ME-465: Design Project Progress Report #2

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1. Background

This project involves the design of a proposed winch system which must meet some given minimum requirements. From the provided project description:

> *A winch, schematically shown in the figure below* (Figure 1)*, is operated by a worm-gear mesh. The input torque to the worm is provided by an electric motor that rotates at 1,500 rpm. The expected speed of the worm gear is between 30 and 35 rpm. The peak torque requirement for the winch is about 4,000 lbf·in, the operating ambient temperature is 120 °F, and the average output power doesn't have to exceed 1.2 hp. Knowing that the winch drum radius is 8 inch and that the winch operates 4 to 5 hours a day, perform the following tasks, which combined lead to the overall design project.*

2: Problem Statement

The second component of designing the winch is to select an appropriate shaft which will support the worm gear. From the provided project description:

> *Design the shaft (from bearing 1 to bearing 2 in figure "side view"), for stress and deflection constraints, that will support the worm gear. Your report should incorporate, but not be limited to, the following… A free body diagram of the shaft holding the worm gear. The bending moment, shear, and torque diagrams for the shaft holding the worm gear. A computer code that calculates the factor of safety and deflections on the shaft that you design to support the worm gear and the drum. A manufacturing part drawing for the designed shaft including appropriate GD&T specifications.*

3. Objectives

The problem statement requests that the worm gear shaft is capable of delivering on the following objectives:

- Deliver 4,000 in·lbf torque from the worm gear to the winch drum
- Perform without excessive deformation in bending or torsion

Per our gear analysis and design in Progress Report #1, the motor does not supply sufficient power to drive the winch drum at the desired speed and torque. As such, the input and output speed was reduced to 1,200 rpm, and 15-20 rpm respectively, which reduced the demand on the motor, so that it could function with the given power.

4. Approach

Using the gears selected in the previous report along with the known inputs, values will be calculated for the stresses the shaft will need to undergo. Most sizes of the shaft will be dictated by the accompanying gears, but the length can be varied more easily. A simple design will be created to meet all given parameters, which will then be analyzed with use of a free body diagram. The shear and bending will be calculated, and then the design will be modified as needed to create a more complete, usable design. Once the shaft has met all the needs given, the design will then be completed by adding the bearing and additional gear part locations. A final analysis will be conducted to verify that the shaft meets all parameters. Should any sections require further in depth analysis or discussion, that will be done so at this point.

5. Assumptions

To generate a preliminary shaft design, we must make assumptions about all required shaft components. Bearings will be located axially against a step in shaft diameter on one side, and a retaining ring on the other side. The worm gear will be keyed to the shaft using a square key and located axially using retaining rings. Retaining rings provide advantages over other locating methods, such as simple assembly and ease of disassembly for performing maintenance, at the cost of requiring flat-bottom grooves in the shaft which increase stresses. If these stress concentrations prove too great for the shaft design, another locating feature could be implemented instead.

Bearings will be selected in a future report, but because their dimensions are required to form an acceptable preliminary shaft design, initial dimensions must be assumed. Because of the relatively high axial loading of worm gears, bearings will be assumed to be tapered roller bearings. They will be assumed to be 0.75 in. wide, since this is a typical approximate width of tapered roller bearings in the expected range of shaft diameters.

The shaft length should be kept to a minimum to reduce deflection. The shaft must be at least as wide as the gearcase plus the width of the two bearings, plus enough length beyond Bearing 2 to couple to the winch drum shaft. The minimum internal width of the gearcase for the gears selected in Progress Report #1, as motivated by Figure 2, is:

$$W_{case,\,int} = \tfrac{1}{2}(OD_{worm} + F_G) + L_{G,hub} W_{case\,wall} = 1.165\ in + 0.5\ in + 1.25\ in = 2.915\ in$$

Including some clearance and the width of the bearings, a shaft with a length between bearing centers of about 4 in. should suffice.

The gear shaft must also extend some distance beyond Bearing 2 to allow coupling to the winch drum shaft. This distance is not specified in the problem statement, and obtaining it would require designing the entire winch assembly which is outside the scope of this project. Because this segment of the shaft only experiences pure torsion, it will be largely excluded from the analyses below and will be assumed to be on the order of 2 in. long beyond the Bearing 2 retaining ring.

The gear (WB696) selected in Progress Report #1 has a bore diameter of 1.375 in. This bore can be expanded by machining out some material, but it cannot easily be made smaller. As such, the shaft diameter at the gear must be at least 1.375 in. minus fit clearance.

The selected gear has a pitch diameter of 16 in. and a pressure angle of 14.5°. These values will be assumed correct, and affect the outcomes of the analyses below. If other gears are selected, these analyses must be performed again with updated pitch diameter and pressure angle.

Right- and left-handed versions of the selected worm and gear are available from the gear supplier. The worm and gear must be of the same handedness, but the handedness is otherwise relatively unimportant to the analysis and design process. Because the gearset was designed to self-lock, the motor must be reversible to permit raising and lowering loads. As a result, regardless of the handedness of the gears, the load will be raised with one motor direction and lowered with the other. However, in order to more clearly analyze forces acting on the shaft, the worm and gear will be assumed to be right-handed as shown in the problem statement (Figure 1). Given this, it is further assumed that the winch is lifting the load when the gear is rotating counter-clockwise as viewed in the front view of Figure 1, implying clockwise rotation of the worm as viewed in the side view of Figure 1.

6. Calculations

Analysis will be performed on the shaft under the conditions required to lift the load. The forces acting on the shaft while lifting are greater than those during lowering, so the shaft must be designed to adequately withstand these greater forces.

6.1 Simplified shaft design

The simplified shaft can be seen in Figure 3. It was generated using the starting assumptions listed above. The distance between bearing centers is 4.25 in. with the center of the gear face positioned 1.8 in. from the center of Bearing 1, with the hub positioned on the Bearing 2 side. From this basic setup, a free body analysis may be conducted to find the forces acting on the shaft. The purpose of this design is to obtain a basic starting point to create a free body diagram and perform shear and bending moment analyses. Once this information is obtained, a preliminary design of the “real” shaft may be created.

6.2 Free body diagram

The axes used in the free body analysis provide direction to forces. The positive z-axis extends toward Bearing 1 from Bearing 2. The positive y-axis extends toward the worm from the gear shaft. The positive x-axis extends radially from the gear shaft axis, perpendicular to the y- and z-axes following the right hand rule. For instance, in the front view of Figure 1, the x-axis is positive to the right, the y-axis is positive up, and the z-axis is positive coming out of the plane.

In order to output the required 4,000 in·lbf torque to the winch drum, the force acting tangentially to the gear at the pitch radius of 8 in. must be at least 500 lbf. Using the analysis performed for the selected gear in Progress Report #1 and a required tangential force $W_G{}^t = F_x = 500\ lbf$, the axial and radial forces acting on the gear are $W_G{}^a = F_z = 53.265\ lbf$ and $W_G{}^r = F_y = -130\ lbf$, respectively.

A right-view free body diagram is shown in Figure 4. The y-direction reaction force at B_2 can be calculated by taking the moment about the x-axis at B_1:

$$\Sigma M_{B1,x} = 0 = F_z \cdot \overline{CD} + F_y \cdot \overline{B_1C} + F_{B2,z} \cdot \overline{B_1B_2}$$

$$F_{B2,y} = -\frac{F_z \cdot \overline{CD} + F_y \cdot \overline{B_1C}}{\overline{B_1B_2}} = -\frac{53.265\ lbf \cdot 8.0\ in - 130\ lbf \cdot 1.80 in}{4.25\ in} = -45.205\ lbf$$

With this, the y-direction reaction force at B_1 can be found by summing forces in the y-direction:

$$\Sigma F_y = 0 = F_{B1,y} + F_{B2,y} + F_y$$

$$F_{B1,y} = -F_{B2,y} - F_y = 45.205\ lbf + 130\ lbf = 175.205\ lbf$$

A top-view free body diagram is used to find the reaction forces in the x-direction, shown in Figure 5. Taking the sum of moments about the y-axis at point B_1, we find:

$$\Sigma M_{B1,y} = 0 = F_x \cdot \overline{B_1C} + F_{B2,x} \cdot \overline{B_1B_2}$$

$$F_{B2,x} = -\frac{F_x \cdot \overline{B_1C}}{\overline{B_1B_2}} = -\frac{500\ lbf \cdot 1.80\ in}{4.25\ in} = -211.76\ lbf$$

The x-direction reaction force at B_1 is found by summing forces in the x-direction:

$$\Sigma F_x = 0 = F_{B1,x} + F_{B2,x} + F_x$$

$$F_{B1,x} = -F_{B2,x} - F_x = 211.76\ lbf - 500\ lbf = -288.24\ lbf$$

If points B_1 and B_2 are both to accept axial load, the shaft loading is statically indeterminate because reaction forces at B_1 and B_2 occur along the same line of action. An ideal, perfectly rigid and sized shaft would indeed distribute the axial load over both bearings. However, with an axial load of only about 50 lbf, any slight deviation from this ideal shaft (due, for instance, to machining tolerances, thermal expansion, or material properties) will result in most or all of the axial load being supported by only one bearing. As it was earlier assumed that tapered roller bearings would be used, this result is perfectly acceptable and will be explored further in a future report. In addition, because the relatively low magnitude of the axial force (contributing only about 10% of the total force acting on the shaft), the impacts of this force on the analyses below are minimal. This report will continue under the assumption, when applicable, that the full axial force on the shaft will be supported by Bearing 2.

In conclusion, the results of this analysis show that the shaft experiences resultant forces of $(-288.24\hat{\imath} + 175.205\hat{\jmath})$ lbf at the center of Bearing 1 and $(-211.76\hat{\imath} - 45.205\hat{\jmath} - 53.265\hat{k})$ lbf at the center of Bearing 2.

6.3 Shear and bending analysis

To analyze the shear and bending moment applied over the length of the shaft, the loading of the shaft will be represented as a singularity function:

$$q = 175.2\text{ lbf}\,\langle x\rangle^{-1} - 130.0\text{ lbf}\,\langle x-1.8\rangle^{-1} + 426.12\text{ in}\cdot\text{lbf}\,\langle x-1.8\rangle^{-2} - 45.2\text{ lbf}\,\langle x-4.25\rangle^{-1}$$

Integrating the loading function once to obtain the shear function:

$$V = \int q\,dx = 175.2\text{ lbf}\,\langle x\rangle^{0} - 130.0\text{ lbf}\,\langle x-1.8\rangle^{0} + 426.12\text{ in}\cdot\text{lbf}\,\langle x-1.8\rangle^{-1} - 45.2\text{ lbf}\,\langle x-4.25\rangle^{0}$$

Integrating the shear function to obtain the bending moment function:

$$M = \int V\,dx = 175.2\text{ lbf}\,\langle x\rangle^{1} - 130.0\text{ lbf}\,\langle x-1.8\rangle^{1} + 426.12\text{ in}\cdot\text{lbf}\,\langle x-1.8\rangle^{0} - 45.2\text{ lbf}\,\langle x-4.25\rangle^{1}$$

With these functions, the shear and bending moment diagrams can be easily generated, and are shown in Figures 6 and 7.

A similar process is carried out for loading in the X-Z plane:

$$q = -288.24\text{ lbf}\,\langle x\rangle^{-1} + 500\text{ lbf}\,\langle x-1.8\rangle^{-1} - 211.76\text{ lbf}\,\langle x-4.25\rangle^{-1}$$

$$V = -288.24\text{ lbf}\,\langle x\rangle^{0} + 500\text{ lbf}\,\langle x-1.8\rangle^{0} - 211.76\text{ lbf}\,\langle x-4.25\rangle^{0}$$

$$M = -288.24\text{ lbf}\,\langle x\rangle^{1} + 500\text{ lbf}\,\langle x-1.8\rangle^{1} - 211.76\text{ lbf}\,\langle x-4.25\rangle^{1}$$

Shear and bending moment diagrams are shown in Figures 8 and 9.

Because the shaft is supported by bearings with negligible friction, there is no torque delivered between $x = 0$ in. and $x = 1.8$ in. Torque is added to the shaft at the center of the gear face, $x = 1.8$ in. and is removed by the winch drum, which is beyond the end of the gear shaft. A torque diagram is shown in Figure 10.

6.4 Initial shaft design

The above analyses provide key information about the stresses the shaft must endure during lifting operation of the winch. A more detailed shaft design may now be generated, which can then be analyzed with the above results to check for failure. If this initial design is unable to withstand the required forces or if it provides an unnecessarily high factor of safety, it will be iterated to improve performance.

Initial shaft design begins with assumptions made up to this point. The diameter of the shaft at the gear location may not be under 1.375 in. minus the fit clearance. The distance between bearing centers must be 4.25 in. for the results of the above free body, shear, and bending moment analyses to be accurate.

Bearings will be located against steps in shaft diameter on one side, and retaining rings on the other side. The gear will be keyed to the shaft to transmit torque, and will be located on both sides with retaining rings. The shaft will need flat-bottom grooves to support the retaining rings.

A shaft design meeting these specifications is shown in Figure 11. It features a maximum diameter of 1.375 in. at the gear location, and steps down to 1.125 in. at the bearing locations under the assumption that a 0.125 in. step is sufficient to support the bearings. Bearing retaining ring grooves are 0.056 in. wide and 0.033 in. deep, and gear retaining ring grooves are 0.042 in. deep and 0.056 in. wide, per manufacturer recommendations. The gear key is 0.3125 in. wide, 0.25 in. deep, and 1.25 in. long. The keyway cut into the shaft is 1.25 in. long between radius centers, 0.3125 in. wide, and 0.125 in. deep. Fillets at steps in shaft diameter are designed to be 0.05 in. radius, while retaining ring grooves have 0.005 in. radius fillets along the internal edges. The shaft extends 2.0 in. beyond the Bearing 2 retaining ring groove to provide space to couple the shaft to the winch drum shaft. AISI 1020 cold-rolled steel was chosen for shaft material.

6.5 Initial shaft design analysis

This initial design must now be analyzed to check for sufficient performance. The static and fluctuating stresses in the shaft will be calculated first, followed by deflection, and finally the critical speed. As before, shaft locations will be identified by their distance from the center of Bearing 1.

6.5.1 Stress analysis

Stress analysis will be performed at five critical locations: both steps in diameter (x = 0.375 in. and 3.875 in.), both gear retaining ring grooves (x = 1.262 in. and 3.568 in.), and at the center of the gear face width (x = 1.79 in.). The shaft loading during operation is assumed to be fully reversed, and torsion is assumed to be steady. The Distortion-Energy Goodman criteria will be used to check for failure due to fatigue. All equations for stress analysis are taken from *Shigley's Mechanical Engineering Design* [2].

6.5.1.1 Bearing 1 locating step, x = 0.375 in.

Because the shaft loading is completely reversed, the mean bending moment M_m and the alternating torsion T_a are assumed to be 0. In addition, because there is no torque transmitted in this area, the mean torsion T_m is also assumed to be 0. To use the DE-Goodman, the following values are required:

$$d = 1.125\ in.$$

$$S_{ut} = 60.9\ kpsi$$

$$S_e' = 0.5\ S_{ut} = 30.45\ kpsi$$

$$k_a = a\ (S_{ut})^b = 2.70 \cdot (60.9\ kpsi)^{-0.265} = 0.90874\ \ \textit{(machined surface)}$$

$$k_b = 0.879\ d^{-0.107} = 0.86799$$

$$k_c = 1\ \ \textit{(loaded in bending)}$$

$$k_d = 0.975 + 0.432\ (10^{-3})\ T_F - 0.115\ (10^{-5})\ T_F^2 + 0.104\ (10^{-8})\ T_F^3 - 0.595\ (10^{-12})\ T_F^4 = 1.020$$

(175°F sump temperature)

$$k_e = 0.814 \qquad \textit{(99\% reliability)}$$

$$S_e = k_a k_a k_a k_a k_a S_e' = 0.90874 \cdot 0.86799 \cdot 1 \cdot 1.020 \cdot 0.814 \cdot 30.45\ kpsi = 19.94\ kpsi$$

$$K_f = 2.23,\ K_{fs} = 1.7$$

$$M_a = \sqrt{65.7^2 + 108.09^2} = 126.5\ in \cdot lbf$$

Plugging these into the DE-Goodman equation:

$$\tfrac{1}{n} = \tfrac{16}{\pi d^3} \left\{ \tfrac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \tfrac{1}{S_{ut}}[4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} = \ldots$$

$$\ldots = \tfrac{16}{\pi d^3} \left\{ \tfrac{1}{S_e}[4(K_f M_a)^2]^{1/2} \right\} = \tfrac{16}{\pi(1.125 in)^3} \left\{ \tfrac{1}{19.94 kpsi}[4\ (2.23 \cdot 126.5\ in \cdot lbf)^2]^{1/2} \right\} = 0.205$$

$$n = 0.205^{-1} \approx 4.87$$

The factor of safety is sufficient, so the shaft should not fail in fatigue in this location. Using the von Mises maximum stress to check for yielding:

$$\sigma_{max}' = \left[\left(\frac{32\ K_f\ (M_m + M_a)}{\pi d^3} \right)^2 + 3 \left(\frac{16\ K_{fs}\ (T_m + T_a)}{\pi d^3} \right)^2 \right]^{1/2} = \frac{32\ K_f\ M_a}{\pi d^3} = 2018\ psi$$

$$n_f = \frac{S_y}{\sigma_{max}'} = \frac{50.8\ kpsi}{2.018\ kpsi} \approx 25$$

The shaft should not yield at this location.

6.5.1.2 Bearing 2 locating step, x = 3.875 in.

The same analysis is performed at this location. All values for the DE-Goodman equation are the same except for the alternating bending moment M_a, which is:

$$M_a = \sqrt{79.43^2 + 16.97^2} = 81.22\ in \cdot lbf$$

Plugging values into the DE-Goodman equation:

$$\frac{1}{n} = \frac{16}{\pi(1.125in)^3} \left\{ \frac{1}{19.94kpsi} [4\,(2.23 \cdot 81.22\ in \cdot lbf)^2]^{1/2} \right\} = 0.06498$$

$$n = 0.06498^{-1} = 15.4$$

Checking for yield:

$$\sigma_{max}' = \frac{32\,K_f\,M_a}{\pi d^3} = 1295.7\ psi$$

$$n_f = \frac{S_y}{\sigma_{max}'} = \frac{50.8\ kpsi}{1.2957\ kpsi} \approx 39$$

This location also appears to avoid failure due to fatigue or yield.

6.5.1.3 Gear retaining ring 1, x = 1.262 in.

The following values are needed to perform the stress analysis:

$$d = 1.291\ in$$

$$k_b = 0.879\ d^{-0.107} = 0.8553$$

$$S_e = 19.648\ kpsi$$

$$K_f = K_{fs} = 5.25$$

$$M_a = \sqrt{221.1^2 + 363.76^2} = 425.7\ in \cdot lbf$$

All other values are the same as those listed in section 6.5.1.1.

Plugging these values into the DE-Goodman equation:

$$\frac{1}{n} = \frac{16}{\pi(1.291in)^3} \left\{ \frac{1}{19.648kpsi} [4\,(5.25 \cdot 425.7\ in \cdot lbf)^2]^{1/2} \right\} = 0.5385$$

$$n = 0.5638^{-1} \approx 1.86$$

This factor of safety is quite small, but the analysis will continue to check for any other potential problems.

Checking for yield:

$$\sigma_{max}' = \frac{32\,K_f\,M_a}{\pi d^3} = 10580.0\ psi$$

$$n_f = \frac{S_y}{\sigma_{max}'} = \frac{50.8\ kpsi}{10.58\ kpsi} \approx 4.8$$

This location also appears to avoid failure due to fatigue or yield.

6.5.1.4 Gear retaining ring 2, x = 3.568 in.

Values for the DE-Goodman equation are the same as the previous analysis, except for the alternating bending moment M_a:

$$M_a = \sqrt{30.8464^2 + 144.44^2} = 147.697\ in \cdot lbf$$

Using the DE-Goodman equation:

$$\frac{1}{n} = \frac{16}{\pi(1.291 in)^3} \left\{ \frac{1}{19.648 kpsi} [4\ (5.25 \cdot 147.697\ in \cdot lbf)^2]^{1/2} \right\} = 0.1868$$

$$n = 0.1868^{-1} \approx 5$$

Checking for yield:

$$\sigma_{max}' = \frac{32\ K_f\ M_a}{\pi d^3} = 3670.7\ psi$$

$$n_f = \frac{S_y}{\sigma_{max}'} = \frac{50.8\ kpsi}{3.671\ kpsi} \approx 14$$

This location should not fail in fatigue or yield.

6.5.1.5 Center of gear face width, x = 1.79

The point of the shaft at the center of the gear face width is the last critical location to examine. The tangential and radial forces acting on the shaft originate from the intersection of the worm pitch cylinder and the gear pitch circle, causing the bending moment in both the X-Z and Y-Z plane at this location to be the highest along the entire length of the shaft. In addition to the high magnitude of forces, the keyway cut into the shaft to transmit torque from the gear creates stress concentrations in this region. Assuming a keyway cut with a fillet radius of 0.0275 in. along the bottom of the groove, and thus a ratio $\frac{r}{d} = 0.02$, stress concentration factors for the keyway are:

$$K_f = 2.14,\ K_{fs} = 3.0 \qquad [2, pp. 384]$$

Other factors needed for the DE-Goodman equation are:

$$d = 1.375\ in.$$

$$k_b = 0.879\ d^{-0.107} = 0.8496$$

$$S_e = 19.518\ kpsi$$

$$M_a = \sqrt{315.36^2 + 518.832^2} = 607.16\ in \cdot lbf$$

Plugging values into the DE-Goodman equation:

$$\frac{1}{n} = \frac{16}{\pi(1.375in)^3} \left\{ \frac{1}{19.518kpsi} [4\,(2.14 \cdot 607.16\ in \cdot lbf)^2]^{1/2} \right\} = 0.261$$

$$n = 0.261^{-1} \approx 3.8$$

Check for yielding:

$$\sigma_{max}' = \frac{32\,K_f\,M_a}{\pi d^3} = 5091.1\ psi$$

$$n_f = \frac{S_y}{\sigma_{max}'} = \frac{50.8\ kpsi}{5.091\ kpsi} \approx 10$$

This location also avoids failure due to fatigue and yielding.

6.5.2 Stress analysis continued

The results of the stress analyses show that the shaft is capable of withstanding the required forces. However, the results for fatigue strength at gear retaining ring 1 warrants some discussion.

The fatigue factor of safety at this location was found to be about 1.8. Being greater than 1, this factor of safety does not indicate failure of the shaft. If all calculations up to this point are accurate, this would be an effective shaft design. In addition, this factor of safety is for fatigue strength rather than yielding, so the consequence of a low factor of safety (even a factor of less than 1) is not immediate catastrophic failure of the winch, but rather diminished lifetime of the device. With that being said, this winch is designed to lift 500 lbf loads, and failure of the device in any manner poses risk of property damage and significant dangers to health at best, and life-threatening harm at worst. Proper safety precautions should be taken by users of the device, but it is the responsibility of engineers to minimize the risks involved as much as possible during the design process. If this was a “real” design project rather than an educational exercise, tests should be conducted on the device to ensure expected performance. If factors such as friction, temperature, shock loading, or alignment of components deviate from expected values, factors of safety for *all* components and shaft locations would be affected. Alterations to values used in calculations would be updated, the design of components would be iterated, and more tests would be conducted until a design performs as expected with satisfactorily high factors of safety. For the purposes of this academic exercise, though, this report will move forward under the assumption that all calculations are correct and that the shaft’s minimum factor of safety of 1.8 is sufficient for the application.

6.5.3 Deflection analysis

To compute the deflection along the length of the shaft, the Young's modulus E of the shaft material and the second moment of area I of the shaft's cross section at various points are required.

$$E = 27\ kpsi$$

$$I_B = \frac{\pi d^4}{64} = \frac{\pi(1.125\ in.)^4}{64} = 0.0786285\ in.^4$$

$$I_G = \frac{\pi d^4}{64} = \frac{\pi(1.375\ in.)^4}{64} = 0.175461\ in.^4$$

The net force acting on the shaft is:

$$F =- 130\ lbf,\ M = 426.12\ in \cdot lbf$$

A plot of shaft deflection is shown in Figure 12. The maximum deflection is -0.02778 in. and occurs about 1.625 in. from the centerline of Bearing 1. The slope at Bearing 1 is -0.03444 $^{in}/_{in}$ and the slope at Bearing 2 is 0.00178 $^{in}/_{in}$, which equate to shaft angles of -0.034 rad and 0.0018 rad, respectively. The transverse deflection of the shaft at the center of the gear face (x = 1.79 in.) is about -0.0213 in., and the shaft angle at this location is about 0.028 rad.

Recommended maximum slope for tapered roller bearings is 0.0012 rad, and maximum transverse deflection at gears is approximately 0.003 in. This shaft design fails to meet every one of these recommendations—the slope at Bearing 1 is almost 30 times too large, the slope at Bearing 2 is about 1.5 times too large, and the transverse deflection of the gear is about 7 times too large. The shaft requires design iteration to improve these values.

6.6 Shaft design iteration

The diameter of the shaft will be increased to accommodate these restrictions. The new diameter should be at least:

$$d_{new} = d_{old}\left|\frac{n_d\, y_{old}}{y_{allow}}\right|^{1/4} = 1.375\ in.\left|\frac{1.1 \cdot 0.0213\ in.}{0.003\ in.}\right|^{1/4} \approx 2.3\ in. \qquad [2,\ pp.\ 373]$$

The hub diameter of the gear selected in the previous report has a hub outer diameter of 3.0 in. Expanding the inner diameter of the gear bore to accept a larger shaft removes material from this hub, which prevents the gear from bending due to axial forces. Expanding the bore to the size determined above will leave the hub only 0.35 in. thick walls. A full analysis to determine if this is suitable to support the gear is outside the scope of this project, but would be examined in detail in a real design process. This report will move forward under the assumption that the remaining hub wall thickness is sufficient.

The new shaft design has a diameter of 2.3 in. at the gear and 1.875 in. at the bearings. All retaining ring grooves will be assumed 0.056 in. wide and 0.05 in. deep. The keyway will be 0.625 in. wide and 0.21875 in. deep. All other features of the shaft will remain the same as the prior design. This shaft design is shown in Figure 13, and mechanical drawings are provided alongside this report as a PDF.

6.6.1 Deflection analysis revisited

With the larger shaft diameter, the second moments of area of the shaft are:

$$I_B = \frac{\pi d^4}{64} = \frac{\pi (2.3\ in.)^4}{64} = 1.9175\ in.^4$$

$$I_G = \frac{\pi d^4}{64} = \frac{\pi (1.875\ in.)^4}{64} = 0.7854\ in.^4$$

This shaft design has a transverse deflection at the center of the gear face width of −0.00195 in., which is within recommended values. However, while the shaft angle at Bearing 2 (about 0.00017 rad) is within recommended values, the angle at Bearing 1 (0.0032 rad) is still almost double the acceptable maximum shaft angle for tapered roller bearings. The moment created by the relatively small 53 lbf axial force is nonetheless substantial due to the 8.0 in. moment arm created by the pitch radius of the gear. This moment combined with the 130.0 lbf radial force on the shaft causes the relatively large shaft angle at Bearing 1. A smaller diameter gear would reduce this, but calculations from the previous report found that the gear selected was one of only two acceptable gears, and the other's pitch radius was larger still. Reducing the shaft angle at Bearing 1 would require a shaft diameter approaching or exceeding the hub diameter of the gear:

$$d_{new} = d_{old}\left|\frac{n_d\,(dy/dx)_{old}}{(slope)_{allow}}\right|^{1/4} = 1.375\ in.\left|\frac{1.0 \cdot 0.0032\ rad}{0.0012\ rad}\right|^{1/4} \approx 2.94\ in.$$

The design process has hit a wall, it seems. In a real design situation, there would be a few possible solutions:

1. Choose a different gear supplier if possible, select new gears with smaller pitch diameters or larger hub diameters according to the process from the previous report, and design a shaft that is appropriate for one of these new gears.
2. Remove this gear's hub, machine a new larger hub which is bored for a larger shaft diameter, and affix it to the gear to replace the original hub.
3. Support the shaft in a different manner than described in the problem statement, perhaps extending the shaft far enough beyond Bearing 1 that a deep-groove ball bearing can be used to relieve some of the force acting on the shaft near Bearing 1.
4. Transmit the axial force using a component other than tapered roller bearings (a bronze thrust bearing, for instance) so that bearings with larger shaft angle tolerances may be used.

The first three options are beyond the control of this report. For now, it will be assumed that a solution to the Bearing 1 shaft angle can be reached in the following report covering bearing selection. This report will move forward under the assumption that this shaft design performs within acceptable deflection limits.

6.6.1 Critical speed analysis

To compute the critical speed of the shaft, it is split into discrete sections. The contribution of the retaining ring grooves, fillets, and sections of shaft beyond the bearing centers is assumed to be negligible. The first section extends from x = 0 in. to 0.375 in., the second is between x = 0.375 in. and 3.875 in., and the last is between x = 3.875 in. and 4.25 in. The density of the shaft is assumed to be 0.284 lbf / in^3.

The weights of these sections are:

$$w_1 = w_3 = \frac{\pi d^2}{4} l\rho = \frac{\pi(1.875\ in)^2}{4} \cdot 0.375\ in. \cdot 0.284\ \frac{lbf}{in.^3} = 0.2941\ lbf$$

$$w_2 = \left(\frac{\pi d^2}{4} l - V_{key}\right)\rho = \left(\frac{\pi(2.3\ in)^2}{4} \cdot 3.5\ in. - 0.238\ in.^3\right) \cdot 0.284 \frac{lbf}{in.^3} = 4.06\ lbf$$

Deflection at the centers of these elements is:

$$y_1 = 0.00059\ in.$$

$$y_2 = 0.00121\ in.$$

$$y_3 = 0.0000299\ in. \approx 0\ in.$$

The critical speed of the redesigned shaft is given by the following:

$$\omega_1 = \sqrt{\frac{g\,\Sigma(w_i y_i)}{\Sigma(w_i y_i^2)}} = \left(\frac{386\ in/s^2 \cdot (0.2941 \cdot 0.00059 + 4.06 \cdot 0.00121 + 0)}{0.2941 \cdot 0.00059^2 + 4.06 \cdot 0.00121^2}\right)^{1/2} \approx 570\ s^{-1} \approx 5443\ rpm$$

The shaft outputs the required torque at a motor speed of about 1420 rpm. The gear shaft, then, rotates at about $1420 \cdot \frac{1}{96} = 14.8$ rpm. The critical speed of the shaft is two orders of magnitude larger, so the shaft design is quite safe from failure due to critical speed.

7. Results

After creating the simple design specified in section 6.1, an analysis of the free body diagram was performed, which gave resultant forces of $(-288.24\hat{\imath} + 175.205\hat{\jmath})$ lbf at the center of Bearing 1 and $(-211.76\hat{\imath} - 45.205\hat{\jmath} - 53.265\hat{k})$ lbf at the center of Bearing 2. After then performing the shear and bending moment analysis, more details were added to the design. The design was given a maximum diameter of 1.375 in. at the gear location, and steps down to 1.125 in. at the bearing locations under the assumption that a 0.125 in. step is sufficient to support the bearings.

Bearing retaining ring grooves were 0.056 in. wide and 0.033 in. deep, and gear retaining ring grooves were 0.042 in. deep and 0.056 in. wide. The gear key was 0.3125 in. wide, 0.25 in. deep, and 1.25 in. long. The keyway cut into the shaft was 1.25 in. long between radius centers, 0.3125 in. wide, and 0.125 in. deep. Fillets at steps in shaft diameter were designed to be 0.05 in. radius, while retaining ring grooves had 0.005 in. radius fillets along the internal edges. The shaft extended 2.0 in. beyond the Bearing 2 retaining ring groove to provide space to couple the shaft to the winch drum shaft. AISI 1020 cold-rolled steel was chosen for shaft material. Various calculations were then performed to determine if the selected design met all the outlined requirements, which is detailed throughout section 6.5. To perform this analysis, it was assumed that tapered rollers would be used as the bearings, though these will be decided on finally in the next report. It was determined that none of the parts of the design would fail due to fatiguing or yielding, and with factors of safeties all within acceptable parameters. A deflection analysis was then performed, which indicated that further iterations of the shaft design would be necessary. The new shaft design had a diameter of 2.3 in. at the gear and 1.875 in. at the bearings. All retaining ring grooves were assumed 0.056 in. wide and 0.05 in. deep. The keyway will be 0.625 in. wide and 0.21875 in. deep. All other features of the shaft remained the same as the prior design. Finally, a critical speed analysis was performed, which determined the critical speed was 5443 rpm.

8. Discussion

The results showed that the current iteration of the shaft should be able to meet all the requirements of the project, with a few points of note. Firstly, the factor of safety of the gear retaining ring 1 (shown section 6.5.1.3) was lower than ideal. This would not lead to a catastrophic failure, but would shorten the lifespan of the design, and potentially cause problems longer term in real life applications. Further testing beyond the scope of the project would be required to determine if this would need to be fixed, so it was assumed that it would be acceptable for the sake of continuing with the project. All other factors of safety were high enough to move on, with the next lowest being 3.8 (at the center of gear facing width, section 6.5.1.5), and the highest being 15.4 (at bearing 2, section 6.5.1.2). The next point of note is that, after performing the deflection analysis and iterating the shaft design, it became clear that the shaft was deflecting far further than would be acceptable, with a required diameter wider than that of the gear hub to function. Several options were considered (listed in section 6.6.1), and it was decided that to continue the project, the 4th option would be chosen. This allowed the analysis to continue, and the problem to potentially be fixed later in the design of the shaft. Other than these two points, all other aspects of the design met a standard we found acceptable. The final shaft diameter was tentatively set to 2.3 in. with potential to be changed later should the in depth analysis and selection of the bearings not be enough to overcome the deflection issues mentioned above. Finally, the critical speed analysis showed that the critical speed was two orders of magnitude higher than would be necessary during operation, and so was well within the needs set by the project.

9. Conclusions

To design the shaft, it was stated that it would need to deliver 4,000 in·lbf torque from the worm gear to the winch drum, and be able to do so without excessive bending or torsion, and without failing. Using the gears selected in the previous report, diameters for sections of the shaft were selected, along with lengths that would meet the needs outlined for this project. After the initial design was created using the assumptions outlined above, a simple force analysis of the free body diagram was performed, which allowed for a fully detailed shaft to be created. Analyses were then performed on each point of interest, to determine the factors of safety at each point, and if any of these points would fail due to fatigue or yielding. After determining that they would not, a deflection analysis was performed, which revealed an issue with the thickness of the shaft. It was decided that the issue could be potentially resolved with the selection of bearings, and so was put on hold until the bearings were selected. The team then moved to the final critical speed analysis, which showed that there were no further issues with the design. This concluded the design of the shaft, with a potential for the diameter to be altered in the bearing selection stage of the design, if they could not resolve the diameter issue alone.

10. Appendices

A1. Figures

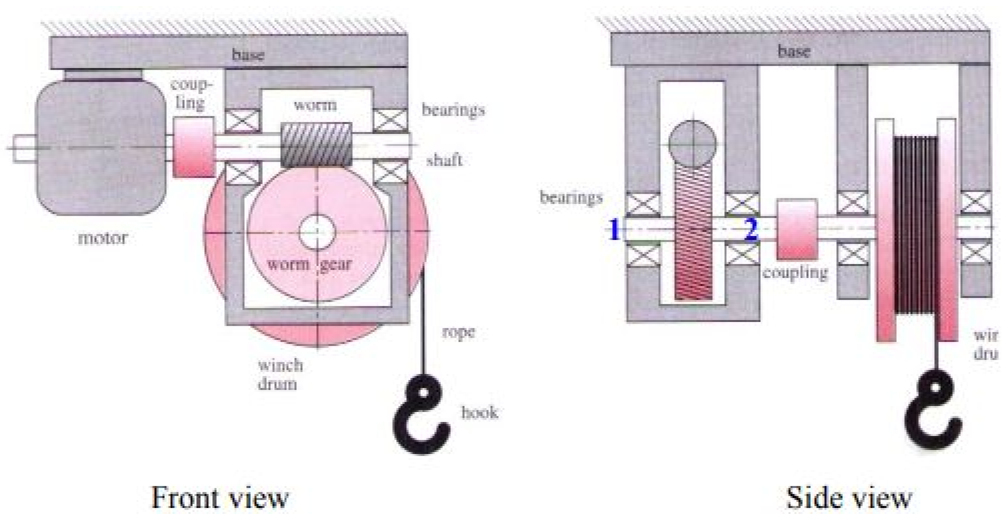

Figure 1: Diagram of the basic winch setup, provided in the project description

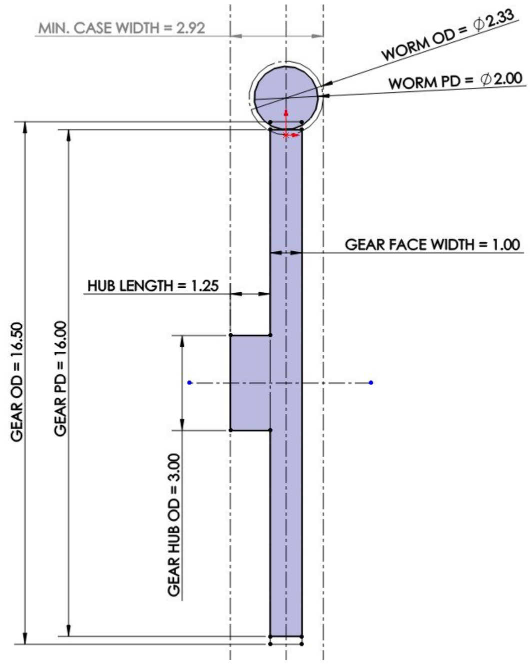

Figure 2: Minimum gearcase width

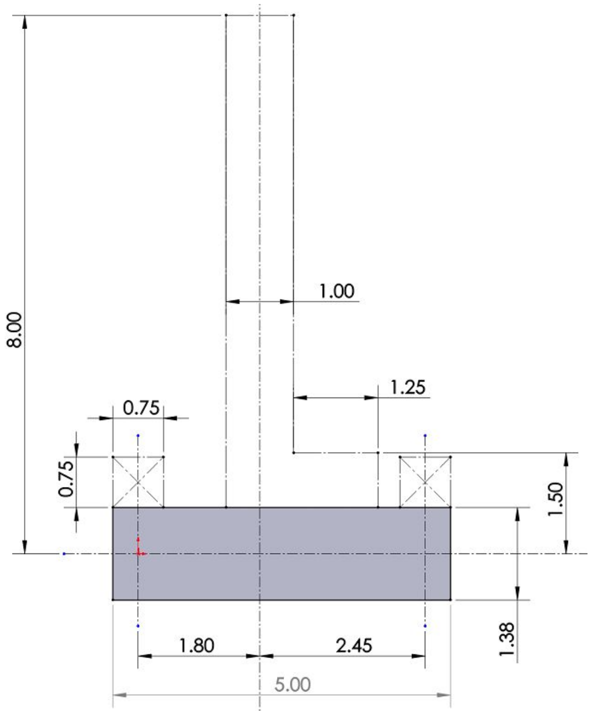

Figure 3: Preliminary shaft design

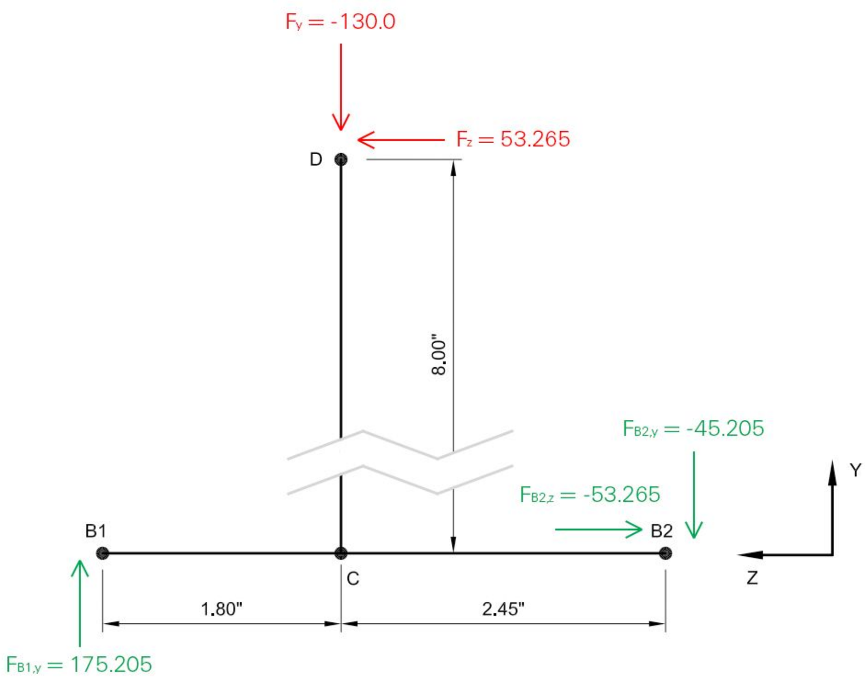

Figure 4: Right-view free body diagram

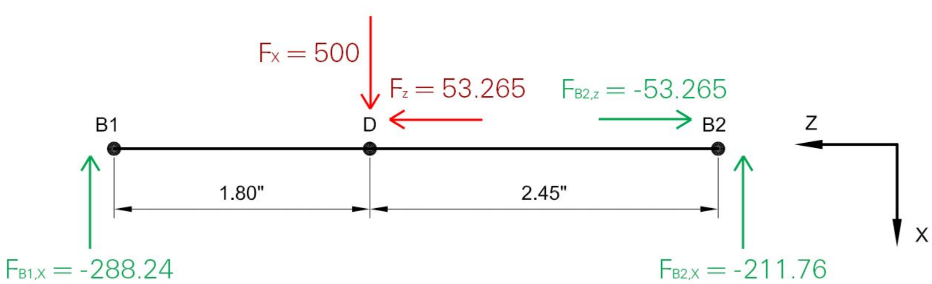

Figure 5: Top-view free body diagram

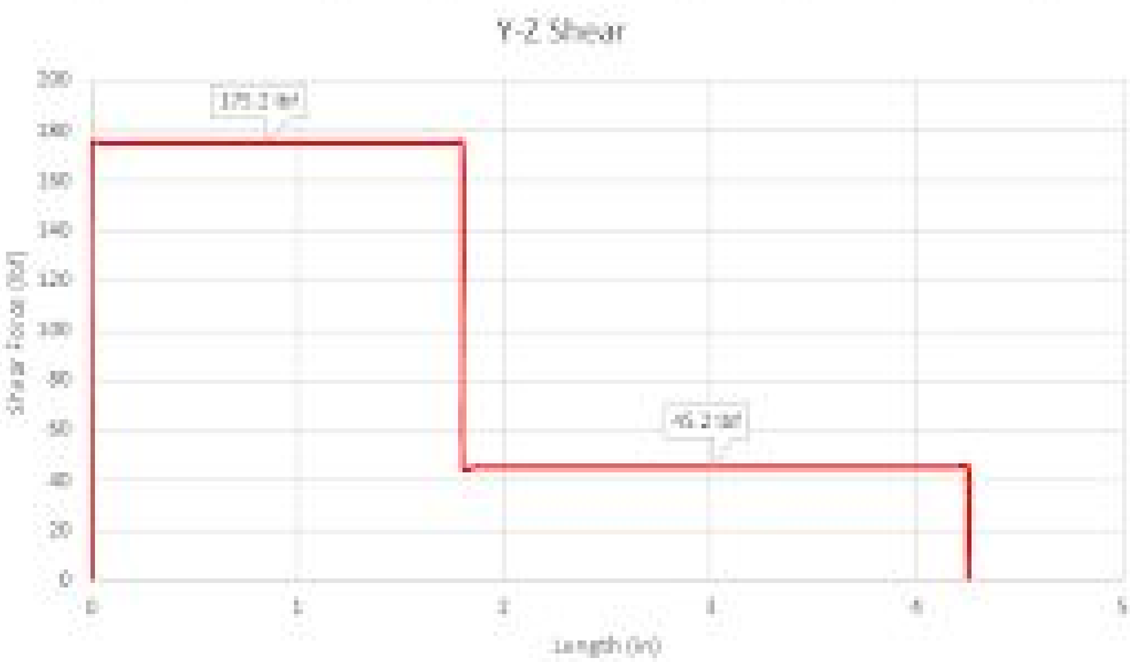

Figure 6: Shear diagram, Y-Z plane

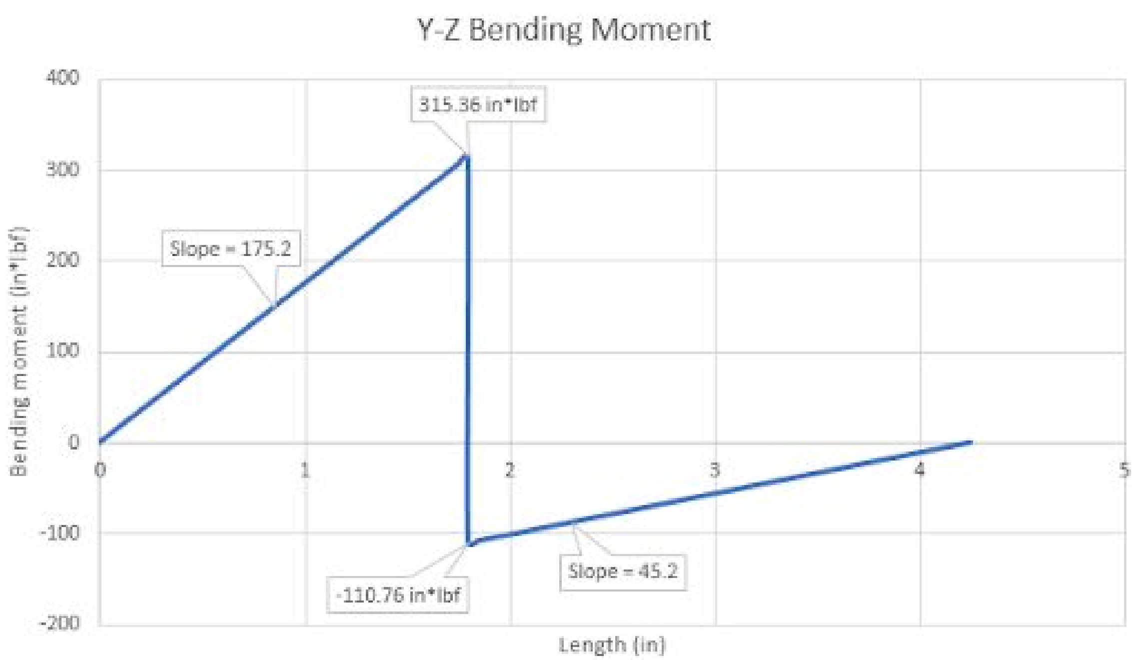

Figure 7: Bending moment diagram, Y-Z plane

Figure 8: Shear diagram, X-Z plane

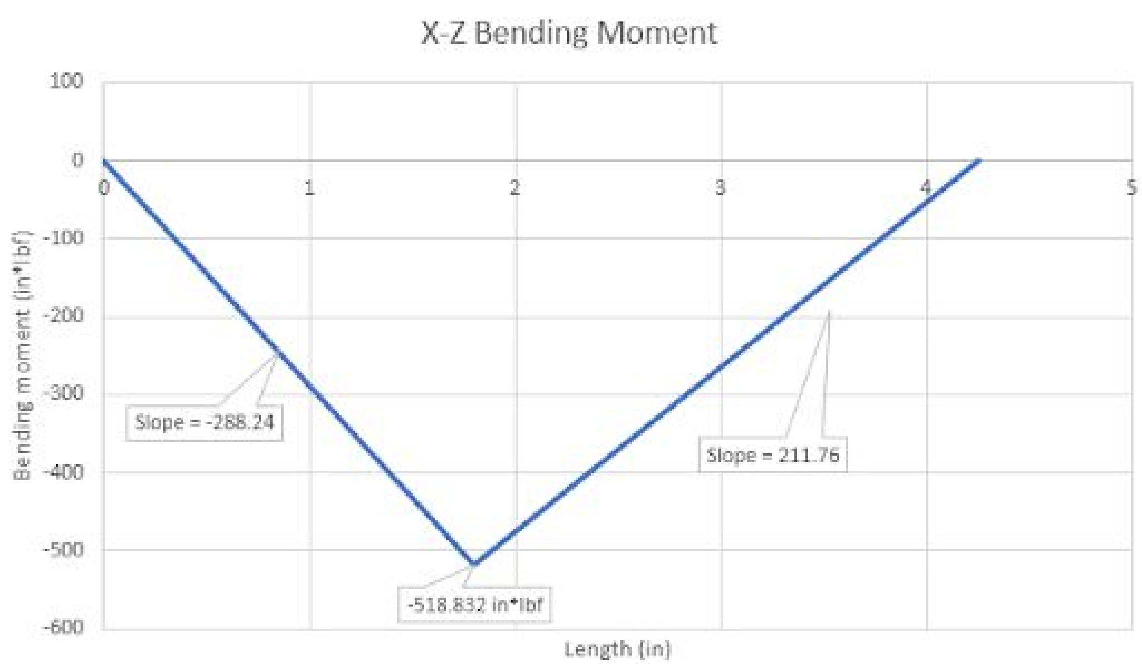

Figure 9: Bending moment diagram, X-Z plane

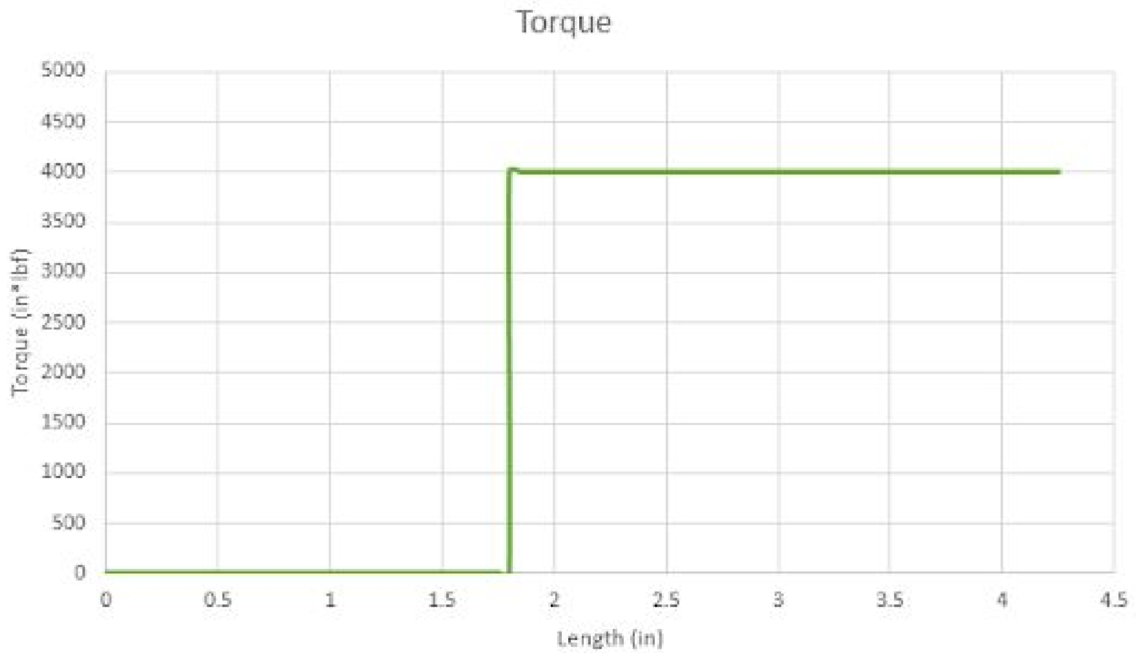

Figure 10: Torque diagram

Figure 11: Initial shaft design

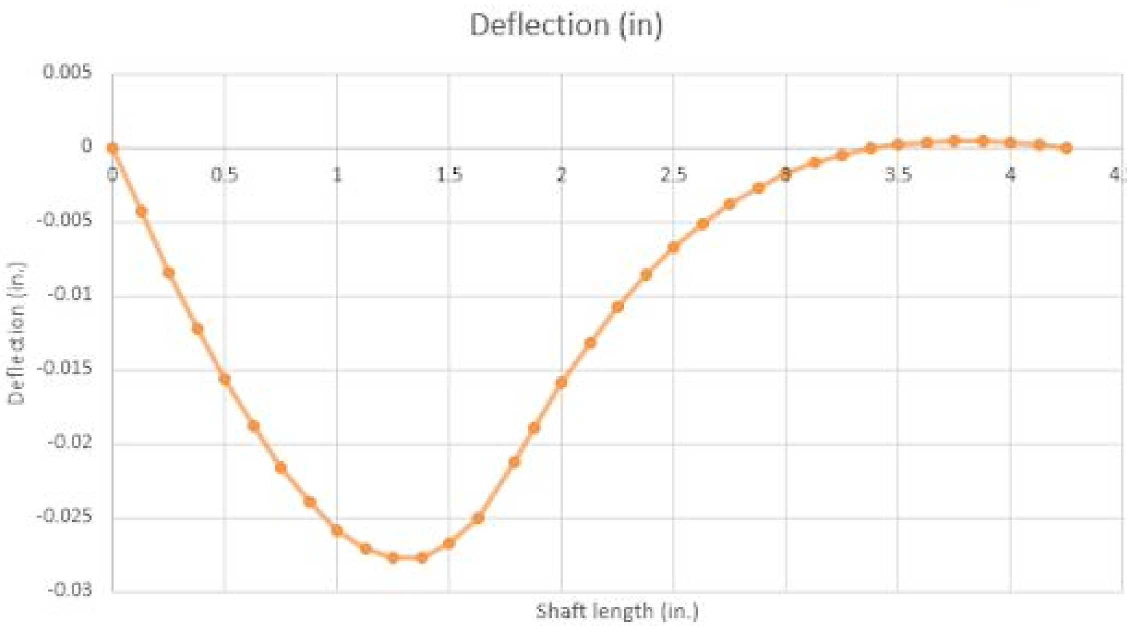

Figure 12: Shaft deflection

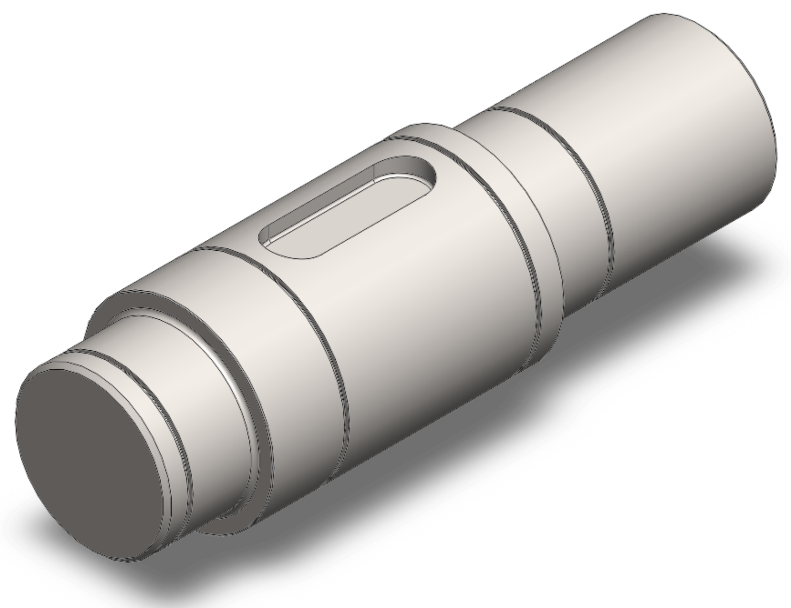

Figure 13: Improved shaft design

A2. References

[1] "AISI 1020 Steel, cold rolled," MatWeb. [Online.] Available: http://matweb.com/search/datasheet_print.aspx?matguid=10b74ebc27344380ab16b1b69f1cffbb (accessed Oct. 28, 2020).

[2] R. G. Budynas and J. K. Nisbett, *Shigley's Mechanical Engineering Design*, 5th ed. New York, NY, US: McGraw-Hill, 2015.