# Project 03: Numerical Estimation of π

**Author:** Sean McMeans

**Date:** 12 February 2026

**Course:** CS4130 - HPC

# 1. Optimization of Sample Size (N)

**Objective:** Determine the optimal value of N that balances accuracy with computational cost.

- **Observation:** As N increases, the change in the estimated value of π decreases.
- **Optimal Value:** Based on the results, the optimal N is approximately 10,000. At this point, the error is within acceptable bounds without decreasing much with incrementing n.

| N Value | Iterations | Time | Pi Estimate |
|---|---|---|---|
| 10 | 301919 | 2 seconds | 3.841329 |
| 100 | 107082 | 2 seconds | 3.356590 |
| 1000 | 46008 | 5 seconds | 3.210519 |
| 10000 | 20291 | 17 seconds | 3.176233 |
| 100000 | 6941 | 51 seconds | 3.169112 |

**Figure 1:** *Convergence of π as a function of N. Testing using an ε of $10^{-9}$.*

## 2. Convergence Thresholds (ε)

**Objective:** Analyze the lower bounds of the tolerance parameter ε before convergence fails or becomes computationally prohibitive.

- **Minimum ε:** The algorithm successfully converges for ε values as small as **$10^{-12}$**.
- **Behavior at Limit:** Below this threshold, the time to run exceeds 90 minutes using 4 processors.

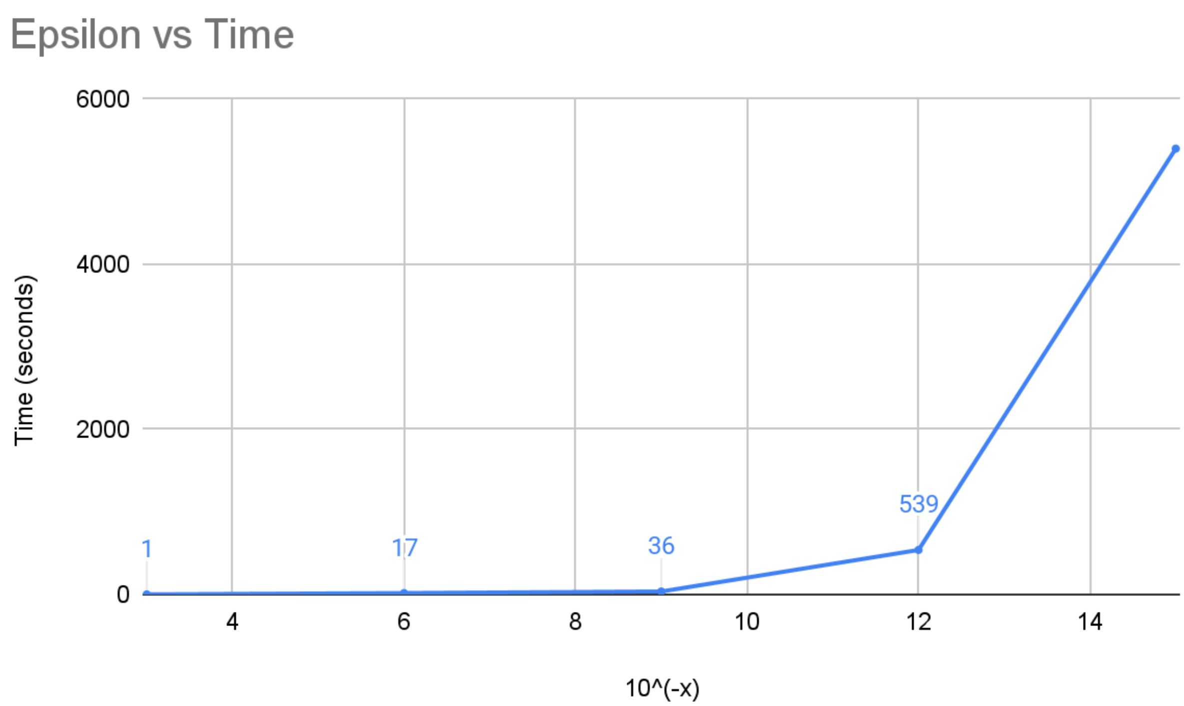


**Figure 2:** *The time to complete the calculation with each ε. The x axis represents the negative power of ε, so an x of 6 represents ε being $10^{-6}$. Note the exponential increase in computational time as the exponent x increases (representing a smaller ε).*

# 3. Divergence Analysis

**Question:** Does the approximation of π ever converge initially and then diverge ("blow up")?

- **Observation:** No, the method remained stable.

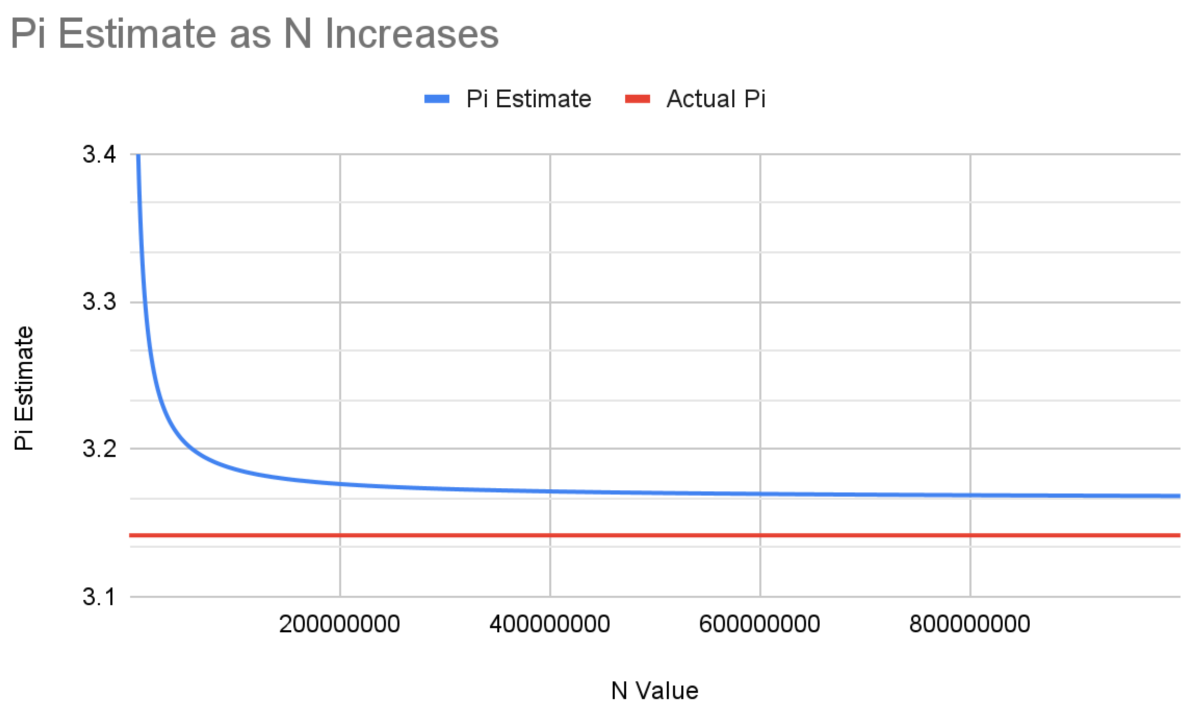


**Figure 3:** *Convergence of π as a function of N. The horizontal line represents π.*

# 4. Parallelization (ncpu)

**Objective:** Assess the impact of core count on performance.

| Core count | Time Taken (Seconds) | Speed Up Factor |
|---|---|---|
| 2 | 23 | 1.0x |
| 4 | 15 | 1.53x |
| 8 | 15 | 1.53x |
| 16 | 11 | 2.09x |

**Figure 4:** *How long it takes each number of cores to complete the calculation with an ε of $10^{-6}$.*

- **Reasonable Core Count:** The best number of cores appears to be **4** cores. Beyond this, the overhead of putting all the calculations on 1 core outweighed the benefits.

# 5. Non-Convergence

**Investigation:** Reasons why the code might fail to converge to the exact value of π.

- **Observation:** The code consistently converged with a value slightly higher than π. See figure 1.
- **Reason:** Without changing the method to calculate M and P, the if statement does not include 1 as being within the circle, despite it being on the border of the circle. Additionally, the algorithm relies on randomness to achieve a value, which will become more precise as the algorithm runs, with current runs not being long enough to get an exact answer.