2020 Summer JMO (Day I)

July 27, 2020 to August 24, 2020

Note: For any geometry problem whose statement begins with an asterisk ($*$), the first page of the solution must be a large, in-scale, clearly labeled diagram. Failure to meet this requirement will result in an automatic 1-point deduction.

SJMO 1. Find all positive integers $k \geq 2$ for which there exists some positive integer n such that the last k digits of the decimal representation of $10^{10^n} - 9^{9^n}$ are the same.

SJMO 2. Consider all possible infinite sequences $a_1, a_2, \ldots$ of positive integers where the sum of any 8 consecutive terms is at most 16. Prove that for all such sequences, given any positive integer k, there exists some consecutive terms $a_i, a_{i+1}, \ldots, a_j$ that sum to $16k$.

SJMO 3. ($*$) Let O and Ω denote the circumcenter and circumcircle, respectively, of scalene triangle $\triangle ABC$. Furthermore, let M be the midpoint of side BC. The tangent to Ω at A intersects BC and OM at points X and Y, respectively. If the circumcircle of triangle $\triangle OXY$ intersects Ω at two distinct points P and Q, prove that PQ bisects $\overline{AM}$.

Time: 4 hours and 30 minutes.
Each problem is worth 7 points.