Life of a Particle : Quiz 3

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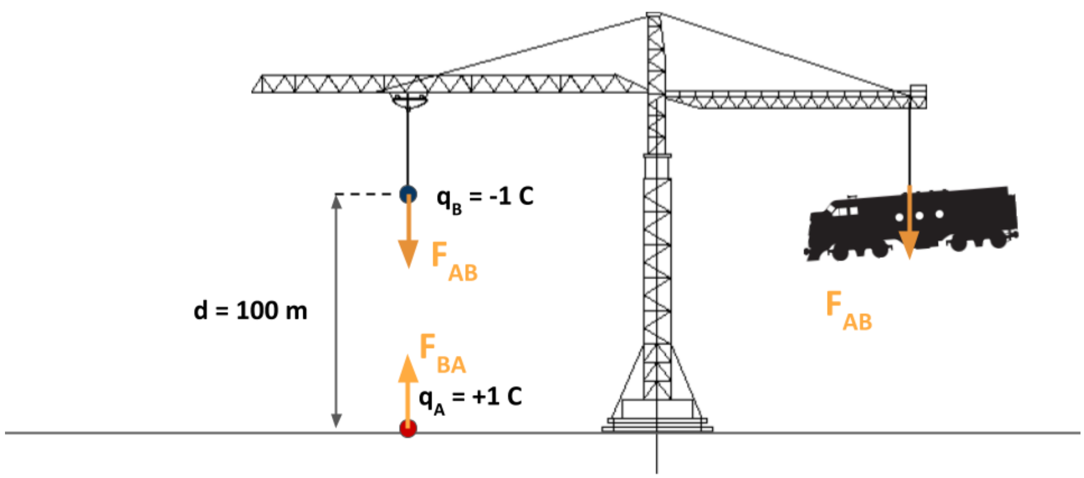
Solutions

1 The unit of charge: the Coulomb



Question A: What is the mass to be suspended on the other side of the crane to balance this system out? Compare this to a day-to-day life object.

The two charges will attract each other due to the electrostatic force. Let's compute $\vec{F}_{AB}$, the force exerted by the ground charge on the upper one. It points downwards and we get the magnitude using Coulomb's law:

$$\mathbf{F_{AB}} = k_e \frac{q_A\, q_B}{|\,\mathbf{r_{AB}}|^2}\, \hat{\mathbf{r}}_{\mathbf{AB}} \tag{1}$$

With the values we obtain:

$$F_{AB} = 8.99 \times 10^9\ \frac{1.00 \times 1.00}{100^2} = 899\,\mathrm{kN} \tag{2}$$

That's a lot of force. To know the corresponding mass to hang on the other side, we need to have a weight hanging there with the same magnitude as F_{AB}. With Newton's first law of universal gravitation, we know the weight of an object of mass m on Earth is defined as the force of gravity on the object. It is obtained using the relation:

$$\vec{F}_{\text{grav}} = \vec{g}.m \tag{3}$$

For the crane to be balanced, we must have $\vec{F}_{AB} = \vec{F}_{\text{grav}}$. Thus the mass is:

$$m = \frac{F_{\text{grav}}}{g} = \frac{F_{AB}}{g} = \frac{899 \times 10^3}{9.81} = 91.6 \times 10^3 \text{ kg} = 91.6 \text{ tons} \tag{4}$$

We have to hang a big locomotive to balance two charges of 1 Coulomb separated by 100 metres!

Question B: A typical lightning strike is about 40 coulombs of charge, consisting of separate "strokes" (that's why lightning usually looks flickery). Each stroke lasts about 30 microseconds. What is the current?

The current is, by definition, a flow (amount) of charge per unit of time. We have for our strike:

$$I = \frac{Q}{\Delta t} = \frac{40}{30 \times 10^{-6}} = 1.33 \times 10^6 \text{A} \tag{5}$$

Our lightning strike has a current of 1.33 million amperes!
To compare, when you charge your smartphone you are using roughly 1 ampere.

2 The electron volt

Question A: How does the energy of the proton-proton collisions at LHC compare with respect to the energy stored in a cocoa chocolate bar?

Let's convert the energy in the Kingsbite Milk Chocolate Bar in eV:

$$\frac{274\text{Calories} \times 4184\text{J/Calorie}}{1.6 \cdot 10^{-19}\text{J/eV}} = 7.2 \cdot 10^{24}\text{eV} \tag{6}$$

The ratio with the energy in the LHC is:

$$\frac{7.2 \cdot 10^{24}}{14 \cdot 10^{12}} = 513 \cdot 10^9 \tag{7}$$

It would take a 500 billions proton-proton collisions at the LHC top energy to get the same amount of energy as in a Kingsbite Milk Chocolate Bar.

Question B: Make the same comparison with energy density.

The Kingsbite Milk Chocolate Bar has an energy density of about 274 Cal/100 cm^3, so 2.74 Cal/cm^3. A proton-proton collision at 14 TeV has an energy density around 14 $\cdot 10^{12}$ eV/10^{-39} cm^3 $\cdot$ 1.6 $\cdot 10^{-19}$ J/eV $\times$ 4184 Joules. This is 9.4 $\cdot 10^{35}$ Cal/cm^3.

Proton-proton collisions at the LHC have an energy density about a billion billion billion billion times a Kingsbite Milk Chocolate Bar.