The Joy of Volatility

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Michael A. H. Dempster
University of Cambridge

Igor Evstigneev
The University of Manchester

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Michael A.H. Dempster   Igor V. Evstigneev
Klaus Reiner Schenk-Hoppé

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The Joy of Volatility

Michael A.H. Dempster[1], Igor V. Evstigneev[2], Klaus Reiner Schenk-Hoppé[3]

[1] *Judge Business School, University of Cambridge, UK,*

[2] *Economic Studies, School of Social Sciences, University of Manchester, UK,*

[3] *School of Mathematics and Leeds University Business School, University of Leeds, UK.*

Modern portfolio theory regards the return of an asset as its upside, while volatility is seen as its downside. This view is shared by the majority of investors who dislike volatile markets. Recent results in financial mathematics, however, show that volatility is actually good, rather than bad, for financial growth. Very simple active portfolio management opens up this profitable opportunity to generate growth from volatility.

Can there be any profitable investment when all assets in a market destroy, rather than create, value? Thanks to volatility, the answer is yes – even if one does not wish to risk bankruptcy by going short (i.e. by selling what one does not own). This result can be illustrated with the aid of a simple model of a financial market with only two risky assets, whose returns in each period are determined by flipping a fair coin (Figure 1).

Placing one's money in Asset 1 will, on average, reduce one's investment by one-tenth within 10 periods. Buying and holding Asset 2 will result in losing as much as one-third of one's investment within the same time span. Since growth rates depend on the logarithm of the (gross) return, investing in the apparently profitable Asset 1, with net returns of +40% and -30%, actually results in a loss of money. Any buy-and-hold investment, which purchases both assets, but does not update the positions, can only do as well, in the long term, as the best-performing asset. Poverty is the inevitable fate of the passive investor.

Consider making an investment according to a simple active management style: buying or selling assets so as to always maintain an equal investment in both. On average, wealth will double in 80 periods and grow without limits. This investment style rebalances wealth according to a constant proportions strategy. It succeeds, where buy-and-hold fails, because of the volatility of asset returns.

It has recently been proved mathematically[1,2,3] that, with stationary asset returns, *every* constant proportions rebalancing strategy beats the corresponding market index (defined as the weighted average of individual asset growth rates). In particular, if one only invests in the assets growing at maximum rate, or if the market is volatile enough, *any* such strategy will beat the best buy-and-hold portfolio. Though examples of this phenomenon have been reported for quite some time[4], and the best rebalancing strategy is well-known for performing at least as well as any buy-and-hold portfolio[5], it is surprising to learn that no conjecture has ever been made as to the validity of a general growth-volatility link.

The power of rebalancing strategies is often claimed to be the result of "buying low and selling high." However this is the gambler's fallacy, arguing that the longer the run of black numbers, the higher the odds of red numbers at the next spin of the roulette wheel. When returns are determined by the flip of a coin, an asset's upside and downside potential does not change over time. Such an asset is not cheap or expensive at any point in time. Nor does arbitrage (the opportunity to get 'something for nothing') drive this phenomenon – the market in the example is free of arbitrage. Finally, all investors have equal opportunities, unlike in Parrondo's paradox[6] where some investors (depending on their wealth) are given favorable odds, this excludes the dynamics of the wealth distribution as an explanation.

The engine that generates growth from volatility is in fact an elementary mathematical relation, the Jensen inequality. It describes the effect of interchanging concave (or convex) functions and weighted averages, i.e. expected values. Constant proportions rebalancing strategies combine random returns in fixed proportions. The logarithm of this financially engineered return is higher than the combination of the assets' individual growth rates, because the logarithm is a strictly concave function. For fixed, deterministic returns, both quantities are equal, and no excess growth can be achieved.

This financial market phenomenon closely resembles observations on stochastic resonance[7], a theory in physics that has various applications, e.g. in biology and neurophysiology. Similar to amplifying a weak signal by adding noise to a nonlinear system, constant proportions strategies combine two random processes to achieve an increase in the growth rate. The required nonlinearity is provided by the compound return on the investment and the logarithmic function that appears in the growth rate of wealth.

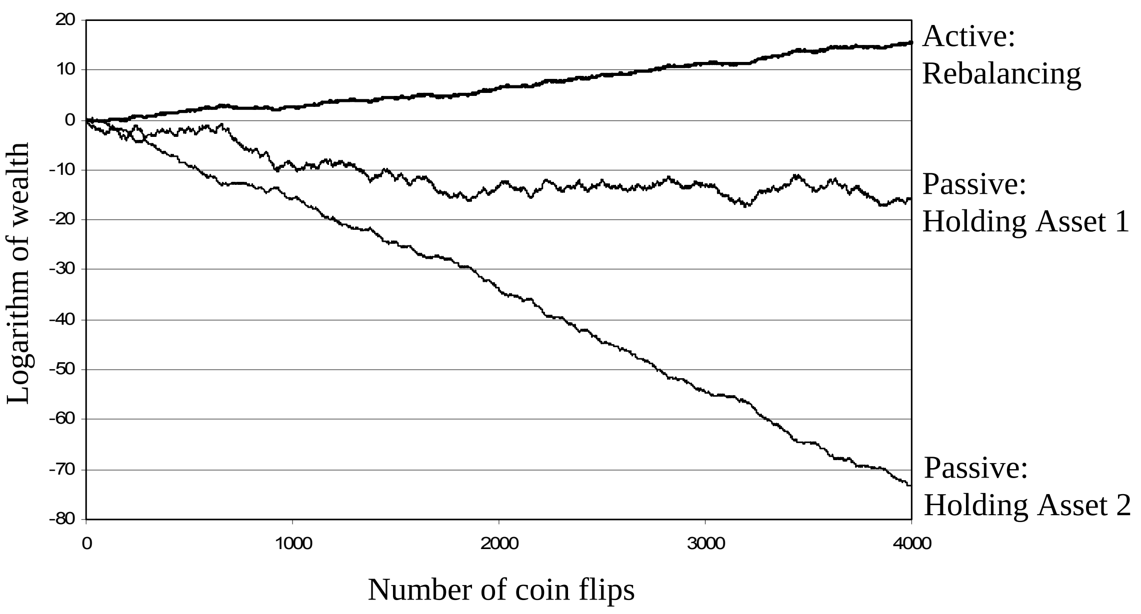
While the implications of the growth-volatility link for asset pricing and portfolio theory are still being explored, active investors should enjoy the bumpy ride of volatility. However, as with any investment advice, a word of caution is in order: Constant proportions strategies do well in the long term but, over short time horizons, their superior performance cannot be guaranteed!

Figure 1. Performance of passive and active investment styles

	Asset 1		Asset 2	
	Heads	Tails	Heads	Tails
Net return on assets	+40%	-30%	-20%	+15%
Growth rate of assets	½ ln(1.4) + ½ ln(0.7) ≈ -0.010		½ ln(0.8) + ½ ln(1.15) ≈ -0.042	
Buy-and-hold strategies cannot do better than Asset 1 in the long-term. All passive styles lose.				

	Heads	Tails
Net return on active investment (1:1 mix)	½ 40% + ½ (-20%) = +10%	½ (-30%) + ½ 15% = -7.5%
Growth rate of investment	½ ln(1.10) + ½ ln(0.925) ≈ +0.0087	
The constant proportions strategy generates positive growth. It beats all passive styles.		

Simulation of wealth dynamics



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Correspondence and requests for materials should be addressed to K.R. Schenk-Hoppé (k.r.schenk-hoppe@leeds.ac.uk).

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