Linear Control Systems

(EE-379)

DE-43 Mechatronics

Syndicate – A

Project Report

Control System Analysis of a Quarter Bus Suspension System

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Abstract:

This project entails a study of the suspension system through deriving transfer functions and testing their response with disturbances. PID and lead compensators are developed using the respective MATLAB tools PID Tuner and SISO for disturbance rejection improvement options. The controllers will be tuned optimally according to their stability, overshoot, settling time, etc. Evaluation of the controllers with respect to disturbances is then performed through simulation to show how better performance can be brought to the system.

Introduction:

A vehicle suspension system is a crucial system in automobiles whose purpose is to ensure passenger comfort and safety. The suspension system is found between the wheel and the vehicle's body and is responsible for absorbing various disturbances caused by road irregularities such as road bumps, speed breakers, potholes etc.

Compensator:

Compensators are used in control systems to improve the output response of the system. We often require changing or modifying the parameters of the system. A compensator in such cases helps in improving the control systems performance. The additional component called compensator is added to the structure of the control system while redesigning it. It is added to compensate for the deficient performance of the system. The types of a compensator can be hydraulic, electrical, mechanical, etc [1].

Disturbance:

Disturbance is any form of input to the system that may cause the desired output response of the system to change. Disturbances may arise from [2]:

- **External Forces:** Unexpected inputs like wind gusts, vibrations, or load changes.
- **Noise**: Random fluctuations in sensors or actuators that degrade measurement accuracy.
- **Modelling Errors:** Differences between the mathematical model and the actual system, due to approximations or unknown dynamics.

Importance of compensators in mitigating disturbances:

Compensators play a vital role in mitigating disturbances. Some of the ways compensators can be used for minimizing disturbances are listed below:

- **Noise filtering:** Compensators can filter out unwanted high or low frequency noise from the systems to enhance measurement accuracy and control.
- **Rejection of Disturbances:** Compensators help the system maintain its desired output by reducing the influence of external disturbances [2].
- **Improved Robustness:** They ensure that the system performs consistently, even in the presence of uncertainties or variations [2].
- **Addressing modelling errors:** By addressing modeling errors, compensators ensure that the system achieves its objectives reliably.

For example:

In active vehicle suspension systems, a compensator can further minimize the effect of external forces caused due to road irregularities.

In communication systems, compensators (e.g., filters) reduce noise and interference for clearer signal transmission.

In vehicle cruise control systems, compensators can be used to reduce steady state errors ensuring the vehicle continuous its journey at the desired speed irrespective of road irregularities.

Proposed System:

Quarter Bus Suspension System:

The system proposed in this project is 1/4th of the suspension system of a bus which can be modelled as a collection of mass spring damper systems. The parameter values were taken from a website to ensure realistic modelling of an actual bus suspension system

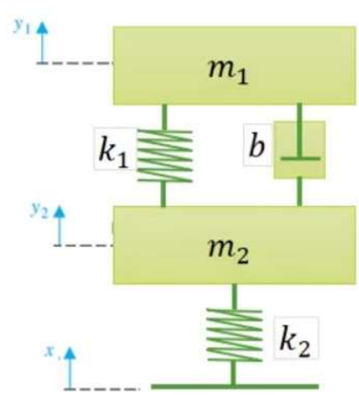


Figure 1: Quarter bus suspension system model [4].

In ***figure 1***, the mass m_2 and the spring k_2 represent the model of a wheel and tire whereas k_1 and b represent the model of a shock absorber with m_2 being the mass of the vehicle. 'X' is any disturbance caused by road irregularities and y_1 is the displacement output caused due to it which is felt by the passengers inside the vehicle. The goal of shock absorber is to remove or reduce road irregularities providing comfort to passengers inside.

Mathematical Modelling:

$$\boldsymbol{Mathematical\ Modeling\ of\ a\ Quarter-Bus\ Suspension\ System}$$

$\boldsymbol{The\ system\ consists\ of}$:

(m_1): $1/4thmassofvehicle'sbody = 2500kg$

(m_2): $mass\ of\ wheel\ = 320kg$

(k_1): $spring\ constant\ of\ suspension\ system\ = 80\ kN/m$

(k_2): $Spring\ constant\ of\ wheel\ and\ tire$: $500\ kN/m$

$(\ b\ )$: $damping\ coefficient\ = 350\ N.s/m$

(y_1): $displacement\ of\ (m_1)$.

(y_2): $displacement\ of\ (m_2)$.

(x): $road\ displacement\ input$.

$$\boldsymbol{Equations\ of\ Motion}$$

$\boldsymbol{For\ (m_1)}$:

$$m_1\ddot{y}_1 = -k_1(y_1 - y_2) - b(\dot{y}_1 - \dot{y}_2)$$

$\boldsymbol{TakingtheLaplacetransform(assumingzeroinitialconditions)}$:

$$m_1s^2Y_1 = -k_1(Y_1 - Y_2) - b(sY_1 - sY_2)$$

$\boldsymbol{Rearranging}$:

$$m_1s^2Y_1 + (k_1 + bs)Y_1 = (k_1 + bs)Y_2$$

$$Y_1(m_1s^2 + k_1 + bs) = Y_2(k_1 + bs) \qquad (1)$$

$\boldsymbol{For(m_2)}$:

$$m_2\ddot{y}_2 = -k_2(y_2 - x) + k_1(y_1 - y_2) + b(\dot{y}_1 - \dot{y}_2)$$

$\boldsymbol{Taking\ the\ Laplace\ transform}$:

$$m_2s^2Y_2 = -k_2(Y_2 - X) + k_1(Y_1 - Y_2) + b(sY_1 - sY_2)$$

$\boldsymbol{Rearranging}$:

$$m_2s^2Y_2 + (k_1 + k_2 + bs)Y_2 = k_2X + (k_1 + bs)Y_1$$

$$Y_2(m_2s^2 + k_1 + k_2 + bs) = k_2X + (k_1 + bs)Y_1 \qquad (2)$$

$\boldsymbol{Solving\ for\ Transfer\ Function(\frac{Y_1}{X})}$

$\boldsymbol{FromEquation(1)}$:

$$Y_2 = \frac{Y_1(m_1 s^2 + k_1 + bs)}{k_1 + bs} \quad (3)$$

$\boldsymbol{Substitute Equation(3) into Equation(2)}$:

$$\frac{Y_1(m_1 s^2 + k_1 + bs)}{k_1 + bs}(m_2 s^2 + k_1 + k_2 + bs) = k_2 X + (k_1 + bs)Y_1$$

$\boldsymbol{Multiply\ through\ by\ (k_1 + bs)\ to\ eliminate\ the\ fraction}$:

$$Y_1(m_1 s^2 + k_1 + bs)(m_2 s^2 + k_1 + k_2 + bs) = (k_1 + bs)^2 Y_1 + (k_1 + bs)k_2 X$$

$\boldsymbol{Rearrange\ terms}$:

$$Y_1[(m_1 s^2 + k_1 + bs)(m_2 s^2 + k_1 + k_2 + bs) - (k_1 + bs)^2] = (k_1 + bs)k_2 X$$

$\boldsymbol{Solve\ for\ (\frac{Y_1}{X})}$:

$$\frac{Y_1}{X} = \frac{k_2(k_1 + bs)}{(m_1 s^2 + k_1 + bs)(m_2 s^2 + k_1 + k_2 + bs) - (k_1 + bs)^2}$$

$\boldsymbol{Final\ Transfer\ Function}$

$$\frac{Y_1}{X} = \frac{k_1 k_2 + b k_2 s}{(m_1 s^2 + k_1 + bs)(m_2 s^2 + k_1 + k_2 + bs) - (k_1 + bs)^2}$$

$\boldsymbol{Putting\ in\ the\ values}$:

$$\frac{Y_1}{X} = \frac{4 \times 10^{10} + 1.75 \times 10^8 s}{(2500s^2 + 80000 + 350s)(320s^2 + 580000 + 350s) - (80000 + 350s)^2}$$

$\boldsymbol{Further\ simplification}$:

$$\frac{Y_1}{X} = \frac{4 \times 10^{10} + 1.75 \times 10^8 s}{8 \times 10^5 s^4 + 987000s^3 + 1.476 \times 10^9 s^2 + 1.75 \times 10^8 s + 4 \times 10^{10}}$$

Addition of Disturbance

To assess the system's performance, it was subjected to different types of disturbances, which are as follows:

Step Disturbance

This model includes the disturbance that arises from the surface of a road having anomalies at some specific points and edges, as when one can physically feel the edges of riding surface changes due to crossing over features such as curbs, sidewalks and other vertical displacements on the road.

Input in Laplace Domain:

$$U(s) = 1s. U(s) = \frac{1}{s}.$$

Output in Laplace Domain:

$$Y(s) = H(s) \cdot U(s) = H(s) \cdot 1sY(s) = H(s) \cdot U(s) = H(s) \cdot \frac{1}{s}$$

$$Y(s) = \frac{(4 \times 10^{10} + 1.75 \times 10^8 s)}{s \cdot (8 \times 10^5 s^4 + 987000s^3 + 1.476 \times 10^9 s^2 + 1.75 \times 10^8 s + 4 \times 10^{10})}.$$

Impulse Disturbance

An adjustment to the balance condition of the suspension system is the result of an impulse disturbance due to change in position of the spring and damper. This can be in the form of a hammer strike to the wheel or the wheel hitting a pothole.

Input in Laplace Domain:

$$U(s) = 1. U(s) = 1.$$

Output in Laplace Domain:

$Y(s) = H(s) \cdot U(s) = H(s). Y(s) = H(s) \cdot U(s) = H(s).$

$$Y(s) = \frac{4 \times 10^{10} + 1.75 \times 10^8 s}{8 \times 10^5 s^4 + 987000 s^3 + 1.476 \times 10^9 s^2 + 1.75 \times 10^8 s + 4 \times 10^{10}}.$$

Sinusoidal Disturbance

The surface of the road is sometimes not completely smooth, even the beams of the suspension bridge or the structure of switching noise changes smoothly and has bumps or waves, which can reflect the welded seam of the drawings in measuring switching noise. This can be represented usually with the sinusoidal function.

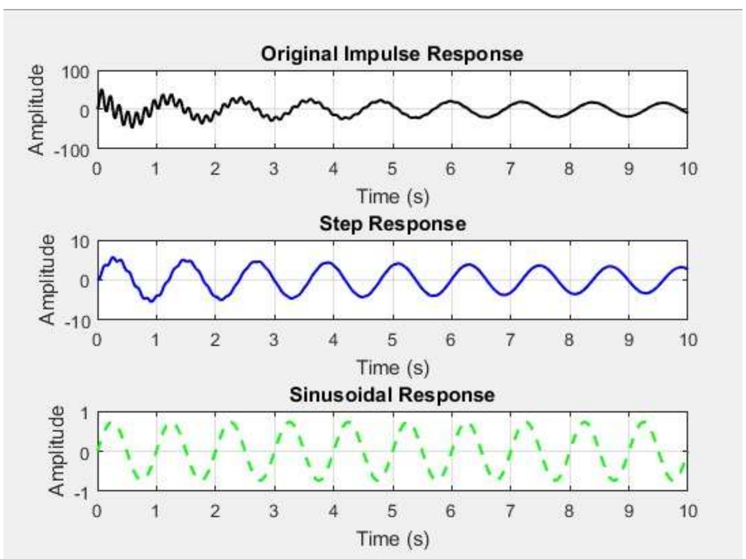
Input in Laplace Domain:

$$U(s) = \omega s2 + \omega 2. U(s) = \frac{\omega}{s^2 + \omega^2}.$$

Output in Laplace Domain:

$Y(s) = H(s) \cdot U(s). Y(s) = H(s) \cdot U(s).$

$$Y(s) = \frac{(4 \times 10^{10} + 1.75 \times 10^8 s) \cdot \omega}{(s^2 + \omega^2) \cdot (8 \times 10^5 s^4 + 987000 s^3 + 1.476 \times 10^9 s^2 + 1.75 \times 10^8 s + 4 \times 10^{10})}.$$



Compensator Design

Compensators alter a system's behaviour to accomplish intended results, especially when there are disruptions present. Among their principal responsibilities are:

- **Enhancing Stability:** Ensure the system reacts consistently in a range of scenarios.
- **Improving Transient Response:** Modify how quickly or slowly the system reacts to inputs.
- **Minimizing Steady-State Error:** Reduce the discrepancies between the intended and actual outputs once transients have subsided.
- **Disturbance Rejection:** Lessen the effect of outside disruptions on the output of the system.
- **Shaping Frequency Response:** Modify the bandwidth and resonant peak of the system's response.

Step Response Root Locus

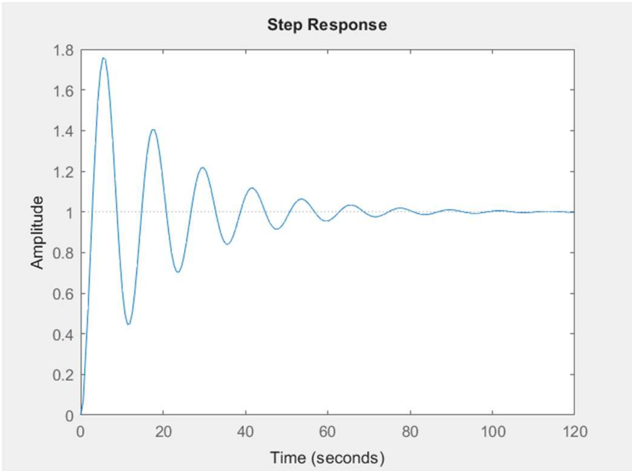


Figure2.

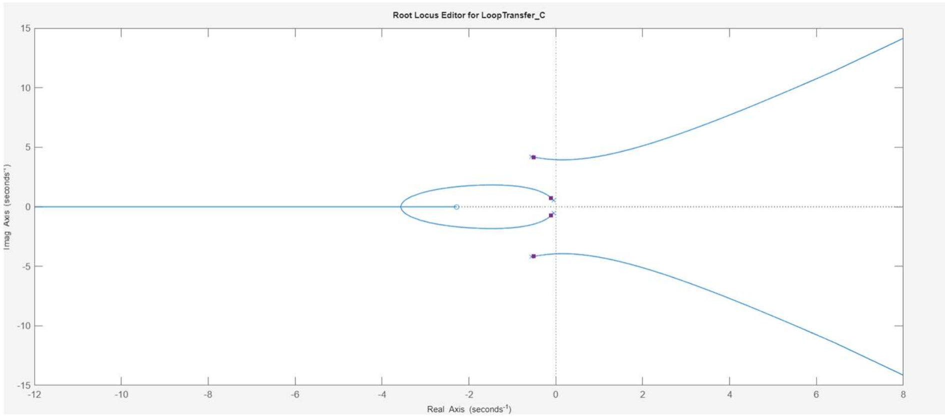


Figure3.

Rise Time: 1.9082 Transient Time: 72.8291 Settling Time: 72.8291 Settling Min: 0.4467
Settling Max: 1.756 Overshoot: 75.6932 Undershoot: 0 Peak Time: 1.7569
Peak Value: 5.3722

PID Tuning

Step response of the system is tuned with *pidTuner()* function. Further disturbances were discussed in previous section.

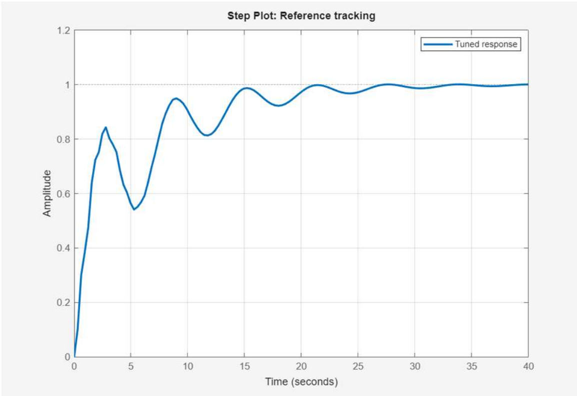


Figure4.

Compensator Design

The system requires an increase in stability; shift the poles to the left. Hence lead compensator is required.

Lead Compensator

Key uses of this compensator would be:

- Improves stability.
- Speeds up the response by shifting poles left.

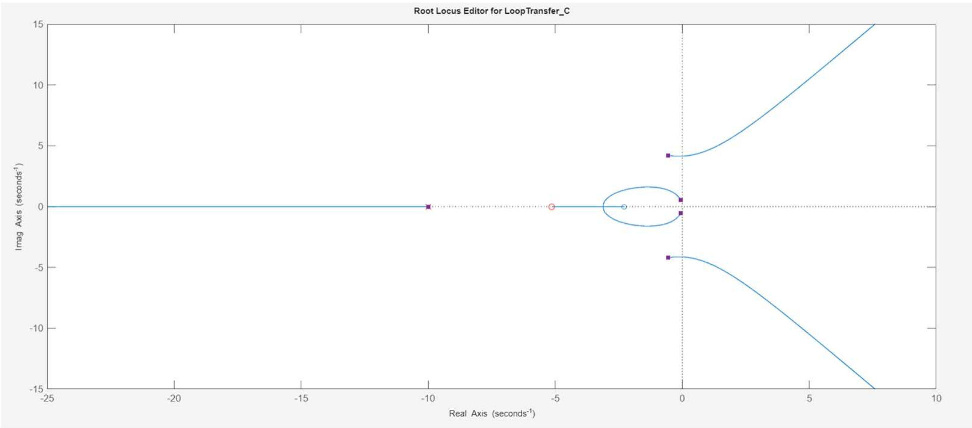


Figure5.

A zero is placed closer to the origin, at -5, and a pole at -10 which is further left. Compensator equation is as follows:

$$C = 0.0001959 \times \frac{(1+0.19s)}{(1+0.1s)}$$

As a result, the amplitude greatly decreases:

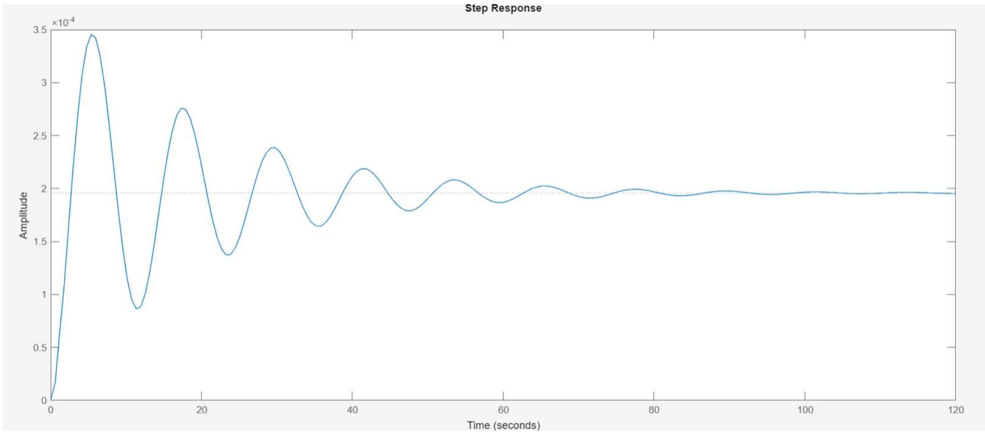


Figure 6

Comparing with original function (orange), the compensated response (blue) seems insignificant.

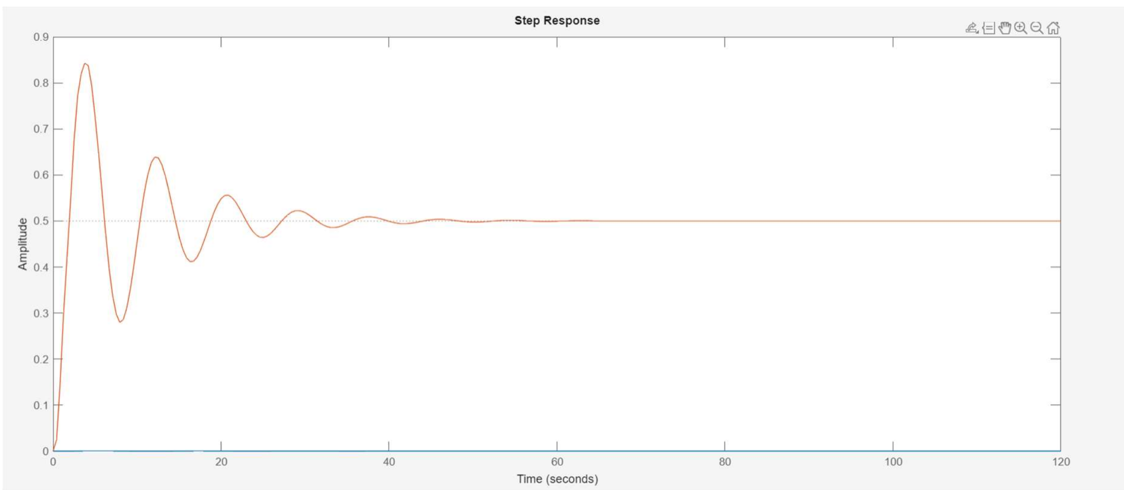


Figure 7

Disturbance Rejection

A damping disturbance is added to a system using SISOTOOL. Its rejection is shown as follows:

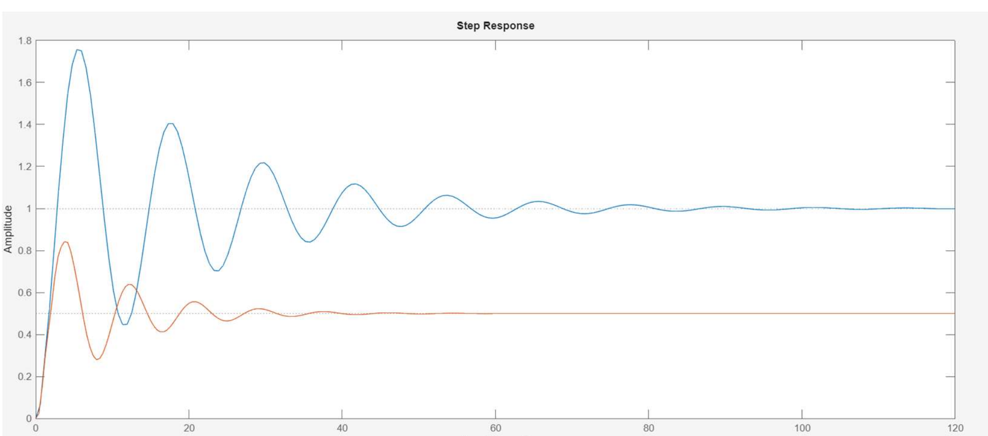


Figure8.

Performance based comparison of different controller types

PID Controller Performance

Pros:

- Lowers steady-state error (due to integral action).
- Balances stability with response speed (due to derivative action).

Cons:

- Longer settling time and overshoot (potential due to integral action).
- More complexity.
- Noise sensitivity (from the derivative).

PD Controller Performance

Pros:

- Enhances transient response (decreases settling time and overshoot).
- Quicker reaction.

Cons:

- No impact on steady-state error.
- Noise sensitivity (from the derivative).

Lead Compensator Performance

Pros:

- Enhances phase margin (increases stability and faster reaction).
- Potential for enhanced transient response without significantly altering steady-state error.

Cons:

- Does not lower steady-state error.
- May increase peak overshoot.

1. **Lag Compensator Performance**

Pros:

- Improves steady-state error (due to integral-like action)

Cons:

- Slows system response (slows down transient response).
- Reduced system bandwidth (may result in slower overall dynamics).

Effectiveness of Disturbance Rejection

Comparison of Step Responses

- **Plant:** Exhibits significant oscillations and a slow settling time, indicating an underdamped system with a sluggish response to step inputs.
- **PID Control:** Significantly improves the response with reduced oscillations and faster settling time compared to the plant. However, some overshoots remain.
- **Lead Compensator:** Further enhances the response with reduced overshoot and faster settling time compared to PID control. Demonstrates a more rapid and stable response.
- **Lag Compensator:** Has minimal impact on the transient response compared to the plant. Oscillations and settling time are comparable to the plant, suggesting that the lag

compensator primarily focuses on improving steady-state error, which is not evident in the step response plot.

Applying these on the system, the responses are displayed:

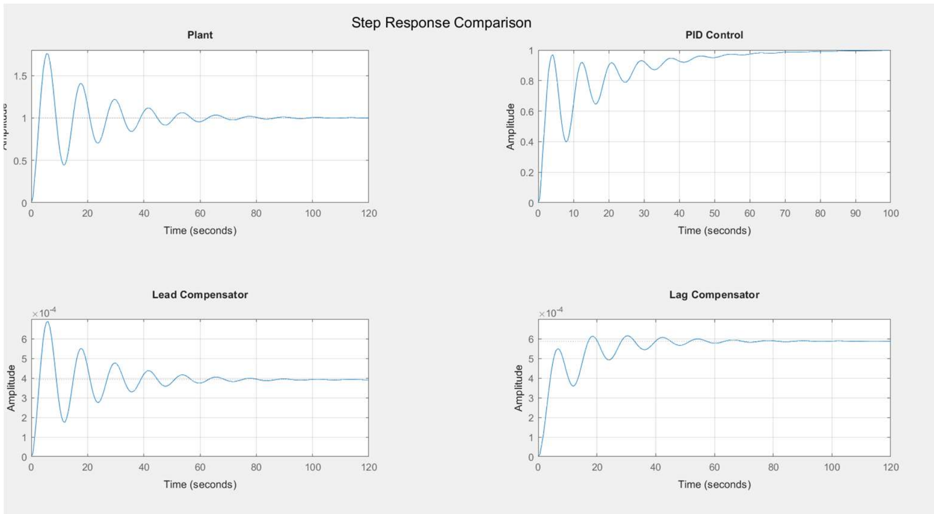


Figure 9

Resulting comparison of response correction of various compensators based on different input disturbances:

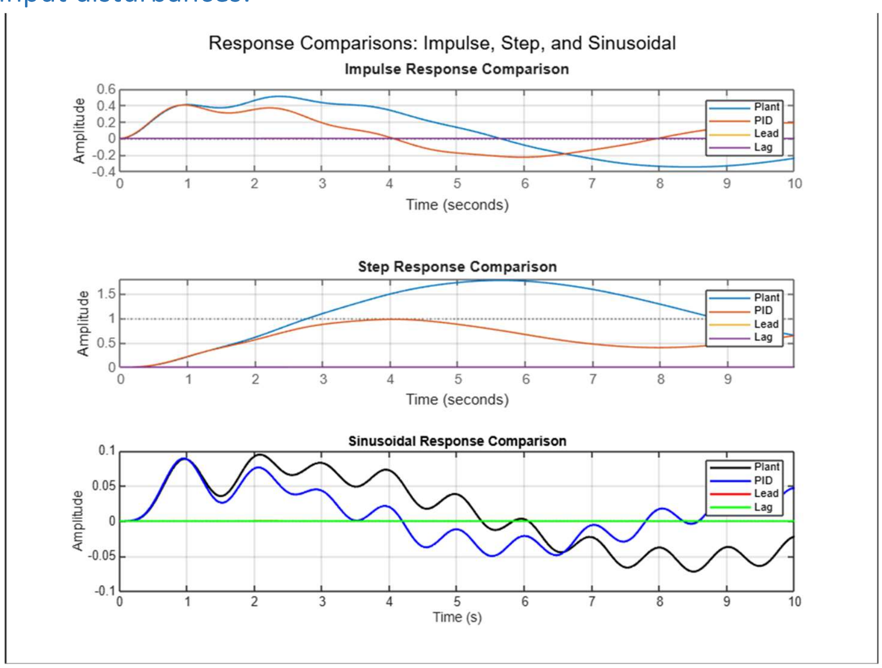


Figure 10

The code for the graphs obtained is given in appendix A.5 The results show the following findings:

- **PID Controller:** Offers a well-rounded performance, balance stability and speed across all tests. Suitable for general purpose applications requiring both accuracy and fast response.
- **Lead Compensator:** Optimizes the speed of the system's response, making it ideal for applications prioritizing quick adjustments, but may tolerate slight overshoot.

- **Lag Compensator:** Focuses on stability and steady-state accuracy, ideal for systems where minimizing oscillations is crucial, even at the cost of slower dynamics.

Results and Conclusion:

In conclusion, the output response to disturbance of each compensator obtained, Lead compensator is the most suitable compensator that provides faster damping and transient response characteristics most suitable for further improving the bus suspension system.

References:

[1] "Control system Compensators - javatpoint," *www.javatpoint.com*. https://www.javatpoint.com/control-system-compensators

[2] OpenAI, "ChatGPT," *Openai.com*, Nov. 30, 2022. https://openai.com/index/chatgpt/

[3] "Control Tutorials for MATLAB and Simulink - Suspension: System Modeling," *ctms.engin.umich.edu*. https://ctms.engin.umich.edu/CTMS/index.php?example=Suspension&section=SystemModeling

[4] MAFarooqi, "LCS - 06b - Modelling of Quarter Car Suspension System," *YouTube*, Jul. 20, 2021. https://www.youtube.com/watch?v=nPn7L8-l_Lk (accessed Dec. 24, 2024).

Appendix

Appendix A

Two column format.

```
% System Parameters

m1 = 2500;

m2 = 320;

k1 = 800;

k2 = 5000;

b = 350;

% Define the Plant

s = tf('s');

numerator = k2 * (k1 + b * s);

denominator = (m1 * s^2 + k1 + b * s) * (m2 *
s^2 + k1 + k2 + b * s) - (k1 + b * s)^2;

plant = numerator / denominator;

% Design Controllers

Kp = 1;

Ki = 0.1;

Kd = 0.01;

pid_controller = pid(Kp, Ki, Kd);

lead_zero = 10;

lead_pole = 5;

lead_compensator = 0.0001959*tf([1,
lead_zero], [1, lead_pole]);

lag_zero = 0.3;

lag_pole = 0.1;

lag_compensator = 0.0001959*tf([1,
lag_zero], [1, lag_pole]);

% Create Closed-Loop Systems

closed_loop_pid = feedback(pid_controller *
plant, 1);

closed_loop_lead =
feedback(lead_compensator * plant, 1);

closed_loop_lag = feedback(lag_compensator
* plant, 1);

% Time range for responses
```

```
t = linspace(0, 10, 500);

% Impulse Responses

figure;

subplot(3, 1, 1);

impulse(plant, t);

hold on;

impulse(closed_loop_pid, t);

impulse(closed_loop_lead, t);

impulse(closed_loop_lag, t);

title('Impulse Response Comparison');

legend('Plant', 'PID', 'Lead', 'Lag');

grid on;

% Step Responses

subplot(3, 1, 2);

step(plant, t);

hold on;

step(closed_loop_pid, t);

step(closed_loop_lead, t);

step(closed_loop_lag, t);

title('Step Response Comparison');

legend('Plant', 'PID', 'Lead', 'Lag');

grid on;

% Sinusoidal Responses

subplot(3, 1, 3);

omega = 2 * pi; % Frequency for sinusoidal
input

sin_input = sin(omega * t);

y_plant = lsim(plant, sin_input, t);

y_pid = lsim(closed_loop_pid, sin_input, t);

y_lead = lsim(closed_loop_lead, sin_input, t);

y_lag = lsim(closed_loop_lag, sin_input, t);

plot(t, y_plant, 'k', 'LineWidth', 1.5);

hold on;

plot(t, y_pid, 'b', 'LineWidth', 1.5);

plot(t, y_lead, 'r', 'LineWidth', 1.5);

plot(t, y_lag, 'g', 'LineWidth', 1.5);

title('Sinusoidal Response Comparison');

xlabel('Time (s)');

ylabel('Amplitude');

legend('Plant', 'PID', 'Lead', 'Lag');

grid on;

sgtitle('Response Comparisons: Impulse, Step,
and Sinusoidal');
```