



July 27, 2020 to August 24, 2020

Note: For any geometry problem whose statement begins with an asterisk (*), the first page of the solution must be a large, in-scale, clearly labeled diagram. Failure to meet this requirement will result in an automatic 1-point deduction.

SMO 4. Let $p > 2$ be a fixed prime number. Find all functions $f : \mathbb{Z}_p \rightarrow \mathbb{Z}_p$, where the symbol \mathbb{Z}_p denotes the set $\{0, 1, \dots, p-1\}$, for which p divides $f(f(n)) - f(n+1) + 1$ for all positive integers n .

SMO 5. (*) In triangle $\triangle ABC$, let E and F be points on sides AC and AB , respectively, such that $BFEC$ is cyclic. Let lines BE and CF intersect at point P , and M and N be the midpoints of \overline{BF} and \overline{CE} , respectively. If U is the foot of the perpendicular from P to BC , and the circumcircles of triangles $\triangle BMU$ and $\triangle CNU$ intersect at second point V different from U , prove that A, P , and V are collinear.

SMO 6. We say that a number is *angelic* if it is greater than 10^{100} and all of its digits are elements of $\{1, 3, 5, 7, 8\}$. Suppose P is a polynomial with nonnegative integer coefficients such that over all positive integers n , if n is angelic, then the decimal representation of $P(s(n))$ contains the decimal representation of $s(P(n))$ as a contiguous substring, where $s(n)$ denotes the sum of digits of n .

Prove that P is linear and its leading coefficient is 1 or a power of 10.

*Time: 4 hours and 30 minutes.
Each problem is worth 7 points.*