Ex 6.1

$u = (1,1)$ $v = (3,2)$
$w = (0,-1)$ $k = 3$

Q No 1 (a) $\langle u, v \rangle$

$\langle u, v \rangle = u_1 v_1 + u_2 v_2$
$= (1)(3) + (1)(2)$
$= 3 + 2$
$= 5$

(b) $\langle kv, w \rangle$

$kv = 3(3,2)$
$= (9,6)$

$= (9)(0) + (6)(-1)$
$= 0 - 6$
$= -6$

(c) $\langle u + v, w \rangle$

$u + v = 1+3,\ 1+2 = (4,3)$
$\langle u + v, w \rangle = 4(0) + (-1)(3)$
$= -3$

(d) $\|v\|$

$\|v\| = \sqrt{\langle v, v \rangle} = \sqrt{(3)(3)+(2)(2)}$
$= \sqrt{9+4}$
$= \sqrt{13}$

(e) $d(u,v)$

$v_1 - u_1 = 3 - 1 = 2$ $(v_1-u_1)^2 = (2)^2 = 4$
$v_2 - u_2 = 2 - 1 = 1$ $(v_2-u_2)^2 = (1)^2 = 1$
$4 + 1 = 5$
$d(u,v) = \sqrt{5}$

(f) $\|u - kv\|$

$$kv = 3(3,2) = (9,6)$$

$u - kv$

$(1,1) - (9,6)$

$(-8, -5)$

$$\|u - kv\| = \sqrt{(-8)(-8) + (-5)(-5)}$$
$$= \sqrt{64 + 25}$$
$$= \sqrt{89}$$

QNO2:

$$\langle u, v \rangle = 2u_1v_1 + 3u_2v_2$$

(a) $\langle u, v \rangle$

$$\langle u, v \rangle = 2u_1v_1 + 3u_2v_2$$
$$= 2(1)(3) + 3(1)(2)$$
$$= 6 + 6$$
$$= 12$$

(b) $\langle kv, w \rangle$

$kv = (9,6)$

$$\langle kv, w \rangle = 2kv_1w_1 + 3kv_2w_2$$
$$= 2(9)(0) + 3(6)(-1)$$
$$= -18$$

(c) $\langle u+v, w \rangle$

$u+v = (4,3) \quad w = (0,-1)$

$\langle kv+v, w \rangle = 2(4)(0) + 3(3)(-1)$

$= -9$

(e) $d(u,v)$

$(u,v) = 2u_1v_1 + 3u_2v_2$

solve from Q1 part e.

$= 2(4) + 3(1)$

$= 8+3$

$= \sqrt{11}$

(d) $\|v\|$ $= \sqrt{2\cdot3\cdot3 + 3\cdot2\cdot2}$

$\sqrt{18+12}$

$= \sqrt{30}$

(f) $\|u-kv\|$

$\|u-kv\| = \sqrt{2(-8)(-8) + 3(-5)(-5)}$

$= \sqrt{128+75}$

$= \sqrt{203}$

QNO3:

(a) $\langle u, v \rangle = \langle v, u \rangle$

$\langle u, v \rangle = u_1v_1 + u_2v_2$

$= (3)(4) + (-2)(5)$

$= 12-10$

$= 2$

$\langle v, u \rangle = v_1u_1 + v_2u_2$

$= (4)(3) + (5)(-2)$

$= 12-10$

$= 2$ proved.

(b) $\langle u+v, w\rangle = \langle u, w\rangle + \langle v, w\rangle$

L.H.S

$\langle u+v, w\rangle$

$u+v = 3+4, -2+5 = (7, 3)$

$\langle u+v, w\rangle = (7)(-1) + 3(6)$

$= -7 + 18$

$= 11$

R.H.S

$\langle u, w\rangle + \langle v, w\rangle$

$\langle u_1 w_1 + u_2 w_2\rangle + \langle v_1 w_1 + v_2 w_2\rangle$

$\langle (3)(-1) + (-2)(6)\rangle + \langle (4)(-1) + (5)(6)\rangle$

$(-3-12) + (-4+30)$

$-15 + 26$

11

proved.

(c) $\langle u, v+w\rangle = \langle u, v\rangle + \langle u, w\rangle$

L.H.S

$\langle u, v+w\rangle$

$v+w = (4-1, 5+6) = (3, 11)$

$\langle u, v+w\rangle = (3)(3) + (-2)(11)$

$= 9 - 22$

$= -13.$

$\langle u, v\rangle + \langle u, w\rangle$

$u_1 v_1 + u_2 v_2 + u_1 w_1 + u_2 w_2$

$\{(3)(4) + (-2)(5)\} + \{(3)(-1) + (-2)(6)\}$

$\{12 - 10\} + \{-3 - 12\}$

$2 - 15$

-13

proved.

(d) $\langle ku, v\rangle = k\langle u, v\rangle = \langle u, kv\rangle$

• $\langle ku, v\rangle$

$-4(3, -2) = (-12, 8)$

$\langle ku, v\rangle = (-12)(4) + (8)(5)$

$= -48 + 40$

$= -8 \longrightarrow 1$

• $k\langle u, v\rangle$

$\langle u, v\rangle = (3)(4) + (-2)(5)$

$= 12 - 10$

$= 2$

$k\langle u, v\rangle = (2)(-4) = -8 \longrightarrow 2$

• $\langle u, kv\rangle$

$kv = -4(4, 5) = (-16, -20)$

$\langle u, kv\rangle$

$= 3(-16) + (-2)(-20)$

$= -48 + 40$

$= -8 \longrightarrow 3$

proved

(e) $\langle 0, v\rangle = \langle v, 0\rangle = 0$

$\langle 0, v\rangle = 0(4) + 0(5) = 0$

$\langle v, 0\rangle = 4(0) + 5(0) = 0$

Hence, proved

QNO4: $\langle u, v\rangle = 4u_1v_1 + 5u_2v_2$

(a) $\langle u, v\rangle = \langle v, u\rangle$

$$\langle u, v\rangle = 4u_1v_1 + 5u_2v_2$$
$$= 4(3)(4) + 5(-2)(5)$$
$$= 48 - 50$$
$$= -2$$

$$\langle v, u\rangle = 4v_1u_1 + 5v_2u_2$$
$$= 4(4)(3) + 5(5)(-2)$$
$$= 48 - 50$$
$$= -2$$

(b) $\langle u+v, w\rangle = \langle u, w\rangle + \langle v, w\rangle$

$u+v = 3+4 = 7 \quad u+v = -2+5 = 3$

$$\langle u+v, w\rangle = 4(7)(-1) + (5)(3)(6)$$
$$= -28 + 90$$
$$= 62$$

$$\langle u, w\rangle + \langle v, w\rangle$$

$$= \{4(3)(-1) + 5(-2)(6)\} + \{4(4)(-1) + 5(5)(6)\}$$
$$= \{-12 - 60\} + \{-16 + 150\}$$
$$= -72 + 134$$
$$= 62$$

proved

(c) $\langle u, v+w \rangle = \langle u, v \rangle + \langle u, w \rangle$

$v_1 + w_1 = 4 - 1 = 3$

$v_2 + w_2 = 5 + 6 = 11$

$\langle u, v+w \rangle = 4(3)(3) + 5(-2)(11)$

$= 36 - 110$

$= -74$

$\langle u, v \rangle = 4(3)(4) + 5(-2)(5)$

$= 48 - 50$

$= -2$

$\langle u, w \rangle = 4(3)(-1) + 5(-2)(6)$

$= -12 - 60$

$= -72$

$\langle u, v \rangle + \langle u, w \rangle = -2 - 72 = -74$

(d) $\langle ku, v \rangle = k\langle u, v \rangle = \langle u, kv \rangle$

$ku_1 = -4(3)$ $\quad ku_2 = -4(-2)$

$= -12$ $\quad = 8$

$\langle ku, v \rangle = 4(-12)(4) + 5(8)(5)$

$= -192 + 200$

$= 8$

$k\langle u, v \rangle = -4\left(4(3)(4) + 5(-2)(5)\right)$

$= -4(48 - 50)$

$= -4(-2)$

$= 8$

$kv_1 = -4(4) = -16$

$kv_2 = -4(5) = -20$

$\langle u, kv \rangle = 4(3)(-16) + 5(-2)(-20)$

$= -192 + 200$

$= 8$

Hence, proved.

(e) $\langle 0, v \rangle = \langle v, 0 \rangle = 0$

$\langle 0, v \rangle = 4(0)(4) + 5(0)5$

$= 0$

$\langle v, 0 \rangle = 4(0)(4) + 5(0)(5)$

$= 0$

proved

QNO5: $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$\langle x, y \rangle = (Ax) \cdot (Ay)$

$u = (2, 1) \quad v = (-1, 1) \quad w = (0, -1)$

$Au = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$= (5, 3)$

$Av = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$Aw = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

(a) $\langle u, v \rangle = Au \cdot Av$

$= 5(-1) + (3)(0)$

$= -5$

(b) $\langle v, w \rangle = Av \cdot Aw$

$= -1(-1) + 0(-1)$

$= 1$

(c) $\langle u+v, w \rangle$

$u + v = (5 - 1 ; 3 + 0) = (4, 3)$

$\langle u+v, w \rangle = 4(-1) + 3(-1)$

$= -4 - 3$

$= -7$

(d) $\|v\| = \sqrt{Av \cdot Av}$

$\|v\| = \sqrt{(-1)^2 + (0)}$

$= 1$

(e) $d(v, w)$

$d(v, w) = \|v - w\|$

$v - w = (-1 + 1, 0 - (-1))$

$= (0, 1)$

$d(v, w) = \sqrt{(0)^2 + (1)^2}$

$= 1$

(f) $\|v - w\|^2$

$\|v - w\|^2 = (0)^2 + (1)^2$

$= 1$

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QNO6: $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

$u = (2, 1) \quad v = (-1, 1) \quad w = (0, -1)$

$$Au = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = (2, 3)$$

$$Av = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = (-1, -3)$$

$$Aw = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = (0, 1)$$

(a) $\langle u, v \rangle = Au \cdot Av$

$= 2(-1) + 3(-3)$

$= -2 - 9$

$= -11$

(b) $\langle v, w \rangle = Av \cdot Aw$

$= -1(0) + (-3)(1)$

$= -3$

(c) $\langle u+v, w \rangle$

$u + v = 2 - 1, \; 3 - 3$

$= 1, 0$

$\langle u+v, w \rangle = 1(0) + 1(0)$

$= 0$

(d) $\|v\| = \sqrt{Av \cdot Av}$

$= \sqrt{(-1)^2 + (-3)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$

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(e) $d(v, w) = \|v - w\|$

$v - w = -1, -4$

$\|v-w\| = \sqrt{(-1)^2 + (-4)^2}$

$= \sqrt{17}$

(f) $\|v-w\|^2 = \sqrt{(-1)^2 + (-4)^2}$

$= 17$

Ex 5.1

QNO 7:

(a) $u = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}$ $v = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$

$$\langle u, v \rangle = 3(-1) + (-2)(3) + 4(1) + 8(1)$$
$$= -3 - 6 + 4 + 8$$
$$= 3$$

(b) $u = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$ $v = \begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix}$

$$\langle u, v \rangle = 1(4) + 2(6) - 3(0) + 5(8)$$
$$= 4 + 12 + 40$$
$$= 56$$

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QNO8:

(a) $p = -2 + x + 3x^2$ $q = 4 - 7x^2$

$\langle p, q \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2$

$p = -2 + x + 3x^2$ $q = 4 - 7x^2$

$a_0 = -2$ $b_0 = 4$

$a_1 = 1$ $b_1 = 0$

$a_2 = 3$ $b_2 = -7$

$\langle p, q \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2$

$= (-2)(4) + (1)(0) + (3)(-7)$

$= -8 - 21$

$= -29.$

QNO 8 (b) $p = -5 + 2x + x^2$ $q = 3 + 2x - 4x^2$

$a_0 = -5$ $b_0 = 3$

$a_1 = 2$ $b_1 = 2$

$a_2 = 1$ $b_2 = -4$

$\langle p, q \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2$

$= (-5)(3) + (2)(2) + 1(-4)$

$= -15 + 4 - 4$

$= -15$

QNO9: $9u_1 v_1 + 4u_2 v_2$

$A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$\langle u, v \rangle = u^T A v$

$= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$= \begin{bmatrix} 3u_1 + 2u_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$= 3u_1 v_1 + 2u_2 v_2$

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$\langle u, v \rangle = (3u_1)(3v_1) + (2u_2)(2v_2)$

$\langle u, v \rangle = 9u_1v_1 + 4u_2v_2$

(b) $u = (-3, 2)$ $\quad v = (1, 7)$

$\langle u, v \rangle = 9(-3)(1) + 4(2)(7)$

$= -27 + 56$

$= 29.$

QNO 10:

(a) $\langle u, v \rangle = 5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2$

$\langle u, v \rangle = u^T A v$

$= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$= \begin{bmatrix} 2u_1 - u_2 & u_1 + 3u_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$= (2u_1 - u_2)v_1 + (u_1 + 3u_2)v_2$

$= 2u_1v_1 - u_2v_1 + u_1v_2 + 3u_2v_2$

$\langle u, v \rangle = (2u_1)(2v_1) + (-u_2)(v_1) + (u_1v_2) + (3u_2)(3v_2)$

$= 4u_1v_1 - u_2v_1 + u_1v_2 + 9u_2v_2$

The matrix does not generate the given inner product

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(b) $u = (0, -3) \quad v = (6, 2)$

$\langle u, v \rangle = 5u_1 v_1 - u_1 v_2 - u_2 v_1 + 10 u_2 v_2$

$= 5(0)(6) - (0)(2) - (-3)(6) + 10(-3)(2)$

$= 18 - 60$

$= -42.$

Ex 6.1

QNO12:

(a) $p = -2 + 3x + 2x^2$

$a_0 = -2$

$a_1 = 3$

$a_2 = 2$

$\|p\| = \sqrt{a_0^2 + a_1^2 + a_2^2}$

$= \sqrt{(-2)^2 + (3)^2 + (2)^2}$

$= \sqrt{4 + 9 + 4}$

$= \sqrt{17}$

(b) $p = 4 - 3x^2$

$a_0 = 4 \quad a_1 = 0 \quad a_2 = -3$

$\|p\| = \sqrt{(4)^2 + (0)^2 + (-3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

QNO14: $p = 3 - x + x^2 \quad q = 2 + 5x^2$

$d(p, q) = \|p - q\|$

$(p - q) = 3 - 2 + (-1 + 0)x + (1 - 5)x^2$

$= 1 - x - 4x^2$

$a_0 = 1 \quad a_1 = -1 \quad a_2 = -4$

$d(p, q) = \sqrt{(1)^2 + (-1)^2 + (-4)^2} = \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

QNO13:

$\|A\| = \sqrt{\langle A, A \rangle} = \sqrt{a_{11}^2 + a_{12}^2 + a_{21}^2 + a_{22}^2}$

(a) $A = \begin{bmatrix} -2 & 5 \\ 3 & 6 \end{bmatrix}$

$= \sqrt{(-2)^2 + (5)^2 + (3)^2 + (6)^2}$

$= \sqrt{4 + 25 + 9 + 36}$

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$= \sqrt{74}$

(b) $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\|A\| = \sqrt{(0)^2 + (0)^2 + (0)^2 + (0)^2}$

$= 0$

QNO 15:

(a) $A = \begin{bmatrix} 2 & 6 \\ 9 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 7 \\ 1 & 6 \end{bmatrix}$

$d(A-B)$

$\begin{bmatrix} 2+4 & 6-7 \\ 9-1 & 4-6 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 8 & -2 \end{bmatrix}$

$\|A-B\| = \sqrt{(6)^2 + (-1)^2 + (8)^2 + (-2)^2}$

$= \sqrt{105}$

(b) $A = \begin{bmatrix} -2 & 4 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -5 & 1 \\ 6 & 2 \end{bmatrix}$

$d(A-B) = \begin{bmatrix} -2+5 & 4-1 \\ 1-6 & 0-2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -5 & -2 \end{bmatrix}$

$\|A-B\| = \sqrt{(3)^2 + (3)^2 + (-5)^2 + (-2)^2}$

$= \sqrt{47}$

QNO 16:

Let $x_0 = -2$ $x_1 = 0$ $x_2 := 2$

$p(-2) = 1 + (-2) + (-2)^2 = 1 - 2 + 4 = 3$

$q(-2) = 1 - 2(-2)^2 = -7$

$p(0) = 1 + (0) + (0)^2 = 1$

$q(0) = 1 - 0 = 1$

$p(2) = 1+2+4 = 7$
$q(2) = 1-2(4) = -7$

(a) $\langle p, q \rangle$

$\langle p, q \rangle = (3)(-7) + (1)(1) + 7(-7)$
$= -21 + 1 - 49$
$= -69.$

(b) $\|p\| = \sqrt{p(-2)^2 + p(0)^2 + p(2)^2}$
$= \sqrt{3^2 + 1^2 + 7^2}$
$= \sqrt{9 + 1 + 49}$
$= \sqrt{59}.$

(c) $d(p, q)$
$= 1-1 + (1+0)x + (1+2)x^2$
$= x + 3x^2$
$(p-q)-2 = -2 + 3(-2)^2$
$= -2 + 12 = 10$

$(p-q)\,0 = 0$
$(p-q)2 = 2 + 3(2)^2$
$= 2 + 12 = 14$

$d(p, q) = \sqrt{(10)^2 + (0)^2 + (14)^2}$
$= \sqrt{296}$

QNO 17: $x_0 = -1$ $x_1 = 0$ $x_2 = 1$ $x_3 = 2$

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$p = x + x^3$ $q = 1 + x^2$

$x_0 = -1$

$-1 + (-1)^3$ $= 1 + (-1)^2$

$-1 - 1$ $= 1 + 1$

$= -2$ $= 2$

$x_1 = 0$

$= 0 + (0)^3$ $= 1 + (0)^2$

$= 0$ $= 1$

$x_2 = 1$

$= 1 + (1)^3$ $= 1 + (1)^2$

$= 2$ $= 2$

$x_3 = 2$

$= 2 + (2)^3$ $1 + (2)^2$

$= 10$ 5

$\{p, q\}$

$(p, q) = (-2)(2) + 0(1) + 2(2) + 10(5)$

$= -4 + 4 + 50$

$= 50$

$\|p\|$

$\|p\| = \sqrt{(-2)^2 + (0)^2 + (2)^2 + (10)^2}$

$= \sqrt{108}$

Q NO 18: find $\|w\|$ $w = (-1, 3)$

(a) $\|w\| = \sqrt{(-1)^2 + (3)^2} = \sqrt{1+9} = \sqrt{10}$

(b) $3u_1v_1 + 2u_2v_2$

$\|w\| = \sqrt{3(-1)^2 + 2(3)^2}$

$= \sqrt{3 + 18} = \sqrt{21}$

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(c) $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$$Aw = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1+6 \\ 1+9 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\langle w, w \rangle = (Aw)^T (Aw) = \begin{bmatrix} 5 & 10 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$= 25 + 100$$
$$= 125$$
$$\|w\| = 5\sqrt{5}$$

Q NO 19: $u = (-1, 2)$ $v = (2, 5)$

$w = u - v = (-3, -3)$

(a)

$d(u, v) = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

(b) $3u_1v_1 + 2u_2v_2$

$d(u, v) = \sqrt{3(-3)^2 + 2(-3)^2} = \sqrt{27 + 18} = \sqrt{45} = 3\sqrt{5}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$$Aw = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ -3 \end{bmatrix} = \begin{bmatrix} -3-6 \\ 3-9 \end{bmatrix} = \begin{bmatrix} -9 \\ -6 \end{bmatrix}$$

$d(u, v) = \sqrt{(-9)^2 + (-6)^2} = \sqrt{81 + 36} = \sqrt{117} = 3\sqrt{13}$

Q NO 20: (a) $\langle u+v, v+w \rangle$

$\langle u+v, v+w \rangle = \langle u, v \rangle + \langle u, w \rangle + \langle v, v \rangle + \langle v, w \rangle$

$= 2 + 5 + \cancel{4} - 3$

$= 8$

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(b) $\langle 2v - w, 3u + 2w \rangle$

$= 6\langle u, v\rangle + 4\langle v, w\rangle - 3\langle w, u\rangle$
$- 2\langle w, w\rangle$

$= 6(2) + 4(-3) - 3(5) - 2(49$
$= 12 - 12 - 15 - 98$
$= -113$

(c) $\langle u - v - 2w, 4u + v\rangle$

$= 4\langle u, u\rangle + \langle u, v\rangle - 4\langle v, u\rangle - \langle v, v\rangle$
$- 8\langle w, u\rangle - 2\langle w, v\rangle$

$= 4(1) + 2 - 4(2) - 4 - 8(5) - 2(-3)$
$= -40$

(d) $\|u + v\| = \sqrt{\langle u+v\rangle, \langle u + v\rangle}$
$\sqrt{\langle u, u\rangle + \langle u, v\rangle + \langle v, u\rangle \langle v, v\rangle}$
$\sqrt{1 + 2 + 2 + 4}$
$= \sqrt{9}$
$= 3$

(e) $\|2w - v\|$
$\|2w - v\| = \sqrt{\langle 2w - v\rangle \langle 2w - v\rangle}$
$= \sqrt{4\langle w, w\rangle - 2\langle w, v\rangle - 2\langle v, w\rangle + \langle v, v\rangle}$
$= \sqrt{4(49) - 2(-3) - 2(-3) + 2}$
$= \sqrt{212}$

(f) $\|u - 2v + 4w\|$

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QNO 20:

(f) $\|u - 2v + 4w\|$

$$= \sqrt{\langle u - 2v + 4w, u - 2v + 4w \rangle}$$

$$= \sqrt{\langle u,u\rangle - 2\langle u,v\rangle + 4\langle u,w\rangle - 2\langle v,u\rangle + 4\langle v,v\rangle - 8\langle v,w\rangle + 4\langle w,u\rangle - 8\langle w,v\rangle + 16\langle w,w\rangle}$$

$$= \sqrt{\langle u,u\rangle + 4\langle v,v\rangle - 4\langle u,v\rangle - 16\langle w,v\rangle + 8\langle u,w\rangle + 16\langle w,w\rangle}$$

$$= \sqrt{1 + 4(4) - 4(2) - 16(-3) + 8(5) + 16(49)}$$

$$= \sqrt{881}$$

QNO 23:

$u = \langle u_1, u_2 \rangle \quad v = \langle v_1, v_2 \rangle$

(a) $\langle u, v \rangle = 3u_1v_1 + 5u_2v_2$

(i) For all $u = \langle u_1, u_2 \rangle \in R^2$

$$\langle u, u \rangle = 3u_1u_1 + 5u_2u_2$$
$$= 3u_1^2 + 5u_2^2 \geq 0$$
$$3u_1^2 + 5u_2^2 \geq 0$$

Equality holds if $u_1 = u_2 = 0$ i.e $u = 0$

$$u = \langle u_1, u_2 \rangle = \langle 0, 0 \rangle$$
$$u = 0$$

(ii)

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(ii) $\langle u, v \rangle = 3u_1 v_1 + 5u_2 v_2$

$\langle v, u \rangle = 3v_1 u_1 + 5v_2 u_2$

$\langle u, v \rangle = \langle v, u \rangle$

(iii) $\langle au + bv, w \rangle$

$= a\langle u, w \rangle + b\langle v, w \rangle$

$= \langle (au_1 + bv_1, au_2 + bv_2), (w_1, w_2) \rangle$

$\langle au + bv, w \rangle = 3(au_1 + bv_1)w_1 + 5(au_2 + bv_2)w_2$

$= 3au_1w_1 + 3bv_1w_1 + 5au_2w_2 + 5bv_2w_2$

$a(3u_1w_1 + 5u_2w_2) + b(3v_1w_1 + 5v_2w_2)$

$\langle au + bv, w \rangle = a\langle u, w \rangle + b\langle v, w \rangle$

So $(R^2, \langle , \rangle)$ is an inner product

(b) $\langle u, v \rangle = 4u_1v_1 + u_2v_1 + u_1v_2 + 4u_2v_2$

(i) for all $u = \langle u_1, u_2 \rangle \in R^2$

$\langle u, u \rangle = 4u_1u_1 + u_2u_1 + u_1u_2 + 4u_2u_2$

$= 4u_1^2 + 2u_2u_1 + 4u_2^2$

$= (2u_1)^2 + 2 \cdot 2u_1 \cdot \frac{u^2}{2} + \left(\frac{u_2}{2}\right)^2 - \left(\frac{u_2}{2}\right)^2 + 4u_2^2$

$= \left(2u_1 + \frac{u_2}{2}\right)^2 - \frac{u_2^2}{4} + 4u_2^2$

$= \left(2u_1 + \frac{u_2}{2}\right)^2 + \frac{15u_2^2}{4}$

$\left(2u_1 + \frac{u_2}{2}\right)^2 \geq 0$ & $\frac{15u^2}{4} \geq 0$

(i) $2u_1 + \frac{u_2}{2} = 0 \quad \Rightarrow u_2 = 0$

$2u_1 = 0$

$u_1 = 0$

$u, u = (0, 0)$

$u = 0$

(ii) $\langle v, u \rangle$

$= 4v_1u_1 + v_1u_2 + v_2u_1 + 4v_2u_2$

$\langle u, v \rangle = \langle v, u \rangle$

(iii) $\langle au + bv, w \rangle$

$= a\langle u, w \rangle + b\langle v, w \rangle$

$= \langle (au_1 + bv_1, au_2 + bv_2), (w_1, w_2) \rangle$

$= \langle au + bv, w \rangle = 4\langle au_1 + bv_1 \rangle w_1 + \langle au_2 + bv_2 \rangle w_2$
$+ \langle au_1 + bv_2 \rangle w_1 + 4\langle au_2 + bv_2 \rangle w_2$

$= 4au_1w_1 + 4bv_1w_1 + au_2w_2 + bv_1w_2 +$
$au_1w_1 + bv_2w_1 + 4au_2w_2 + 4bv_2w_2$

$= a(4u_1w_1 + u_2w_1 + u_1w_2 + 4u_2w_2) +$
$b(4v_1w_1 + v_1w_1 + v_2w_1 + 4v_2w_2)$

$\langle au + bv, w \rangle = a\langle u, w \rangle + b\langle v, w \rangle$

So $R^2 \langle\ \rangle$ is an inner product.

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a)

$\langle u, v \rangle = u_1 v_1 + u_3 v_3$

i)

$\langle u, u \rangle = u_1 v_1 + u_3 v_3$

$= v_1 u_1 + v_3 u_3$

$\langle u, u \rangle = \langle v, u \rangle$

ii)

$\langle au + bw, v \rangle = (au_1 + bw_1)v_1 + (au_3 + bw_3) v_3$

$= a(u_1 v_1 + u_3 v_3) + b(w_1 v_1 + w_3 v_3)$

$= a\langle u, v \rangle + b \langle w, v \rangle$

iii)

$\langle u, v \rangle \geq 0$ and $\langle u, u \rangle = 0 \iff u = 0$

$\langle u, u \rangle = u_1^2 + u_3^2 \geq 0$

However, for $u = (0, 1, 0) \neq 0$

$\langle u, u \rangle = 0^2 + 0^2 = 0$

axiom three fail so, not an inner product

b)

$\langle u, v \rangle = u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$

$\langle u, v \rangle = u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$

$\langle u, v \rangle = \langle v, u \rangle$

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Axiom ii)

Test with $u = (1,0,0)$

$v = (1,0,0)$, $a = 2$

$\langle 2u, v\rangle = (2\cdot 1)^2 \cdot 1^2 = 4 \neq 2 \cdot \langle u,v\rangle$

$= 2 \cdot 1^2 \cdot 1^2$

$= 2$ fail (not inner product

c)

$\langle u, v\rangle = 2u_1v_1 + u_2v_2 + 4u_3v_3$

i) $\langle u, v\rangle = 2u_1v_1 + u_2v_2 + 4u_3v_3$

$\langle u, v\rangle = \langle v, u\rangle$

ii)

$\langle au + bv, w\rangle = a\langle u, w\rangle + b\langle v, w\rangle$

$\langle au + bv, w\rangle = 2(au_1 + bv_1)w_1 + (au_2 + bv_2)w_2 + 4(au_3 + bv_3)w_3$

$= 2au_1w_1 + 2bv_1w_1 + au_2w_2 + bv_2w_2 + 4au_3w_3$

$+ 4bv_3w_3$

$= a(2u_1w_1 + u_2w_2 + 4u_3w_3) + b(2u_1w_1 + v_2w_2 + 4v_3w_3)$

$= a\langle u, w\rangle + b\langle v, w\rangle$

iii)

$\langle u, u\rangle = 2u_1^2 + u_2^2 + 4u_3^2$

$\because$ $u_1^2, u_2^2, u_3^2 \geq 0$

$2u_1^2 + u_2^2 + 4u_3^2 \geq 0$

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if $u = 0$ then $\langle u, u \rangle = 0$

if $\langle u, u \rangle = 0$ then

$$2u_1^2 + u_2^2 + 4u_3^2 = 0$$

$$u_1 = u_2 = u_3 = 0$$

$$\Rightarrow \quad u = 0$$

d)

$$\langle u, v \rangle = u_1 v_1 - u_2 v_2 + u_3 v_3$$

i)

$$\langle u, v \rangle = \langle v, u \rangle$$

$$\langle u, v \rangle = u_1 v_1 - u_2 v_2 + u_3 v_3$$

$$= v_1 u_1 - v_2 u_2 + v_3 u_3$$

$$\langle u, v \rangle = \langle v, u \rangle$$

ii)

$$\langle au + bv, w \rangle = a \langle u, w \rangle + b \langle v, w \rangle$$

$$\langle au + bv, w \rangle = (au_1 + bv_1) w_1 - (au_2 + bv_2) w_2 + (au_3 + bv_3) w_3$$

$$= au_1 w_1 + bv_1 w_1 - au_2 w_2 - bv_2 w_2 + au_3 w_3 + bv_3 w_3$$

$$= a(u_1 w_1 - u_2 w_2 + u_3 w_3) + b(v_1 w_1 - v_2 w_2 + v_3 w_3)$$

$$= a \langle u, w \rangle + b \langle v, w \rangle$$

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iii)

$\langle u, u \rangle = u_1^2 - u_2^2 + u_3^2$

for non-negative test

for $u = (0, 1, 0) \neq 0$

$\langle u, u \rangle = 0^2 - 1^2 + 0^2 = -1 < 0$ fail

so, not inner product

QNO 27:

$$\langle u, v\rangle = u_1v_1 + u_2v_3 + u_3v_2 + u_4v_4$$

(i) $\langle u, u\rangle = u_1u_1 + u_2u_3 + u_3u_2 + u_4u_4$

$$= u_1^2 + 2u_2u_3 + u_4^2$$

does not hold.

(ii) $\langle v, u\rangle = v_1u_1 + v_3u_2 + v_2u_3 + v_4u_4$

$$\langle v, u\rangle = \langle u, v\rangle$$

(iii) $(au_1 + bw_1)v_1 + (au_2 + bw_2)v_3 + (au_3 + bw_3)v_2 + (au_4 + bw_4)v_4$

$= auv_1 + bw_1v_1 + au_2v_3 + bw_2v_3 + au_3v_2 + bw_3v_2$
$au_4v_4 + bw_4v_4$

$= a(uv_1 + u_2v_3 + u_3v_2 + u_4v_4) +$
$b(w_1v_1 + w_2v_3 + w_3v_2 + w_4v_4)$

Not an inner product.