BAHRIA UNIVERSITY
ISLAMABAD CAMPUS

Theory of Automata
Assignment 3

This assignment will not be graded, but it is important for your Exam practice and for Assignment 4 which covers CLO-3 and will carry 15% marks.

1 Language to Context-Free Grammar [10 Points] (CLO)

For the following languages over the alphabet a, b, c, give context-free grammars that generate these languages. Briefly explain why your grammar generates the given language

a) $L_1 := \{w \in a, b, c^* \mid\mid w \mid_a = \mid w \mid_c\}$

b) $L_2 := \{a^n b^{2m} c^{n+m} \mid n, m \geq 0\}$

2 Context-Free Grammar to Language [10 Points] (CLO)

For the following grammars G_1 and G_2, which languages $L(G_1)$ and $L(G_2)$ are generated. Use set notation to provide langugaes.

- $G_1 = (\{S, B, C\}, \{a, b, c\}, P, S)$ where P has following rules

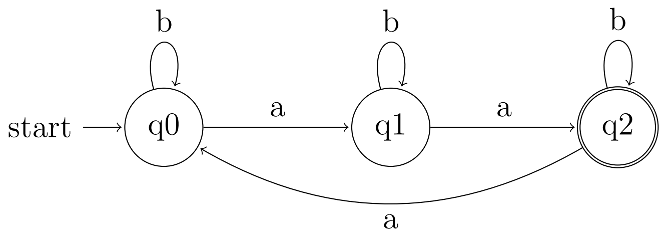
$$\begin{aligned} S &\rightarrow aBC \\ B &\rightarrow bB \mid \epsilon \\ C &\rightarrow cC \mid \epsilon \end{aligned}$$

- $G_2 := (\{S, A, B\}, \{a, b, c\}, P, S)$ where P has following rules

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow ab \mid aAb \\ B &\rightarrow bc \mid bBc \end{aligned}$$

3 Automata to CFG [5 Points] (CLO)

For the following NFA, provide a context-free grammar that produces the same language.



4 CFL closure properties 5 Points]

(CLO) Discuss that the class of context-free languages is closed under union.

5 Pumping Lemma for CFG [5 Points]

Describe the pumping lemma for CFGs

6 Chomsky Normal Form [10 Points]

(CLO) Provide CNF for the following Grammar

$$\begin{array}{rcl} S & \rightarrow & AB \mid BA \mid C \\ A & \rightarrow & DAD \mid a \\ B & \rightarrow & DBD \mid b \\ C & \rightarrow & A \mid B \mid abC \mid D \\ D & \rightarrow & a \mid b \end{array}$$

7 CYK Algorithm [10 Points] (CLO)

Let $\mathcal{G}$ be the following context-free grammar:

$$\begin{array}{rcl} S & = & SA \mid a \\ A & = & BS \\ B & = & BB \mid BS \mid b \mid c \end{array}$$

Use CYK Algorithm to show if $c = abcaa$ is in $L(\mathcal{G})$

8 PDA [10 Points]

Give a PDA that accepts the language of all non-palindromes

$$L = \{w \in \{a,b\}^* \mid w \neq w^{\mathcal{R}}\}$$

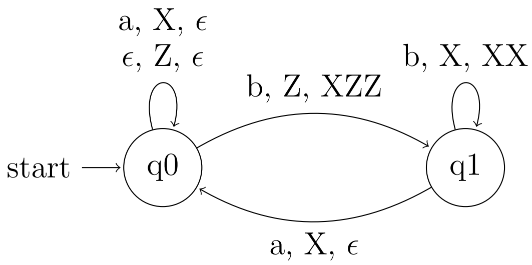
9 Grammar to PDA [15 Points]

Provide PDA for the following Grammar and show that *ccbacc* is an acceptance configuration run (use left derivation)

$$\begin{array}{rcl} S & \rightarrow & CX \\ X & \rightarrow & TC \mid SC \\ T & \rightarrow & AB \mid BA \\ A & \rightarrow & AA \mid a \\ B & \rightarrow & BB \mid b \\ C & \rightarrow & c \end{array}$$

10 PDA to Grammar [10 Points]

State the grammar GA. Simplify the resulting grammar as in the lecture by removing the rules whose right-hand sides contain nonterminal symbols that never appear on the left side of a rule. Do not simplify the grammar any further.



11 Marking algorithm [10 Points]

Use the marking algorithm to show which non-terminals are terminating and discuss if Language of this grammar is empty

$$\begin{array}{rcl} S & \to & ABCd \mid DCB \mid aFC \\ A & \to & DaC \mid bAB \mid cCd \\ B & \to & aSC \mid bCaS \mid cFG \\ C & \to & dE \mid ad \\ D & \to & BBc \mid CbE \mid CbA \\ E & \to & bBb \\ F & \to & aBd \\ G & \to & aAC \end{array}$$

12 Context Sensitive Grammar [10 Points]

Consider the context-sensitive grammar $G = (N, \{a, b\}, P, S)$ with the following rules in P :

$$\begin{array}{rcl} S & \to & AbBC \\ Ab & \to & ab \mid aAb \\ bB & \to & bA \mid bBC \\ C & \to & bC \mid b \end{array}$$

a) Give a derivation of the word $aababbb$.

b) Give a $w \in L(G)$ with minimal length and its derivative.

13 Turing Machine [10 Points]

a) Design a Turing machine that accepts exactly the palindromes over the alphabet $\{0, 1\}$.

b) Provide a three track truing machine that performs eXclusive-Or operation on two binary numbers where length of each number is three.

c) Give a description of the behavior of the following Turing machine M. Does M halt on all inputs? If so, which language is decided by M

δ	0	1	B
q_0	(q_0, B, R)	(q_1, B, R)	(q_3, B, R)
q_1	(q_2, B, R)	(q_1, B, R)	(q_3, B, R)
q_2	(q_0, B, R)	(q_4, B, R)	(q_3, B, R)
q_3	(q_3, B, R)	(q_3, B, R)	$(\overline{q}, 0, N)$
q_4	(q_4, B, R)	(q_4, B, R)	$(\overline{q}, 1, N)$

d) provide state transition diagram for table

e) Describe formally a Turing machine that decides the language $\{q \in \{0,1\}^* \mid\mid w \mid_0 = \mid w \mid_1\}$

f) Provide Gödle number of following. Also show machine's computations for $w = 00111$

δ	0	1	B
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	$(q_2, 0, L)$
q_2	$(q_2, 0, L)$	$(q_2, 1, L)$	(q_3, B, R)