

# 1 Laplace Transform Pairs

$f(t)$	$F(s)$
Step function $u(t)$	$\frac{1}{s}$
$e^{-at}$	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$t^n$	$\frac{n!}{s^{n+1}}$
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \dots - f^{(k-1)}(0^-)$
$\int_{-\infty}^t f(t) dt$	$\frac{F(s)}{s} + \frac{1}{s} \int_{-\infty}^0 f(t) dt$
Impulse function $\delta(t)$	1
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{\omega} [(\alpha - a)^2 + \omega^2]^{1/2} e^{-at} \sin(\omega t + \phi)$ $\phi = \arctan \frac{\omega}{\alpha - a}$	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t, \zeta < 1$	$\frac{\omega_n^2}{s^2+2\zeta \omega_n s+\omega_n^2}$
$\frac{1}{a^2+\omega^2} + \frac{1}{\omega \sqrt{a^2+\omega^2}} e^{-at} \sin(\omega t - \phi)$ $\phi = \arctan \frac{\omega}{-a}$	$\frac{1}{s[(s+a)^2+\omega^2]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi)$ $\phi = \arccos \zeta, \zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta \omega_n s+\omega_n^2)}$
$\frac{\alpha}{a^2+\omega^2} + \frac{1}{\omega} \left[ \frac{(\alpha-a)^2+\omega^2}{a^2+\omega^2} \right]^{1/2} e^{-at} \sin(\omega t + \phi)$ $\phi = \arctan \frac{\omega}{\alpha-a} - \arctan \frac{\omega}{-a}$	$\frac{s+a}{s[(s+a)^2+\omega^2]}$