Note: For any geometry problem whose statement begins with an asterisk ($*$), the first page of the solution must be a large, in-scale, clearly labeled diagram. Failure to meet this requirement will result in an automatic 1-point deduction.

SJMO 1. Find all positive integers $k \geq 2$ for which there exists some positive integer n such that the last k digits of the decimal representation of $10^{10^n} - 9^{9^n}$ are the same.

SJMO 2. Anthony writes the $(n+1)^2$ distinct positive integer divisors of 10^n, each once, on a whiteboard. On a move, he may choose any two distinct numbers a and b on the board, erase them both, and write $\gcd(a, b)$ twice. Anthony keeps making moves until all of the numbers on the board are the same. Find the minimum possible number of moves Anthony could have made.

SJMO 3. ($*$) Let O and Ω denote the circumcenter and circumcircle, respectively, of scalene triangle $\triangle ABC$. Furthermore, let M be the midpoint of side BC. The tangent to Ω at A intersects BC and OM at points X and Y, respectively. If the circumcircle of triangle $\triangle OXY$ intersects Ω at two distinct points P and Q, prove that PQ bisects $\overline{AM}$.