11/1/2019

Simulation Project

Assessing the longevity of two competing semiconductor circuit designs through simulation

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The Problem

An electrical engineering firm is trying to differentiate between two possible semiconductor circuit designs to be used in their next big product. One of the most important factors for the product is the longevity of the individual parts due to skyrocketing repair and replacement costs. Based on manufacturer specifications, Circuit A's longevity (in hours) is exponentially distributed with λ = 10-4 and Circuit B's longevity (in hours) is lognormally distributed with μ = 6 and σ2 = 5.4. The Company needs answers to two scenarios before deciding:

1. Use a simulated sample of size 1000 to estimate the probability that a Circuit A lasts longer than Circuit B.
2. Estimate the probability Circuit A lasts more than twice as long as Circuit B.

Assumptions & Expectations

Based on the specifications, Circuit A is designed to last for an expected value of 10,000 hours, while Circuit B is expected to last for 6,002 hours (*see fig. 1*).

Circuit A

$$\mu = \frac{1}{\lambda} = \frac{1}{10^{-4}} = 10{,}000 \text{ hours}$$

$$\sigma^2 = \frac{1}{\lambda^2} = \frac{1}{10^{-8}} = 100{,}000{,}000 \text{ hours}$$

$$\sigma = \frac{1}{\lambda} = \frac{1}{10^{-4}} = 10{,}000 \text{ hours}$$

Circuit B

$$\mu = e^{\mu+\sigma^2/2} = e^{6+5.4/2} = 6002\ hours$$

$$\sigma^2 = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} = e^{2*6+2*5.4} - e^{2*6+5.4} = 7{,}942{,}335{,}309\ hours$$

$$\sigma = \sqrt{\sigma^2} = 89{,}120\ hours$$

Fig 1: expected value, variance, and standard deviation for Circuit A & Circuit B

This indicates the simulated longevity expected values, or means, should be somewhere in the range of 10,000 hours and 6,000 hours for Circuit A and Circuit B, respectively. Circuit A & Circuit B are also independent of each other since there is not a relationship between the two competing circuit designs.

Simulation & Distributions

Using the parameters given from the manufacturer specifications, SAS code can be written to simulate the longevity of 1,000 random instances of each circuit (Appendix A). From that code, a simulation is achieved, which gives random values for 1,000 instances of each circuit design. In order to ensure the simulation is accurate, a comparison can be made from the expectation above to the distribution analysis below.

Circuit A’s simulation appears to be accurate and exponentially distributed based on 3 factors:

1. The mean is close to 10,000 and is slightly greater than the median (*see fig. A1*)

Basic Statistical Measures			
Location		Variability	
Mean	9565.740	Std Deviation	9263
Median	6983.026	Variance	85794496
Mode	.	Range	61590
		Interquartile Range	10967

Fig A1: Basic Statistical Measure for Circuit A

2. The distribution of values appears to fit the exponential distribution model (*see fig. A2*)

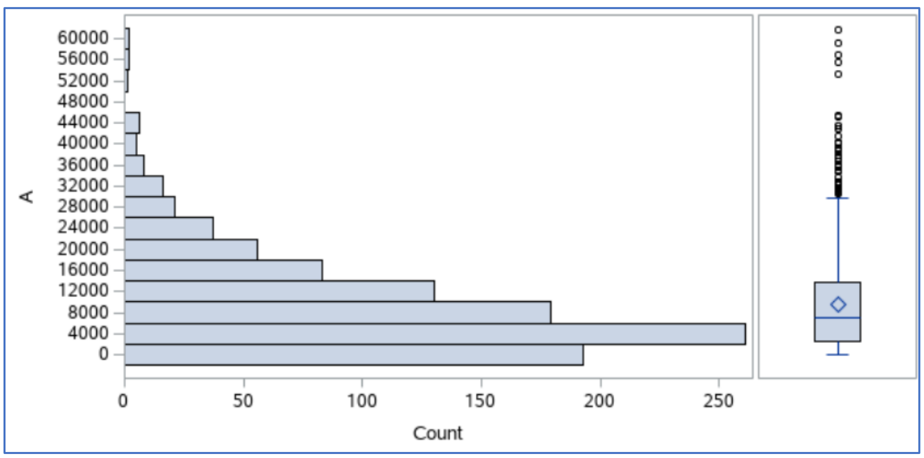

Fig A2: Histogram distribution and box and whisker plot for Circuit A

3. The Q-Q model is concave with a high concentration at the left and middle, and a few outliers at the right end (*see fig. A3*)

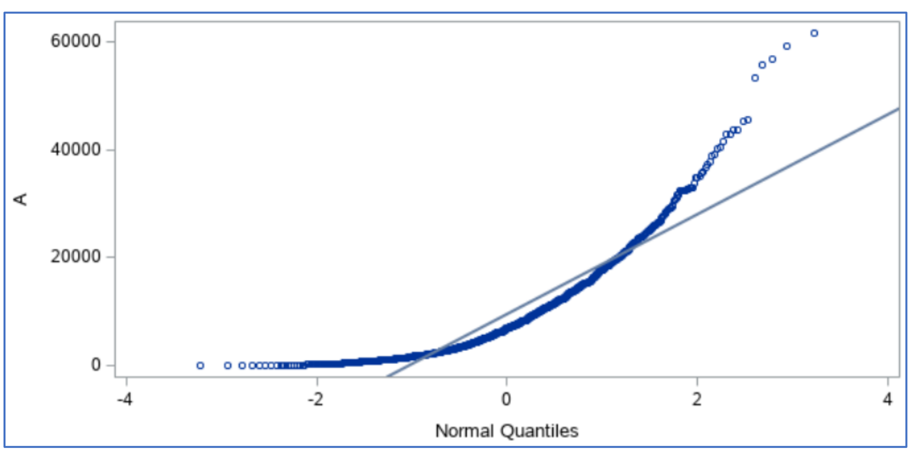

Fig A3: Q-Q Plot for Circuit A

Circuit B's simulation appears to be exponentially distributed based on 3 factors:

1. The mean is near 6,000 and is significantly greater than the median (*see fig. B1*)

Basic Statistical Measures			
Location		Variability	
Mean	4987.872	Std Deviation	24003
Median	392.346	Variance	576147464
Mode	.	Range	538280
		Interquartile Range	2067

Fig B1: Basic Statistical Measure for Circuit B

2. The distribution of values appears to fit the lognormal distribution model (*see fig. B2*)

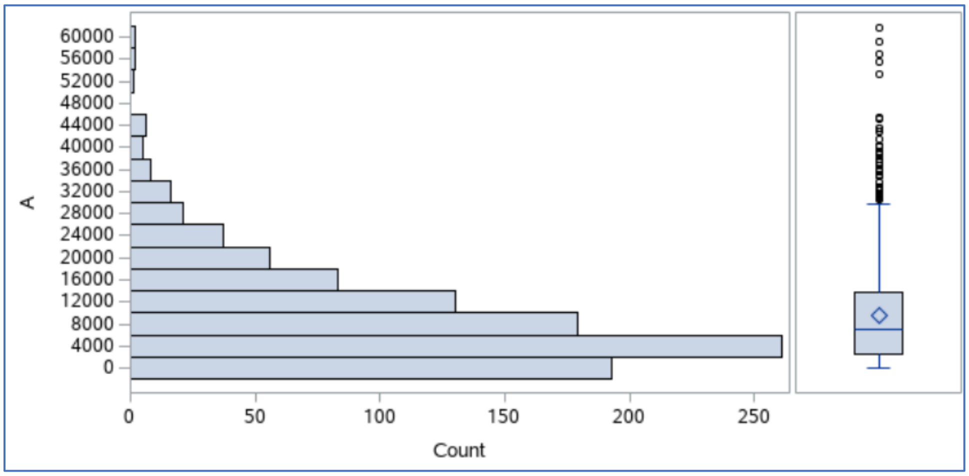

Fig B2: Histogram distribution and box and whisker plot for Circuit B

3. The Q-Q model is flat with a few outliers at the right end (*see fig. B1*)

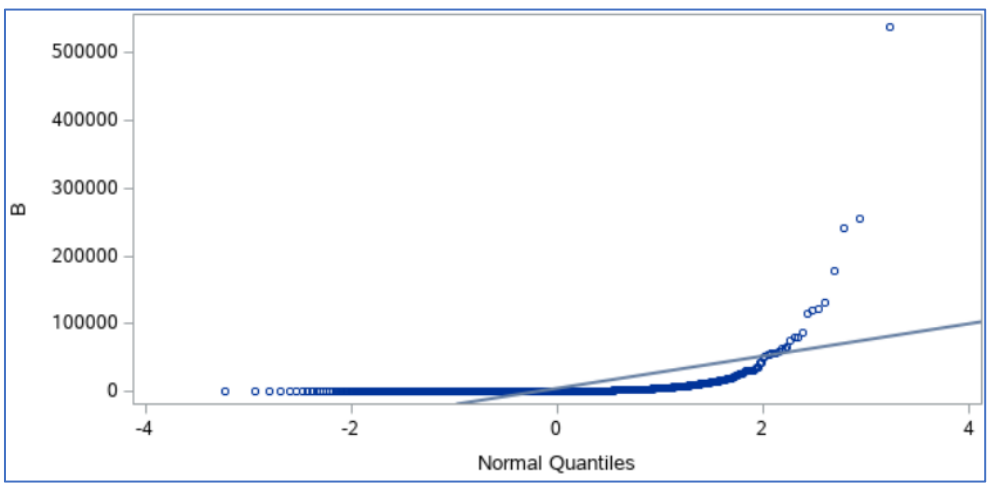

Fig B3: Q-Q Plot for Circuit B

Simulation probability

Based on the SAS simulation (*Appendix A*), the probability that Circuit A will outlast Circuit B is approximately 83.40% (*see fig. C1)*.

P	Frequency	Percent
0	166	16.60
1	834	83.40

Fig C1: Frequency of Circuit A's longevity greater than Circuit B's longevity (P=1) and Circuit A's not greater than Circuit B's (P=0)

This is calculated through the simulation by subtracting Circuit B's longevity from Circuit A's longevity for each instance. If the difference is positive, the instance is flagged as a 1, otherwise it is a 0. The flags are then counted, and a percentage is determined based on 1,000 instances.

Based on the SAS simulation (*Appendix A*), the probability that Circuit A will last twice as long as Circuit B is approximately 76.10% (*see fig. C2)*.

P	Frequency	Percent
0	239	23.90
1	761	76.10

Fig C2: Frequency of Circuit A's longevity is twice as long as Circuit B's longevity (P=1) and Circuit A's less than Circuit B's (P=0)

This is calculated through the simulation by subtracting two times Circuit B's longevity from Circuit A's longevity for each instance. If the difference is positive or zero, the instance is flagged as a 1, otherwise it is a 0. The flags are then counted, and a percentage is determined based on 1,000 instances.

Conclusion

Based on the manufacturer specifications, the expected longevity for Circuit A is 10,000 hours, or ~4,000 more hours than Circuit B. Through a simulation comparing 1,000 instances of each Circuit based on manufacturer specifications, Circuit A had a probability of 83.40% to last longer than Circuit B, and a probability of 76.10% to last twice as long.

Appendix A: SAS Code

```
/*Simulate 1000 samples of longevity from product A and product B*/

%let N = 1000; /*Number of samples*/
data circuits;
	call streaminit (4321); /* Seed to make experiment replicable*/
	do i = 1 to &N; /*loop from 1 -> 1000 for each variable*/
	A = rand ("exponential", (1/.0001));	/* A ~ exp(1/λ)	*/
	B = rand ("lognormal", 6, SQRT(5.4));  /* B ~ log(μ,σ)	*/
	Diff = A-B; /*Calculate the difference of B from A. This will be used to calculate probability below*/
	Diff2 = A-2*B; /*Calculate the difference of (2*B) from A. This will be used to calculate probability below*/
	output;
	end;
	drop i;
run;

/* Determine mean and variance of simulated plate areas*/
proc univariate data = circuits plots;
	var a;
run;

proc univariate data = circuits plots;
	var b;
run;

/* Determine the probability that circuit A will last longer than circuit B*/
data prob1;
	set circuits;
	if Diff > 0 then P = 1; else P = 0; /* If the value of "Diff" calculated above is greater than 0, flag the
item as a 1, otherwise 0*/
run;

proc freq data = prob1; /* This will count the frequency of P values that were calculated in the step above,
giving the probability that Circuit A will last longer than Circuit B */
	tables P / nocum;
run;

/* Determine the probability that circuit A will last long twice as long as circuit B*/
data prob2;
	set circuits;
	if Diff2 => 0 then P = 1; else P = 0; /* If the value of "Diff2" calculated above is greater than or
equal to 0, flag the item as a 1, otherwise 0*/
run;

proc freq data = prob2; /* This will count the frequency of P values that were calculated in the step above,
giving the probability that Circuit A will last twice as long as Circuit B */
	tables P / nocum;
run;
```