Homework for Chapter 2

1. Is an oil pipeline a simplex system, a half-duplex system, a full-duplex system, or none of the above? What about a river or a walkie-talkie-style communication?

   Solution:

   An oil pipeline: half-duplex;

   A walkie-talkie-style communication: half-duplex;

   A river: simplex.

2. How much bandwidth is there in 0.1 microns of spectrum at a wavelength of 1 micron?

   Solution:

$$\begin{aligned} & c &=& \lambda f \\ \Rightarrow\quad & f &=& \frac{c}{\lambda} \\ \Rightarrow\quad & \frac{\mathrm{d}f}{\mathrm{d}\lambda} &=& -\frac{c}{\lambda^2} \\ \Rightarrow\quad & \Delta f &=& \frac{c}{\lambda^2}\,\Delta\lambda \\ & &=& \frac{3\times 10^8}{10^{-6}\times 10^{-6}}\,0.1\times 10^{-6} \\ & &=& 3\times 10^{13} = 30000\text{ GHz.} \end{aligned}$$

3. Compute the Fourier coefficients for the function $f(t) = t$ $(0 \le t \le 1)$.

   Solution:

$$\begin{aligned} a_n &= \tfrac{2}{T}\int_0^T g(t)\sin(2\pi nft)\,dt = 2\int_0^1 t\sin(2\pi nt)\,dt = \frac{-1}{\pi n}\int_0^1 t\,d\cos(2\pi nt) = \frac{-1}{\pi n}. \\ b_n &= \tfrac{2}{T}\int_0^T g(t)\cos(2\pi nft)\,dt = 2\int_0^1 t\cos(2\pi nt)\,dt = \frac{1}{\pi n}\int_0^1 t\,d\sin(2\pi nt) = 0. \\ c &= \tfrac{2}{T}\int_0^T g(t)\,dt = 2\int_0^1 t\,dt = 1. \end{aligned}$$

4. A noiseless 3-kHz channel is sampled every 1 msec. (i) What is the maximum data rate? (ii) How does the maximum data rate change if the channel is noisy, with a signal-to-noise ratio of 30 dB?

   Solution:

   The sampling frequency is 1000 Hz, its cutoff is 500Hz.

   (i) For binary signals over a noiseless 3-kHz channel,

$$2B\log_2 V = 2\times 500\log_2 2 = 1\,\text{kbps}.$$

   For 4-level signals over a noiseless 3-kHz channel,

$$2B\log_2 4 = 2\times 500\log_2 4 = 2\,\text{kbps}.$$

   For 2^n-level signals over a noiseless 3-kHz channel,

$$2B\log_2 2^n = 2\times 500\log_2 2^n = n\,\text{kbps}.$$

   So, the maximum data rate over a noiseless 3-kHz channel can be infinite.

   (ii) With a signal-to-noise ratio of 30 dB, over a noiseless 3-kHz channel sampled every 1 msec,

$$2B\log_2\sqrt{1+S/N} = 2\times 500\log_2\sqrt{1001} = 4.984\,\text{kbps}.$$

5. If a binary signal is sent over a 3-kHz channel whose signal-to-noise ratio is 20 dB, what is the maximum achievable data rate?

   Solution:

$$\begin{aligned} 2B \log_2 V &= 2 \times 3 \times 10^3 \log_2 2 = 6 \text{ kbps}, \\ 2B \log_2 \sqrt{1 + 100} &= 2 \times 3 \times 10^3 \log_2 \sqrt{101} = 6 \times 10^3 \times 3.329 = 19.975 \text{ kbps}. \end{aligned}$$

   6 kbps.

6. What is the minimum bandwidth needed to achieve a data rate of B bits/sec if the signal is transmitted using NRZ, MLT-3, and Manchester encoding? Explain your answer.

   Solution:

$$\text{DataRate} = B = 2 \times \text{bandwidth} \times \log_2 v \qquad \Rightarrow \qquad \text{bandwidth} = \frac{B}{2 \log_2 v}$$

   (i) NRZ: bandwidth $= \dfrac{B}{2 \log_2 v} = \dfrac{B}{2 \log_2 2} = \dfrac{B}{2}$.

   (ii) MLT-3: bandwidth $= \dfrac{B}{2 \log_2 v} = \dfrac{B}{2 \log_2 4} = \dfrac{B}{4}$.

   (iii) Manchester encoding: bandwidth $= \dfrac{B}{2 \log_2 v} = \dfrac{B}{2 \log_2 0.5} = B$.

7. Prove that in 4B/5B mapped data with the NRZI encoding, a signal transition will occur at least every four bit times.

   Solution:

   (i) Assertion: The 4B/5B mapping ensures that a sequence of consecutive 0s cannot be longer than 3.

      no codeword has more than 2 consecutive zeros;

      the longest leading sequence is 1 zero;

      the longest trailing sequence is 2 zeros.

   So, in the worst case, the transmitted bits will have a sequence 10001.

   (ii) Since 4B/5B encoding uses NRZI, there is a signal transition every time a 1 is sent.

   From (1) and (ii), a signal transition will occur at least every four bit times.

8. In the discussion about orthogonality of CDMA chip sequences, it was stated that if $\mathbf{S} \cdot \mathbf{T} = 0$ then $\mathbf{S} \cdot \overline{\mathbf{T}}$ is also 0. Prove this.

   Solution:

$$\mathbf{S} \cdot \overline{\mathbf{T}} + \mathbf{S} \cdot \mathbf{T} = \mathbf{S} \cdot \mathbf{0} = 0 \qquad \Rightarrow \qquad \mathbf{S} \cdot \overline{\mathbf{T}} = -\mathbf{S} \cdot \mathbf{T}.$$

9. A CDMA receiver gets the following chips: $(-1\ +1\ -3\ +1\ -1\ -3\ +1\ +1)$. Assuming the chip sequences defined in Fig. 2-22(a), which stations transmitted, and which bits did each one send?

Solution:

$$
\begin{aligned}
\mathbf{S}\cdot\mathbf{A} &= (-1\ +1\ -3\ +1\ -1\ -3\ +1\ +1)\cdot \\
&\quad\ (-1\ -1\ -1\ +1\ +1\ -1\ +1\ +1)/8 \\
&= (+1\ -1\ +3\ +1\ -1\ +3\ +1\ +1)/8 \\
&= 1, \\
\mathbf{S}\cdot\mathbf{B} &= (-1\ +1\ -3\ +1\ -1\ -3\ +1\ +1)\cdot \\
&\quad\ (-1\ -1\ +1\ -1\ +1\ +1\ +1\ -1)/8 \\
&= (+1\ -1\ -3\ -1\ -1\ -3\ +1\ -1)/8 \\
&= -1, \\
\mathbf{S}\cdot\mathbf{C} &= (-1\ +1\ -3\ +1\ -1\ -3\ +1\ +1)\cdot \\
&\quad\ (-1\ +1\ -1\ +1\ +1\ +1\ -1\ -1)/8 \\
&= (+1\ +1\ +3\ +1\ -1\ -3\ -1\ -1)/8 \\
&= 0, \\
\mathbf{S}\cdot\mathbf{D} &= (-1\ +1\ -3\ +1\ -1\ -3\ +1\ +1)\cdot \\
&\quad\ (-1\ +1\ -1\ -1\ -1\ -1\ +1\ -1)/8 \\
&= (+1\ +1\ +3\ -1\ +1\ +3\ +1\ -1)/8 \\
&= 1.
\end{aligned}
$$

10. A regional telephone company has 15 million subscribers. Each of their telephones is connected to a central office by a copper twisted pair. The average length of these twisted pairs is 10 km. How much is the copper in the local loops worth? Assume that the cross section of each strand is a circle 1 mm in diameter, the density of copper is 9.0grams/cm^3, and that copper sells for \$6 per kilogram.

    Solution:

$$
\begin{aligned}
&\left(15\times 10^6\right)\times\left(10\times 10^5\right)\times\left(\left(\pi\times 0.05^2\right)\times 2\right)\times\left(9.0\times 10^{-3}\right)\times 6 \\
=\ & 1.5\times 10^{13}\times 3.14159\times 5\times 10^{-3}\times 9\times 10^{-3}\times 6 \\
=\ & 1.272\times 10^{10}\ \$
\end{aligned}
$$

11. If a T1 carrier system slips and loses track of where it is, it tries to resynchronize using the first bit in each frame. How many frames will have to be inspected on average to resynchronize with a probability of 0.001 of being wrong?

    Solution:

$$\left(\frac{1}{2}\right)^n \le 0.001 \Rightarrow n \ge 10. \qquad \text{(One bit per superframe is used for frame synchronization)}$$

12. What is the available user bandwidth in an OC-12c connection?

    Solution:

$$(12\times 90 - 12\times 3 - 1)\times 9\times 8\times 8000 = 600.768\times 10^6 \text{ bps}.$$

13. Suppose that x bits of user data are to be transmitted over a k-hop path in a packet-switched network as a series of packets, each containing p data bits and h header bits, with $x \gg p+h$. The bit rate of the lines is b bps and the propagation delay is negligible. What value of p minimizes the total delay?

    Solution:

$$y = \frac{\frac{x(p+h)}{p}}{b} + (k-1)\frac{p}{b} \quad\Rightarrow\quad \frac{dy}{dp} = \frac{x}{b}\frac{-h}{p^2} + \frac{k-1}{b} = 0 \quad\Rightarrow\quad p = \sqrt{\frac{xh}{k-1}}.$$

14. How fast can a cable user receive data if the network is otherwise idle? Assume that the downstream cable channel works at 27 Mbps and that the user interface is

    (a) 10 Mbps Ethernet

    (b) 100 Mbps Ethernet

(c) 54 Mbps Wireless.

Solution:

The downstream data rate of the cable user will be

(a) 10 Mbps Ethernet

(b) 27 Mbps Ethernet

(c) 27 Mbps Wireless.

15. Calculate the transmit time in the previous problem if packet switching is used instead. Assume that the packet size is 64 KB, the switching delay in the satellite and hub is 10 microseconds, and the packet header size is 32 bytes.

Solution:

$$\frac{\left(2^{30} + \frac{2^{30}}{64\times 2^{10}} \times 32\right) \times 8}{10^6} + \frac{4 \times 35800 \times 10^3}{3 \times 10^8} + 3 * 10^{-6} = 8594.128896 + 0.477333 + 0.000003 = 8594.606232 \text{ sec.}$$