HW6

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1. At Western University the historical mean of scholarship examination scores for freshman applications is 900. A historical population standard deviation $\sigma = 180$ is assumed known. Each year, the assistant dean uses a sample of applications to determine whether the mean examination score for the new freshman applications has changed.

a. State the hypotheses.

$$H_0 : \mu_0 = 900; H_1 : \mu_0 \neq 900$$

b. What is the 95% confidence interval estimate of the population mean examination score if a sample of 200 applications provided a sample mean of 935?

$$P\left(-\tau_\gamma < \sqrt{n}\frac{\bar{X} - \mu}{\sigma} < \tau_\gamma\right) = \gamma$$

$$P\left(\bar{X} - \tau_\gamma \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + \tau_\gamma \cdot \frac{\sigma}{\sqrt{n}}\right) = \gamma$$

$$P\left(935 - 1.96 \cdot \frac{180}{\sqrt{200}} < \mu < 935 + 1.96 \cdot \frac{180}{\sqrt{200}}\right) = \gamma$$

$$(910, 960)$$

c. Use the confidence interval to conduct a hypothesis test. Using $\alpha = 0.05$, what is your conclusion?

Так как $\mu_0 \notin (910, 960)$ - доверительный интервал с $\alpha = 0.05$, гипотезу $H_0 : \mu = 900$ нужно отвергнуть.

2. A federal funding program is available to low-income neighborhoods. To qualify for the funding, a neighborhood must have a mean household income of less than \$15,000 per year. Neighborhoods with mean annual household income of \$15,000 or more do not qualify. Funding decisions are based on a sample of residents in the neighborhood. A hypothesis test with a 0.02 level of significance is conducted. If the funding guidelines call for a maximum probability of $\alpha = 0.05$ of not funding a neighborhood with a mean annual household income of \$14,000, what sample size should be used in the funding decision study? Use $\sigma =$ \$4000 as a planning value.

$$H_0 : \mu_0 = 15000; H_1 : \mu_0 < 15000$$

Пусть (c_-, c_+) - доверительный интервал для гипотезы H_0.
Тогда $c_- = \mu_0 - \tau_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}}$, с другой стороны при доходе в $\mu_1 = 14000$ и уровень ошибки $\alpha_{II} = 0.05$ получаем $c_- = \mu_1 + \tau_{1-\alpha_{II}} \cdot \frac{\sigma}{\sqrt{n}}$

$$\mu_0 - \tau_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}} = \mu_1 + \tau_{1-\alpha_{II}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\mu_0 - \mu_1 = \frac{\sigma}{\sqrt{n}}(\tau_{1-\alpha} + \tau_{1-\alpha_{II}})$$

$$n = \sigma^2 \frac{(\tau_{1-\alpha} + \tau_{1-\alpha_{II}})^2}{(\mu_0 - \mu_1)^2}$$

$$n = 4000^2 \cdot \frac{(2.05 + 1.65)^2}{1000^2} = 219$$

Стоит использовать выборку размера 219.

3. According to the federal government, 24% of workers covered by their company's health care plan were not required to contribute to the premium (Statistical Abstract of the United States: 2006). A recent study found that 81 out of 400 workers sampled were not required to contribute to their company's health care plan.

a. Develop hypotheses that can be used to test whether the percent of workers not required to contribute to their company's health care plan has declined.

$$H_0 : p = 0.24; H_1 : p < 0.24$$

b. What is a point estimate of the proportion receiving free company-sponsored health care insurance?

$$\hat{p} = \frac{81}{400} = 0.2025$$

c. Has a statistically significant decline occurred in the proportion of workers receiving free company-sponsored health care insurance? Use $\alpha = 0.05$.

$$P\left(-\tau_{1-\alpha} < \sqrt{n}\frac{\hat{p} - p}{\sigma} < \tau_{1-\alpha}\right) = 1 - \alpha$$

$$P\left(\hat{p} - \tau_{1-\alpha} \cdot \sqrt{\frac{p(1-p)}{n}} < p < \hat{p} + \tau_{1-\alpha} \cdot \sqrt{\frac{p(1-p)}{n}}\right) = 1 - \alpha$$

$$P\left(0.2025 - 1.96 \cdot \sqrt{\frac{0.24 \cdot 0.76}{400}} < p < 0.2025 + 1.96 \cdot \sqrt{\frac{0.24 \cdot 0.76}{400}}\right) = 0.95$$

$$(0.161, 0.244)$$

Статистически значимых свидетельств о снижении доли работников со страховкой от компании нет.

4. A study by the Centers for Disease Control (CDC) found that 23.3% of adults are smokers and that roughly 70% of those who do smoke indicate that they want to quit (Associated Press, July 26, 2002). CDC reported that, of people who smoked at some point in their lives, 50% have been able to kick the habit. Part of the study suggested that the success rate for quitting rose by education level. Assume that a sample of 100 college graduates who smoked at some point in their lives showed that 64 had been able to successfully stop smoking.

a. State the hypotheses that can be used to determine whether the population of college graduates has a success rate higher than the overall population when it comes to breaking the smoking habit.

$$H_0 : p = 0.5; H_1 : p > 0.5$$

b. Given the sample data, what is the proportion of college graduates who, having smoked at some point in their lives, were able to stop smoking?

$$\hat{p} = \frac{64}{100} = 0.64$$

c. At $\alpha = 0.01$, what is your hypothesis testing conclusion?

$$P\left(\hat{p} - \tau_{1-\alpha} \cdot \sqrt{\frac{p(1-p)}{n}} < p < \hat{p} + \tau_{1-\alpha} \cdot \sqrt{\frac{p(1-p)}{n}}\right) = 1 - \alpha$$

$$P\left(0.64 - 2.6 \cdot \sqrt{\frac{0.25}{100}} < p < 0.64 + 2.6 \cdot \sqrt{\frac{0.25}{100}}\right) = 0.99$$

$$(0.51, 0.77)$$

Для уровня доверия 0.99 можно сказать, что гипотеза H_0 неверна, и среди выпускников колледжа доля бросивших курить выше средней по популяции.

5. On Friday, Wall Street traders were anxiously awaiting the federal government's release of numbers on the January increase in nonfarm payrolls. The early consensus estimate among economists was for a growth of 250,000 new jobs (CNBC, February 3, 2006). However, a sample of 20 economists taken Thursday afternoon provided a sample mean of 266,000 with a sample standard deviation of 24,000. Financial analysts often call such a sample mean, based on late-breaking news, the whisper number. Treat the "consensus estimate" as the population mean. Conduct a hypothesis test to determine whether the whisper number justifies a conclusion of a statistically significant increase in the consensus estimate of economists. Use $\alpha = 0.01$ as the level of significance.

$$H_0 : \mu = 250000; H_1 : \mu > 250000$$

$\tau_{1-\alpha}$ - квантили распределения Стьюдента.

$$P\left(-\tau_{1-\alpha} < \sqrt{n}\frac{\bar{X} - \mu}{\bar{\sigma}} < \tau_{1-\alpha}\right) = 1 - \alpha$$

$$P\left(\bar{X} - \tau_{1-\alpha} \cdot \frac{\bar{\sigma}}{\sqrt{n}} < \mu < \bar{X} + \tau_{1-\alpha} \cdot \frac{\bar{\sigma}}{\sqrt{n}}\right) = 1 - \alpha$$

$$P\left(266000 - 2.85 \cdot \frac{24000}{\sqrt{20}} < \mu < 266000 + 2.85 \cdot \frac{24000}{\sqrt{20}}\right) = 0.99$$

$$(250700, 281300)$$

Для уровня доверия 0.99 гипотеза H_0 должна быть отвергнута.