue length denoted by x satisfies

$$V^b = \eta(x, k_m)(1 - p). \tag{4}$$

A buyer choosing a price p that has an expected queue x is served and obtains $1-p$ with probability $\eta(x,k_m)$. The situation is the same for all the other buyers. In our large market setting, any individual agents are negligible and so have no influence on the value of V^b; hence, it is treated as given in anyone's decision problem. As $\eta(\cdot)$ is strictly decreasing in x, Equation (4) determines $x = x(p, k_m|V^b) \in (0, \infty)$ as a strictly decreasing function of price p given k_m and V^b.

Middlemen's price. Given the buyers' directed search described above, the next step is to describe the optimal price of a middleman. In any equilibrium where V^b is buyers' expected utility, the optimal price of a middleman that has capacity k_m, denoted by $p_m(V^b)$, is given by

$$p_m(V^b) = \arg\max_p px(p, k_m|V^b)\eta(x(p, k_m|V^b), k_m).$$

The expected number of buyers is x for a given price p, and each buyer is served with probability $\eta(x,k_m)$. Hence, the expected number of sales is given by $x\eta(x, k_m)$. The expected profits of the middleman are price times the expected number of sales. Using Equation (4) to substitute out price p yields an objective function, denoted by $\pi(x)$, given by:

$$\pi(x) = x\eta(x, k_m) - xV^b.$$

Setting $\frac{\partial \pi(x)}{\partial x} = 0$ and rearranging it using Equation (4), we have

[6] This property can be generalized in some class of directed search models (see e.g. Holzner and Watanabe, 2016) but does not hold true in a more general class of models (see e.g. Moraga-Gonzales and Watanabe, 2017).

$$p_m(V^b) = -\frac{\partial \eta(x,k_m)/\partial x}{\eta(x,k_m)/x} = \frac{k_m\left(1-\frac{\Gamma(k_m+1,x)}{\Gamma(k_m+1)}\right)'}{x\eta(x,k_m)}, \tag{5}$$

where $x = x(p_m(V^b), k_m|V^b)$ satisfies Equation (4). Note this objective function has a unique maximum because $\pi(x)$ is strictly concave in $x \in (0,\infty)$:

$$\frac{\partial^2 \pi(x)}{\partial x^2} = 2\frac{\partial \eta(x,k_m)}{\partial x} + x\frac{\partial^2 \eta(x,k_m)}{\partial x^2} = -\frac{x^{k_m-1}e^{-x}}{\Gamma(k_m)} < 0.$$

Middlemen equilibrium. The analysis above has established the equilibrium price of middlemen $p_m(V^b)$ given V^b. In the middlemen equilibrium, each period buyers must be indifferent between visiting the sellers' market and the middlemen's market:

$$V^b = \lambda^b = \eta(x_m, k_m)(1-p_m). \tag{6}$$

This indifference condition determines the equilibrium V^b, where $x_m = x(p_m, k_m|V^b)$ is the equilibrium queue in the middlemen's market and x_s in the sellers' market, satisfying Equation (1). In the retail market, buyers are successfully served by a middleman with probability $\eta(x_m, k_m)$ at the equilibrium price $p_m = p_m(V^b)$ given by Equation (5), and by a seller with probability λ^b and full surplus. A middleman obtains equilibrium expected profits given by

$$\Pi^m = x_m \eta(x_m, k_m) p_m. \tag{7}$$

At the end of each period, each middleman restocks the necessary units from sellers at zero price in the wholesale market according to Equation (2), determining the selling capacity of each middleman $k_m \geq 1$. Sellers obtain zero profits both in the retail market and the wholesale market. Identifying this equilibrium is now reduced to a standard fixed point problem. [7]

Theorem 1: *For any* M $\in$ *(0,*S*),* $\lambda \in$ *(0,1), a middlemen equilibrium exists and is unique given* $S \in [\underline{S}, \bar{S})$ *with* $\lambda < \underline{S} < \bar{S} < 1 + \lambda^b$*, satisfying* $x_m, x_s \in (0,\infty)$*,* $p_m \in (0,1)$ *and* $k_m \geq 1$.

The configuration of middlemen, in terms of their scale and quantity, matters in the middlemen equilibrium. For a smaller population of middlemen M, the retail markets will become tighter and each operating middleman attracts more buyers, sells more units and restocks more inventory. Hence, the middlemen equilibrium features a negative relationship between the scale k_m and the quantity M of middlemen: it encompasses an economy with many middlemen each having a small scale and an economy with few middlemen each having a large scale.

[7] In the theorem, the parameter restriction, $S \in (\underline{S}, \bar{S})$, is needed to guarantee that the steady-state equilibrium is sustainable. This is a modified version of theorem 2 in Watanabe (2010), but for the purpose of the present paper the steady-state condition is stated here in terms of S rather than M.

Corollary 1: *In the middlemen equilibrium established in Theorem 1, a decrease in the population of middlemen M leads to an increase in the units of each operating middleman* k_m *and the queue of buyers to each middleman* x^m.

3. Frictionless limit and Walrasian price

This section presents the main results on the frictionless limit and the Walrasian price. The following proposition shows the necessary and sufficient condition for the frictionless limit to emerge in our middlemen economy.

Proposition 1: *In the limit as* $M \to 0$*, it holds in the middlemen equilibrium that* $k_m \to \infty$, $x_m = k_m$ *and* $\eta(x_m, k_m) \to 1$.

As shown above, each middleman can stock more inventory when fewer middlemen are operating. As the proportion of middlemen becomes sufficiently small, $M \to 0$, each middleman stocks a sufficiently large amount of inventory and is of sufficiently large scale, $k_m \to \infty$. In this situation, all buyers anticipate that stockouts will never occur at any given middleman Their expectations are self-fulfilling in equilibrium only when the operating middlemen restock sufficiently large units. This is, indeed, the case here, and the middlemen's market clears: at the individual level, each middleman guarantees consumers that trade will occur and there will be no stockouts, $\eta \to 1$, while at the aggregate level the middlemen's stocking in total generates the supply Mk_m just equal to the total demand Mx_m in the middlemen's market.

In the above limiting equilibrium, the equilibrium price of middlemen is still positive,[8]

$$p_m = 1 - \frac{\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)}}{\eta(x_m, k_m)} \to 1 - \lambda^b > 0,$$

as $M \to 0$, because there exist frictions in the sellers' market, which prevent buyers from expecting the full surplus, and buyers must be indifferent between the sellers' market and the middlemen's market in our equilibrium. This implies that if free entry of middlemen were allowed, this frictionless limit would emerge as the entry costs required to operate as a middleman becomes prohibitively high.

Corollary 2: *Suppose in the middlemen equilibrium that free entry determines the population of operating middlemen M with entry costs* $c > 0$*. Then, the frictionless limit established in Proposition 1 occurs in the limit as* $c \to \infty$.

An interesting question would be to ask if a competitive limit or in general Walrasian pricing is possible. The next subsection will address this issue.

Walrasian price. The objective of this subsection is to illustrate the role of middlemen in the price formation. Suppose now that each period sellers choose one market to

[8] To obtain the price equation, we apply the equilibrium condition $V^b = \lambda^b$ that is reduced to $\Gamma(k_m, x_m)/\Gamma(k_m) = \lambda^b$.

operate, either in the retail market or in the wholesale market. Then, because sellers would make zero profits, they do not have a strict incentive to operate in the retail market. A usual tie-breaking assumption guarantees that each seller sells only to a middleman; thus, buyers have no option of visiting the sellers' market.[9] The condition describing search frictions (1) is modified to:

$$Mx_m = 1. \tag{8}$$

A measure $S > 0$ of goods are produced and supplied to a measure $M > 0$ of middlemen. Thus, condition (2) is now modified to:

$$Mk_m = S. \tag{9}$$

The units of each middleman is determined by Equation (9). The rest of the setups are exactly the same as before. A symmetric steady-state equilibrium exists and is unique, satisfying: $x_m = 1/M$ by Equation (8); $0 < p_m < 1$ by (5); and $k_m = S/M$ by Equation (9).

In this simplified situation, aggregate supply (aggregate demand) is vertical line S (one). Hence, buyers (middlemen) are on the short-side of the retail market if $S > 1$ (if $S < 1$). The Walrasian price in this economy, denoted as p_w, defines:

$$p_w = \begin{cases} 0 & \text{if } S > 1 \\ \tilde{p}_w & \text{if } S = 1 \\ 1 & \text{if } S < 1, \end{cases}$$

where $\tilde{p}_w$ is some point in [0,1]. For any $0 < M < S$, the retail market does not clear on either side of the market (i.e. $0 < \eta < 1$ and $0 < x_m\eta/k_m < 1$) in the equilibrium, and, hence, $p_m \neq p_w$. However, the following proposition shows that the equilibrium price that middlemen set in this simplified economy can be Walrasian in the limit as the proportion of middlemen becomes sufficiently small.

Proposition 2: *Suppose that each period sellers choose one market to operate, either in the retail market or in the wholesale market. Then in the limit as* M $\rightarrow$ *0, the equilibrium price of middlemen in this economy approaches the Walrasian price,* $p_m \rightarrow p_w$.

Within this simplified economy, expected trade imbalances at the individual level $x_m \gtreqless k_m$ are determined solely by the aggregate condition $S \gtreqless 1$, as evident from comparing Equations (8) and (9). Given that sellers do not operate in the retail market, Proposition 2 claims that middlemen can carry out the short-side principle for the equilibrium price determination in the limit as $M \rightarrow 0$. If there exists an aggregate excess supply $S > 1$, the probability that stockouts occur approaches zero at the individual level $\eta^m \rightarrow 1$, and the proportion of trading surplus that any given buyer receives in each meeting approaches one, $p_m \rightarrow 0$ in the limit as $M \rightarrow 0$. This case corresponds to the competitive limit. Conversely, if there exists an aggregate excess demand $S < 1$, the stockout probability approaches one at the individual level, and the proportion of trading surplus that any given buyer receives approaches zero, $p_m \rightarrow 1$ in the limit as $M \rightarrow 0$.

[9] Notice that now buyers only have an option to visit the middlemen's market, and so the parameter λ^b will never appear in this modified setup (i.e. the sellers' market is not available for buyers anymore).

In this case, the probability of clearing out the goods approaches one at any given middleman (i.e. $x_m \eta / k_m \rightarrow 1$ as $M \rightarrow 0$).

These results suggest that when market frictions disappear on either side of the market, the short-side of the agents at the individual level coincides with the aggregate counterpart *and* the short-side of the agents take all the trade surplus.

4. Discussion

The present framework contributes to the literature on intermediation by integrating the key functions of middlemen into a unified framework, each of which has been separately studied elsewhere. Middlemen emerge with their ability to deal with many agents simultaneously.

There are two closely related papers. Watanabe (2010) presents a similar model but considers directed search with price posting in both of the retail markets. A middlemen equilibrium is established where buyers are indifferent between searching in the sellers' market, where both the price and the likelihood of obtaining the good are low, and in the middlemen's market, where both the price and the likelihood are high. Watanabe then studies the turnover behaviours of sellers to become middlemen under a simplifying assumption of infinite discounting, like in the current paper. Multiple turnover equilibria exist: one is stable and has many middlemen, each with few units and a high price, and the other is unstable and has few middlemen, each with many units and a low price. Watanabe (2013) relaxes the infinite-discounting assumption, and shows that a middlemen equilibrium can be sustained with forward-looking agents. The middlemen's high selling-capacity enables them to serve many buyers at a time, thus to lower the likelihood of stock-out, which generates a retail premium of inventories. Assuming away the turnover issue, it investigates the effect of middlemen's inventory capacities on the market outcomes, in particular, how the behaviour of intermediation premia is related to economic welfare and market efficiency.

To renovate the theory, the following two aspects seem to be important. First, while delivering an intuitive and valid insight, the meeting probability derived here is not very tractable when it is used for applied research works. Hence, one important issue is to increase the tractability of the model without loosing the essence of the original theory. In addition, the current model is designed to address some theoretical issues and is kept intentionally simple. This is fine for its own purpose, but clearly it is too abstract to study tangible economic problems related to real-life phenomena. Hence, the other important task is to increase the applicability of the theory.

Demonstrating recent progress, Gautier *et al.* (2016) study the choice of intermediation modes. An intermediary can operate as a middleman (a merchant or a dealer) to pursue buying and selling, and/or as a market-maker (a platform or a broker) to be engaged in fee business by offering a marketplace. Just like in the current model, a middleman has a relatively high selling capacity in order to serve many buyers at a time, and to offer a relatively low likelihood of stock-out. Hence, if intermediation fees were not available, then this new model would be a simplified version of the original model, adding an outside market.

Notice that in the current setup the middleman's inventory is modelled as an indivisible unit (i.e. a positive integer), so that the middlemen face a non-degenerate distribution of their selling units as other sellers do. In contrast, Gautier *et al.* model the inventory as a mass, assuming more flexible inventory technologies, so that the middleman faces a degenerate distribution of sales. This simplification allow us to characterize the middleman's profit-maximizing choice of inventory holdings, which seems like major progress given that the inventory level of middlemen is determined by aggregate demand-supply balancing in the current paper and in Watanabe (2010); it is treated as an exogenous parameter in Watanabe (2013). With this simplification, they show that while the middleman mode is more efficient for resource allocation, using it exclusively is not necessarily good for rent-seeking purposes, especially when agents have improved search technologies (e.g. online versus off-line shopping) or when agents are exposed to markets for a relatively long time (e.g. durable goods and assets). This result may help explain the emergence of market-making middlemen, such as Amazon and Expedia or specialists in financial and real estate markets, who do both the middlemen business and the market-making business simultaneously, in accordance with the large volume of Internet-based transactions.

Applications to the labour market are relatively scarce in the intermediation literature. Holzner and Watanabe (2016) study a job-brokering service offered by Public Employment Agencies (PEA). They find empirically that mitigating coordination frictions is the primary role of the PEA. Indeed, the PEA's ability to facilitate the matching between the registered job seekers and vacancies is similar to the middlemen's high capability to deal with many agents simultaneously. Given this observation, they use the same approach and build a tractable model with a simplified matching function to explore the implication of having a marketplace (i.e. the PEA) with less severe coordination frictions. They show that the PEA creates a matching opportunity for difficult-to-fill vacancies, and for discouraged job-seekers who have a lower prospect of being successful in finding a suitable job.

In contrast to the literature, however, note that the PEA does not act as a private agent, who charges a premium for their service. This raises the question of why not all agents use the middlemen. Their answer is that while the PEA is able to coordinate job applications, its free service attracts all the job seekers so that its proposal to firms is less likely to be a selective enough candidate. Instead, private markets are costly to search and so only those who have a high enough prospect will participate. Hence, firms have to trade off the lower wage and the lower degree of coordination frictions with the disadvantage of facing a less suited pool of applicants in the PEA. Helzner and Watanabe point out that even if it has no ability to screen workers, the very existence of the PEA as a middleman can facilitate the selection mechanism of the overall market.

5. Conclusion

In this paper, I have presented the idea that middlemen can arise to coordinate transactions. They announce prices publicly and take advantage of their high trading capacity. The approach in the present study is to consider together the key functions of middlemen that are typically studied separately in the literature. I have also discussed the recent progress that has been made to enhance the tractability and the applicability of the original model.

From a broader perspective, the paper is related to the classic issue of how the market economy coordinates transactions. It is, indeed, surprising how well the market economy can coordinate enormously interdependent and complex decisions and activities of individuals, each with a very specialized role in production and consumption. Debating on the economic organization of Greek city states, Plato asks: "If the farmer or any other craftsman brings what he has produced to the market, and he doesn't arrive at the same time as those who need what he has to exchange, will he sit idle, his craft unattended?" (Book II of *The Republic*, at 371c). One answer I gave in this paper is the emergence of middlemen. For further development of this idea, we will have to await further research.

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Appendix I

Proof of Theorem 1. First, applying the equilibrium price p^m in Equation (5) with $x = x^m$ to the indifference condition Equation (6), we obtain:

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} = \lambda^b. \tag{A1}$$

The left-hand side of this equation is strictly increasing in k_m, which can be seen from $\Gamma(k_m, x_m)/\Gamma(k_m) = \sum_{j=0}^{k_m-1} x_m^j e^{-x_m}/j!$ (i.e. the series definition of the cumulative gamma function) and is strictly decreasing in x_m, which can be seen from $\Gamma(k_m, x_m) = \int_{x_m}^{\infty} t^{k_m-1} e^{-t} dt$ (i.e. the integral definition of the incomplete gamma function). Hence, this equation determines $x_m = x_m(k_m)$, which is monotone increasing in k_m, satisfying $x_m(1) = \ln(1/\lambda^b)$. Given the $x_m(k_m)$ defined above, applying (1) to (2), we obtain the fixed point condition,

$$\Phi(k_m, M) \equiv M x_m(k_m)(\eta(x_m(k_m), k_m) - \lambda^b) + \lambda^b = S, \tag{A2}$$

which should identify the aggregate demand–supply balancing value of $k_m \geq 1$. Observe that: $\Phi(1, M) = M(1 - \lambda^b - \ln(1/\lambda^b)\lambda^b) + \lambda^b \equiv \underline{S}$; $\Phi(k_m, M) < 1 + \lambda^b$ for all $k_m \geq 1$;

$$\frac{d\Phi(k_m, \cdot)}{dk_m} = M \frac{dx_m}{dk_m} \left(\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} - \lambda^b \right) + M x_m \frac{\partial \eta(x_m, k_m)}{\partial k_m} = M x_m \frac{\partial \eta(x_m, k_m)}{\partial k_m} > 0,$$

where in the second equality we apply (10). Hence, for $S \in [\underline{S}, \bar{S})$, with some $\bar{S} \in (\underline{S}, 1 + \lambda^b)$, there exists a unique $k_m \geq 1$ that satisfies the fixed point condition. Given this value, $x_m = x_m(k_m) > 0$ is uniquely pinned down by (A1), $x^s > 0$ by (1) and $p^m \in (0, 1)$ by (5) with $x = x^m$. This solution then satisfies the equilibrium requirements (1), (2), (5), (6) and so describes a middlemen equilibrium. ■

Finally, we prove the property of $\eta(\cdot)$ with respect to k_m described in the main text. To show that $\eta(\cdot)$ is increasing in k_m, observe that for any given value of $x^m \in (0, \infty)$,

$$\begin{aligned}\eta(x^m, k_m+1) - \eta(x^m, k_m) &= \sum_{j=0}^{k_m} \frac{x_m^j e^{-x_m}}{j!} + (k_m+1)\left[\sum_{j=0}^{\infty} \frac{x_m^j e^{-x_m}}{(j+1)!} - \sum_{j=0}^{k_m} \frac{x_m^j e^{-x_m}}{(j+1)!}\right] \\ &\quad - \sum_{j=0}^{k_m-1} \frac{x_m^j e^{-x_m}}{j!} + k_m\left[\sum_{j=0}^{\infty} \frac{x_m^j e^{-x_m}}{(j+1)!} - \sum_{j=0}^{k_m-1} \frac{x_m^j e^{-x_m}}{(j+1)!}\right] \\ &= \frac{x_m^{k_m} e^{-x_m}}{k_m!} - k_m \frac{x_m^{k_m} e^{-x_m}}{(k_m+1)!} + \left[\sum_{j=0}^{\infty} \frac{x_m^j e^{-x_m}}{(j+1)!} - \sum_{j=0}^{k_m} \frac{x_m^j e^{-x_m}}{(j+1)!}\right] \\ &= \frac{1}{x_m}\left(1 - \frac{\Gamma(k_m+2, x_m)}{\Gamma(k_m+2)}\right) > 0.\end{aligned}$$

The limiting value, $\eta(\cdot) \to 1$ as $k_m \to \infty$, is shown in the proof of Proposition 1. ■

Proof of Corollary 1. The fixed point condition Equation (A2) defines a continuous and differentiable function $k_m = k_m(M)$ for all the admissible values of M. Observe that:

$$\frac{\partial \Phi(\cdot, M)}{\partial M} = x_m(k_m)\left(\eta(x_m(k_m), k_m) - \lambda^b\right) = Mk_m\left(1 - \frac{\Gamma(k_m+1, x_m)}{\Gamma(k_m+1)}\right) > 0,$$

where in the second equality we apply (3) and (A1). Because $d\Phi(k_m, \cdot)/dk_m > 0$ as shown above, we have $\partial k_m/\partial M < 0$. ■

Proof of Proposition 1. First of all, Equation (A2) implies that we must have:

$$k_m \to \infty \text{ as } M \to 0.$$

Because the following property holds in the limit (see Temme, 1996):

$$\frac{\Gamma(k, x)}{\Gamma(k)} \to D \text{ as } k \to \infty \quad \text{[Property T]},$$

where $0 \leq D \leq 1$, and $D = 1$ iff $x < k$ and $D = 0$ iff $x > k$, we should have $x_m = k_m$ as $M \to 0$. Furthermore, because

$$\frac{\Gamma(k, x)}{\Gamma(k)} \to \frac{\Gamma(k+1, x)}{\Gamma(k+1)}$$

as $k \to \infty$, we apply $x_m = k_m$ to obtain

$$\eta(x_m, k_m) = \frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} + \frac{k_m}{x_m}\left(1 - \frac{\Gamma(k_m+1, x_m)}{\Gamma(k_m+1)}\right) \to 1$$

as $k_m \to \infty$. Hence, $\eta \to 1$ as $M \to 0$. The limiting value of $Mx_m \in (0, 1)$ is determined by Equation (A2), $Mx_m \to (S - \lambda^b)/(1 - \lambda^b)$ as $M \to 0$. ■

Proof of Corollary 2. In the free entry condition,

$$\Pi^m = c,$$

because $\eta \to 1, k_m = x_m \to \infty$ as $M \to 0$, we must have $\Pi^m \to \infty$ in (7) for $p_m > 0$ as $M \to 0$. ■

Proof of Proposition 2. The equilibrium conditions (8) and (9) imply that: $x_m \to \infty$ and $k_m \to \infty$ as $M \to 0$, and $x_m \gtreqless k_m$ if $S \gtreqless 1$. Notice that unlike in the previous setup, now buyers only have an option to visit the middlemen's market, and so the parameter λ^b will never appear in this modified setup (i.e. the sellers' market is not available for buyers anymore). Keeping these in mind, we show that there are three cases. ■

Case 1 (S > 1): $p_m \to 0$, $\eta \to 1$, and $0 < x_m\eta/k_m < 1$ as $M \to 0$.

Proof of Case 1. Given that $k_m \to \infty$ as $M \to 0$, and $x_m < k_m$ with $S > 1$, [Property T] implies:

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} \to 1 \text{ and } 1 - \frac{\Gamma(k_m+1, x_m)}{\Gamma(k_m+1)} \to 0 \text{ as } M \to 0.$$

This implies $\eta \to 1$ and $p_m \to 0$ as $M \to 0$ and, hence, $x_m\eta/k_m < 1$ as $M \to 0$. ■

Case 2 (S < 1): $p_m \to 1$, $0 < \eta < 1$, and $x_m\eta/k_m \to 1$ as $M \to 0$.

Proof of Case 2. Given that $k_m \to \infty$ as $M \to 0$ and $x_m > k_m$ with $S < 1$, [Property T] implies:

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} \to 0 \quad \text{and} \quad 1 - \frac{\Gamma(k_m+1, x_m)}{\Gamma(k_m+1)} \to 1 \quad \text{as} \quad M \to 0,$$

which further imply that: $\eta \to k_m/x_m = S < 1$ as $M \to 0$ and, hence, $x_m\eta/k_m \to 1$ as $M \to 0$. Applying these limiting values into Equation (5) yields: $p_m \to 1$ as $M \to 0$. ■

Case 3 (S = 1): $0 < p_m < 1$, $\eta \rightarrow 1$, and $x_m\eta/k_m \rightarrow 1$ as $M \rightarrow 0$.

Proof of Case 3. Given that $k_m \rightarrow \infty$ as $M \rightarrow 0$ and $x_m = k_m$ with $S = 1$, [Property T] implies:

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} \rightarrow D \quad \text{and} \quad 1 - \frac{\Gamma(k_m + 1, x_m)}{\Gamma(k_m + 1)} \rightarrow 1 - D \quad \text{as} \quad M \rightarrow 0,$$

where $0 < D < 1$. Given $x_m = k_m$, this further implies that: $\eta \rightarrow 1$ as $M \rightarrow 0$ and, hence, $x_m\eta/k_m \rightarrow 1$ as $M \rightarrow 0$. Applying these limiting values into Equation (5) yields: $0 < p_m < 1$ as $M \rightarrow 0$. ■

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REFERENCES

Burstein, A. T., J. C. Neves and S. Rebelo (2003) "Distribution Costs and Real Exchange Dynamics during Exchange-rate-based-stabilizations", *Journal of Monetary Economics*, Vol. 50, No. 6, pp. 1189–1214.
Butters, G. (1977) "Equilibrium Distribution of Sales and Advertising Prices", *Review of Economic Studies*, Vol. 44, pp. 465–491.
Clower, R. and A. Leijonhufvud (1975) "The Coordination of Economic Activities: A Keynesian Perspective", *American Economic Review*, Vol. 65, No. 2, pp. 182–188.
Gautier, P. A., B. Hu and M. Watanabe (2016) "Marketmaking Middlemen", Working Paper, Tinbergen Institute, Amsterdam.
Holzner, C. and M. Watanabe (2016) "Understanding the Role of the Public Employment Agency", Working Paper, Tinbergen Institute, Amsterdam.
Moraga-Gonzales, J. L. and M. Watanabe (2017) "Price Posting in Search Markets for Inspection Goods", mimeo., Tinbergen Institute.
Rubinstein, A. and A. Wolinsky (1987) "Middlemen", *Quarterly Journal of Economics*, Vol. 102, No. 3, pp. 581–593.
Spulber, D. F. (1996) "Market Making by Price Setting Firms", *Review of Economic Studies*, Vol. 63, No. 4, pp. 559–80.
——— (1999) *Market Microstructure*, Cambridge: Cambridge University Press.
Temme, N. M. (1996) *Special Functions: An Introduction to the Classical Functions of Mathematical Physics*, New York: Wiley.
Watanabe, M. (2010) "A Model of Merchants", *Journal of Economic Theory*, Vol. 145, No. 5, pp. 1865–1889.
——— (2013) "Middlemen: A Directed Search Equilibrium Approach", Working Paper, Tinbergen Institute, Amsterdam.