



Introduction to Neural Networks

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2019 Beginners Workshop Machine Learning

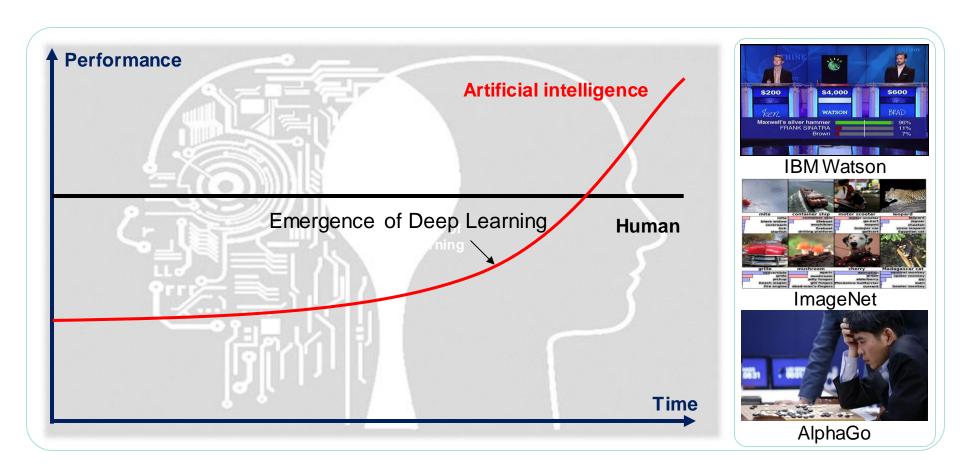
Contents

History of NN

- Rosenblatt's Perceptron
 - Cost (= Loss) function
 - Gradient descent algorithm

- Multi-Layer Perceptron (MLP)
 - Activation function
 - Backpropagation

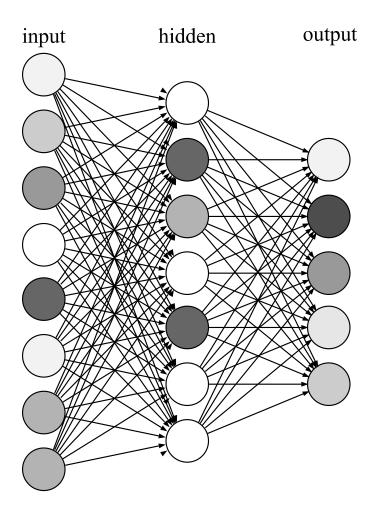
Emergence of Deep Learning



 The technology of AI has been remarkably developed and now has surpassed human intelligence

(Artificial) Neural Network

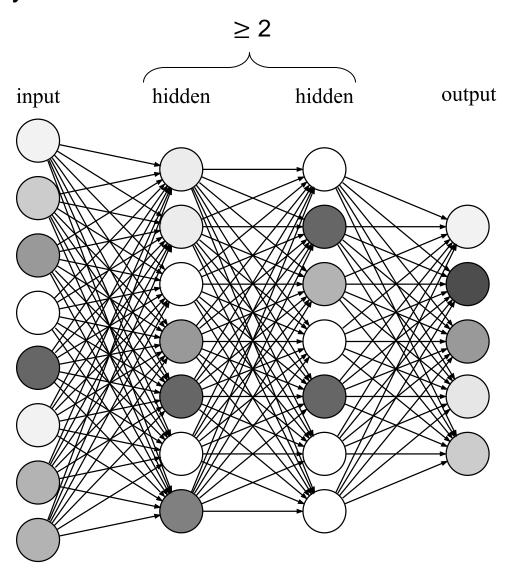
- Deep learning is one of the ML research tool which uses NN
- Pieces of math!



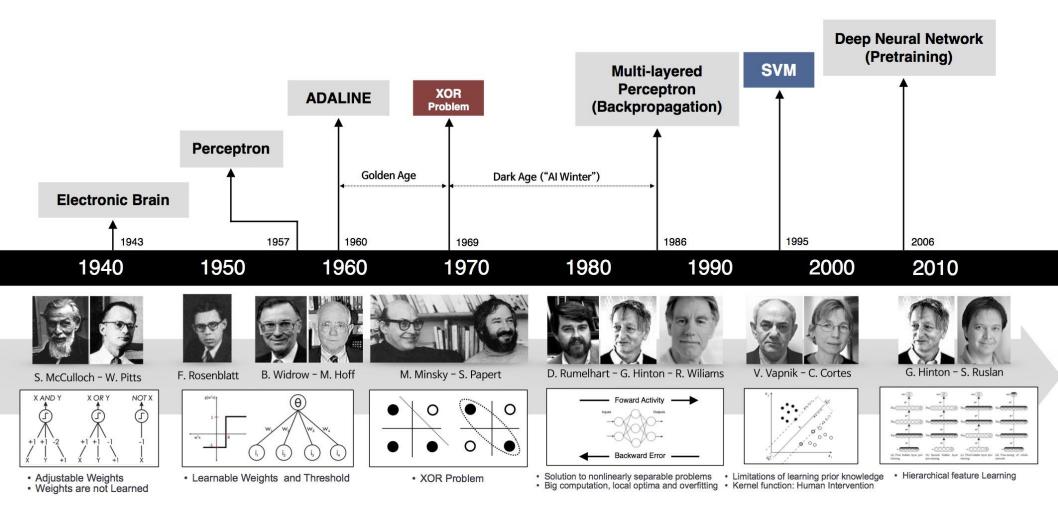
Multi-layer Perceptron 4/92

Deep Neural Network

• # of hidden layer ≥ 2

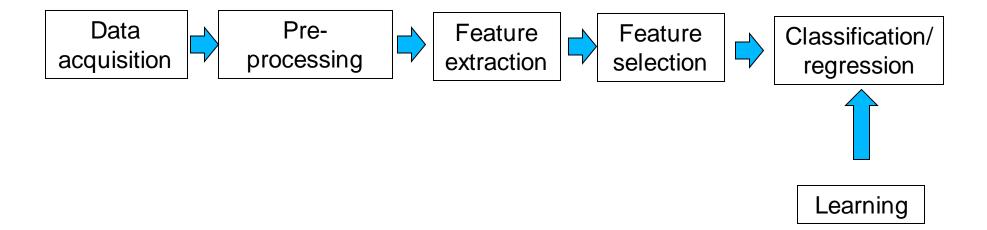


Milestones in the Development of Neural Networks



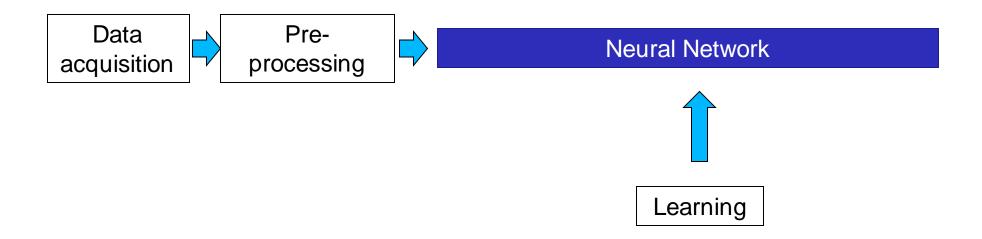
Core Idea: Feature Learning

Classical pattern recognition pipeline



Core Idea: Feature Learning

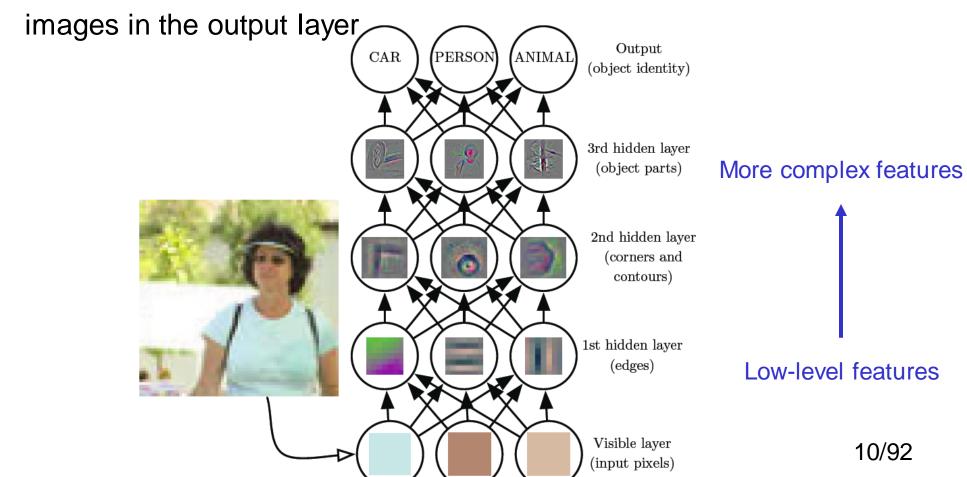
Neural networks pipeline



Deep Neural Network in action

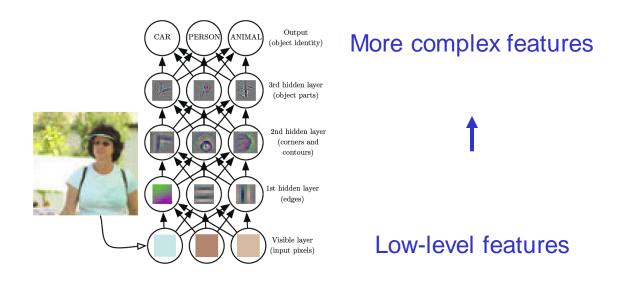
Learning representations with increasing level of abstraction

By passing it with several layers hierarchically, we can classify the

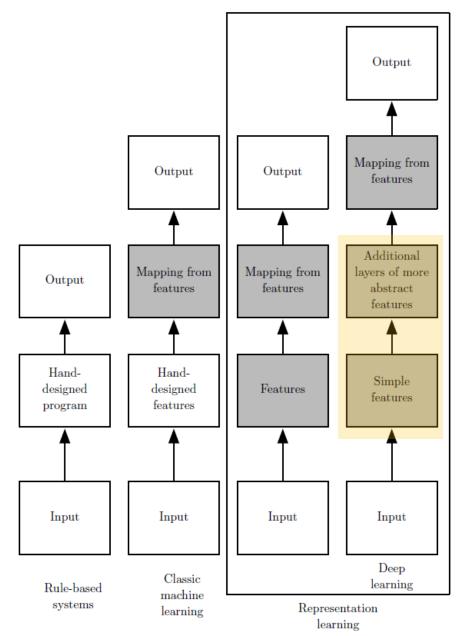


Deep Neural Network in action

- Image recognition
 - pixel → edge → Texton → motif → part → object
- Text
 - Character → word → word group → clause → sentence → story
- Speech
 - sample → spectral band → sound → ... → phone → phoneme → word

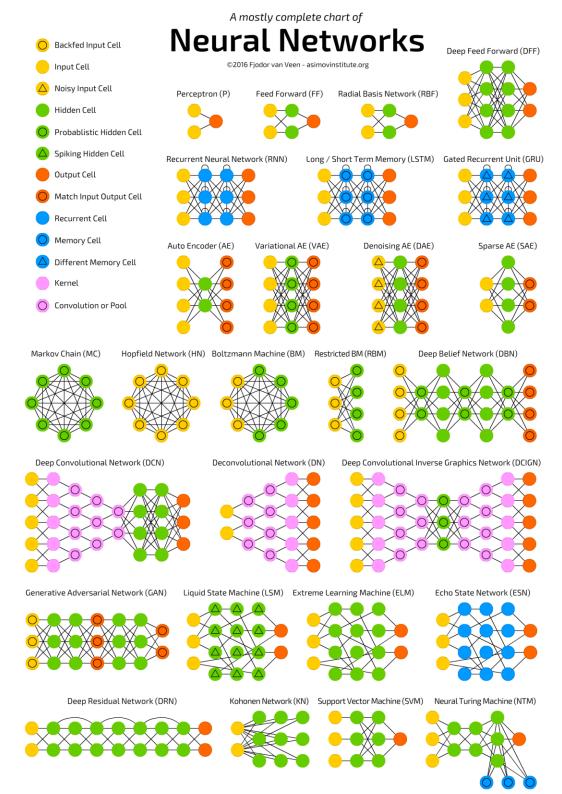


Learning Multiple Components



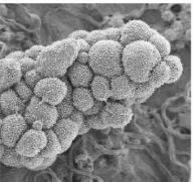
Automatic feature extraction

Neural Network Zoo



Deep Neural Networks: Applications and Models











Internet & Cloud

Image classification
Speech recognition
Language Translation
Language processing
Sentiment analysis
Recommendation

Medicine & Biology

Cancer cell detection
Diabetic grading
Drug discovery

Media & Entertainment

Video captioning
Video search
Real time translation

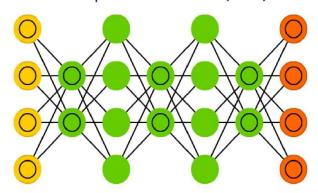
Security & Defense

Face detection Video surveillance Satellite imagery

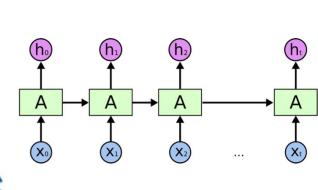
Autonomous Machines

Pedestrian Detection
Lane Tracking
Traffic sign recognition

Deep Belief Network (DBN)



input feature maps feature maps



Recurrent Neural Network

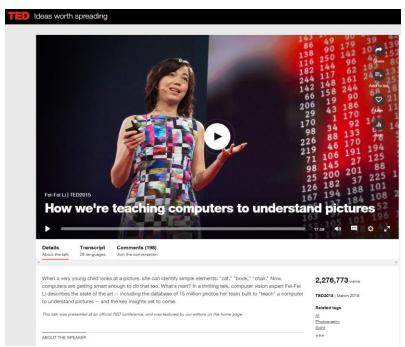
Deep Belief Network

References on Neural Networks

- Andrew Ng's and other ML tutorials
 - https://class.coursera.org/ml-003/lecture
 - http://www.holehouse.org/mlclass/
 - http://deeplearning.stanford.edu/tutorial/
- Stanford lecture videos
 - CS231n: Convolutional Neural Networks for

 Visual Recognition (http://cs231n.stanford.edu/syllabus.html)
 - CS224d: Deep Learning for Natural Language Processing (http://cs224d.stanford.edu/syllabus.html)
- Ted by Fei-Fei Li (How we're teaching computers to understand pictures

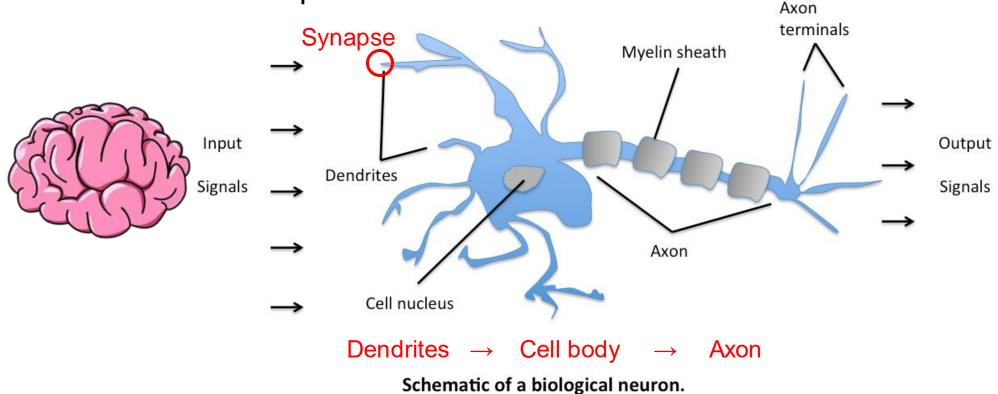
(https://www.ted.com/talks/fei_fei_li_how_we_re_teaching_computers_to_understand_pictures)



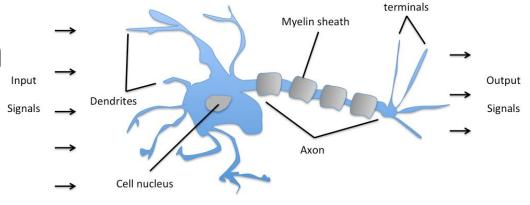
Perceptron

Neuronal Activity in the Brain

10¹¹ neurons of > 20 types, 10¹⁴ synapses with very complex connections, 1ms–10ms cycle time Signals are noisy "spike trains" of electrical potential

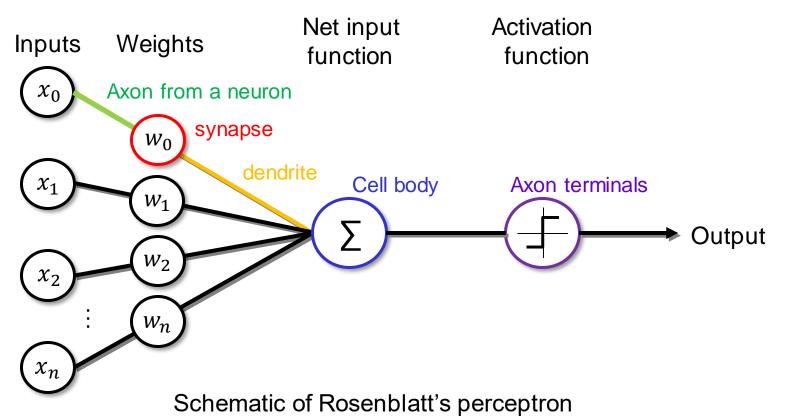


Rosenblatt's Perceptron

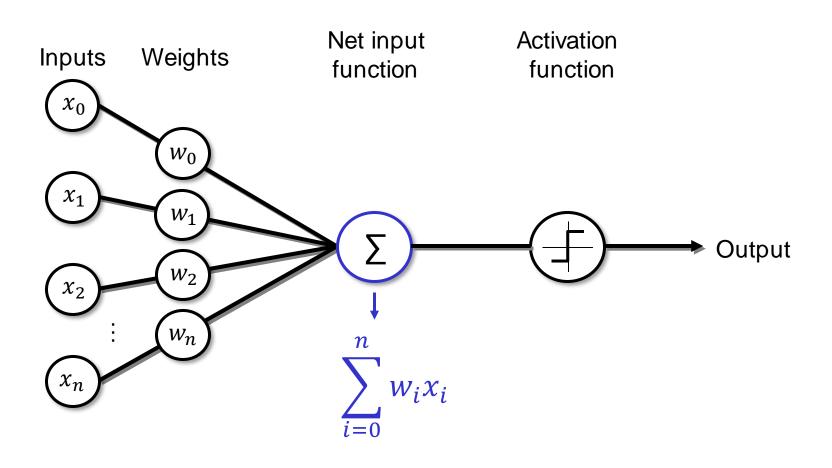


Axon

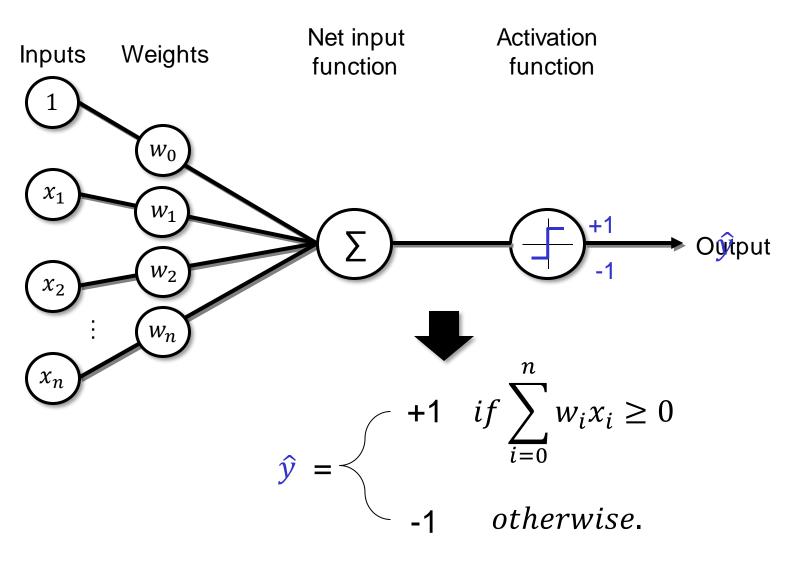
Schematic of a biological neuron.



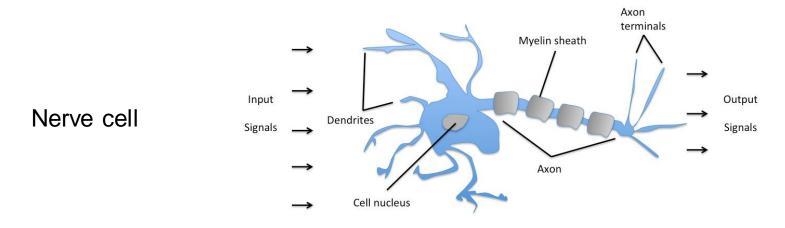
Rosenblatt's Perceptron: Cell Body



Rosenblatt's Perceptron: Activation Function



Perceptron



Schematic of a biological neuron.



Linear Threshold Unit

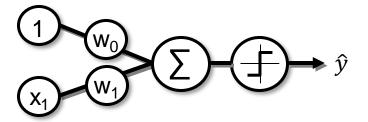
(simple) Perceptron
$$\hat{y}(x_1, \dots, x_n) = \begin{cases} +1 & if \ \mathbf{w}^T \mathbf{x} \geq 0 \\ -1 & otherwise. \end{cases}$$

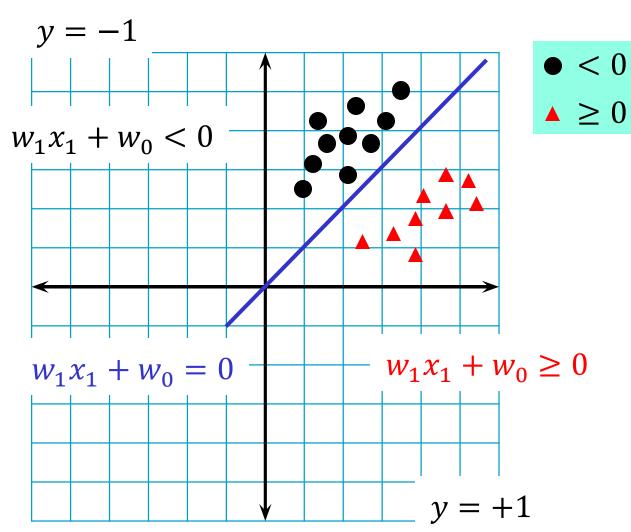
Perceptron

$$\hat{y}(x_1, \dots, x_n) = \begin{cases} +1 & if \ \mathbf{w}^T \mathbf{x} \ge 0 \\ -1 & otherwise. \end{cases}$$

Perceptron on 2-D coordinate

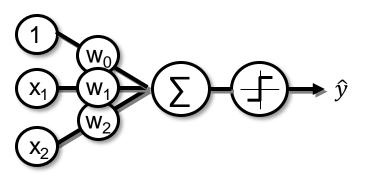
In case if i = 1

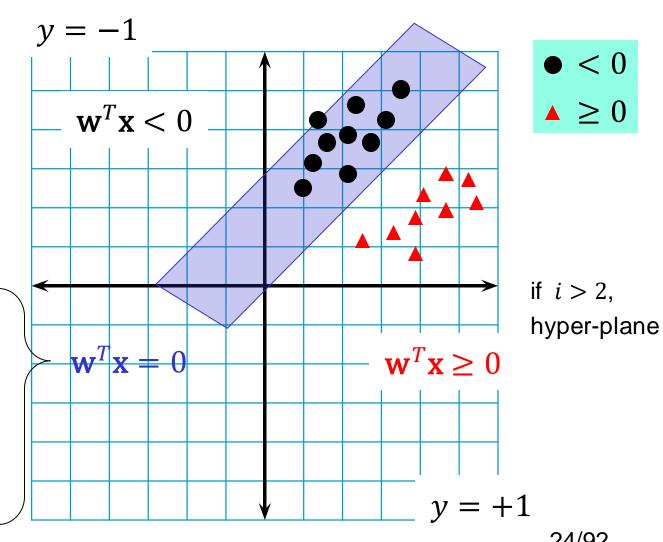




Perceptron on 2-D coordinate

In case if i = 2





 $(w_0 w_1) {x_0 \choose x_1} = 0$ $\mathbf{w}^T = (w_0 \, w_1)$ $\mathbf{x} = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$

 $w_1 x_1 + w_0 x_0 = 0$

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Learning on Perceptron

$$\hat{y} = \mathbf{w}^T \mathbf{x}$$

$$= \mathbf{w}_0 x_0 + \mathbf{w}_1 x_1 + \dots + \mathbf{w}_n x_n$$

Delta rule of Rosenblatt's perceptron

- 1. Initialize the weights
- 2. For each training sample $(\mathbf{x}^{(i)}, y^{(i)})$,
 - 1. Compute the output \hat{y}
 - 2. Update the weight using Δw_i
- 3. Repeat 2nd step

Delta Update (=Training) Rule

- 1. Initialize the weights
- 2. For each training sample $(\mathbf{x}^{(1)}, y^{(1)}), ..., (\mathbf{x}^{(d)}, y^{(d)}) \in D$, perform the following:
 - 1. Compute the prediction output $\hat{y} = \mathbf{w}^T \mathbf{x}$
 - 2. Update the weight $w_i^{new} \leftarrow w_i^{old} + \Delta w_i$

$$\left(\Delta w_i = -\eta \times \sum_{d \in D} \left(y^{(d)} - \hat{y}^{(d)} \right) \times x_i \right)$$

3. Repeat until the error is less than threshold

$$\Delta w_i = -\eta \times \sum_{d \in D} (y^{(d)} - \hat{y}^{(d)}) \times x_i$$

 $y^{(d)}$: Target (correct) output for sample d (0 or 1)

 $\hat{y}^{(d)}$: Perceptron output (continuous value)

 η : Learning rate (range [0 1])

Perceptron: Cost(= Loss) Function

Loss:
$$E(\mathbf{w}) \equiv \frac{1}{2} \sum_{d \in D} (\underline{y^{(d)} - \hat{y}^{(d)}})^2 \quad y^{(d)} : \text{ target (0 or 1)} \\ \hat{y}^{(d)} : \text{ output} \\ \hat{y} = w_0 + w_1 x_1 + \dots + w_1 x_1$$

Difference between target value $y^{(d)}$ and output value $\hat{y}^{(d)}$ for training sample d

$\min_{\mathbf{W}} \text{minimize } E(\mathbf{W})$

Our objective is to find w which can minimize cost function

$$E(\mathbf{w}) = \frac{1}{2} \sum_{d \in D} (y^{(d)} - \hat{y}^{(d)})^2 \qquad \qquad E(\mathbf{w}) = \frac{1}{2} \sum_{d \in D} (y^{(d)} - \mathbf{w}^T \mathbf{x}^{(d)})^2$$

X	у
1	1
2	2
3	3

•
$$w = 1, E(\mathbf{w})$$
?

$$E(\mathbf{w}) \equiv \frac{1}{2} \sum_{d \in D} (y^{(d)} - \mathbf{w}^T \mathbf{x}^{(d)})^2$$

X	у
1	1
2	2
3	3

•
$$\mathbf{w} = 1$$
, $E(\mathbf{w}) = 0$

$$\frac{1}{2} ((1 - 1 \times 1)^2 + (2 - 1 \times 2)^2 + (3 - 1 \times 3)^2)$$

$$E(\mathbf{w}) \equiv \frac{1}{2} \sum_{d \in D} (y^{(d)} - \mathbf{w}^T \mathbf{x}^{(d)})^2$$

x	у
1	1
2	2
3	3

•
$$\mathbf{w} = 1$$
, $E(\mathbf{w}) = 0$

$$\frac{1}{2} ((1 - 1 \times 1)^2 + (2 - 1 \times 2)^2 + (3 - 1 \times 3)^2)$$

•
$$w = 0, E(w) = ?$$

•
$$\mathbf{w} = 2, E(\mathbf{w}) = ?$$

$$E(\mathbf{w}) \equiv \frac{1}{2} \sum_{d \in D} (y^{(d)} - \mathbf{w}^T \mathbf{x}^{(d)})^2$$

X	у
1	1
2	2
3	3

•
$$\mathbf{w} = 1$$
, $E(\mathbf{w}) = 0$

$$\frac{1}{2} ((1 - 1 \times 1)^2 + (2 - 1 \times 2)^2 + (3 - 1 \times 3)^2)$$

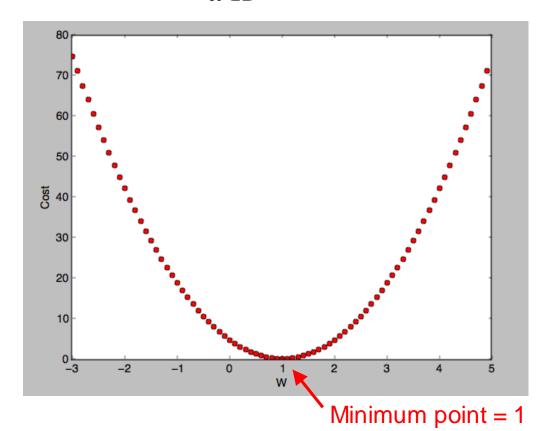
•
$$\mathbf{w} = 0$$
, $E(\mathbf{w}) = 4.5$

$$\frac{1}{2} ((1 - 0 \times 1)^2 + (2 - 0 \times 2)^2 + (3 - 0 \times 3)^2)$$

•
$$\mathbf{w} = 2$$
, $E(\mathbf{w}) = 4.5$

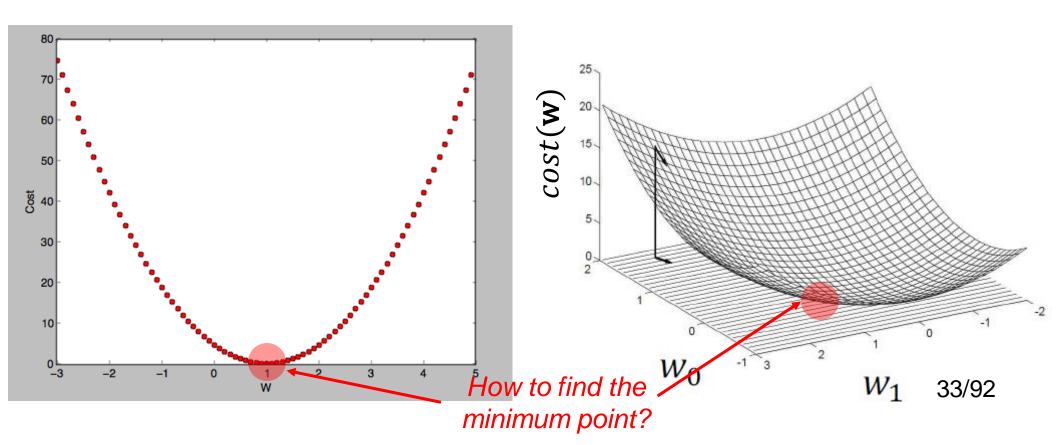
$$\frac{1}{2} ((1 - 2 \times 1)^2 + (2 - 2 \times 2)^2 + (3 - 2 \times 3)^2)$$

$$E(\mathbf{w}) \equiv \frac{1}{2} \sum_{d \in D} (y^{(d)} - \hat{y}^{(d)})^2$$



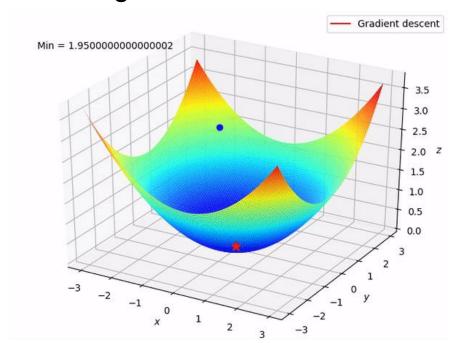
How to minimize cost?

$$E(\mathbf{w}) \equiv \frac{1}{2} \sum_{d \in D} (y^{(d)} - \hat{y}^{(d)})^2$$



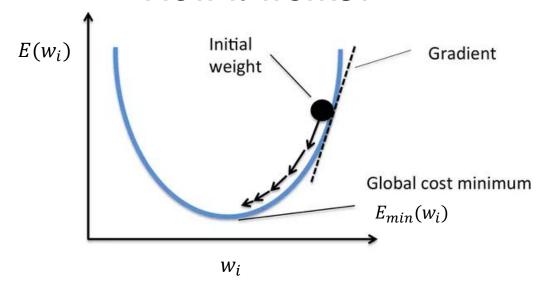
Gradient Descent Algorithm

- Minimize cost function
- Gradient descent is used for many minimization problems
- For a given cost function, $E(\mathbf{w})$, it will find w to minimize cost
- Repeat until you converge to a local minimum



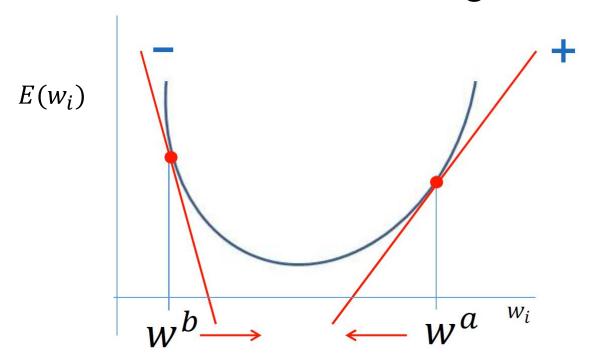
Gradient Descent Algorithm

How it works?



- 1. Start with initial guesses
 - Start at random value
- 2. Each weight is updated by taking a step into the opposite direction of the gradient $\Delta w_i = -\eta \times \frac{\partial E}{\partial w_i}$
 - Compute the partial derivative of the cost function $\frac{\partial E}{\partial w_i}$ for each weight
- 3. Repeat until you converge to a local minimum

Gradient Descent Algorithm



$$\Delta w_i = -\eta \, rac{\partial E}{\partial w_i}$$
 $\qquad \frac{\eta: \text{ learning rate}}{\partial w_i}: \text{ gradient}$

 η : learning rate (e.g. 0.001)

$$\frac{\partial E}{\partial w_i}$$
: gradient

Gradient descent algorithm

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d} (y^{(d)} - \hat{y}^{(d)})^2$$

$$= \frac{1}{2} \sum_{d} \frac{\partial}{\partial w_i} \left(y^{(d)} - \hat{y}^{(d)} \right)^2$$

$$= \frac{1}{2} \sum_{d} 2(y^{(d)} - \hat{y}^{(d)}) \frac{\partial}{\partial w_i} (y^{(d)} - \hat{y}^{(d)})$$

$$= \sum_{d} (y^{(d)} - \hat{y}^{(d)}) \frac{\partial}{\partial w_i} (y^{(d)} - w_0 x_0 - w_1 x_1 - \dots - w_i x_i)$$

$$= \sum_{d} (y^{(d)} - \hat{y}^{(d)}) \times (-x_i) \Delta w_i = \eta \times \sum_{d \in D} (y^{(d)} - \hat{y}^{(d)}) \times (-x_i)$$

Delta Rule based on Gradient Descent Algorithm (Summary)

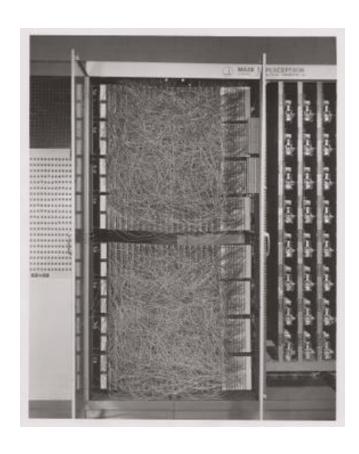
$$\hat{y} = w_0 \mathbf{1} + w_1 x_1 + \dots + w_n x_n$$

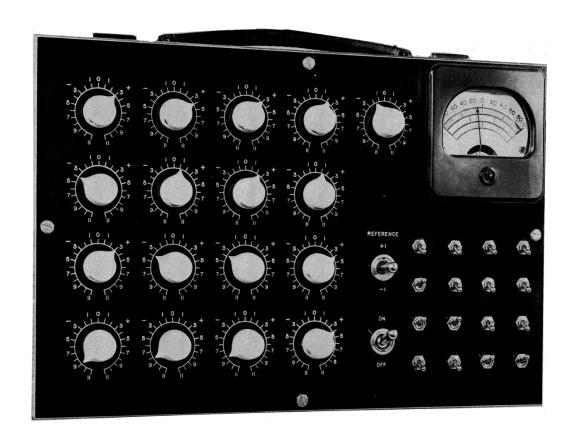
$$E(\mathbf{w}) \equiv \frac{1}{2} \sum_{d} (y^{(d)} - \hat{y}^{(d)})^2 \longrightarrow \text{Squared}_{\text{loss function}}$$

$$w_i^{new} \leftarrow w_i^{old} + \Delta w_i$$

$$\Delta w_i = -\eta \times \sum_{d \in D} (y^{(d)} - \hat{y}^{(d)}) \times x_i$$

Hardware implementations





Frank Rosenblatt, ~1957: Perceptron

Widrow and Hoff, ~1960: Adaline/Madaline

False Promises

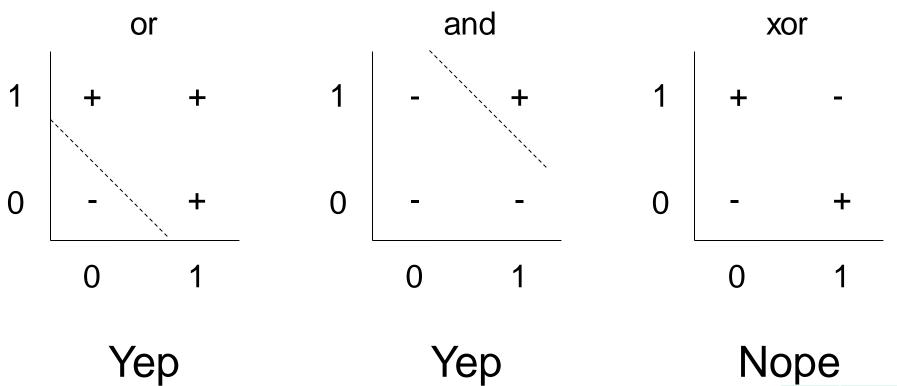
The New York Times

NEW NAVY DEVICE LEARNS BY DOING

July 8, 1958

"The Navy revealed the embryo of an electronic computer today that it expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence... Dr. Frank Rosenblatt, a research psychologist at the Cornell Aeronautical Laboratory, Buffalo, said Perceptrons might be fired to the planets as mechanical space explorers"

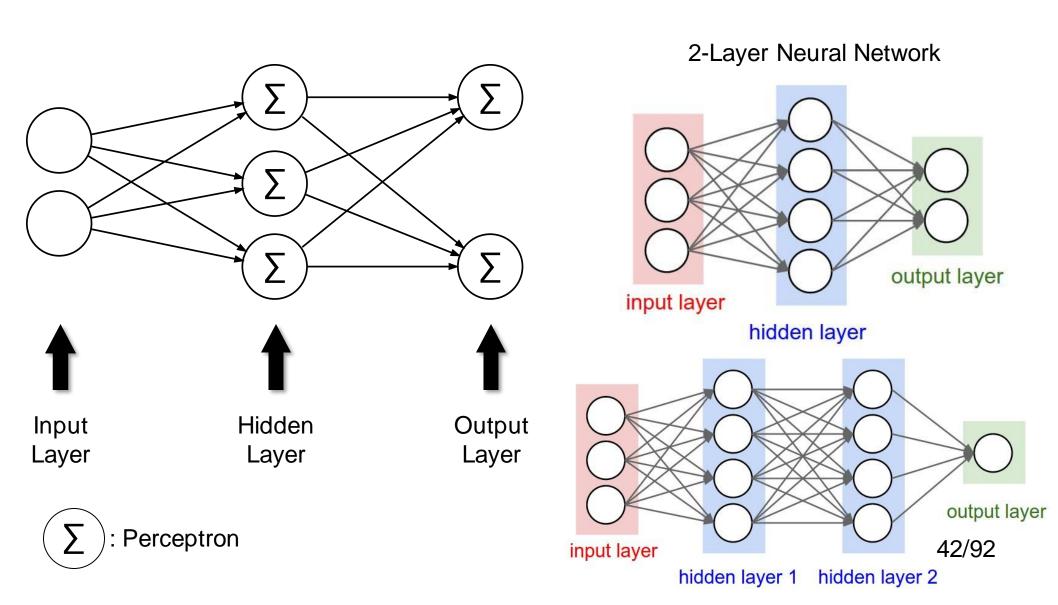
(Simple) AND/OR problem: linearly separable?



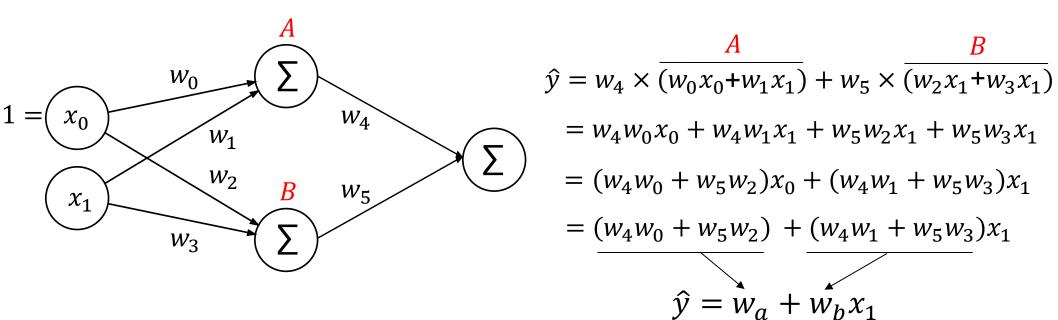
Nope

X ₁	X ₂	у
0	0	0
0	1	1
1	0	1
1	1	0

Multi-Layer Perceptron (by M. Minsky)



Multi-Layer Perceptron: Limitation

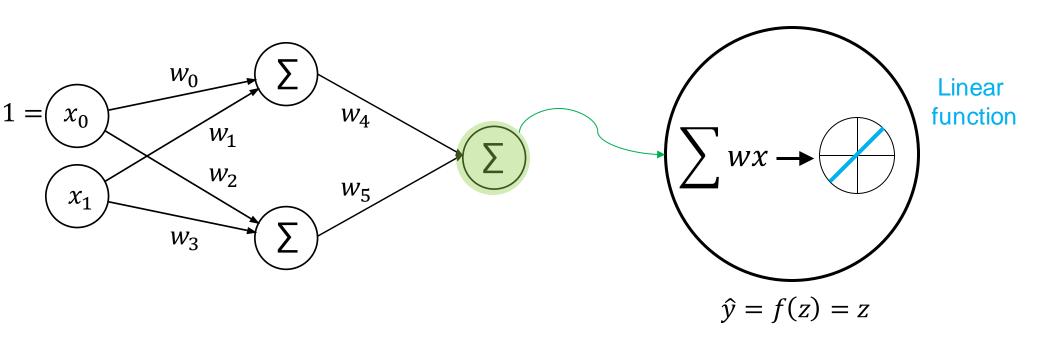


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Still Linear equation

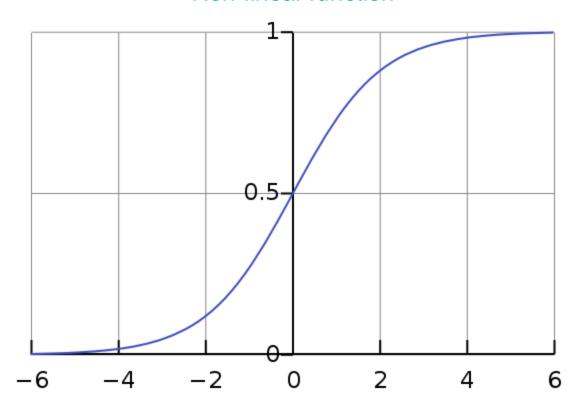
(Line, plane, or hyper-plane)

Multi-Layer Perceptron: Limitation



Multi-Layer Perceptron: Activation Function

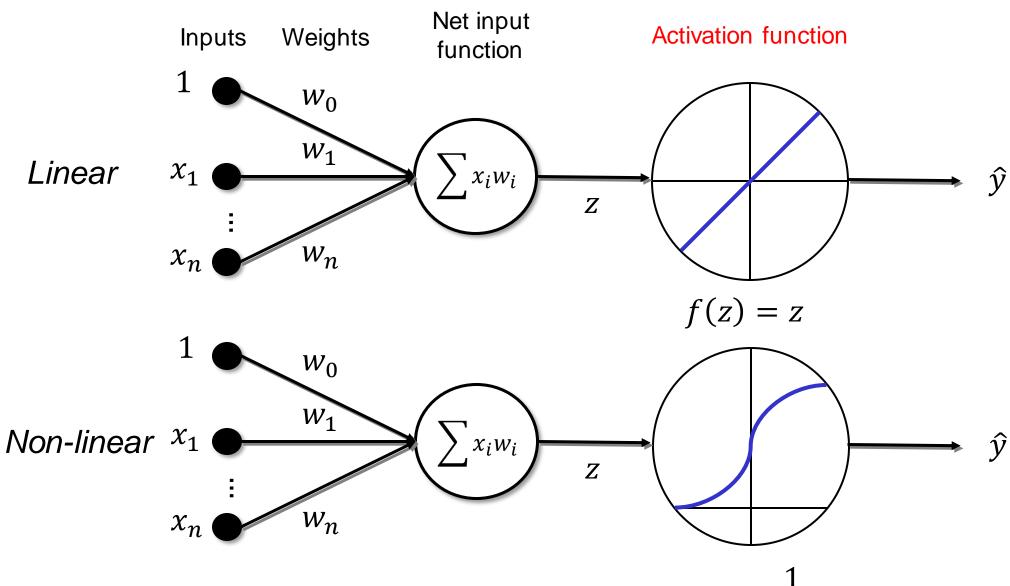
Non-linear function



Sigmoid function

$$f(z) = \frac{1}{1 + e^{-z}} = \frac{e^z}{1 + e^z}$$

Activation Functions

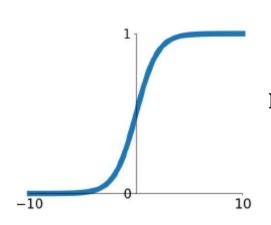


Note:
$$\frac{d}{dz}f(z) = f(z)(1 - f(z))$$
 $f(z) = \frac{1}{1 + e^{-z}}$ 46/92

Activation Functions

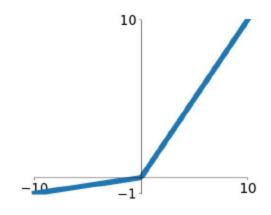
Sigmoid

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



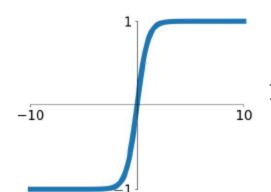
Leaky ReLU

 $\max(0.1x, x)$



tanh

tanh(x)



10

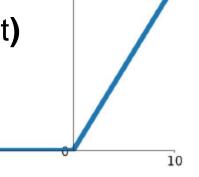
-10

Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ReLU (Rectified Linear Unit)

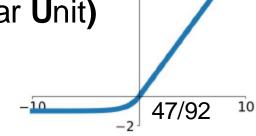
 $\max(0, x)$



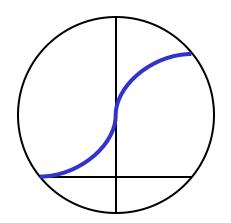
ELU

(Exponential Linear Unit)

$$\begin{cases} x & x \ge 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Derivative of Sigmoid Function



$$f(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{1 + e^x}$$

$$\frac{d}{dx}f(x) = f(x)(1 - f(x))$$

$$\frac{d}{dx}f(x) = \frac{d}{dx}\frac{1}{1+e^{-x}} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$\left(\frac{d}{dx}e^x = e^x\right) = \frac{1-1+e^{-x}}{(1+e^{-x})^2} = \frac{1+e^{-x}}{(1+e^{-x})^2} - \frac{1}{(1+e^{-x})^2}$$

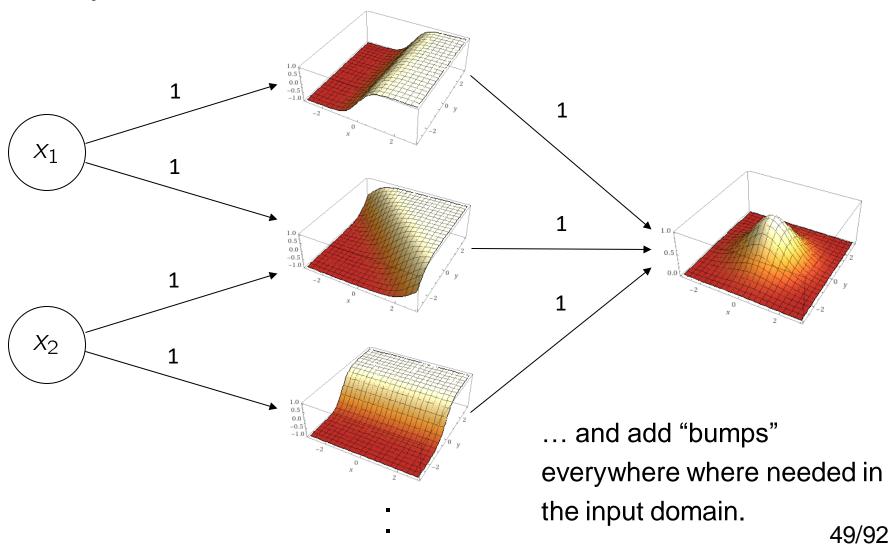
$$= \left(\frac{1}{1+e^{-x}}\right) - \left(\frac{1}{1+e^{-x}}\right)^2 = \left(\frac{1}{1+e^{-x}}\right)\left(1-\frac{1}{1+e^{-x}}\right)$$

$$= f(x)(1-f(x))$$

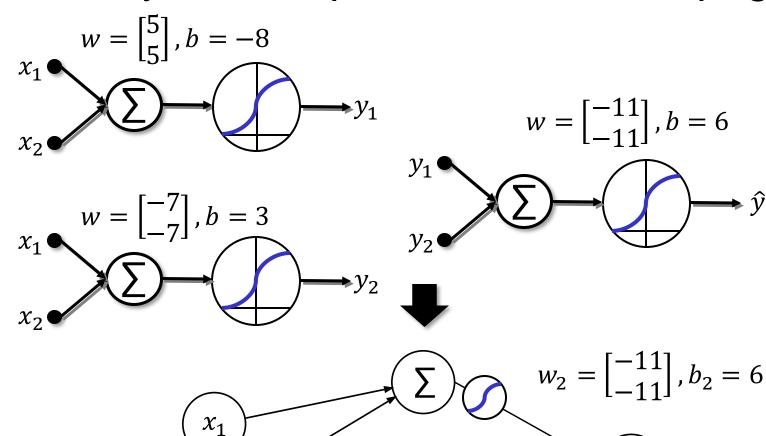
Link: http://www.derivative-calculator.net/

Multi-layer Perceptron is Universal Approximator

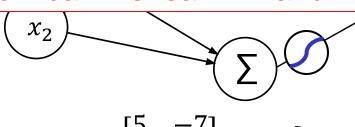
"Proof" by construction:



Multi-Layer Perceptron: Forward Propagation

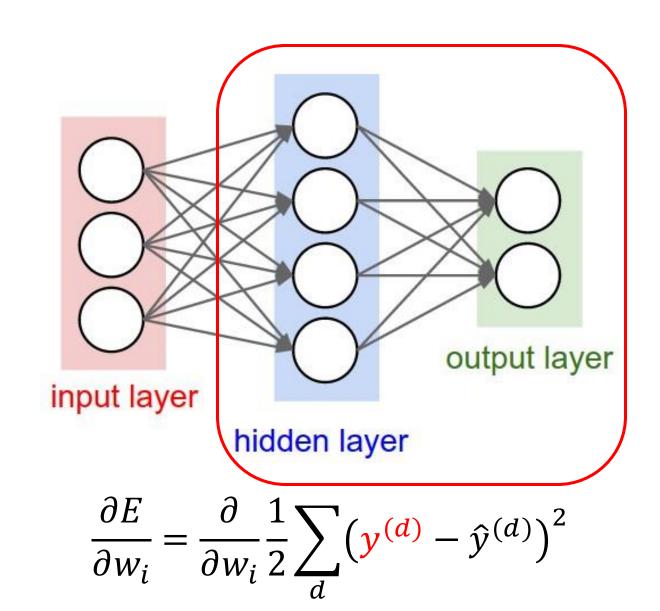


Question: How can we learn W and B from training data?



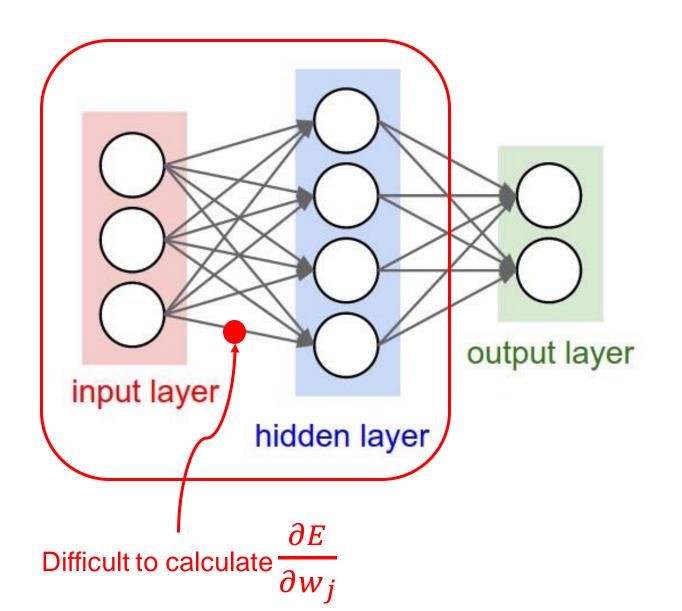
$$w_1 = \begin{bmatrix} 5 & -7 \\ 5 & -7 \end{bmatrix}, b_1 = \begin{bmatrix} -8 & 3 \end{bmatrix}$$

Learning on Multi-Layer Perceptron

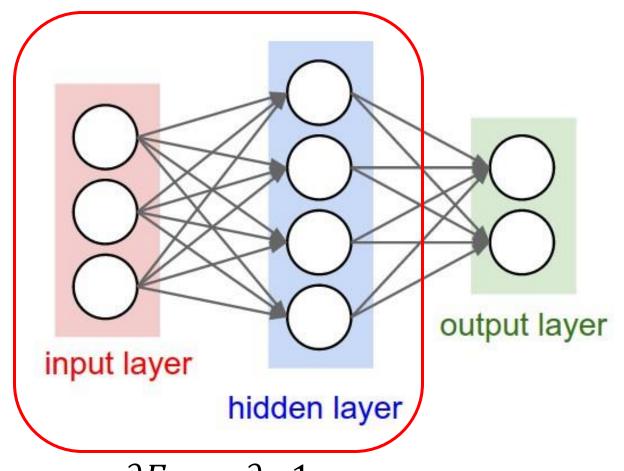


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Learning on Multi-Layer Perceptron



Learning on Multi-Layer Perceptron

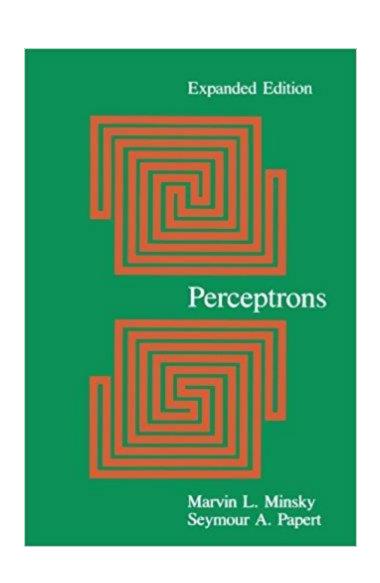


Unsolved problem for 20 years

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d} \left(? - \hat{y}^{(d)} \right)^2$$

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Limitation of Multi-layer Perceptron By Marvin Minsky, founder of the MIT AI Lab.

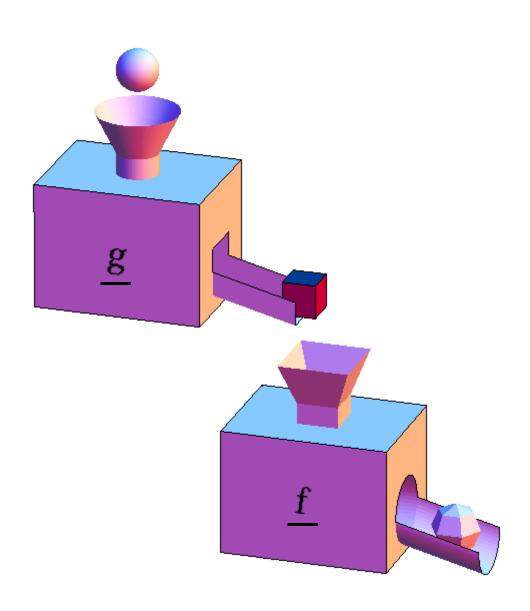


- We need to use MLP, multilayer perceptrons (multilayer neural nets)
- No one on earth had found a viable way to train MLPs good enough to learn such simple functions

Backpropagation (1974, 1982 by Paul Werbos, 1986 by Hinton)

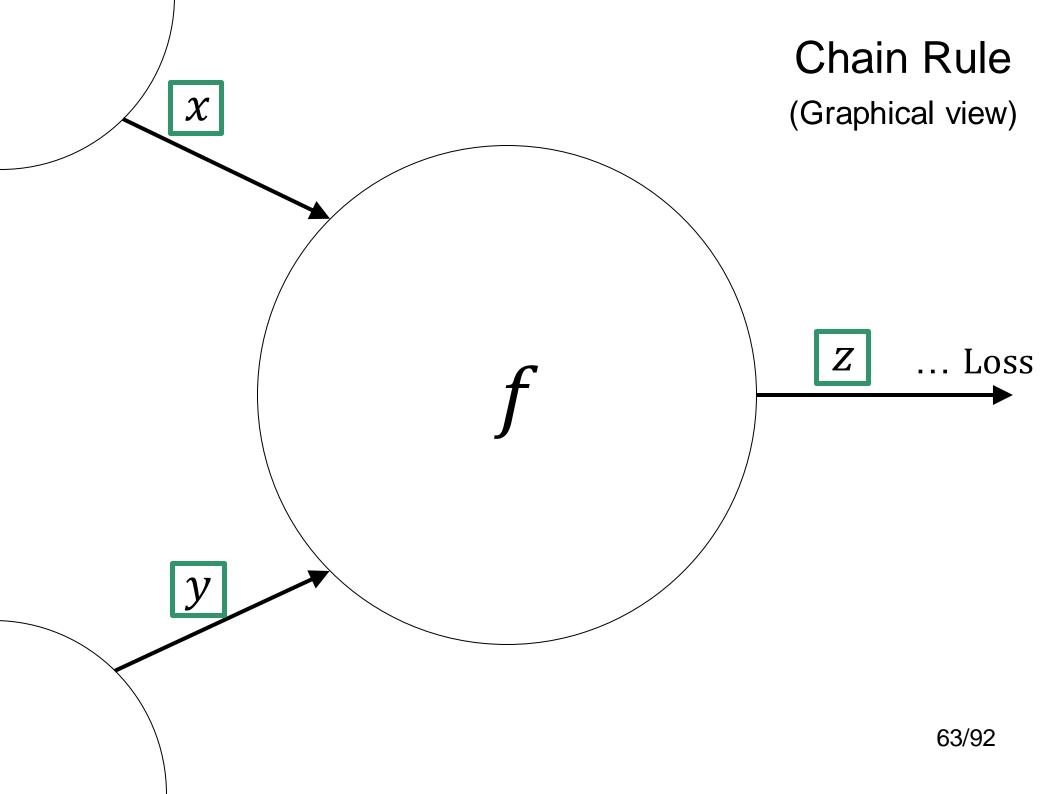
Training forward "dog" labels =? "human face" error

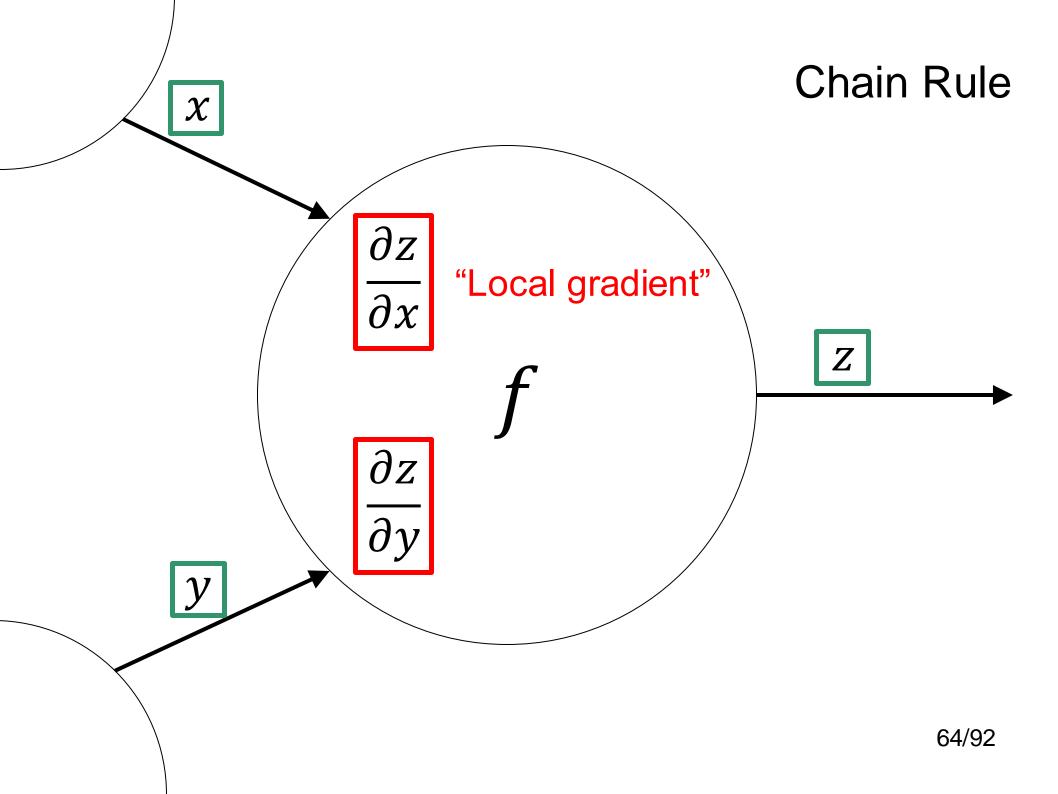
Chain Rule

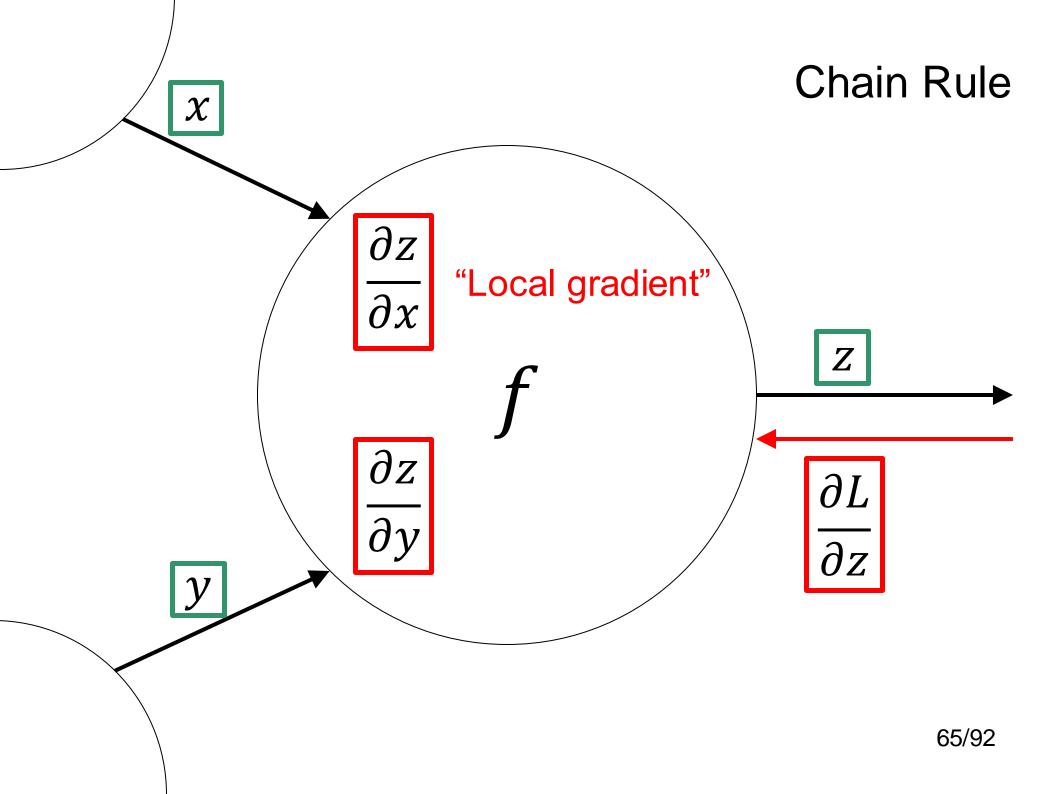


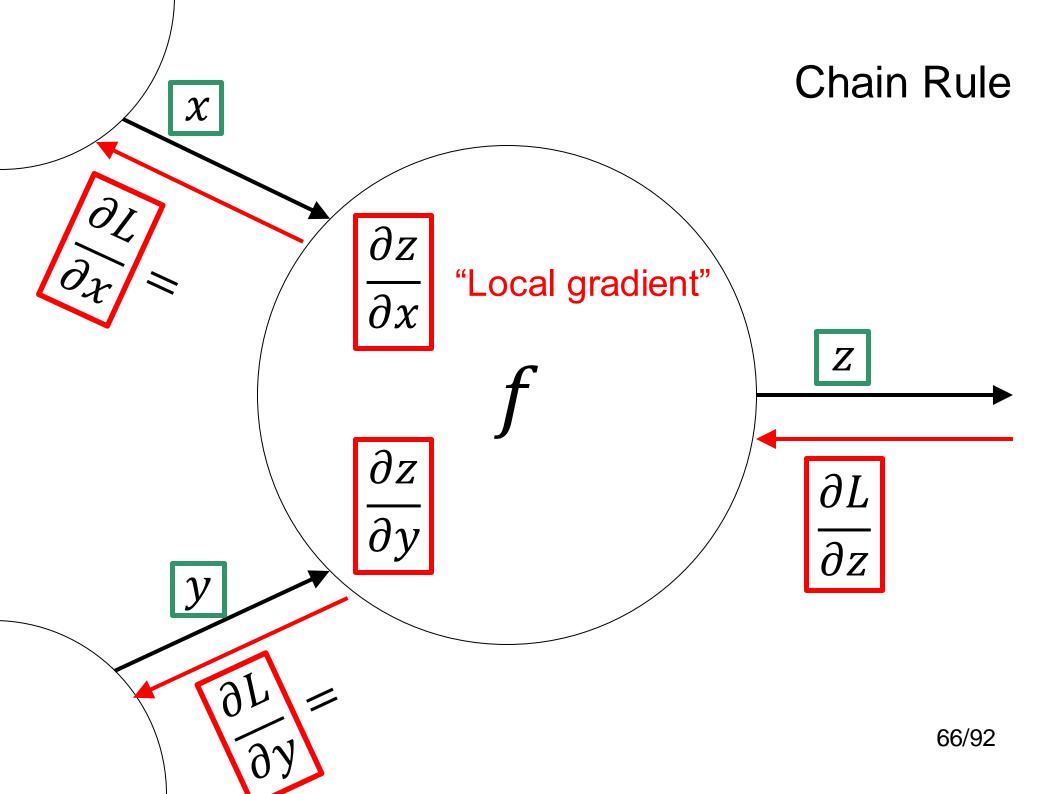
$$f = \underline{f}(g); g = \underline{g}(x)$$

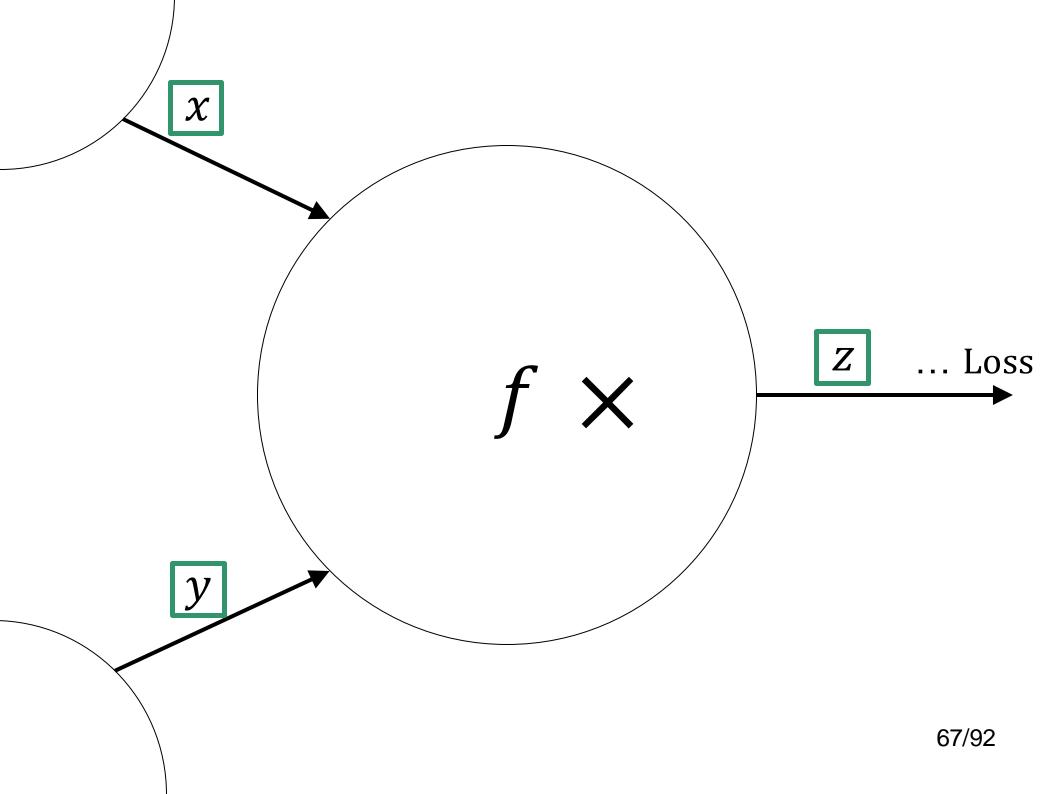
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x}$$

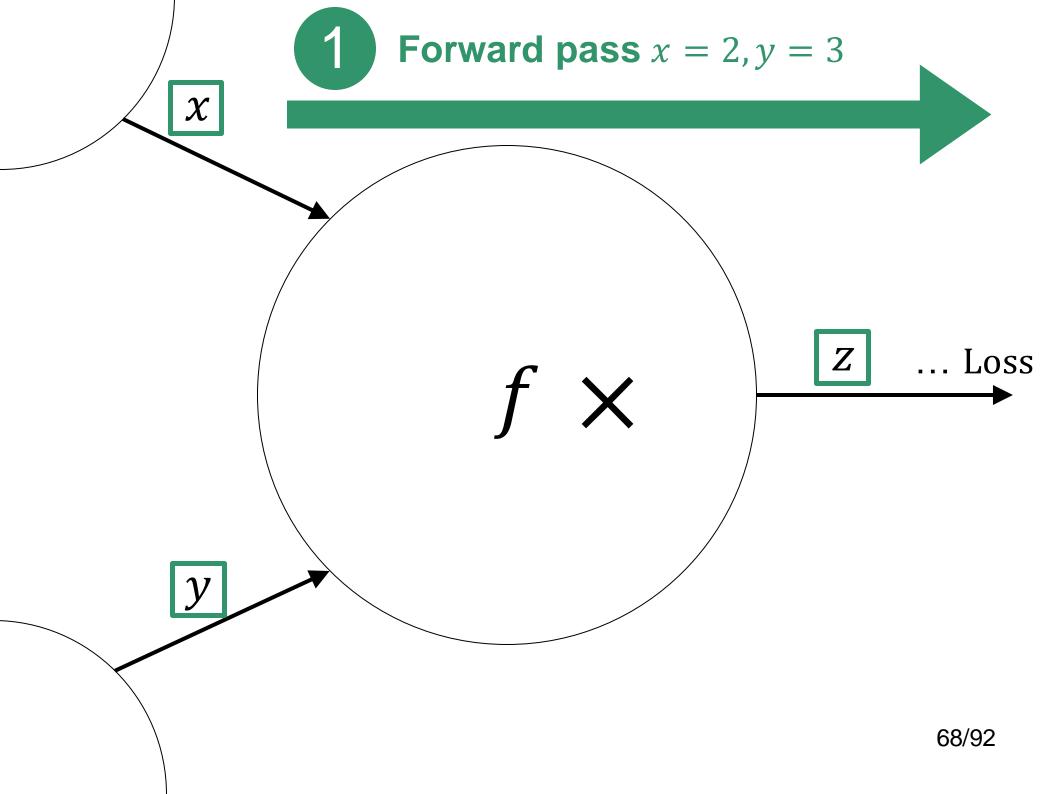


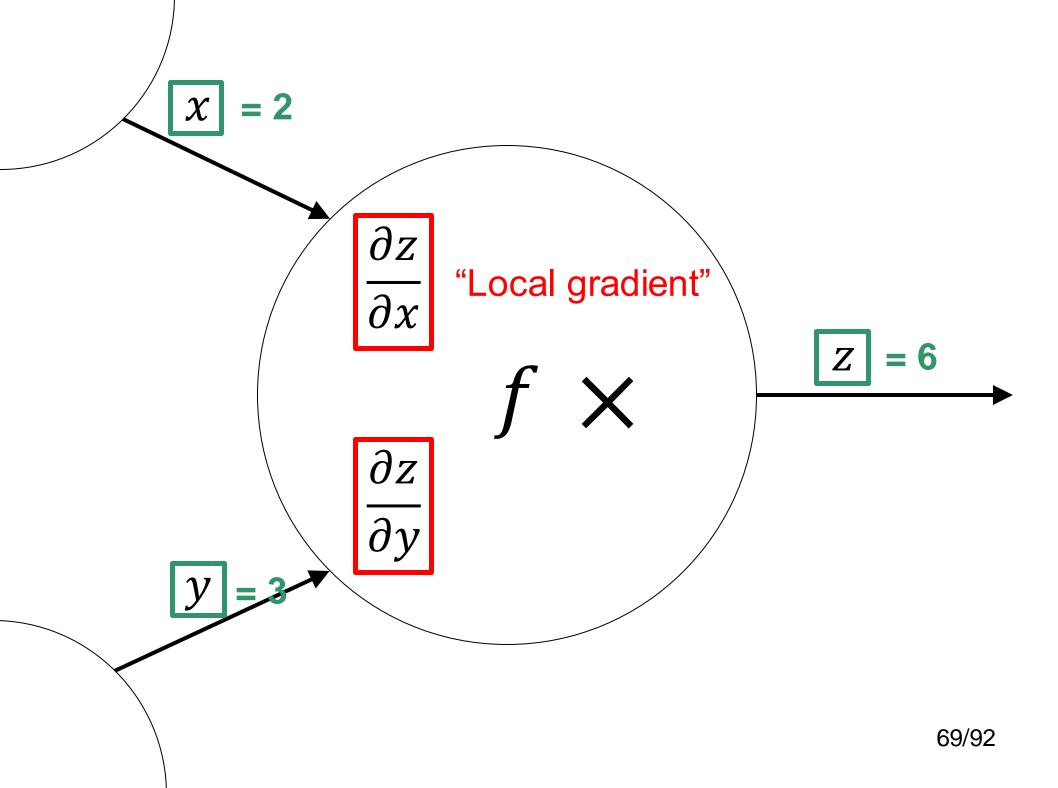


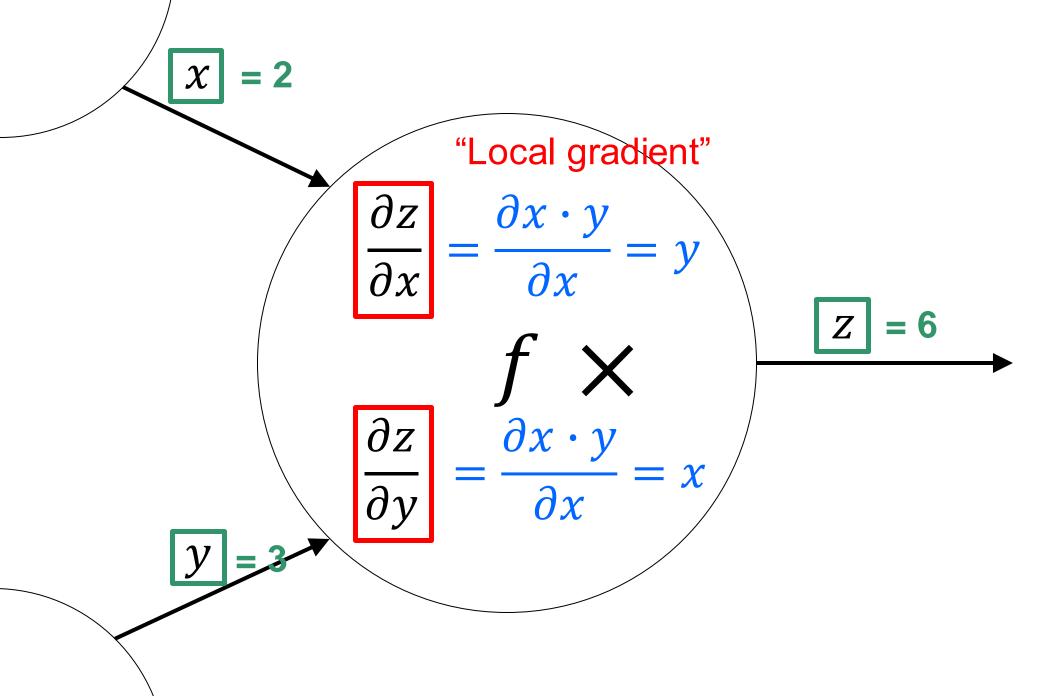


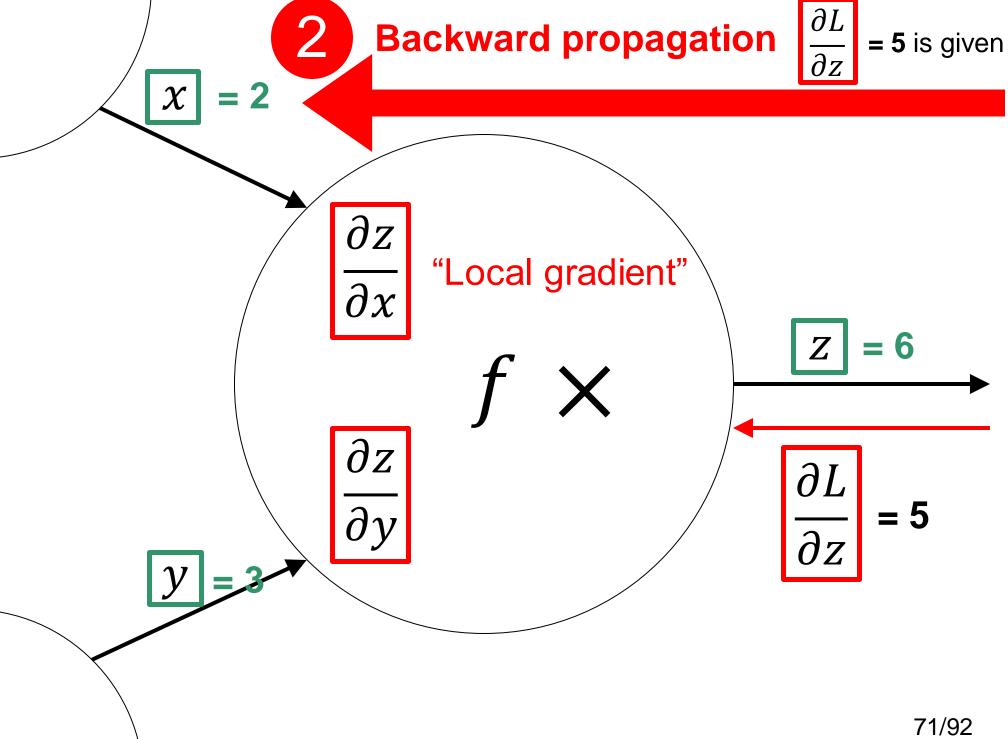


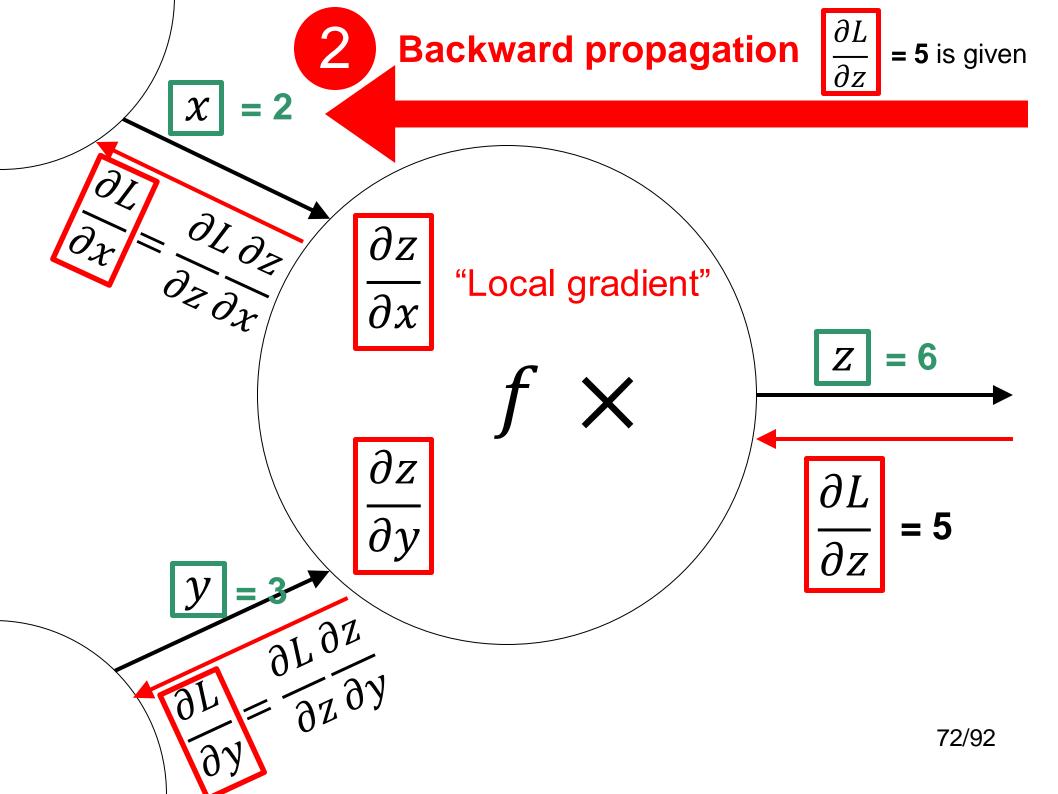












$$\frac{\partial L}{\partial z}$$
 = **5** is given

$$|\mathcal{X}| = 2$$

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}$$

$$= 5.$$

$$y = 15$$

$$\frac{\partial z}{\partial x}$$

"Local gradient"

$$Z = 6$$

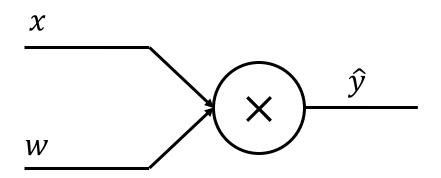
$$\frac{\partial z}{\partial y}$$

$$\left| \frac{\partial L}{\partial z} \right| = 5$$

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial x} = 10$$

$$\hat{y} = x \times w$$

$$\hat{y} = x \times w$$



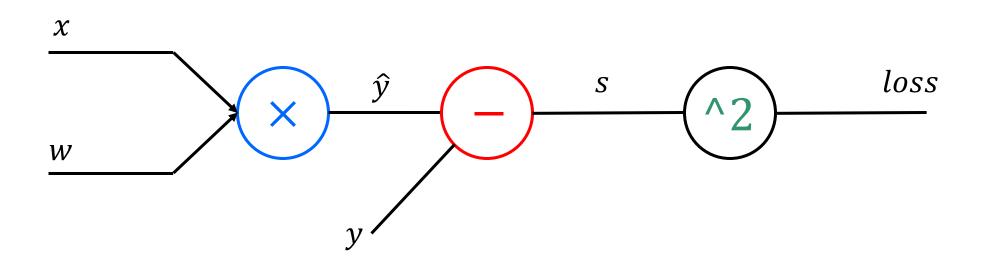
$$\hat{y} = x \times w \qquad loss = (\hat{y} - y)^2 = (x \times w - y)^2$$

$$\frac{1}{2} \sum_{d \in D} (\hat{y}^{(d)} - y^{(d)})^2$$

 $\frac{x}{w}$

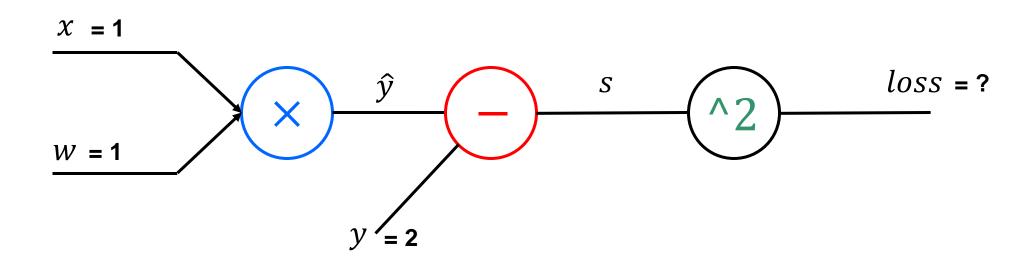
Our loss function

$$\hat{y} = x \times w$$
 $loss = (\hat{y} - y)^2 = (x \times w - y)^2$



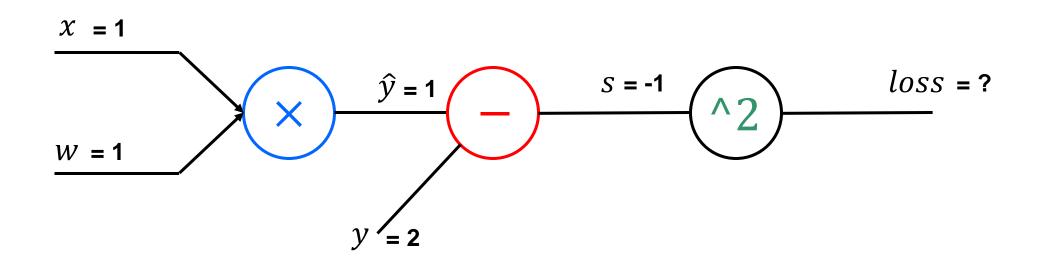
Forward pass x = 1, y = 2 where w = 1

$$\hat{y} = x \times w$$
 $loss = (\hat{y} - y)^2 = (x \times w - y)^2$



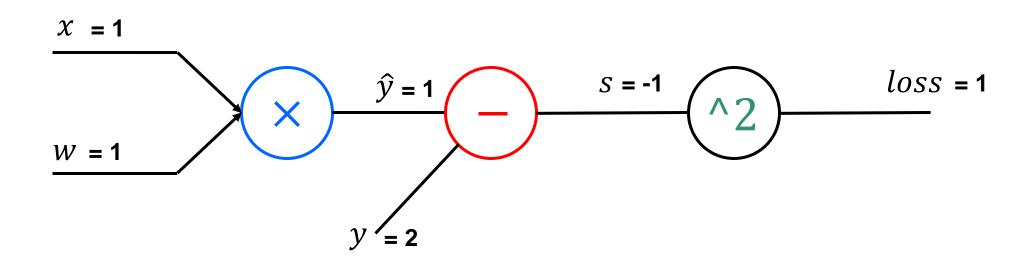
Forward pass x = 1, y = 2 where w = 1

$$\hat{y} = x \times w$$
 $loss = (\hat{y} - y)^2 = (x \times w - y)^2$



Backward propagation

$$\hat{y} = x \times w$$
 $loss = (\hat{y} - y)^2 = (x \times w - y)^2$



Backward propagation

$$\hat{y} = x \times w$$
 $loss = (\hat{y} - y)^2 = (x \times w - y)^2$

$$\frac{x = 1}{\partial w} = x$$

$$\frac{\hat{y} = 1}{\sqrt{2}} \qquad \frac{\hat{y} = 1}{\sqrt{2}} \qquad \frac{\partial \hat{y} - y}{\partial \hat{y}} = 1$$

$$\frac{\partial \hat{y} - y}{\partial \hat{y}} = 1$$

$$\frac{\partial \hat{y} - y}{\partial \hat{y}} = 1$$

$$\frac{\partial \hat{y} - y}{\partial \hat{y}} = 2s$$

Backward propagation

 $\hat{y} = x \times w$ $loss = (\hat{y} - y)^2 = (x \times w - y)^2$

82/92

$$\frac{\partial loss}{\partial w} = \frac{\partial loss}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w} = -2 \cdot x = -2 \cdot 1 = -2$$

$$x = 1$$

$$y = 1$$

$$y = 2$$

$$\frac{\partial \hat{y} - y}{\partial \hat{y}} = 1$$

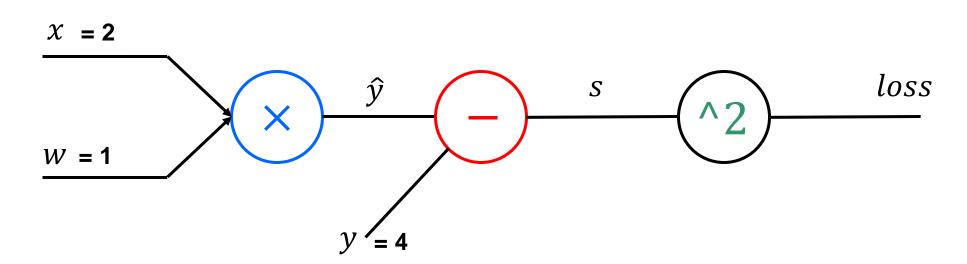
$$\frac{\partial s^2}{\partial s} = 2s$$

$$\frac{\partial loss}{\partial \hat{y}} = \frac{\partial loss}{\partial s} \frac{\partial \hat{y}}{\partial \hat{y}} = -2 \cdot 1 = -2$$

$$\frac{\partial loss}{\partial s} = 2s = -2$$

$$82/92$$

Backpropagation: Exercise



$$\frac{\partial loss}{\partial w} = 2$$

Backpropagation: Summary

- 1: Initialize all weights to small random values.
- 2: repeat
- 3: **for** each training example do
- 4: <u>Forward propagate</u> the input features of the example to determine the MLP's outputs.
- 5: Back propagate error to generate Δw_i for all weights Δw_i
- 6: Update the weights using Δw_i
- 7: end for
- 8: until stopping criteria reached.

Summary

- We learned what a perceptron and multilayer perceptron is
- We have some intuition about using gradient descent on an error function
- We know a learning delta rule for updating weights in order to minimize the error: $\Delta w_i = -\eta \times \frac{\partial E}{\partial w_i}$
- We know activation function for non-linearity
- We can use this rule to learn an MLP using the backpropagation algorithm

Optimizer: How to Optimize?

Gradient Descent (GD) vs. Stochastic GD (SGD)

Objective to Minimize:

$$E(\mathbf{w}) \equiv \frac{1}{D} \sum_{d \in D} E^{(d)}(\mathbf{w})$$

<u>GD:</u>

while True:

$$\mathbf{w}^{new} \leftarrow \mathbf{w}^{old} - \eta \frac{\partial}{\partial \mathbf{w}} \frac{1}{D} \sum_{d \in D} E^{(d)}(\mathbf{w})$$

$$O(N)$$

<u>SGD:</u>

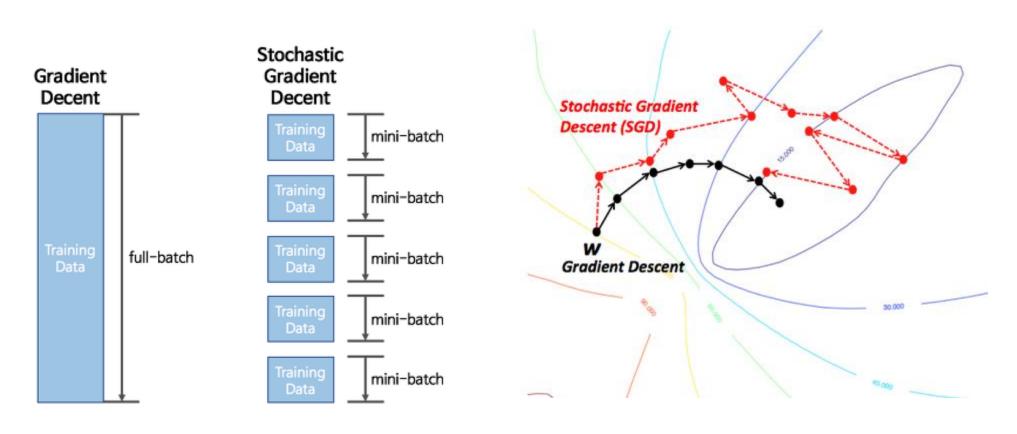
while True:

$$d \leftarrow mini\ batch(1,N)$$

$$\mathbf{w}^{new} \leftarrow \mathbf{w}^{old} - \eta \frac{\partial E^{(d)}(\mathbf{w})}{\partial \mathbf{w}}$$

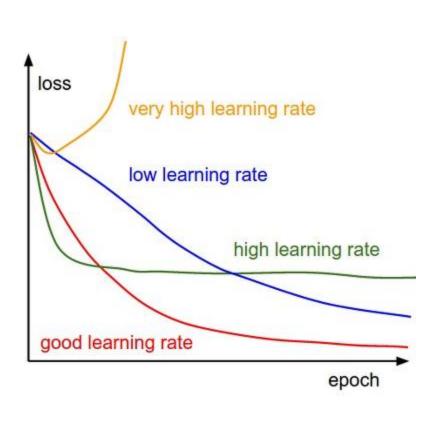
Optimizer: How to Optimize?

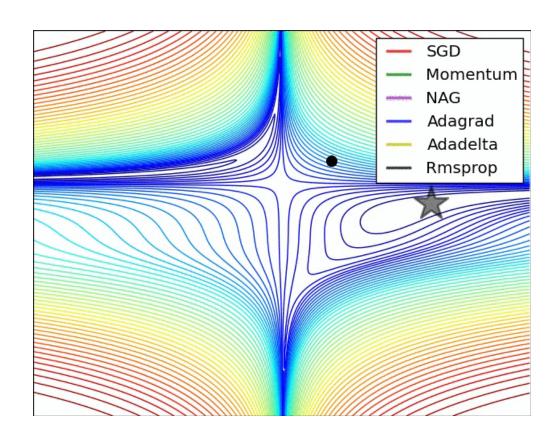
Gradient Descent vs. SGD



GD vs. SGD

Limitations of SGD and its Solutions





Tutorial on Neural Network: Zero to All!

- Perceptron from scratch
 - Based and basic algebra



- Perceptron based on pytorch
- Multi-layer perceptron

Step	Method	Data	Model	Forward	Loss	Backward	Update
1	Perceptron	X	X	X	X	X	X
2		X	X	X	X	0	X
3		X	0	0	0	0	0
4	MLP	0	0	0	0	0	0

Deep Neural Network Toolbox



Artificial Intelligence and Machine Learning SkillsFuture Courses and Training

- Why PYTÖRCH?
 - More pythonic
 - Flexible, intuitive and cleaner code, and easy to debug
- More neural networkic

Write code as the network works and forward/backward

90/92

Multi-Layer Perceptron: Programming

1. Training data

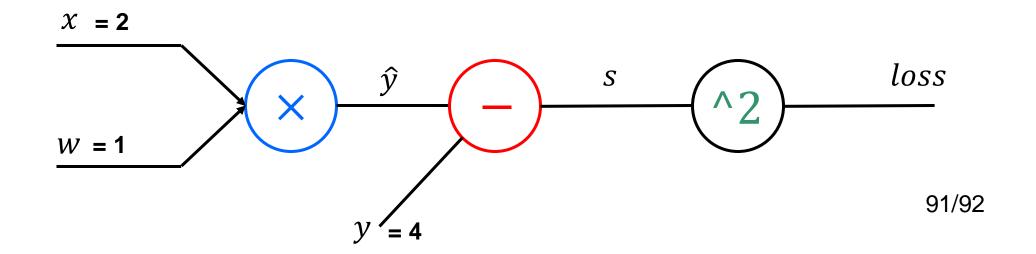
x	у
1	2
2	4
3	6

2. Model (forward pass)

$$\hat{y} = x \times w$$

3. Loss function

$$loss = (\hat{y} - y)^2 = (x \times w - y)^2$$



Multi-Layer Perceptron: Programming

1. Training data

Х	у
1	2
2	4
3	6

2. Model (forward pass)

$$\hat{y} = x \times w$$

3. Loss function

$$loss = (\hat{y} - y)^2 = (x \times w - y)^2$$

92/92

Learning Process (Very important !!!!)

