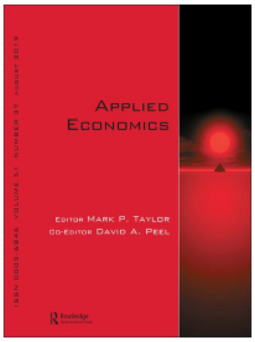

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Are betting markets efficient? Evidence from European Football Championships

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Are betting markets efficient? Evidence from European Football Championships

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This article investigates the degree of efficiency of the European Football online betting market by using odds quoted by 12 bookmakers on 21 European championships over 11 years. We show that systematically picking out odds inferior to a threshold delivers a rate of return of 4.45% if best odds are selected across bookmakers and 2.78% if mean odds are used. This amounts to backing overwhelmingly favourites whose probability of winning exceeds 90%. Our results only exploit information contained in odds, are robust to the use of real-time data and different sample periods and hold under risk neutrality and expected utility preferences for realistic degrees of risk aversion. Transaction costs reduce profitability but only for small stake bets.

Keywords: decision-making under risk; information and market efficiency; betting markets; asset allocation

JEL Classification: D81; G14

I. Introduction

Online betting markets are a flourishing industry that has been gaining in popularity worldwide for the last decade. All sorts of sport events can be bet on, and attractive redistribution rates are offered, sometimes greater than 90%. Betting markets also provide an interesting field to scholars who study the working of information markets and implications for participants' risk preferences and degree of rationality. By offering bets whose payoffs are contingent to the occurrence of uncertain sporting events, betting markets produce information about the likelihood of these events in a way similar to how financial markets aggregate information about future and uncertain assets' payoffs. A natural way to investigate the functioning of betting markets is consequently to study their information efficiency.

Markets are efficient if prices reflect information to the point where the marginal benefit of acting on information does not exceed marginal costs (Jensen, 1978). This property can be tested by checking that the use of historical prices does not yield abnormal returns (Fama, 1991). Market efficiency has been originally formulated for financial markets but applies in a natural way to betting markets where asset prices are replaced by betting odds. Weak-form efficiency requires that bettors cannot earn a risk-adjusted better return or undergo a smaller loss by selecting a class of bets on the basis of their odds. Semi-strong-form efficiency tests expand the information set to all types of public information

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like teams' past performance or whether they benefit from home advantage.

A related and crucial issue is whether the best rules identified deliver a positive return. A mere rate of return differential between alternative betting rules invalidates the information revelation property of betting markets but does not necessarily challenge participants' rationality whose motivation to chase better return is weakened by the absence of profitable opportunities. To the contrary, the presence of profitable rules that are not arbitraged away strongly challenges the hypothesis of rational behaviour which is key to the existence of efficient markets. Conditions under which a profitable betting rule is proved to exist are however stringent. The computed excess return should be positive with a sufficient level of confidence. It should not depend on unrealistic assumptions about risk preferences and transaction or information costs. Last, the betting rule should be based on readily observable information, robust to out-of-sample testing, and simple enough so that a large number of bettors may be able to detect and exploit it.

This article investigates the degree of weak-form efficiency of betting markets by analysing the statistical relationship between odds and return in the European Football online betting market, which is the most developed market in Europe. The data cover 21 championships played in 11 European countries over a period of 11 years (2000–2011). Each year, odds of home wins, away wins and draws posted by six to 10 online bookmakers, depending on the years, are recorded. Overall, the dataset includes 79 446 football matches and around 1 800 000 betting odds. Our data are an order of magnitude larger than any other dataset previously examined in online betting markets. We show that systematically picking out betting odds inferior to a threshold delivers a rate of return of 4.45% if best odds are selected across bookmakers and 2.78% if mean odds are used. Annualized rates of return are 106% and 52% respectively. This amounts to backing overwhelmingly favourites whose probability of winning exceeds 90%. These results only exploit information contained in odds, do not rely on complex econometric models, and are robust to out-of sample tests and different risk preferences assumptions. Transaction costs are shown to reduce profitability but only for small stake bets. Overall, the evidence indicates that profitable opportunities are not exploited away by bettors and that bookmakers do not set odds in a rational way.

This article is linked to the literature that documents the presence of a favourite-longshot bias in betting markets (see Sorensen and Ottaviani (2008) for a survey). This bias refers to the observation that the expected return on longshot bets (or bets on outsiders) tends to be systematically lower than on favourite bets. It is mostly observed in racetrack betting data (Snowberg and Wolfers, 2010). Our data reveals the presence of a favourite-longshot bias in the European Football Championships although the relationship between odds and return is noisier than in racetrack betting data. Hence the result that backing outcomes with the shortest odds is a profitable strategy can be seen as the exploitation of a favourite-longshot bias.

There are however important differences with previous results found in racetrack betting data. First, horse racing betting markets are pari-mutuel that is the money bet on all outcomes is pooled and then shared proportionally among those who picked the winning outcome. Under this sharing rule, a favourite-longshot bias means that bettors tend to underbet on favourites and to overbet on longshots. Hence, deviations from market efficiency could only be attributable to bettors' behaviour. The European Football betting market is not pari-mutuel but a fixed-odds market in which bookmakers set odds several days in advance and then rarely change them during the betting period. Since they do not balance the books, odds do not reflect supply and demand for each bet. If bettors tend to overbet on longshots, bookmakers may take advantage of this bias by skewing odds against outsiders.[1] Hence a decreasing function between odds and return may come from optimal pricing by bookmakers dealing with bettors characterized by a favourite-longshot bias.

Second, although existing studies generally find that backing favourites yields a better return than betting on outsiders, they do not document profit opportunities, contrary to our study. This is true in horse racing (Snowberg and Wolfers, 2010) and in other sports as well. After a comprehensive review of the literature on sport forecasting in horse racing and several team sports, Stekler *et al.* (2010) come to the conclusion that there is no evidence that either statistical systems or experts consistently outperform the market. Our article reaches a different conclusion by showing that a simple betting rule is able to beat bookmakers' odds.

Many empirical studies dedicated to European Football betting markets have developed forecasting models that serve as a basis to the elaboration of

[1] Levitt (2004) provides evidence that bookmakers are better at predicting games outcomes than bettors in the American National Football League gambling market, and exploit this advantage by distorting odds.

betting strategies (Pope and Peel, 1989; Cain *et al*., 2000; Kuypers, 2000; Asimakopoulos and Goddard, 2004; Dixon and Pope, 2004; Deschamps and Gergaud, 2007). Some of these articles suggest profitable yet fragile betting opportunities. In particular, they rely on complex statistical models and lack the kind of robustness tests performed in this study. None of them provide evidence of profitable rules based solely on odds. The main reason why previous studies have failed to uncover the availability of odd-based profitable rules is that they focus on one or a few championships observed during one or a few seasons and consequently dispose of a relatively small database. Our dataset is on average 10 times larger than theirs and allows us to assess the profitability of betting strategies involving infrequent bets with high accuracy.[2]

The result that a simple and profitable price-based betting rule is left unexploited is also of interest for the efficiency literature in finance. There is an ongoing debate whether return anomalies are genuine deviations from market efficiency (see e.g. Shleifer (2000) and Schwert (2003) for two contrasting views). A wealth of empirical studies has failed to bring forth a consensus about this issue. One major reason is that tests of market efficiency rely on a particular asset-pricing model. Hence deviations from efficiency can always be rejected on the ground that the theoretical model at hand is inadequate. Efficiency tests in betting markets face the same difficulty, but require a much simpler model of asset returns. Bets have only two payoffs delivered a few days forward so that time is not an issue. The probability of winning is easier to estimate than the stochastic behaviour of financial assets. If any, betting markets have also a better chance of being efficient as betting is a repeated activity with immediate feedbacks that facilitate learning (Thaler and Ziemba, 1988; Erev and Haruvy, 2010). Hence the presence of unexploited profit opportunities is instructive about the degree of efficiency and participants' rationality in information markets.

This article is organized as follows. The next section describes the data, Section III presents the expected return to bets at different odds. Section IV investigates the profitability of a simple price-based betting rule. Section V analyses to what extent profitability is affected by transaction costs. Next, Section VI tests the robustness of the betting rule and Section VII estimates which fraction of their wealth risk-averse bettors would dedicate to wagering. Section VIII concludes.

II. Description of the Dataset

The dataset comprises the results of 79 446 football matches played in national championships of 11 European countries between 2000 and 2011, together with the odds against each possible outcome (home win, away win, even) and the date of the matches. All data can be freely downloaded from the website football.data.co.uk. The championships covered by this resource are the top four leagues of England and Scotland, the top two leagues of Germany, Italy, Spain and France, and the top leagues of the Netherlands, Belgium, Portugal, Turkey and Greece. All championships are present in the dataset as soon as 2000.

To avoid selection bias and to dispose of the largest dataset, all championships and years recorded by the website are used in the sequel (except the fifth league of England whose data only begins in 2005). Betting odds data are quoted by 12 online bookmakers accessible to British gamblers.[3] Appendix A indicates the number of matches present in the dataset broken down by seasons and bookmakers. Five online bookmakers are recorded the first season 2000–2001. Their number gradually increases to 10 for more recent seasons. The records of several competing bookmakers allow us to use two types of odds for every match outcome: the best odds recorded in the dataset and the mean odd which is the arithmetic mean of all odds quoted for a given outcome.

III. The Mean Return to Bets at Different Odds

This section investigates the expected return to bets placed at various ranges of odd. All seasons, championships and odds status are pooled. For every match outcome the best odd available in the market is selected. Figure 1 plots average rates of return of bets with similar odds along with their 95%

[2] As a comparison, Kuypers (2000) exploit 3382 matches in his study, Dixon and Pope (2004) 6629 matches and Asimakopoulos and Goddard (2004) 8144 matches, whereas the present dataset comprises 79 446 matches.

[3] These bookmakers are Bet365, Blue Square, Bwin, Gamebookers, Interwetten, Ladbrokes, Sportingbet, Sporting Odds, Stan James, Stanleybet, Victor Chandler and William Hill. They are all well-established online bookmakers except Sporting Odds which has ceased its activity.

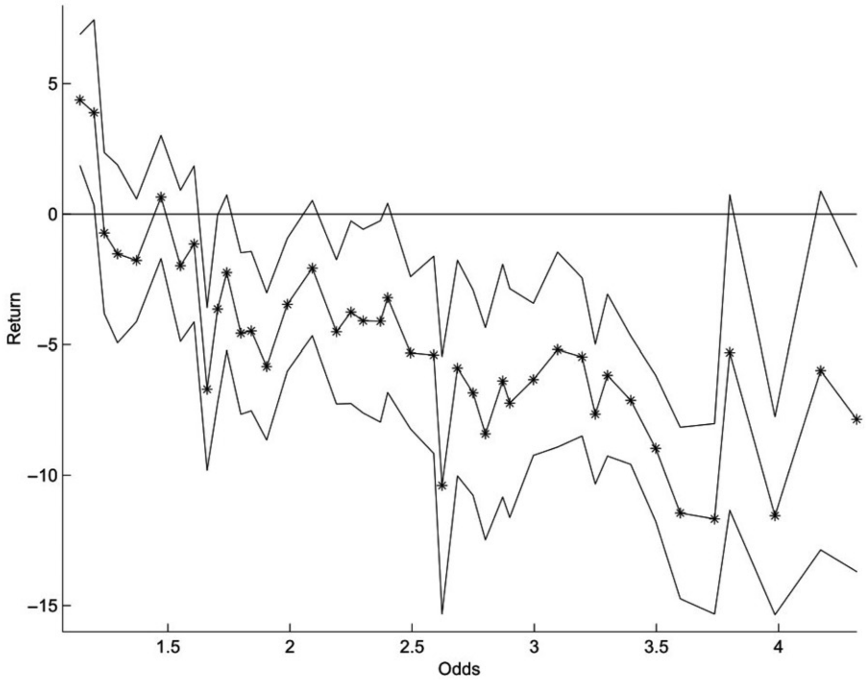

Fig. 1. Rate of return at different odds

Notes: Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011. A 95% confidence interval is added between each estimate.

confidence interval. Appendix B provides detailed information about how the graph is constructed.

Returns displayed in Fig. 1 are computed with best odds but a similar profile translated downward would obtain if mean odds were used instead. Two important observations are in order. First, a favourite longshot bias is observed. Return tends to decrease with odds although the relationship is somewhat noisy and seem to disappear for odds greater than 3.5. This pattern is consistent with several theories. If bettors have expected utility preferences, the odd-return relationship reveals risk loving (Quandt, 1986). A similar pattern would arise if bettors depreciate small probabilities as in Prospect Theory (e.g. Snowberg and Wolfers, 2010). Last, if a subset of bettors benefit from private information, bookmakers may attempt to protect themselves by lowering the return to betting on longshots. Shin (1991, 1992) shows that the presence of insider traders generates a favourite-longshot bias in fixed odds markets. Note however that a decreasing relationship between odds and return for the longest odds is not a feature of our data.

Second, the first odds intervals display a positive return. Bets with odds less than 1.21 yield a return of around 4% (see Table A2 in Appendix B for more details). The confidence interval indicates that the true return is positive with a probability greater than 95%. In addition, Table A2 in Appendix B shows that the probability is actually greater than 99% for bets whose odds is smaller than 1.17. Hence backing overwhelmingly favourites whose probability of winning is around 90% guarantees a positive and statistically robust rate of return. If mean odds are selected instead of best odds, average returns are still around 2% for odds less than 1.21 but are less statistically robust (Table A2 in Appendix B). The 95% confidence interval includes strictly positive returns only for odds smaller than 1.17.

The presence of profitable bets casts serious doubt on the ability of the market to aggregate all relevant information about the likelihood of match outcomes. To the contrary, the evidence suggests that profitable opportunities are not exploited away by bettors and that bookmakers do not fix odds in a rational way, contrary to what Levitt (2004) shows in the American National Football League gambling market. By severely underpricing bets with the smallest odds, bookmakers expose themselves to potentially large losses. From a bettor's perspective, it is however not straightforward how easy this pricing bias may be exploited. The rest of the article investigates this

Table 1. Odd threshold and return

Odd threshold	Number of bets	Number of wins	Win frequency	Return with best odds (%)	Return with mean odds (%)
1.13	204	185	90.69	0.14	−1.09
1.14	241	220	91.29	1.32	0.00
1.15	409	373	91.20	2.71	1.21
1.16	444	407	91.67	3.50	1.94
1.17	614	562	91.53	4.37	2.72
1.18	659	602	91.35	4.40	2.73
1.19	661	604	91.38	4.45	2.78
1.20	1111	993	89.38	4.16	2.32
1.21	1118	999	89.36	4.15	2.33
1.22	1417	1243	87.72	3.19	1.31
1.23	1452	1277	87.95	3.47	1.58
1.24	1455	1280	87.97	3.61	1.72
1.25	2107	1791	85.00	1.86	−0.16
1.26	2115	1798	85.01	1.90	−0.12
1.27	2179	1845	84.67	1.64	−0.39

Notes: This table indicates various statistics for betting rules consisting in backing all bets whose odds are less or equal to a threshold. Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011.

question by analysing the profitability of a simple betting strategy which consists in backing all results whose odds is smaller than a threshold.

IV. A Simple Betting Rule

Table 1 shows the rates of return when bettors back all bets whose odds are less than or equal to various thresholds. Statistics are computed for the whole sample. The threshold applies to best odds regardless bettors select the best odd or the mean odd. For mean odds, one may think of bettors who bet upon observing the best odds in the market, yet use their regular bookmaker to place a stake.

The return profile is hump-shaped both for best odds and mean odds. The rate of return is increasing for odds between 1.14 and 1.19 due to a stable win frequency around 91.5%. Betting all outcomes whose odds are inferior to the cut-off 1.19 yields a maximum return of 4.45% if best odds are chosen and 2.78% if mean odds are selected. This rule implies backing 661 bets over 11 seasons, or on average 60 bets per season. Notice that all strategies based on a price threshold between 1.17 and 1.21 yield a return greater than 4%. Hence bettors may not choose the best cut-off, yet benefit from a rate of return close to the maximum one.

Table 1 also shows that bets with odds smaller than 1.16 deliver a sub-optimal return. A more sophisticated strategy would therefore consist in backing all bets whose odds are between 1.16 and 1.19. This would yield a return of 7.20% with best odds and 5.26% with mean odds. A one-threshold strategy is however maintained in the following in order to keep the betting rule as simple as possible.

Rates of return in Table 1 apply to a very short time period. Bettors typically place a stake Saturday and can withdraw their money the next week. To ease the comparison with financial investments, annualized rates of return are computed by backtesting the betting rule on historical data. Bettors are assumed to face actual betting opportunities, to stake £1 on every bet whose odd is less than or equal to a given threshold and to reinvest the proceeds whenever possible. The gains are withdrawn from the betting account once in a year at the end of each season. An annual rate of return is then calculated for every season from summer to next summer.

More formally, let us denote s_t the amount of money placed in the account balance until the betting period $t = 1, \ldots, T$ and g_t the account balance which is s_t plus previous gains and losses from past bets. s_t and g_t evolve over a season according to

$$s_t = s_{t-1} + \min(g_{t-1} - n_t, 0)$$

$$g_t = g_{t-1} + \sum_{i=1}^{n_t} q_i \times 1_i - n_t$$

with 1_i equal to 1 if bet i is successful and 0 otherwise. n_t is the number of simultaneous bets that meet the betting criterion and belong to the same betting period. A betting period includes all bets that happen within three consecutive days during which we assume that gains reinvestment is not possible.

Table 2. Odd threshold and annualized rates of return

Odd threshold	Return with best odds (%)	Return with mean odds (%)
1.15	33.71	15.94
1.16	45.79	23.50
1.17	71.96	39.42
1.18	78.73	43.94
1.19	77.47	43.18
1.20	103.54	51.79
1.21	106.11	52.54
1.22	63.11	22.96
1.23	76.66	33.50
1.24	79.91	35.68
1.25	33.78	−4.03
1.26	35.34	−3.19

Notes: This table shows the rates of return with best and mean odds for betting rules consisting in backing all bets whose odds are less or equal to a threshold. Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011.

Bettors contribute to their account the sum $s_t - s_{t-1}$ whenever the balance account g_{t-1} is insufficient to stake the sum n_t. End-of-season rate of return is $g_T/s_T - 1$. Table 2 indicates annualized rates of return for odd threshold varying from 1.15 to 1.26.

For every threshold, we first compute annual rates of return for the 11 seasons and then take the geometric mean. Backing outcomes whose odds are less than or equal to 1.21 maximizes the rate of return both for the strategy involving best odds and the one with mean odds. For this threshold, 1111 bets meet the criterion. Given a unit stake of £1, the average total stake over a season is £4.80 with best odd, and the average end-of-season gain is £9.90. This translates into an average rate of return of 106%. The optimal threshold is slightly different from 1.19 which is the cut-off found with the instant return criterion. Overall, the pattern is similar albeit with a different scale, which comes from the opportunity given to bettors to capitalize instant rates of return by reinvesting the proceeds each week.

V. Information and Transaction Costs

Two betting strategies have been distinguished. Bettors may stick with their regular betting website or they may shop around in search of best odds. The profitability of these two options has been evaluated by computing rates of return with mean odds and best odds respectively. Information and transactions costs might however inhibit bettors from exploiting profit opportunities documented in the previous section.

Table 3. The cost of using two money transfer sites

	Moneybookers	Neteller
Deposits by bank transfer	Free	Free
Withdrawal by bank transfer	£1.48	£5
Transfer of money to bookmakers	1%, up to £0.41	Free
Receiving money from bookmakers	Free	Free

A casual internet search uncovers many websites specialized in free odds comparisons. Hence shopping around for best odds do not entail significant search costs. Transaction costs depend on the payment mode. Bettors may deposit money into their betting account using a bank card, bank transfer or a Money Transfer Site (MTS). A card deposit is credited instantly into the account. Some online betting sites do not charge anything for card deposits, while others charge a fee of a few percent on the amount deposited, which is detrimental to the return to betting.

Bank transfers are free of charge both in cases of deposit and withdrawal. Transferring funds may take from 3 to 5 business days.[4] This is a good solution for bettors who choose to place bets in their regular bookmaker. It is less convenient if they want to shop around for best odds. As odds are typically posted Wednesday for next weekend matches, delays of transfer may be too long to take advantage of the best odds proposed by bookmakers. The use of a MTS is more appropriate in this case.

The MTS are third party intermediaries that make easy and cost effective for bettors to manage their online betting payments. Bettors make a single deposit into their account and then allocate the money to different bookmakers. Deposits into the MTS account by bank transfer is free. Payments to bookmakers are made instantly. Table 3 presents the fee structure of two commonly used MTS, Moneybookers and Neteller.

Assuming that bettors commit to their betting strategy over a complete season, Table 4 shows how annualized rates of return similar to the ones computed in Table 2 for a threshold of 1.21 vary when transaction costs are taken into account.

[4] If the money is transferred to a foreign bank, delays may be longer and transfers are not free of charge.

Table 4. Annualized rates of return for £10 and £100 unit stakes (whole sample)

	No transaction costs	Moneybookers		Neteller	
Unit stake	Irrelevant	£10	£100	£10	£100
Return (%)	106.11	75.06	94.74	93.01	104.81

Bettors are assumed to switch to a different bookmaker and pay the transfer fee every time a bet meeting the criterion appears. In practice several bets appearing in a row may be proposed by the same bookmaker, lowering the payment costs. Hence Table 4 provides an upper bound for the impact of transaction costs to return when bettors use Moneybookers. Flat fees make a significant dent in rates of return for small stakes. They are less visible for a £100 unit stake and would become negligible for higher stakes. They stay in all cases above the annualized rate of return of 52.54% obtained by using mean odds (Table 2). Over a typical season, bettors face 102 bets whose odd is less than or equal to 1.21. With a £100 unit stake and profit reinvestment, they place a total of £440. At the end of the season their gains are up to £819 with Moneybookers and £857 with Neteller. The impact of transaction costs on profitability depends on the size of stakes. We will see in Section VII that even risk-averse bettors stake a significant fraction of their wealth, so that transaction costs are no more an issue.

VI. Robustness Tests

Previous rates of return are computed for the whole period which includes 11 seasons. It is possible that profitability were high during the first seasons and then gradually disappears as more and more gamblers bet on shortest odds. Bookmakers may also have realized by themselves their risk exposure and adjusted downward their smallest odds before incurring significant losses. In both cases, a declining profitability should be observed the latest seasons. Alternatively, positive returns could be for some reasons concentrated in the latest years so that bettors and bookmakers have lacked hindsight to notice and exploit the pattern. Table 5 shows that none of these scenarios are observed.

Rates of return are not decreasing over time. To the contrary, returns follow a slightly increasing trend with the last three seasons among the best ones. The pattern is however visible as soon as the first season. Instant rates of return are positive 10 seasons over 11 with best odds and 9 seasons over 10 with mean odds. Hence nearly 11 years of high profitability have not been consistently exploited by bettors.

High rates of return have been obtained by using a price threshold computed over the 11 seasons. One may ask whether high returns persist if the price threshold is computed by using price information available at the beginning of each season. For a given season, it amounts to finding the threshold that maximizes the return between the beginning of the first season recorded in the sample and the end of the previous season, and then to applying this threshold to next seasons. Table 6 shows for each season the optimal threshold computed over past seasons (third column), the corresponding rate of return for the in-sample period (fourth column), the rate of return if this threshold is used the next season (fifth column) and the rate of return if it is used for the rest of the seasons (sixth column).

Optimal thresholds do not vary much when the in-sample period is extended forward, except when the season 2006–2007 is added with a threshold falling from 1.24 to 1.19. Instant rates of return are equal to 4.33% on average with 1 year of negative return. Interestingly, updating the threshold each year is dominated on average by the strategy which uses the same threshold for the rest of the seasons with a mean return of 5.04%. This result suggests that fully updating the odd threshold after every season is not optimal. As expected, using real-time data instead of *ex post* data to compute the optimal threshold reduces the instant rate of return but only from 5.01% (average return in Table 5) to 4.33%.

Appendix C presents the same statistics with mean odds instead of best odds. The next-season average rate of returns is still 2.90% instead of 4.33% with best odds. Keeping the same threshold for the rest of the seasons yields an instant mean return of 3.29% compared to 3.34% when the entire sample is used to compute the odd threshold (Table 5).

VII. Optimal Staking

Selecting the shortest odds is highly profitable but also highly risky. For the class of bets under consideration, bettors may lose their entire stake with a probability of around 10%. Risk might therefore deter bettors from taking advantage of these return opportunities. To check this possibility, we assume that bettors have Expected Utility (EU) preferences and concave utility. We do not claim that

Table 5. Instant rates of return by seasons (betting threshold of 1.19, no transaction costs)

Seasons	Number of bets	Number of wins	Win frequency	Instant return with best odds	Instant return with mean odds
2000:1	50	45	90.00	3.24	2.01
2001:2	42	39	92.86	5.53	4.22
2002:3	76	68	89.47	1.66	0.30
2003:4	91	84	92.31	4.49	2.93
2004:5	79	68	86.08	−2.05	−3.56
2005:6	64	60	93.75	8.27	6.07
2006:7	74	67	90.54	3.61	1.70
2007:8	42	37	88.10	1.36	−0.38
2008:9	31	30	96.77	12.39	10.55
2009:0	62	59	95.16	8.79	6.81
2010:1	50	47	94.00	7.8	6.08
Mean	60.10	54.91	91.73	5.01	3.34

Notes: This table indicates various statistics broken down by season for betting rules consisting in backing all bets whose odds are less than or equal to 1.19. Mean is unweighted arithmetic mean. Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011.

Table 6. Out-of-sample instant rates of return with best odds, no transaction costs

In-sample seasons	t	Optimal threshold over [2000:t]	In-sample rates of return over [2000:t]	Out-of-sample rates of return over [t:$t+1$]	Out-of-sample rates of return over [t:2011]
2000:1	2001	1.21	5.84	7.50	4.00
2000:2	2002	1.21	6.64	2.54	3.68
2000:3	2003	1.24	5.76	4.14	2.93
2000:4	2004	1.24	5.22	0.57	2.70
2000:5	2005	1.24	4.14	3.60	3.14
2000:6	2006	1.24	4.04	−1.95	3.03
2000:7	2007	1.19	3.28	1.36	7.44
2000:8	2008	1.21	3.22	8.98	7.32
2000:9	2009	1.19	3.65	8.79	8.35
2000:10	2010	1.19	4.17	7.80	7.80
Mean		1.21	4.60	4.33	5.04

Notes: This table shows instant rates of return under various scenarios. Column 1 indicates period over which optimal threshold is computed; column 2 indicates last year of in-sample seasons, column 3 indicates optimal threshold over the specified period; column 4 indicates in-sample average instant rate of return; column 5 indicates average instant rate of return obtained by applying the threshold over the next season; column 6 indicates average instant rate of return by using the same threshold until the last season. Mean is unweighted arithmetic mean. Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011.

such preferences apply to the majority of active bettors. To the contrary, the average bettor may be risk loving and fall prey to the same behavioural biases that are observed in financial markets, like overconfidence (Barber and Odean, 2001) or the hindsight bias (Biais and Weber, 2009). Yet the operation of a minority of rational bettors who detect and exploit return deviations may suffice to guarantee market efficiency. If any, these arbitragers should treat probabilities in a rational manner and consequently be EU decision-makers. Note that assuming risk-averse EU bettors minimizes the risk to reject incorrectly the efficient market hypothesis. High-risk-high-return bets would obviously fit risk-loving bettors' preferences.

EU investors put a strictly positive fraction of their wealth in an asset as soon as its rate of return exceeds the safe interest rate (e.g. Bertaut and Haliassos, 1995). As a result, we do not test whether risk-averse bettors participate to the betting market but rather which fraction of their wealth they should invest in a given bet. A fraction invested close to zero would

Table 7. Summary statistics for various classes of bets grouped by odd similarity

Odd type	Range of odds	Mean odd (q)	Sample size	Win fre-quency (p)	Average return (%)	100*($pq-1$)
Best	[1.13–1.15[	1.137	61	0.9672	9.975	9.985
Best	[1.15–1.17[	1.153	214	0.9159	5.592	5.588
Best	[1.17–1.18[	1.170	154	0.9156	7.123	7.123
Best	[1.18–1.21[	1.198	497	0.8672	3.896	3.901
Mean	[1.11–1.13[	1.120	135	0.9333	4.531	4.508
Mean	[1.13–1.15[	1.140	222	0.9144	4.253	4.264
Mean	[1.15–1.17]	1.159	275	0.8655	0.311	0.307
Mean	[1.17–1.18[	1.174	155	0.8774	3.040	3.045
Mean	[1.18–1.21[	1.195	589	0.8404	0.440	0.445

Notes: This table indicates various statistics for different odd classes. Last column indicates theoretical rate of return if bets were perfectly homogenous in each class. Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011.

support the efficient market argument according to which risk prevents arbitragers from eliminating return anomalies.

A simple asset allocation problem is solved and calibrated using empirical values. Betting opportunities are modelled as follows. Let us consider a bet whose odd is q and probability of success p. Several classes of bets with similar odds are distinguished. Their probabilities of winning are identified to win frequencies. Table 7 displays statistics on p and q for different bet classes.

There are eight classes of odds which range from 1.13 to 1.21 for best odds and 1.11 to 1.21 for mean odds. All classes are profitable with average instant rates of return between 0.31% and 9.97%. The last column indicates the theoretical rate of return if bets were perfectly homogenous in each class. They are not very different from average returns, suggesting that bet classes are close to be homogenous despite different odd values included in each class.

Bets are evaluated by EU bettors with isoelastic utility function: $u(c) = c^{1-\sigma}/(1-\sigma)$, where σ is the Relative Risk Aversion Coefficient (RRAC). Bettor's wealth before gambling is denoted as w. The amount of money placed on a single bet is denoted as α. Given the very short period over which a bet is pending, money is the sole alternative investment considered, which interest rate is zero. In such an environment with a zero risk-free interest rate, no serial correlation of rates of return[5] and an isoelastic utility function, it is rational to bet at each period as if the current betting period were the last one (Mossin, 1968). This property greatly simplifies the investment problem. The amount α maximizes bettor's expected utility

$$\alpha^* = \arg\max pu(w - \alpha + \alpha q) + (1-p)u(w-\alpha)$$

The optimal stake in proportion to wealth α^*/w is derived with p and q replaced by their empirical counterparts documented in Table 7. The problem at hand is however only valid for a single bet. 60 bets meeting the betting criterion in a typical season, many bets are actually placed during the same weekend. Over the 11 seasons, only 190 bets, or 29% of total bets, come in isolation during a betting period, 186 bets come in pair, 147 by three, 76 by four, 50 by five and 12 by six. Backing several bets at once is a natural way to diversify risk. This is why optimal stakes are also calculated when more than one bet are taken at the same time. The corresponding maximization programs are presented in Appendix D.

Table 8 displays optimal stakes in proportion to wealth α^*/w for various degrees of risk aversion. Four odd classes taken from Table 7 are compared which differ in their instant rate of return. Three different values for the RRAC are considered. Although most calibrated studies, notably in the equity premium literature, take a RRAC between 1 and 5, we also consider a value of 10 to test results' robustness. Such a value implies that bettors would be ready to pay as much as 4.42% to escape the risk of gaining or losing 10% of their wealth. For simplicity, it is also assumed that when several bets come together, they share the same characteristics in terms of probability and odd.

As expected, the fraction of wealth dedicated to betting is decreasing with the degree of risk aversion and increasing with the expected rate of return and the number of simultaneous bets. The presence of concomitant bets goes a long way toward mitigating risk. Betting on two bets instead of one roughly doubles the fraction invested in wagering. The fraction of wealth gambled is not trivial. In the

[5] The correlation coefficient between two consecutive return rates is not significantly different from zero at a 10% confidence level.

Table 8. Fraction of wealth gambled in percent as a function of the number of simultaneous bets and for different degrees of risk aversion (no transaction costs)

				Relative risk aversion coefficient		
Odds	Odd type	Instant rate of return	Number of bets	$\sigma = 1.5$	$\sigma = 5$	$\sigma = 10$
[1.13–1.15[	Best odds	9.97%	1	54.47	20.54	10.79
			2	90.24	40.56	21.52
			3	99.08	59.88	32.17
			4	99.91	80.14	44.12
[1.15–1.17[	Best odds	5.92%	1	25.96	8.51	4.33
			2	50.93	16.99	8.67
			3	74.59	25.45	13.00
			4	96.90	36.58	18.66
[1.17–1.18[	Mean odds	3.04%	1	11.97	3.73	1.88
			2	23.86	7.45	3.75
			3	35.67	11.17	5.63
			4	57.72	18.15	9.15
[1.18–1.21[	Mean odds	0.44%	1	3.36	1.02	0.51
			2	6.72	2.04	1.02
			3	10.09	3.06	1.53
			4	26.43	8.06	4.04

Notes: Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011. This table reads as follows. Bettors with a relative risk aversion coefficient of 1.5 invest 54.47% of their wealth in a bet whose odd is 1.137 and probability of winning is 96.72%. Odds and win probabilities are displayed in Table 7 for best odds ranging from 1.13 to 1.15. Likewise, the rate of return of 9.97%, in the third column is the mean return computed in Table 7 for the class of bets [1.13–1.15[. Faced with two simultaneous bets whose odds are 1.137 and probabilities of winning 96.72%, bettors place 90.24% of their wealth in the two bets or 45.12% in each bet.

worst case scenario, a single bet with an expected return of 0.44% and a RRAC of 10, bettors still commit 0.5% of their wealth to gambling. With more than one bet, a smaller RRAC or better rates of return, this fraction rapidly reaches high fractions of wealth for realistic betting opportunities. We conclude from these results that risk should not deter most EU bettors from investing a significant proportion of their wealth in the betting market.

VIII. Conclusion

Online betting markets are becoming a global industry which attracts an increasing number of firms and bettors around the world. Gamblers can choose between many bookmakers and easily shop around for best odds. Yet, despite a highly competitive market, this article provides evidence of large deviations from weak-form efficiency. It is shown that systematically picking out the shortest odds delivers an instant rate of return of 4.45% or an annualized rate of return of 106%. Such a betting rule does not require a complex forecasting model. The return is robust to out-of-sample tests and to the use of real-time data. Bettors with realistic degrees of risk aversion are still willing to devote a significant fraction of their wealth to wagering. Transaction costs take the form of small flat fees which can be recouped by stakes large enough.

Our data confirm the presence of a favourite-longshot bias found in other betting markets. Rates of return to betting fall as the odds rise. The bias is large enough to produce a positive return rate when overwhelmingly favourites are backed. This deviation from efficiency involves both sides of the market. Bettors have failed so far to respond to the presence of profitable bets and bookmakers have put their business at risk by underpricing bets with the smallest odds.

By focusing on simple strategies, these results do not make full use of all available information. We have already noticed that discarding the odds below 1.16 would enhance the return rate. In addition, the odd threshold could be adapted for each country or league. It could also depend on whether the odds hold for the home team or the away team. Other improvements would consist in exploiting possible serial correlations of wins or adopting a betting strategy where the size of the stakes are derived from the optimal staking model developed in Section VII.

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Appendix A: Sample Summary Statistics

Table A1 presents summary statistics about the number of odds present in the data broken down by seasons and bookmakers. The table reads as follows. The results of 7095 football matches have been recorded during the season 2000–2001, among which 5542 have odds from the bookmaker Gamebookers. The same year, we dispose of 25 557 triplets of odds (one for each possible match outcome) quoted by five bookmakers. The total number of odds recorded in 2000–2001 is therefore 76 671.

Appendix B: Computation of Fig. 1

The sample includes 79 446 football matches from 21 national championships played in 12 European countries over the period 2000–2011. Our dataset has a quasi-continuum of odds generally quoted with a two-digit accuracy, except for longer odds. We group together the results with similar odds to produce reliable estimates of return. Our grouping algorithm starts from the smallest odds and add bets with identical or increasing odds in each bracket until the number of bets is greater than a threshold x. The threshold is a balance between statistical significance and keeping groups homogenous. Bets with the shortest odds being less frequent, a smaller x is required to make their profitability apparent. x is equal to 2000 for the whole sample, except in the left-hand part of the spectrum where bets are less numerous: x equals 500 for odds strictly smaller than 1.30. Table A2 summarizes relevant statistics for the computation of returns and confidence intervals. It only indicates the first three bins, which correspond to the first three points in Fig. 1, and which are the

Table A1. Number of matches quoted by bookmakers every year

	Season											
	2000–2001	2001–2002	2002–2003	2003–2004	2004–2005	2005–2006	2006–2007	2007–2008	2008–2009	2009–2010	2010–2011	Total 2
Nb M	7095	7039	7137	7340	7329	7324	7258	7258	7258	7117	7191	79 446
GB	5542	5245	6988	7209	7302	7305	7252	7247	7243	7087	7169	75 589
IW	6153	6297	6378	6797	7172	7189	7193	7121	7195	6980	7113	75 588
LB	3169	3694	4597	5044	6784	7000	7020	7146	7183	7077	7152	65 866
SB	4883	4327	6871	7073	7256	7278	7230	7228	7229	7038	7104	73 517
WH	5810	6047	6466	6959	6980	7257	7235	5166	7211	7061	7183	73 375
ST		5565										5565
BE				6844	7135	7303	7250	7251	7250	7093	7180	57 306
SO			3147	3603								6750
BW					7274	7301	7252	7252	7238	7054	7170	50 541
SJ						7291	7216	7243	7219	7027	7093	43 089
VC						7138	7154	7209	7210	7082	7139	42 932
BS								7236	7235	7070	7116	28 657
Nb B	5	6	6	7	7	9	9	10	10	10	10	12
Total 1	25 557	31 175	34 447	43 529	49 903	65 062	64 802	70 099	72 213	70 569	71 419	59 8775

Notes: Nb M = number of matches; GB = Gamebookers; IW = Interwetten; LB = Ladbrokes; SB = Sportingbet; WH = William Hill; ST = Stanley bet; BE = bet365; SO = Sporting Odds; BW = Bwin; SJ = Stan James; VC = Victor Chandler; BS = Blue Square; Nb B = number of recorded bookmakers; Total 1 = number of match-bookmaker couples by season; Total 2 = number of match-bookmaker couples by bookmaker.

Table A2. Summary statistics for the first three odd brackets displayed in Fig. 1

			Instant return (%)		Confidence interval		
Odd brackets	Number of bets	Win frequency	Best odds	Mean odds	90%	95%	99%
[1–1.17[	614	91.53	4.37	2.72	+/−2.01	+/−2.39	+/−3.13
[1.17–1.21[	504	86.71	3.90	1.84	+/−2.97	+/−3.53	+/−4.62
[1.21–1.25[	989	80.09	−0.73	−2.96	+/−2.58	+/−3.06	+/−4.01

Notes: This table shows summary statistics for backing all bets whose odds are included in odd brackets. Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011.

Table A3. Out-of-sample instant rates of return with mean odds, no transaction costs

In-sample seasons	t	Optimal threshold over [2000:t]	In-sample rates of return over [2000:t]	Out-of-sample rates of return over [t:$t+1$]	Out-of-sample rates of return over [t:2011]
2000:1	2001	1.21	4.32	6.12	2.15
2000:2	2002	1.21	5.19	1.01	1.78
2000:3	2003	1.24	4.21	2.29	0.93
2000:4	2004	1.24	3.57	1.22	0.67
2000:5	2005	1.24	2.45	1.21	1.07
2000:6	2006	1.24	2.24	−3.90	1.04
2000:7	2007	1.17	1.74	−2.14	5.51
2000:8	2008	1.19	1.51	10.54	7.36
2000:9	2009	1.17	2.04	6.54	6.31
2000:10	2010	1.19	2.51	6.10	6.10
Mean		1.21	2.98	2.90	3.29

Notes: This table shows instant rates of return for various scenarios. Column 1 indicates period over which optimal threshold is computed; column 2 indicates last year of in-sample seasons, column 3 indicates optimal threshold over the specified period; column 4 indicates in-sample average instant rate of return; column 5 indicates average instant rate of return obtained by applying the threshold over the next season; column 6 indicates average instant rate of return by using the same threshold until the last season. Mean is unweighted arithmetic mean. Sample includes odds of 79 446 football matches played in 11 European countries from 2000 to 2011.

most important for the purpose of our study. The size of the groups is not constant so that bets with identical odds are not split between adjacent groups.

Average return is $\sum_{j=1}^{m} q_j \times 1_{WIN} - 1$ with m the number of bets in a class, q_j the odd of bet j, and 1_{WIN} an index equal to one if the bet is successful. Confidence interval around average return is given by $\psi \times \overline{q}\sqrt{f_n(1-f_n)}/\sqrt{n}$ with $\overline{q}$ the mean odd of the class, f_n the win frequency, and $\psi = 1.65$ for a 90% confidence level, 1.96 for a 95% confidence level and 2.57 for a 99% confidence level. The confidence interval is the same for best odds and mean odds due to the assumption that a bet is triggered as soon as a best odd is inferior to the threshold regardless bettors earn the best odd or the mean odd. For mean odds, we can think of bettors who compare odds of different bookmakers but keep their regular bookmaker to place a bet. Different grouping methods or different thresholds would produce slightly different results without altering the main properties in Fig. 1.

Appendix C: Out-of-sample Instant Rates of Return with Mean Odds

Table A3 indicates the annual rate or return of backing all bets whose odds are less than or equal to a threshold by using real-time data and mean odds.

Appendix D: Money Allocation Problems with Multiple Bets

This appendix shows money allocation problems when bettors have expected utility preferences and place more than one bet during a betting period. Let p be the probability of winning the bet, q be the odd, w be bettor's wealth and α be the amount of money wagered. The bettor's problem with two simultaneous and independent bets is

$$\alpha^* = \arg\max p^2 u(w - 2\alpha + 2\alpha q) + 2p(1-p)u(w - 2\alpha + \alpha q) + (1-p)^2 u(w - 2\alpha)$$

Bettors face a probability p^2 of winning the two bets, a probability $2p(1-p)$ of winning only one bet and $(1-p)^2$ of losing their entire stake. The money allocation problem with three bets is

$$\alpha^* = \arg\max p^3 u(w - 3\alpha + 3\alpha q) + 3p^2(1-p)u(w - 3\alpha + 2\alpha q) + 3p(1-p)^2 u(w - 3\alpha + \alpha q) + (1-p)^3 u(w - 3\alpha)$$

and with four bets

$$\alpha^* = \arg\max p^4 u(w - 4\alpha + 4\alpha q) + 4p^3(1-p)u(w - 4\alpha + 3\alpha q) + 6p^2(1-p)^2 u(w - 4\alpha + 2\alpha q) + 4p(1-p)^3 u(w - 4\alpha + \alpha q) + (1-p)^4 u(w - 4\alpha)$$