HW7

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1. Periodically, Merrill Lynch customers are asked to evaluate Merrill Lynch financial consultants and services. Higher ratings on the client satisfaction survey indicate better service, with 7 the maximum service rating. Independent samples of service ratings for two financial consultants are summarized here. Consultant A has 10 years of experience, whereas consultant B has 1 year of experience. Use $\alpha = 0.05$ and test to see whether the consultant with more experience has the higher population mean service rating.

a. Consultant A: $n_a = 16$, mean rating $\bar{X_a} = 6.82$, sample dev $\bar{s_a} = 0.64$

b. Consultant B: $n_b = 19$, mean rating $\bar{X_b} = 6.25$, sample dev $\bar{s_b} = 0.75$

$$H_0 : \mu_a = \mu_b; H_1 : \mu_a > \mu_b$$

$$z = \frac{\bar{X_a} - \bar{X_b}}{\sqrt{\frac{\bar{s_a}^2}{n_a} + \frac{\bar{s_b}^2}{n_b}}}$$

$$z = \frac{6.82 - 6.25}{\sqrt{\frac{0.64^2}{16} + \frac{0.75^2}{19}}} = 2.43$$

Используем квантиль стандартного нормального распределения для уровня доверия $\alpha = 0.05$ - $t_\alpha = 1.96$. Оттклоняем гипотезу H_0, т.е. у более опытного консультанта выше средний рейтинг.

2. Airline travelers often choose which airport to fly from based on flight cost. Cost data (in dollars) for a sample of flights to eight cities from Dayton, Ohio, and Louisville, Kentucky, were collected to help determine which of the two airports was more costly to fly from (The Cincinnati Enquirer, February 19, 2006). A researcher argued that it is significantly more costly to fly out of Dayton than Louisville. Use the sample data to see whether they support the researcher's argument. Use $\alpha = 0.05$ as the level of significance.

Destination	Dayton	Louisville	Difference(Dayton - Louisville)
Chicago O'Hare	\$319	\$142	177
Grand Rapids, Michigan	192	213	-21
Portland, Oregon	503	317	186
Atlanta	256	387	-131
Seattle	339	317	22
South Bend, Indiana	379	167	212
Miami	268	273	-5
Dallas–Ft. Worth	288	274	14

Преобразуем выборку, получив разность цен на билеты.

$$H_0 : d = 0, H_1 : d > 0$$

$$z = \sqrt{n} \cdot \frac{\bar{d}}{\bar{S}} = \sqrt{8} \cdot \frac{56.75}{121.5} = 1.32$$

Так как $z \in (-1.96, 1.96)$ у нас нет оснований отклонять H_0 и утверждать, что полеты из Дейтона значимо дороже.

3. An American Automobile Association (AAA) study investigated the question of whether a man or a woman was more likely to stop and ask for directions (AAA, January 2006). The situation referred to in the study stated the following: "If you and your spouse are driving together and become lost, would you stop and ask for directions?" A sample representative of the data used by AAA showed 300 of 811 women said that they would stop and ask for directions, while 255 of 750 men said that they would stop and ask for directions. The AAA research hypothesis was that women would be more likely to say that they would stop and ask for directions. Formulate the null and alternative hypotheses for this study. At $\alpha = 0.05$, test the hypothesis, and what conclusion would you expect AAA to draw from this study?

$$H_0 : p_w = p_m; H_1 : p_w > p_m$$

$$\bar{p_w} = \frac{300}{811} = 0.37$$

$$\bar{p_m} = \frac{255}{750} = 0.34$$

$$\bar{p} = \frac{300 + 255}{811 + 750} = 0.356$$

$$z = \frac{\bar{p_w} - \bar{p_m}}{\sqrt{\bar{p} \cdot (1 - \bar{p}) \left(\frac{1}{n_m} + \frac{1}{n_w}\right)}}$$

$$z = \frac{0.37 - 0.34}{\sqrt{0.356 \cdot 0.644 \left(\frac{1}{811} + \frac{1}{750}\right)}} = 1.24$$

Так как $z \in (-1.96, 1.96)$ у нас нет оснований отклонять H_0 и утверждать, что женщины более склонны сказать, что попросили бы помощи в поиске пути.

4. M&M/MARS, makers of M&M® chocolate candies, conducted a national poll in which more than 10 million people indicated their preference for a new color. The tally of this poll resulted in the replacement of tan-colored M&Ms with a new blue color. In the brochure "Colors," made available by M&M/MARS Consumer Affairs, the distribution of colors for the plain candies is as follows:

Brown	Yellow	Red	Orange	Green	Blue
30%	20%	20%	10%	10%	10%

In a follow-up study, samples of 1-pound bags were used to determine whether the reported percentages were indeed valid. The following results were obtained for one sample of 506 plain candies.

Brown	Yellow	Red	Orange	Green	Blue
177	135	79	41	36	38

Use $\alpha = 0.05$ to determine whether these data support the percentages reported by the company.

$$\rho = \sum_{j=1}^{6} \frac{(v_j - np_j)^2}{np_j} \Rightarrow H_5$$

$$\rho = \frac{(177 - 506 \cdot 0.3)^2}{506 \cdot 0.3} + \frac{(135 - 506 \cdot 0.2)^2}{506 \cdot 0.2} + \frac{(79 - 506 \cdot 0.2)^2}{506 \cdot 0.2} + \\ + \frac{(41 - 506 \cdot 0.1)^2}{506 \cdot 0.1} + \frac{(36 - 506 \cdot 0.1)^2}{506 \cdot 0.1} + \frac{(38 - 506 \cdot 0.1)^2}{506 \cdot 0.1} = 29.5 \quad (1)$$

Для распределения Хи-квадрат с 5 степенями свободы и уровнем значимости $\alpha = 0.05$ этот результат является значимым и данные противоречат процентному соотношению, которе предостваила кампания M&M/MARS.

5. Part variability is critical in the manufacturing of ball bearings. Large variances in the size of the ball bearings cause bearing failure and rapid wearout. Production standards call for a maximum variance of .0001 when the bearing sizes are measured in inches. A sample of 15 bearings shows a sample standard deviation of .014 inches. Use $\alpha = 0.10$ to determine whether the sample indicates that the maximum acceptable variance is being exceeded. Assume that the sizes of ball bearings could be well approximated by a normal distribution.

$$H_0 : \sigma = 0.01; H_1 : \sigma > 0.01$$

$$\chi^2 = (n-1) \cdot \frac{\bar{s_n}^2}{\sigma^2} = 14 \cdot \frac{0.014^2}{0.01^2} = 27.4$$

Для распределения Хи-квадрат с $k = n - 1 = 14$ степенями свободы и уровня значимости $\alpha = 0.10$ этот результат является значимым. Гипотезу H_0 нужно отклонить.