Part (B)

The **check_grad** function is used and the following is the output.

<u>Gradient check using **check_grad**:</u> 6.093988195535878e-06
<u>Gradient check using **approx_fprime**:</u>
Value 0: -0.014803935965964259
Value 1: -0.014803935965964259
Value 2: -0.014978769669034562
Value 3: -0.01480393596598994
Value 4: -0.01480393596598994
.
.
.
Value 59205: 4.5192294351136574e-05
Value 59206: -0.13773177587326604
Value 59207: 0.00014157508498854605
Value 59208: 8.768985537933954e-06
Value 59209: -0.00013831824574581144

```python
print(scipy.optimize.check_grad(
    lambda weights_: fCE(trainX, trainY, weights_,NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT),  # No slicing on trainX
    lambda weights_: gradCE(trainX, trainY, weights_, NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT),
    weights
))

weights, avg_loss = train(X_train, y_train, weights, X_val, y_val, lr,num_epochs,batch_size,alpha,NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT)  #
val_accuracy = validate(X_val, y_val, weights, NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT)
test_accuracy = test(X_test, y_test, weights, NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT)


print(scipy.optimize.approx_fprime(weights, lambda weights_: fCE(trainX, trainY, weights_, NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT), 1e-6))
```

```
✓ 46.9s                                                                 Python
3.5305727448724214e-06
[-1.48039360e-02 -1.48039360e-02 -1.49787697e-02 ...  1.41575085e-04
  8.76898554e-06 -1.38318246e-04]
```

All the values are saved in **Part_B_grad_weight.txt**

Part (C)

```python
training_data = np.reshape(np.load("fashion_mnist_train_images.npy"), (-1, 28*28))
training_labels = np.load("fashion_mnist_train_labels.npy")

num_datapoints = training_data.shape[0]
split_index = int(0.8*num_datapoints)
indices = np.arange(num_datapoints)
np.random.shuffle(indices)

train_indices = indices[:split_index]
val_indices = indices[split_index:]

X_train, X_val = training_data[train_indices], training_data[val_indices]
y_train, y_val = training_labels[train_indices], training_labels[val_indices]

X_train = X_train / 255.0 - 0.5
X_val = X_val / 255.0 - 0.5

best_hyp,best_weights,best_acc= validation(X_train,y_train,X_val,y_val)
```

```
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 16, Learning_rate: 0.005, Unregularise CE: 0.32630, Test accuracy: 88.42%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 16, Learning_rate: 0.005, Unregularise CE: 0.26063, Test accuracy: 88.33%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 32, Learning_rate: 0.005, Unregularise CE: 0.39904, Test accuracy: 88.08%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 32, Learning_rate: 0.005, Unregularise CE: 0.31940, Test accuracy: 88.31%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 64, Learning_rate: 0.005, Unregularise CE: 0.50693, Test accuracy: 87.18%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 64, Learning_rate: 0.005, Unregularise CE: 0.40018, Test accuracy: 87.94%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 16, Learning_rate: 0.01, Unregularise CE: 0.28510, Test accuracy: 87.86%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 16, Learning_rate: 0.01, Unregularise CE: 0.22689, Test accuracy: 88.15%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 32, Learning_rate: 0.01, Unregularise CE: 0.32700, Test accuracy: 88.27%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 32, Learning_rate: 0.01, Unregularise CE: 0.26238, Test accuracy: 87.74%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 64, Learning_rate: 0.01, Unregularise CE: 0.40073, Test accuracy: 87.92%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 64, Learning_rate: 0.01, Unregularise CE: 0.32183, Test accuracy: 88.33%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 16, Learning_rate: 0.05, Unregularise CE: 0.24958, Test accuracy: 88.12%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 16, Learning_rate: 0.05, Unregularise CE: 0.20744, Test accuracy: 87.59%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 32, Learning_rate: 0.05, Unregularise CE: 0.25572, Test accuracy: 87.88%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 32, Learning_rate: 0.05, Unregularise CE: 0.20749, Test accuracy: 87.67%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 50, Batch_size: 64, Learning_rate: 0.05, Unregularise CE: 0.28181, Test accuracy: 87.67%
n_hidden_lrs: 3, n_hidden_units: 30, Num_Epoch 100, Batch_size: 64, Learning_rate: 0.05, Unregularise CE: 0.22720, Test accuracy: 88.20%
n_hidden_lrs: 3, n_hidden_units: 40, Num_Epoch 50, Batch_size: 16, Learning_rate: 0.005, Unregularise CE: 0.32104, Test accuracy: 87.61%
n_hidden_lrs: 3, n_hidden_units: 40, Num_Epoch 100, Batch_size: 16, Learning_rate: 0.005, Unregularise CE: 0.25273, Test accuracy: 87.90%
n_hidden_lrs: 3, n_hidden_units: 40, Num_Epoch 50, Batch_size: 32, Learning_rate: 0.005, Unregularise CE: 0.39352, Test accuracy: 87.65%
n_hidden_lrs: 3, n_hidden_units: 40, Num_Epoch 100, Batch_size: 32, Learning_rate: 0.005, Unregularise CE: 0.31330, Test accuracy: 88.41%
n_hidden_lrs: 3, n_hidden_units: 40, Num_Epoch 50, Batch_size: 64, Learning_rate: 0.005, Unregularise CE: 0.50302, Test accuracy: 86.64%
n_hidden_lrs: 3, n_hidden_units: 40, Num_Epoch 100, Batch_size: 64, Learning_rate: 0.005, Unregularise CE: 0.39749, Test accuracy: 88.07%
n_hidden_lrs: 3, n_hidden_units: 40, Num_Epoch 50, Batch_size: 16, Learning_rate: 0.01, Unregularise CE: 0.27818, Test accuracy: 88.17%
...
n_hidden_lrs: 5, n_hidden_units: 50, Num_Epoch 50, Batch_size: 32, Learning_rate: 0.05, Unregularise CE: 0.24024, Test accuracy: 88.53%
n_hidden_lrs: 5, n_hidden_units: 50, Num_Epoch 100, Batch_size: 32, Learning_rate: 0.05, Unregularise CE: 0.17558, Test accuracy: 87.77%
n_hidden_lrs: 5, n_hidden_units: 50, Num_Epoch 50, Batch_size: 64, Learning_rate: 0.05, Unregularise CE: 0.26747, Test accuracy: 88.11%
n_hidden_lrs: 5, n_hidden_units: 50, Num_Epoch 100, Batch_size: 64, Learning_rate: 0.05, Unregularise CE: 0.19745, Test accuracy: 86.88%
Output is truncated. View as a scrollable element or open in a text editor. Adjust cell output settings...
```

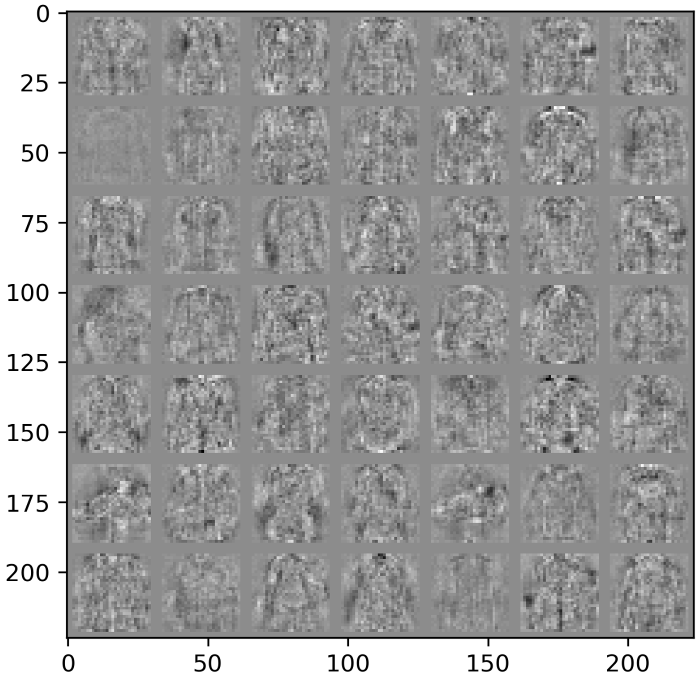
```python
np.save("best_model_weights_bp.npy", best_weights)
np.save("best_model_hyperparameters_bp.npy",best_hyp)
np.save("best_model_accuracy_bp.npy",best_acc)
```

```python
best_acc*100
```

```
88.94166666666666
```

Accuracy: 88.94166666666666
Accuracy for each iteration is saved in **hyperparameter_tune.txt**

Part (D)



In [48]:
```python
# Import the NumPy
import numpy as np

# Import the 'optimize' module from SciPy
import scipy.optimize

# Importing plt from matplotlib.pyplot
import matplotlib.pyplot as plt
```

Unpack Function

Function to unpack a single vector of weights into weight matrices and bias vectors for a multi-layer neural network.

Arguments:

- weights: Flattened array containing all weights and biases.
- NUM_HIDDEN_LAYERS: Number of hidden layers in the neural network.
- NUM_INPUT: Number of input features.
- NUM_HIDDEN: List specifying the number of units in each hidden layer.
- NUM_OUTPUT: Number of output units.

Returns:

- Ws: List of weight matrices for each layer.
- bs: List of bias vectors for each layer.

In [17]:
```python
def unpack(weights, NUM_HIDDEN_LAYERS, NUM_INPUT, NUM_HIDDEN, NUM_OUTPUT):

    # Initialize list to store weight matrices
    Ws = []

    # Unpack the weight matrix for the input layer to the first hidden layer
    start = 0
    end = NUM_INPUT * NUM_HIDDEN[0]  # Calculate the range for the first wei
    W = weights[start:end]  # Slice the weight vector for the first layer's
    Ws.append(W)  # Append the weight vector to the list

    # Unpack weight matrices between hidden layers
    for i in range(NUM_HIDDEN_LAYERS - 1):  # Loop through hidden layers
        start = end  # Update the start index
        end = end + NUM_HIDDEN[i] * NUM_HIDDEN[i+1]  # Calculate the end inc
        W = weights[start:end]  # Slice the weight vector for the current la
        Ws.append(W)  # Append the weight vector to the list

    # Unpack the weight matrix for the last hidden layer to the output layer
    start = end  # Update the start index for the last layer
    end = end + NUM_HIDDEN[-1] * NUM_OUTPUT  # Calculate the end index for t
    W = weights[start:end]  # Slice the weight vector for the output layer's
    Ws.append(W)  # Append the weight vector to the list
```

```python
    # Reshape the weight vectors into matrices for each layer
    Ws[0] = Ws[0].reshape(NUM_HIDDEN[0], NUM_INPUT)  # Reshape the first lay
    for i in range(1, NUM_HIDDEN_LAYERS):  # Loop through hidden layers
        Ws[i] = Ws[i].reshape(NUM_HIDDEN[i], NUM_HIDDEN[i-1])  # Reshape the
    Ws[-1] = Ws[-1].reshape(NUM_OUTPUT, NUM_HIDDEN[-1])  # Reshape the final

    # Initialize list to store bias vectors
    bs = []

    # Unpack bias vector for the first hidden layer
    start = end  # Set start index for biases
    end = end + NUM_HIDDEN[0]  # Calculate the end index for the first bias
    b = weights[start:end]  # Slice the weight vector for the biases
    bs.append(b)  # Append the bias vector to the list

    # Unpack bias vectors for the hidden layers
    for i in range(NUM_HIDDEN_LAYERS - 1):  # Loop through hidden layers
        start = end  # Update the start index for the next bias vector
        end = end + NUM_HIDDEN[i+1]  # Calculate the end index for the next
        b = weights[start:end]  # Slice the weight vector for the biases
        bs.append(b)  # Append the bias vector to the list

    # Unpack the bias vector for the output layer
    start = end  # Set start index for the output layer's bias
    end = end + NUM_OUTPUT  # Calculate the end index for the output layer's
    b = weights[start:end]  # Slice the weight vector for the biases
    bs.append(b)  # Append the bias vector to the list

    # Return the unpacked weight matrices and bias vectors
    return Ws, bs
```

Initializing the weights and biases

Initializes the weights and biases for a multi-layer neural network using a modified Kaiming He Uniform initialization for weights and a small constant for biases.

Arguments:

- NUM_HIDDEN_LAYERS: Number of hidden layers in the neural network.
- NUM_INPUT: Number of input features.
- NUM_HIDDEN: List specifying the number of units in each hidden layer.
- NUM_OUTPUT: Number of output units.

Returns:

- Ws: List of initialized weight matrices.
- bs: List of initialized bias vectors.

In [18]:
```python
def initWeightsAndBiases(NUM_HIDDEN_LAYERS, NUM_INPUT, NUM_HIDDEN, NUM_OUTPU
```

```
    Ws = []  # List to store weight matrices
    bs = []  # List to store bias vectors

    # Strategy:
    # - Weight initialization: Kaiming He Uniform initialization, suitable f
    # - Bias initialization: Small positive constant (0.01).

    # Set random seed for reproducibility
    np.random.seed(0)

    # Initialize weights between input layer and the first hidden layer
    W = 2 * (np.random.random(size=(NUM_HIDDEN[0], NUM_INPUT)).astype(np.flo
    Ws.append(W)  # Add the initialized weight matrix to the list

    # Initialize biases for the first hidden layer
    b = 0.01 * np.ones(NUM_HIDDEN[0], dtype=np.float32)  # Bias initialized
    bs.append(b)  # Add the initialized bias vector to the list

    # Initialize weights and biases for hidden layers
    for i in range(NUM_HIDDEN_LAYERS - 1):
        # Initialize weights between the ith hidden layer and the next hidde
        W = 2 * (np.random.random(size=(NUM_HIDDEN[i+1], NUM_HIDDEN[i])).ast
        Ws.append(W)  # Add the weight matrix to the list

        # Initialize biases for the next hidden layer
        b = 0.01 * np.ones(NUM_HIDDEN[i+1], dtype=np.float32)  # Bias initia
        bs.append(b)  # Add the bias vector to the list

    # Initialize weights for the final output layer (from last hidden layer
    W = 2 * (np.random.random(size=(NUM_OUTPUT, NUM_HIDDEN[-1])).astype(np.f
    Ws.append(W)  # Add the weight matrix to the list

    # Initialize biases for the output layer
    b = 0.01 * np.ones(NUM_OUTPUT, dtype=np.float32)  # Bias initialized to
    bs.append(b)  # Add the bias vector to the list

    # Return the lists of weight matrices and bias vectors
    return Ws, bs
```

In [27]:

```
# Helper functions

def relu(z):

    return np.maximum(0, z)

def relu_derivative(z):
        return (z > 0).astype(float)

def softmax(Z):
    # to avoid large exponent values, we will use the deviation idealogy of

    exponent_z = np.exp(Z - np.max(Z,axis=1,keepdims=True))

    prediction = exponent_z/(np.sum(exponent_z,axis=1,keepdims=True))

    return prediction
```

```python
def fCE (X, Y, weights):
    Ws, bs = unpack(weights)
    H_activation,_= forward_pass(X,Ws,bs,NUM_HIDDEN_LAYERS)
    y_pred = H_activation[-1]
    ce = loss(Y,y_pred,Ws,0)
    return ce

def gradCE(X, Y, weights):
    # Unpack the flattened weights and biases
    Ws, bs = unpack(weights)

    wt_grad = []  # List to store weight gradients
    bi_grad = []  # List to store bias gradients

    # Perform a forward pass through the network
    H_activation, z_score = forward_pass(X, Ws, bs, NUM_HIDDEN_LAYERS)

    # Initialize one-hot encoded labels (y_onehot)
    y_onehot = np.zeros_like(H_activation[-1])

    # Convert Y to integer type if needed
    Y = Y.astype(int)

    # One-hot encode the ground truth labels
    y_onehot[np.arange(len(Y)), Y] = 1

    # Compute the gradient of the loss w.r.t. the output layer (softmax laye
    g = H_activation[-1] - y_onehot  # Difference between prediction and gro

    # Backpropagate through each layer, starting from the output layer
    for i in range(NUM_HIDDEN_LAYERS, -1, -1):
        # Gradient of weights (g^T * H) and biases (sum of gradients over ba
        grad_wt = np.dot(g.T, H_activation[i]) / len(Y)
        grad_bi = np.sum(g, axis=0, keepdims=True) / len(Y)

        # Append the computed gradients to the respective lists
        wt_grad.append(grad_wt)
        bi_grad.append(grad_bi)

        if i > 0:  # If not the first layer (input layer)
            # Backpropagate the gradient to the previous layer using the ReL
            g = np.dot(g, Ws[i]) * relu_derivative(z_score[i-1])

    # Reverse the gradients list so they align correctly with the weight mat
    wt_grad.reverse()
    bi_grad.reverse()

    # Flatten and concatenate all gradients (weights and biases) into a sing
    allGradientsAsVector = np.hstack([W.flatten() for W in wt_grad] + [b.fla

    return allGradientsAsVector
```

Loss Function

Computes the cross-entropy loss between the true labels (y_label) and the predicted probabilities (y_pred) for a batch of data. Optionally, it also supports L2 regularization.

Arguments:

- y_label: Ground truth labels (as integer indices).
- y_pred: Predicted probabilities from the softmax function.
- W: Weight matrix (used for optional L2 regularization, currently not used).
- alpha: Regularization parameter (L2 regularization strength, currently not used).

Returns:

- loss: The average cross-entropy loss for the batch.

In [28]:
```python
def loss(y_label, y_pred, W, alpha):

    # Get the batch size, i.e., number of samples in the current batch.
    batch_s = y_label.shape[0]  # This ensures loss is computed as an averag

    # Ensure that y_label is of integer type, as it represents class indices
    y_label = y_label.astype(int)

    # Compute the negative log-likelihood (cross-entropy loss) for the true
    # For each sample, we access the predicted probability for the true labe
    prob = -np.log(y_pred[range(batch_s), y_label])  # Select the predicted

    # Compute the total loss by summing all the individual losses, and then
    loss = np.sum(prob) / batch_s  # Average loss across the batch.

    # Optionally, L2 regularization (currently commented out).
    # It penalizes large weights to prevent overfitting, and helps to smooth
    # reg_loss = alpha / 2 * np.sum(np.square(W))  # L2 regularization term

    # Compute total batch loss (currently not using regularization).
    # batch_loss = loss + reg_loss  # Total loss (including regularization).

    return loss  # Return the average cross-entropy loss (without regulariza
```

Forward Pass

Performs the forward pass through a neural network, computing activations for each layer.

Arguments:

- x: Input data (batch of samples).
- Weights: List of weight matrices for each layer.
- bias: List of bias vectors for each layer.

- NUM_HIDDEN_LAYERS: Number of hidden layers in the network.

Returns:

- H_activation: A list containing the activations for each layer (including input and output layers).
- z_preactivation_score: A list containing the pre-activation scores (z values) for each layer.

In [29]:
```
def forward_pass(x, Weights, bias, NUM_HIDDEN_LAYERS):

    z_preactivation_score = []  # To store the pre-activation (z) values for
    H_activation = [x]  # List to store activations, starting with the input

    # Loop over each hidden layer to compute pre-activations (z) and activat
    for i in range(NUM_HIDDEN_LAYERS):
        # Compute the pre-activation score for the i-th layer: z = H_prev *
        z = np.dot(H_activation[-1], Weights[i].T) + bias[i]

        # Store the pre-activation score (before applying the activation fun
        z_preactivation_score.append(z)

        # Apply the ReLU activation function to the pre-activation score to
        H = relu(z)

        # Append the activation (H) to the list of activations.
        H_activation.append(H)

    # For the output layer (final layer), apply softmax to get predicted pro
    z = np.dot(H_activation[-1], Weights[-1].T) + bias[-1]  # Pre-activation

    # Store the pre-activation score for the output layer.
    z_preactivation_score.append(z)

    # Apply softmax to the final pre-activation score to compute the predict
    y_pred = softmax(z)

    # Append the output layer activation (y_pred) to the list of activations
    H_activation.append(y_pred)

    # Return both the activations and the pre-activation scores for each lay
    return H_activation, z_preactivation_score
```

Back Propogation

Performs the backpropagation algorithm to compute gradients and update weights and biases using gradient descent with L2 regularization.

Arguments:

- x_batch: Batch of input data.
- y_batch: Batch of true labels (ground truth).

- h_activations: List of activations from the forward pass (including input and output).
- z_score: List of pre-activation scores (z values) from the forward pass.
- Weights: List of weight matrices for each layer.
- bias: List of bias vectors for each layer.
- lr: Learning rate for gradient descent.
- alpha: L2 regularization strength.
- NUM_HIDDEN_LAYERS: Number of hidden layers in the neural network.

Returns:

- Weights: Updated weight matrices after gradient descent.
- bias: Updated bias vectors after gradient descent.

In [30]:
```
def backprop(x_batch, y_batch, h_activations, z_score, Weights, bias, lr, al

    wt_gradient = []  # To store gradients for weight matrices
    bias_gradient = []  # To store gradients for bias vectors
    batch_s = y_batch.shape[0]  # Get the batch size

    # Create a one-hot encoded matrix for the ground truth labels
    y_onehot = np.zeros_like(h_activations[-1])  # Shape is same as the outp
    y_onehot[np.arange(batch_s), y_batch] = 1  # Set the correct label index

    # Compute the initial gradient of the loss with respect to the output (y
    g = h_activations[-1] - y_onehot  # Gradient of cross-entropy loss w.r.t

    # Loop over each layer, starting from the output layer and going backwar
    for i in range(NUM_HIDDEN_LAYERS, -1, -1):
        # Compute the gradient of the weight matrix for the current layer
        grad_wt = np.dot(g.T, h_activations[i]) / batch_s  # Divide by batch
        grad_bi = np.sum(g, axis=0, keepdims=True) / batch_s  # Compute grad

        # Store the gradients for later use
        wt_gradient.append(grad_wt)
        bias_gradient.append(grad_bi)

        # Propagate the gradient to the previous layer (if not the input lay
        if i > 0:
            g = np.dot(g, Weights[i]) * relu_derivative(z_score[i - 1])  # C

    # Reverse the gradients list to maintain the correct order (from first l
    wt_gradient.reverse()
    bias_gradient.reverse()

    # Gradient descent update step for weights and biases
    for i in range(NUM_HIDDEN_LAYERS + 1):
        # Update weights using gradient descent and L2 regularization (alpha
        Weights[i] -= lr * (wt_gradient[i] + alpha * Weights[i])
        # Update bias using gradient descent
        bias[i] -= lr * bias_gradient[i].reshape(-1)
```

```
    return Weights, bias  # Return the updated weights and biases
```

Train Function

Train the neural network using stochastic gradient descent (SGD) with mini-batches.

Arguments:

- X_train: Training data inputs (features).
- y_train: Training data labels (targets).
- weights: Initial flattened weights and biases (single vector).
- X_val: Validation data inputs (optional for validation purposes).
- y_val: Validation data labels (optional for validation purposes).
- lr: Learning rate for gradient descent.
- num_epochs: Number of training epochs.
- batch_size: Size of mini-batches for SGD.
- alpha: L2 regularization strength.
- NUM_HIDDEN_LAYERS: Number of hidden layers in the neural network.
- NUM_INPUT: Number of input features (size of input layer).
- NUM_HIDDEN: List of sizes for each hidden layer.
- NUM_OUTPUT: Number of output classes (size of output layer).

Returns:

- WTS: Flattened final weights and biases after training.
- avg_loss: Average loss over all epochs.

In [40]:
```
def train(X_train, y_train, weights, X_val, y_val, lr, num_epochs, batch_siz

    num_tr_samples = X_train.shape[0]  # Total number of training samples
    tr_indices = np.arange(num_tr_samples)  # Create an array of sample indi

    # Unpack the initial weights and biases from the flattened vector
    Weights, bias = unpack(weights, NUM_HIDDEN_LAYERS, NUM_INPUT, NUM_HIDDEN

    total_loss = 0  # Initialize total loss accumulator

    # Training loop for multiple epochs
    for epoch in range(num_epochs):
        np.random.shuffle(tr_indices)  # Shuffle the training sample indices

        # Iterate over mini-batches
        for num in range(0, num_tr_samples, batch_size):
            # Select a mini-batch of data based on the current indices
            batch_index = tr_indices[num: num + batch_size]
            x_batch = X_train[batch_index]  # Extract the inputs for the mir
            y_batch = y_train[batch_index]  # Extract the labels for the mir
```

```
            # Perform a forward pass to compute activations and pre-activati
            H_activations, z_list = forward_pass(x_batch, Weights, bias, NUM

            # Perform backpropagation and update weights and biases
            Weights, bias = backprop(x_batch, y_batch, H_activations, z_list

        # After each epoch, calculate the loss for the entire training set
        activations, _ = forward_pass(X_train, Weights, bias, NUM_HIDDEN_LAY
        epoch_loss = loss(y_train, activations[-1], Weights, alpha)  # Compu

        total_loss += epoch_loss  # Accumulate the total loss over epochs

    # After training, flatten the final weights and biases into a single vec
    WTS = np.hstack([W.flatten() for W in Weights] + [b.flatten() for b in b

    # Calculate the average loss over all epochs
    avg_loss = total_loss / num_epochs

    return WTS, avg_loss  # Return the final weights and average loss
```

Validate Function

Validate the performance of the neural network on a validation dataset.

Arguments:

- X_val: Validation data inputs (features).
- y_val: Validation data labels (targets).
- weights: Current weights and biases of the model (flattened vector).
- NUM_HIDDEN_LAYERS: Number of hidden layers in the neural network.
- NUM_INPUT: Number of input features (size of input layer).
- NUM_HIDDEN: List of sizes for each hidden layer.
- NUM_OUTPUT: Number of output classes (size of output layer).

Returns:

- accuracy: The accuracy of the model on the validation dataset (fraction of correct predictions).

In [41]:
```
def validate(X_val, y_val, weights, NUM_HIDDEN_LAYERS, NUM_INPUT, NUM_HIDDEN

    # Unpack the weights and biases from the flattened weights vector
    Weights, bias = unpack(weights, NUM_HIDDEN_LAYERS, NUM_INPUT, NUM_HIDDEN

    # Perform forward pass on the validation data to compute activations
    activations, _ = forward_pass(X_val, Weights, bias, NUM_HIDDEN_LAYERS)

    # The last element of activations corresponds to the output layer predic
    y_pred = activations[-1]

    # Get the predicted class labels by taking the index of the maximum pred
    y_pred_labels = np.argmax(y_pred, axis=1)
```

```python
    # Calculate the accuracy by comparing predicted labels with the true lab
    accuracy = np.mean(y_pred_labels == y_val)  # Fraction of correct predic

    # Optionally print the validation accuracy (commented out)
    # print(f'Validation accuracy: {accuracy * 100:.2f}%')

    return accuracy  # Return the computed accuracy
```

In [42]:
```python
def test(X_test, y_test, weights, NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM

    Weights, bias = unpack(weights,NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NU
    # Perform forward pass on validation data
    activations, _ = forward_pass(X_test, Weights, bias, NUM_HIDDEN_LAYERS)

    # Assuming the last activation is the output layer
    y_pred = activations[-1]

    y_pred_labels = np.argmax(y_pred, axis=1)


    # Calculate accuracy
    accuracy = np.mean(y_pred_labels == y_test)
#     print(f'Test accuracy: {accuracy * 100:.2f}%')

    return accuracy
```

In [44]:
```python
if __name__ == "__main__":
    # Load training data.
    # Recommendation: divide the pixels by 255 (so that their range is [0-1]
    # 0.5 (so that the range is [-0.5,+0.5]).

    NUM_HIDDEN_LAYERS = 3
    NUM_INPUT = 784
    NUM_HIDDEN = NUM_HIDDEN_LAYERS * [ 64 ]  # you can customise here
    NUM_OUTPUT = 10

    training_data = np.reshape(np.load("fashion_mnist_train_images.npy"), (-
    training_labels = np.load("fashion_mnist_train_labels.npy")

    num_datapoints = training_data.shape[0]
    split_index = int(0.8*num_datapoints)
    indices = np.arange(num_datapoints)
    np.random.shuffle(indices)

    train_indices = indices[:split_index]
    val_indices = indices[split_index:]

    X_train, X_val = training_data[train_indices], training_data[val_indices
    y_train, y_val = training_labels[train_indices], training_labels[val_inc

    X_train = X_train / 255.0 - 0.5
    X_val = X_val / 255.0 - 0.5

    testing_data = np.reshape(np.load("fashion_mnist_test_images.npy"), (-1,
```

```
    testing_labels= np.load("fashion_mnist_test_labels.npy")

    X_test = testing_data
    y_test = testing_labels
    X_test = X_test / 255.0 - 0.5

#     num_epochs = 100
#     batch_size =64
#     alpha =0.0
#     lr = 0.01


    num_epochs = 10
    batch_size = 8
    alpha = 0.0
    lr = 0.01


    Ws, bs = initWeightsAndBiases(NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM
    weights = np.hstack([ W.flatten() for W in Ws ] + [ b.flatten() for b in
    weights = weights.astype(np.float32)

    trainX, trainY = X_train[:5].astype(np.float32), y_train[:5].astype(np.f

    print(scipy.optimize.check_grad(
        lambda weights_: fCE(trainX, trainY, weights_,NUM_HIDDEN_LAYERS,NUM_
        lambda weights_: gradCE(trainX, trainY, weights_, NUM_HIDDEN_LAYERS,
        weights
    ))

    weights, avg_loss = train(X_train, y_train, weights, X_val, y_val, lr,nu
    val_accuracy = validate(X_val, y_val, weights, NUM_HIDDEN_LAYERS,NUM_INP
    test_accuracy = test(X_test, y_test, weights, NUM_HIDDEN_LAYERS,NUM_INPU


    print(scipy.optimize.approx_fprime(weights, lambda weights_: fCE(trainX,
```

```
3.5305727448724214e-06
[-1.48039360e-02 -1.48039360e-02 -1.49787697e-02 ...  1.41575085e-04
  8.76898554e-06 -1.38318246e-04]
```

Part(B)

Check_grad weights

The output for these weights is saved in a text file named **"Part_B_check_grad_weights.txt"**.

In [47]:
```
with open("output.txt", "w") as file:
    # Gradient check using check_grad
    grad_check = scipy.optimize.check_grad(
        lambda weights_: fCE(trainX, trainY, weights_, NUM_HIDDEN_LAYERS,NUM
        lambda weights_: gradCE(trainX, trainY, weights_, NUM_HIDDEN_LAYERS,
        weights  # Initial weights
    )
```

```
    file.write(f"Gradient check using check_grad: {grad_check}\n\n")

    # Gradient check using approx_fprime
    approx_grad = scipy.optimize.approx_fprime(weights, lambda weights_: fCE

    # Write each value of the approx_fprime output on a new line
    file.write("Gradient check using approx_fprime:\n")
    for i, value in enumerate(approx_grad):
        file.write(f"Value {i}: {value}\n")
```

In [32]:
```
# Next we will perform hyperparameter tuning
# performed iterations around 162 for different hyperparameters
```

In [33]:
```
# Next lets test our model with our best weights and hyperparameters
test_weights = np.load("best_model_weights_bp.npy")
test_hyp = np.load("best_model_hyperparameters_bp.npy", allow_pickle=True)
```

In [34]:
```
testing_data = np.reshape(np.load("fashion_mnist_test_images.npy"), (-1, 28*
testing_labels= np.load("fashion_mnist_test_labels.npy")

X_test = testing_data
y_test = testing_labels
X_test = X_test / 255.0 - 0.5

num_input = 784
num_output = 10
hyperparameters = test_hyp.item()
num_hidden_layers = hyperparameters['num_hidden_layers']
num_hidden_units = hyperparameters['num_hidden_units']
num_epochs = hyperparameters['num_epochs']
learning_rate = hyperparameters['learning_rate']
mini_batch = hyperparameters['mini_batch']

num_hidden = num_hidden_layers * [num_hidden_units]

Best_test_accuracy = test(X_test, y_test, test_weights, num_hidden_layers,nu
```

In [44]:
```
def show_W0 (W,NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT):
    Ws,bs = unpack(W,NUM_HIDDEN_LAYERS,NUM_INPUT,NUM_HIDDEN,NUM_OUTPUT)
    W = Ws[0]
    n = int(NUM_HIDDEN[0] ** 0.5)
    # plt.imshow(np.vstack([
    #     np.hstack([ np.pad(np.reshape(W[idx1*n + idx2,:], [ 28, 28 ]), 2,
    # ]), cmap='gray')

    image_grid = np.vstack([
    np.hstack([
        np.pad(np.reshape(W[idx1 * n + idx2, :], [28, 28]), 2, mode='constar
        for idx2 in range(n)
    ])
    for idx1 in range(n)])

    # plt.show()

    fig, ax = plt.subplots()
```

```
    ax.imshow(image_grid, cmap='gray')

    fig.savefig('weight_photo.png', dpi=300, bbox_inches = 'tight')

    plt.show()
```

Part (D)

In [45]:
```
show_W0(test_weights,num_hidden_layers,num_input,num_hidden,num_output)
```