Fallacy of Per-Weight and Per-Surface Area Standards, and Their Relation to Spurious Correlation

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BOTH IN PHYSIOLOGY and clinical medicine, the results of some measurements, for example, of oxygen consumption and cardiac output, are commonly expressed as per-weight or per-surface area ratios. Normal standards for both these variables, in fact, have been constructed on this basis; and as more similar variables come to be measured on the human being, the number of standards constructed and of results reported in this way, may be expected to increase. It seems useful to point out, therefore, that such standards are theoretically fallacious, and in practice (except under very special circumstances discussed below) misleading. The fallacy involved may be considered as a special case of that well known to statisticians as the spurious correlation of indices.

The writer first became aware of this situation when constructing some new standards for cardiac output in man (1). A cursory review of the literature revealed that the consequences of using these ratios were not at all widely realized. Examples immediately came to light where investigators had drawn positive conclusions not justified by their data, had been confused by a seemingly uninterpretable phenomenon in their results, had proposed a less effective and more biased normal standard in preference to a more effective and less biased one, had reduced a correlation between two physiological functions from a very high to a medium value, and had invented a new clinical syndrome. All these events were wholly or in major part due to the incautious use of ratios. Examples of each are given in the second half of this paper, by way of making the point that the theoretical discussion which now follows has a severely practical upshot. The examples concern cardiac output, basal metabolism standards in adults and children, the relation of body build and oxygen consumption, and the choice of plasma volume standards in man; and in the dog, the statistics of renal plasma flow and glomerular filtration rate.

The theoretical argument is such that the reader whose knowledge of statistics extends merely to the understanding of the words mean, ratio, standard deviation, correlation coefficient and regression line will be able to follow it in detail.

THEORETICAL

Error of the Ratio Standard. The subject can most easily be introduced by reference to cardiac output data on 50 healthy young men reported elsewhere (1). Figure 1 shows the stroke volume of the heart plotted against body weight in the human. Now the use of the ratio per-weight standard implies that in the normal person, the stroke volume is proportional to the weight; in fact that the expression

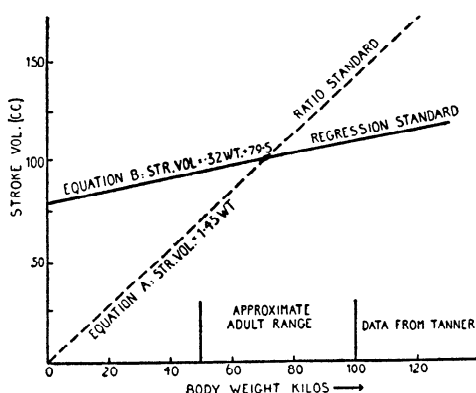$$\text{Str. vol.} = k. \text{ wt.} \tag{A}$$

Fig. 1. RELATION of stroke volume of the heart and body weight.

holds good over the range of values of weight for which the standard is used; that is, all normal adult values. The constant k is determined by the mean values of the series of data on which the standard is founded. Thus the line in figure 1 marked *Equation A* passes through the point of the two means, and by virtue of the form of the equation, through the origin also. If this line were really a newly constructed per-weight standard, we should judge any given stroke volume as normal or abnormal according to how far away from the line our value fell; that is, how far from the figure 1.45 cc/kg. the stroke volume was. This is in fact current clinical practice, using either the per-weight or per-surface area standard. Now actually, this expression by no means represents the mathematically 'best' or 'true' relation between stroke volume and weight. The best relation is that given by the regression equation (we are assuming rectilinearity of regression, which is justified at least as a first approximation) and this equation is

$$\text{Str. vol. (cc.)} = .32 \text{ wt. (kg.)} + 79.5. \tag{B}$$

This is the line called *Equation B* in figure 1, and it will be seen that it only coincides with that of *Equation A* at one point, the means. The ratio standard implies that the line of regression, that is the line fitting most closely the actual data, passes through the origin, and it is here that the fallacy lies. The assumption is unwarranted and obviously untrue, and results from an unjustified extension of the linear relationship into regions where it certainly does not apply, outside the adult range. The somewhat speculative argument that a person of no weight has no stroke volume is quite beside the point.

The difference in normal standards constructed from *equations A* and *B* is considerable. *Equation B* should be our proper standard, and it can be seen from figure 1 that a heavy man of 90 kg. will be given by the ratio per-weight standard a 'normal' stroke volume which is too high, by as much, in this instance, as 21 per cent. It is, indeed, at least partly this fallacious standard which has given rise to the idea that heavy men have relatively low stroke volumes; Starr's statement that "we found that the lowest values were frequently obtained on subjects definitely overweight" (2). Light people, on the contrary, will have too high an output as judged by the current standard, which gives them too low normal values. The error is about 22 per cent for a person of 50 kg. Consequently thin people have been said to have excessively high cardiac outputs and indeed some of the cases described by Starr and Jonas (3) as 'essential hyperkinemia', not those who had very elevated pulse rates, may have been suffering from no more formidable a disease than statistical artefact. The per-surface area standard leads to a similar, though not such a numerically large error.

We must now examine the matter in rather more detail. The actual regression equation, such as B, using X and Y as raw scores, b as the regression coefficient and a as the value of Y when $X = 0$, is

$$Y = bX + a. \qquad\qquad (C)$$

This line will only coincide with the ratio line $Y = kX$ when the two regression coefficients, b and k, are equal, and when a is zero. This condition can be conveniently put in another form for those who are more familiar with the terminology of correlation coefficients. If M_x and M_y are the means of the x and y variables, and r is the coefficient of correlation between them, the regression *equation C* becomes

$$Y = \frac{r\sigma_y}{\sigma_x} X + \left(M_y - \frac{r\sigma_y}{\sigma_x} M_x \right). \qquad\qquad (C1)$$

This line will only pass through the origin when the term in the bracket is zero, i.e. when

$$M_y = \frac{r\sigma_y}{\sigma_x} M_x$$

i.e. when

$$\frac{r\sigma_y}{M_y} = \frac{\sigma_x}{M_x}.$$ $$(C2)$$

The coefficient of variation, v, is given by

$$v = \frac{100\sigma}{M}$$

so that the condition above $(C2)$ reduces to

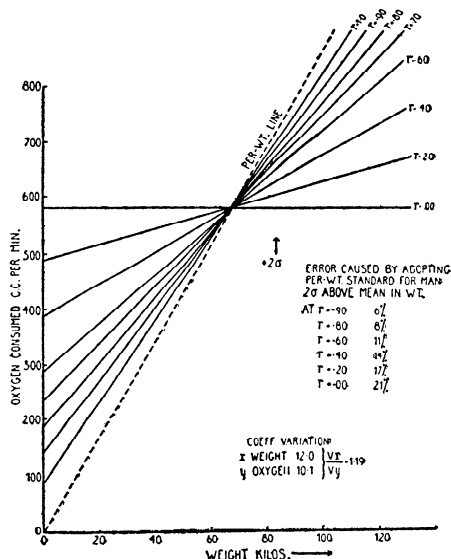$$rv_y = v_x$$

or

$$\frac{v_x}{v_y} = r.$$ $$(D)$$

Fig. 2. RATIO LINE and regression lines to show the varying values of r_{xy}.

This, then is the condition that the regression line should be coincident with the ratio line (since both lines go through the mean always). Figures 2, 3 and 4 illustrate this. In figure 2 data are oxygen consumption per minute following a standard exercise, and body weight, on 74 subjects (4). The dotted line is the per-weight line, and the solid lines show the effect of various values of r between weight and oxygen consumption (the one actually observed being 0.27). In this example the ratio of the coefficients of variation is 1.19, and since r can never be this large, no regression line can exist which would coincide with the ratio-line. But as r increases, the lines do get closer, and the error caused by the use of a ratio standard gets less. The percentage errors for a man of 2σ above the mean in weight are shown for various values of r. For

the real data, supposing the ratio line had been a standard, the *actually* best prediction for such a man would have been *called* about 15 per cent too low.

The actual amount by which the regression equation value differs from the ratio equation value can be very simply obtained from the equations of the two lines *A* and *C*. Thus

$$Y_{regr.} - Y_{ratio} := bX + a - kX$$

$$= (b - k)X + a \qquad (E)$$

When $b = k$, the difference between the equations is independent of X and equal to a; the lines are, in fact, parallel. Under any other circumstances however, $Y_{regr.} - Y_{ratio}$ is dependent upon the value of X, and gets progressively larger numerically as X departs more from its mean value. The lines, coincident at the mean, diverge more and more as we go away from the mean. The ratio equation gives Y/X, stroke volume/wt., as a constant k, but the regression equation leads to

$$\frac{Y}{X} = b + \frac{a}{X} \qquad (E1)$$

that is, stroke volume/wt. not constant, but inversely proportional to the weight. In actual figures, in the data of *equation B*

$$\frac{\text{Str. vol.}}{\text{wt.}} = .32 + \frac{79.5}{\text{wt.}}. \qquad (E2)$$

In other words, and in general, there *actually* exists a correlation between y/x

and x, and this can be shown to be positive if $\dfrac{v_x}{v_y} < r$ and negative if $\dfrac{v_x}{v_y} > r$.

The ratio standard ignores the existence of this correlation.

Relation of the Ratio Standard to Spurious Correlation. This is the crux of the matter. This correlation is merely a special example of a class of correlations very well known to statisticians and described originally by Karl Pearson in 1897, under the heading of 'spurious correlation between indices' (5). The general formula for the coefficient of correlation between two indices formed from the variables 1, 2, 3, 4 (i.e. indices $\frac{1}{3}$ and $\frac{2}{4}$) was shown by Pearson to be

$$r = \frac{r_{12} v_1 v_2 - r_{14} v_1 v_4 - r_{23} v_2 v_3 + r_{34} v_3 v_4}{\sqrt{v_1^2 + v_3^2 - 2r_{13} v_1 v_3} \sqrt{v_2^2 + v_4^2 - 2r_{24} v_2 v_4}} \qquad (F)$$

For the case y/x and $z/1$, variable 4 is a constant, $v_4 = 0$ and we have

$$r = \frac{r_{12} v_1 v_2 - r_{23} v_2 v_3}{\sqrt{v_1^2 + v_3^2 - 2r_{13} v_1 v_3} \sqrt{v_2^2}}$$

$$= \frac{r_{12} v_1 - r_{23} v_3}{\sqrt{v_1^2 + v_3^2 - 2r_{13} v_1 v_3}} \qquad (G)$$

and for our particular case, which is y/x and x, variable 2 $=$ variable 3, and we have

$$r = \frac{r_{12} v_1 - v_2}{\sqrt{v_1{}^2 + v_2{}^2 - 2r_{12} v_1 v_2}} \qquad (H)$$

Thus *expression H* actually measures the degree of departure of the ratio per-weight standard from the true regression standard, in given circumstances of v_x, v_y and r_{xy}. It is, for example, zero when the two standards coincide. In passing, we may note that this expression reduces, if x and y are uncorrelated, to

$$\frac{-v_2}{\sqrt{v_1{}^2 + v_2{}^2}} \qquad (I)$$

and this further reduces to the value $-.71$ if $v_x = v_y$. (This is the figure for our special case corresponding to Pearson's well known figure of 0.5 for the general one).
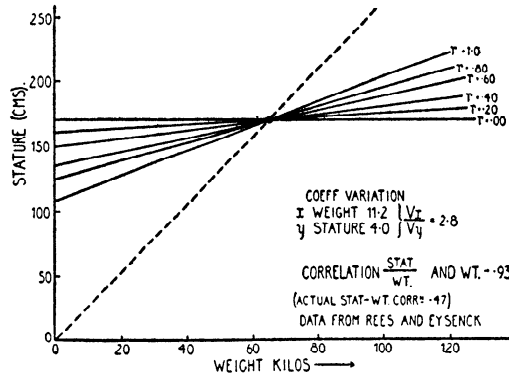

Fig. 3. To show DIFFERENCE between ratio and regression lines when $v_I/v_y > 1$.

In figure 2 the correlation between O_2/wt. and wt. for the actual data is

$$\frac{0.27 \times 10.1 - 12.0}{\sqrt{10.1^2 + 12.0^2 - 2 \times .27 \times 10.1 \times 12.0}} = -0.694$$

Figure 3 illustrates an even more marked case. (The data have for convenience been taken from Rees and Eysenck's study of 200 subjects (6), the present graphs being constructed only for the present occasion: the paper quoted has nothing to do with this fallacy.) Here the stature/wt. and wt. correlation is -0.93, largely because the value of v_x is so much greater than v_y. If the dotted line here represented a standard, that standard would be wildly wrong.

In all these illustrations, the correlation of y/x and x was negative; that is, heavy men were given too high a standard and their 'true' value would thus be said to be too low. With weight as the x-variable, it is rather hard to find an example of the opposite situation, but a somewhat artificial one, be-

tween age and weight, is shown in figure 4 (6). Here v_x is less than v_y and for high correlations of weight (x) and age (y), one would be led to a ratio standard of age for weight where heavy men would be given too low a standard for age. (Such an error introduced into our reckoning of age would lead to the social situation wherein the heavier a man was the younger he would be reckoned, and the lighter, the more aged; the effects on differential mortality from the behavior following such an assumption, and the activities of food firms, can well be imagined; but the example is hypothetical.) For low values of the age-weight correlation, the relation of ratio standard to true standard is reversed, and the two coincide at $r = 0.55$. Since in the actual data the age-weight correlation was zero, the (spurious) correlation of age/wt. and wt. is $-.48$.
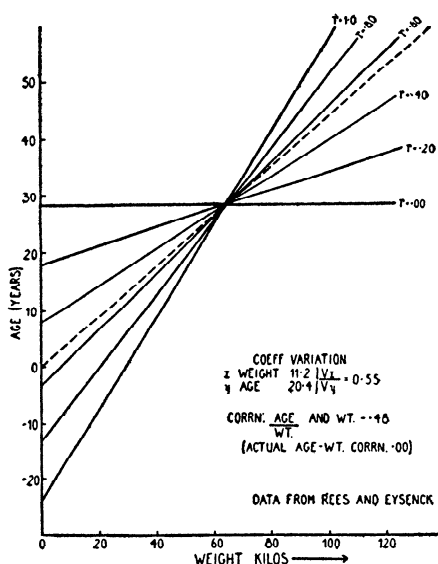

Fig. 4. To show difference between ratio and regression lines when $v_I/v_y < 1$.

EXAMPLES

Basal Metabolism

Adults. The first example of the use of fallacious standards concerned cardiac output and has already been given. That the per-weight or per-surface area standards for this variable are erroneous will not come as any particular shock to the majority of readers, who will have had no direct experience of the use of cardiac output figures. But nearly all must have used the per-surface area standards for basal metabolism and the reader must have had in mind from the beginning a query as to the fallacy in this case. The per-surface area standard is widely used, and appears to be reasonable. It is true that the original standards proposed by Harris and Benedict (7) were regression

ones, for the authors of this classical monograph were very well aware (p. 151) of the fallacy considered here, but Berkson and Boothby (8) showed that the per-surface area standard predicted as well as, or at any rate, not demonstrably worse than, regression standards of the Harris-Benedict type, for their Mayo Clinic data. The reason for this is soon discovered: the oxygen consumption-surface area relationship just happens to come very near satisfying the conditions given above for the ratio and regression standards to coincide. We cannot illustrate this for the data Berkson and Boothby actually tested since they do not give the necessary statistics. But Harris and Benedict give the following figures for 136 male subjects: coefficient of variation of surface area 8.89, of heat production 12.54; correlation coefficient between these two
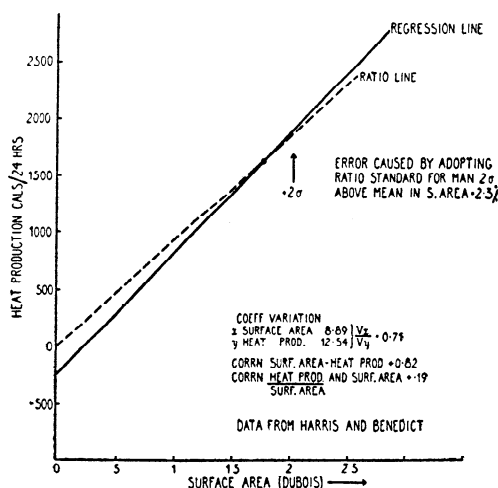

Fig. 5. RELATION BETWEEN the regression and ratio standards for basal metabolism.

variables, .82. For these figures, referring to *equation D*, $\dfrac{v_x}{v_y} = 0.71$ and this value is not very different from *r*. From *equation H* with variable 1 as *y* and 2 as *x*, we find that this measure of the difference between the two standards gives us a correlation of Heat prod/S. area and S. area of only +.19. This difference is too small in relation to the variability of the measurement itself to affect the predicted result. Figure 5 illustrates this. For a man 2σ above the mean in surface area, this difference is only 2.3 per cent of the man's measurement; and the standard deviation of a single individual from day to day about his mean is 3.5 per cent, and of a series of individuals of the same age, 5.8 per cent (9). Galvao's two series (10, 11) both separately and combined, have O_2/S. area ratio lines even more closely approximating the regressions.

In short, the oxygen consumption ratio standard *is*, very nearly, the special case where ratio and regression lines coincide. Even so, because the

lines do not *exactly* coincide, a large man has a very slightly greater chance of being called hyperthyroid than a small man, and this purely artefactually.

Children. In children's standards, however, the effect seems to be important, presumably because the variability of weight and surface area for some single-year age groups considerably exceeds that of adults. In discussing 4 different basal metabolism standards for children, Lewis, Duval and Iliff remark: "The closer any child comes to the mean values for height, weight and surface area of the group with which the standards were established, the less marked will be the discrepancy by the different methods of reference" (12). To emphasize their point they present the deviations of three 8-year-old boys from each of 4 standards. Three of the standards are essentially curvilinear regressions, of basal metabolism on surface area, weight and height, irrespective of age. The fourth is a per-surface area standard covering this single year of age. One boy is of average size and for him the standards agree well. One is small and the ratio standard places him about 14 per cent above the regression standards' estimates; the other is large and the ratio standard places him considerably below the regression standards' estimates. This is precisely the situation depicted in figure 1 of this paper, and the explanation is presumably similar. Probably this is also the explanation of the same authors' finding (13) that over the whole 2- to 12-year age range, the results of other workers average some 7 per cent for boys and 4 per cent for girls above theirs when the per-surface area standards are used, but are practically identical by the regression standards. These authors' children were somewhat larger than most of the others in the literature. They themselves remark in an earlier study that the per-surface area standard produces these positive deviations only in the case of children smaller than theirs (14): their claim that body build affects oxygen consumption needs careful examination by other methods than this before it can be accepted.

The same thing applies to many of the comparisons of basal metabolism between groups, racial and other, where mean differences of a few per cent may be reckoned significant; differences attributable to size must first be allowed for.

Oxygen Consumption and Body Build

The next example concerns this use of ratios in comparisons between groups. Seltzer has reported measurements of oxygen consumption at rest and during moderate and severe exercise for young men who were also measured anthropometrically (15). Oxygen consumption was reported as per-weight or per-surface area, and body measurements chiefly as indices. Considering the resting figures first, Seltzer divides his 34 subjects into two groups, one containing all those below the mean for the measurement or anthropological index in question, the other all those above it. He then compares the O_2/wt.

consumed by each group and considers whether the two O_2/wt. figures differ significantly. The first anthropological index taken is wt/stature, and it is shown that those subjects with a high wt/stature have significantly lower O_2/wt. consumptions than those with a low wt/stature. What effect is to be expected, however, from the spurious correlation involved?

We have seen above that a correlation between O_2/wt. and wt. will always exist (*equations E1* and *E2*) unless the condition $\frac{v_{wt.}}{v_{O_2}} = v_{wt.O_2}$ is satisfied. If $\frac{v_{wt.}}{v_{O_2}} > r$, the correlation between $\frac{O_2}{wt.}$ and wt. is negative, and if $\frac{v_{wt.}}{v_{O_2}} < r$, the correlation is positive. Taking an average value for v_{O_2} for Seltzer's data, we have a $\frac{v_{wt.}}{v_{O_2}}$ in this case of 1.8, far greater than r, which is 0.41. A large negative correlation therefore exists between $\frac{O_2}{wt.}$ and wt. Thus as the index $\frac{wt.}{stature}$ increases, $\frac{O_2}{wt.}$ must be expected to decrease purely as a result of the method used for presenting the data. The same is true for all anthropological indices which correlate positively with weight. Seltzer shows that this occurs for several indices, but his conclusion that more linear people have higher oxygen consumptions than stocky people is not justified by this data. Similar calculations, with similar results, can be made for the oxygen consumptions during exercise.

This is not to say, of course, that linear people do not in fact have higher oxygen consumptions. Such may well be the case, but other methods, of partial correlation or covariance analysis, are needed to demonstrate it. As a matter of a fact, there are two indices, span/stature and leg length/stature which remain significantly associated with oxygen consumption at rest when spurious relation is removed. It seems that men with short limbs and long trunks have higher oxygen consumptions relative to their surface area than do those with the opposite build.

Plasma Volume

The next example concerns the construction of standards and the choice between a per-weight and per-surface area standard for plasma volume. Gregerson has recommended that the per-weight standard be used for this measurement (16). Is this the best ratio standard available, and how great is the bias its use entails? These questions can be answered from the raw data on height, weight, surface area and plasma volume for 41 normal men given by Gibson and Evans (17). From this data the coefficients of variation of these variables can be calculated to be: height 4.9, weight 14.2, surface area 10.4 and plasma volume 15.8. The correlation coefficients are: plasma volume and height .72, weight .68, surface area .74. Does per-height, per-weight or per-

surface area standard most nearly coincide with its equivalent regression standard?

The $\frac{v_x}{v_y}$ for height is $\frac{4.9}{15.8}$ = .31 and r_{xy} for height is .72, a figure very considerably different. *Equation H* gives the correlation between $\frac{\text{plasma volume}}{\text{height}}$ and height to be as much as +.51. For weight, $\frac{v_x}{v_y}$ is .90, and r_{xy} .68; there is still considerable discrepancy between these 2 figures, and the correlation of $\frac{\text{plasma volume}}{\text{weight}}$ and weight is −.29. (This can be seen illustrated in Gibson and Evans' fig. 5B). For surface area $\frac{v_x}{v_y}$ is .66 and r_{xy} .74. The agreement is much better, and the correlation of $\frac{\text{plasma volume}}{\text{surface area}}$ and surface area only +.12. Clearly the per-surface area is the best of these ratio standards, and the bias its use entails is of the same order of magnitude as that present in the basal metabolism standards. The plasma volume data of Mather, Bowler, Crooke and Morris, on 53 normal men (18) gave rise to almost identical figures, and thus to the same conclusion.

There are really two criteria by which to judge the efficiency of ratio standards. One is the closeness with which $\frac{v_x}{v_y}$ approaches r, and the second is the amount of variance of the dependent variable that is accounted for by the independent variable. This is given by the square of the coefficient of correlation between the variables. In our plasma volume example, surface area accounts for $.73^2$ = 55 per cent of the variance of plasma volume, and weight accounts for $.72^2$ = 52 per cent. Again the advantage lies with surface area, but very slightly, and if this was the only consideration, the claims of the per-weight standard, as Gregerson says, would be strong.

Glomerular Filtration Rate and Renal Plasma Flow

A last example may be taken from Houcks' recent study of renal plasma flow and filtration rates in dogs (19). Both these renal functions were calculated per-weight and per-surface area; surface area being calculated either from weight alone or from weight and length combined. No choice is made between the two, yet a very strong reason indeed exists for perferring the per-weight figures. The coefficient of variation of filtration rate was 33.7, of effective renal plasma flow 36.2, of weight 28.2 and of surface area 19.6 (for 75 resting female dogs). $\frac{v_x}{v_y}$ for filtration rate and surface area is .58 and their r is .75,

giving a correlation of $\dfrac{\text{filtration rate}}{\text{surface area}}$ and surface area of $+.28$. The per-weight ratio does much better: $\dfrac{v_x}{v_y} = .84$, $r = .80$, correlation $\dfrac{\text{filtration rate}}{\text{weight}}$ and weight $-.06$. For effective renal plasma flow it is the same, with surface area $\dfrac{v_x}{v_y} = .54$ and $r = .71$ with a correlation of $\dfrac{\text{plasma flow}}{\text{surface area}}$ and surface area $= +.24$; for weight, $\dfrac{v_x}{v_y} = .78$, $r = .80$, correlation $\dfrac{\text{plasma flow}}{\text{weight}}$ and weight $-.03$. By the second criterion weight is the better too: it accounts for more of the variance of each renal function than does surface area, as can be seen from the correlation coefficients.

This last example serves finally to recall our attention to the spurious correlation between indices. The author gives the correlation coefficients of $\dfrac{\text{filtration rate}}{\text{weight}}$ and $\dfrac{\text{plasma flow}}{\text{weight}}$ as $.73$; and between $\dfrac{\text{filtration rate}}{\text{surface area}}$ and $\dfrac{\text{plasma flow}}{\text{surface area}}$ as $.79$. He remarks these figures are relatively high, whichever one is taken, and there is a straight-forward physiological implication of such a fact. But part of this correlation is spurious, and to the detriment of his implied thesis, for the actual straightforward correlation between filtration rate and plasma flow for the data is $.90$. The agreement has been unnecessarily lowered by the use of the indices.

DISCUSSION

There remain two points to be discussed. In considering desirable standards or ways of reporting data it is often implied or explicitly stated that the best ratio standard is that which produces the smallest coefficient of variation of the data, the ratio which gives the least spread. The idea is a widespread one, and I have failed to trace its origin since recent authors take its correctness for granted, without need of a supporting reference. It rests to some extent on the belief that "the fact that the coefficient of variation of the data so calculated (renal functions as per-surface area) has the small magnitude of 13.1 per cent indicates a high degree of correlation (between the renal functions and surface area)" (20).

This statement is not strictly justified. The coefficient of variation of an index, variables $\frac{1}{2}$, is

$$\frac{100 \sqrt{v_1{}^2 - 2r_{12}v_1 v_2 + v_2{}^2}}{1 - r_{12} v_1 v_2 + v_2{}^2} \qquad (J)$$

Thus while it is true that the larger r is the lower is the coefficient of variation of the index, v_1 and v_2 play a larger part than does r. Even when $v_1 = v_2$,

$$v_{\text{index}} = \frac{100v_1 \sqrt{2(1 - r)}}{1 + v_1^2(1 - r)} \tag{K}$$

and thus depends largely on the value of v_1.

There is only one circumstance in which this minimizing of the coefficient of variation of the ratio could actually produce the best standards. That would be if it made them coincide with the regression standard, in other words if minimizing the expression J reduced to the condition $\dfrac{v_1}{v_2} = r_{12}$. This it does not in fact do; the minimal J is actually secured by the rather more complicated condition

$$r_{12} = \frac{1}{2}\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right)$$

Lastly, the relation between linear regression standards and standards of the form per-weight$^\alpha$ should be mentioned. The power standards, recently reviewed in the case of oxygen consumption by Kleiber (21), are more similar to the regression standards than to the ordinary ratio ones, despite first appearances. The reason for this is that the value of α is obtained by fitting a regression line to the logs of the variables. Thus the investigator relating oxygen consumption and body weight plots the log. of oxygen against the log. of weight and fits a regression line to the resulting scatter diagram. The equation of this regression line is

$$\log. O_2 = \alpha \log. \text{wt.} + \beta \tag{L}$$

and this is equivalent to

$$O_2 = \beta' \text{wt.}^\alpha. \tag{M}$$

Is then the power standard as valid statistically as the regression standard and can the two be used replaceably?

The answer to the second of these questions is unequivocally no: and to the first, it depends on a particular circumstance, the nature of which does not seem to be widely realised amongst biologists. It has recently been discussed with masterly clarity by Sholl (22). The point is a simple one. When we fit a regression line to 2 sets of variables, we choose the line about which the sums of squares of deviations are a minimum. We do this because it follows from the method of maximum likelihood, a method for finding so-called efficient statistics which have great advantages over inefficient statistics (of which the ratio standard is an example). But the method of maximum likelihood only leads to this least-squares solution if for a given value of x, the independent

variable, the values of y, the dependent variable, are normally distributed. This is the thing that, as Sholl points out, is often overlooked. Now if we plot the logs. of x and y and fit by least squares a line to this graph, we are tacitly assuming that the logs. of y are normally distributed, not the raw values of y. It is not the case that if y is normally distributed log. y is also normally distributed; on the contrary. But in the usual method of constructing a power standard, α is obtained by *equation L*, which produces the best value for α if, and only if log. y is normally distributed and y itself is skewed. Where this is indeed the case, the power standard is the appropriate standard, and more valid than the linear regression standard. But where it is *not* the case, the power standard obtained this way is erroneous. Sholl has shown that a valid figure for α in *equation M can* be obtained in cases where y is normally distributed, but that it differs (often very considerably) from that produced by the method we have discussed involving *equation L*.

The upshot of this is that the power standard as usually calculated is only valid when there are sound statistical reasons for saying that the logarithms of the dependent variable, such as cardiac output, are normally distributed whereas the raw values of the dependent variable are significantly skewed. When this is not so, the regression standard is the standard which should be used. If, on biological grounds, it is felt that a standard of the form $O_2 = k$ wt.$^\alpha$ is desirable, with α expressing active mass, then, if the variable O_2 is not significantly skewed, α must be found by Sholl's methods.

SUMMARY

The present standards which express physiological functions such as oxygen consumption and cardiac output as per-weight or per-surface area are shown to be theoretically fallacious and in practice (except in a special case) misleading. The fallacy involved in their application may be considered a special example of the spurious correlation between indices. Examples are cited of the undesirable practical effects of using such standards; the examples involve cardiac output, oxygen consumption and plasma volume in man, and glomerular filtration rate and renal plasma flow in the dog. The assumption that the best ratio standard is the one which minimizes the spread of the data concerned is criticized. The conditions are examined under which the power standard (e.g. $O_2 = $ wt.$^\alpha$), as usually obtained, is valid; and the more common conditions where it is invalid unless α is obtained, by Sholl's method.

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