#1 $T(x,y,z) = (x+y,\ x-3,\ z)$

선형변환 조건

상수가 존재하므로 선형 변환X

1) 덧셈 보존 $T(u+v) = T(u) + T(v)$

2) 스칼라곱 보존 $T(cv) = c\,T(v)$

#2 선형 변환 맞음

$$\begin{bmatrix} 3 & 0 & 4 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

#3 $T(1,0,0) = (3,1,2)$ $\begin{bmatrix} 3x \\ x \\ 2x \end{bmatrix}$ $T(0,1,0) = (2,-1,3)$ $\begin{bmatrix} 2y \\ -y \\ 3y \end{bmatrix}$

$T(0,0,1) = (4,0,2)$ $\begin{bmatrix} 4z \\ 0 \\ 2z \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 2 & -1 & 3 \\ 4 & 0 & 2 \end{bmatrix} = (9,0,7)$$

#4

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

#5 $R = I\cos\theta + (1+\cos\theta)\cdot(\hat{n}\times\hat{n}) + \sin\theta\cdot[\hat{n}]_x$

$$I\cos\theta = \frac{\sqrt{3}}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$(1-\cos\theta)\cdot(\hat{n}\times\hat{n}) = \frac{2-\sqrt{3}}{2}\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}\cdot\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right] = \frac{2-\sqrt{3}}{6}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\sin\theta\cdot[\hat{n}]_x = \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\cdot\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \frac{1}{2\sqrt{3}}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} \end{bmatrix} + \frac{2-\sqrt{3}}{6}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{1}{2\sqrt{3}}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

#6

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 0 & -9 & 1 \end{bmatrix}$$

#7

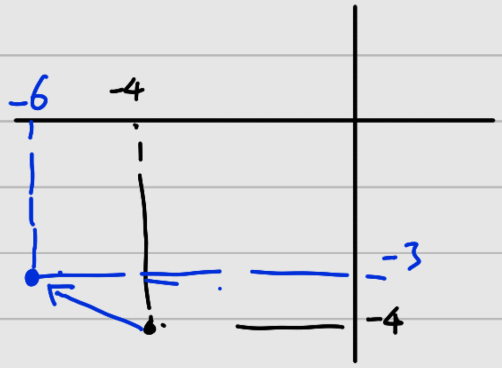
$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & 0 & -9 & 1 \end{bmatrix}$$

#8

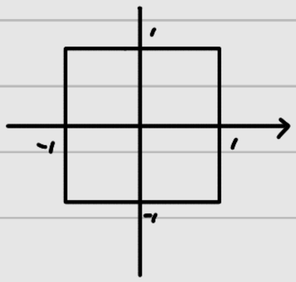
$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 5 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ -2 & 5 & 1 & 1 \end{bmatrix}$$

#9

$$[-4, -4, 0, 0] \cdot \begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



#10



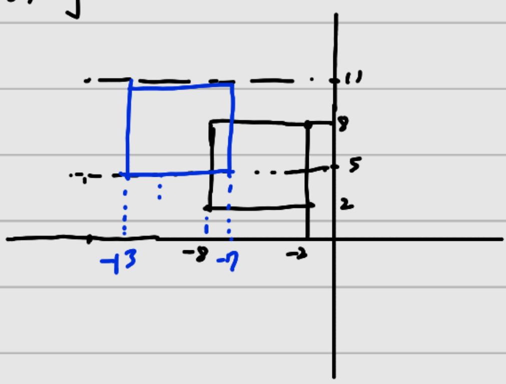
$\cos(-45°) = \frac{\sqrt{2}}{2}$

$\sin(-45°) = -\frac{\sqrt{2}}{2}$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#11

$[-8, 2, 0]$, $[-2, 8, 0]$



#12 $R_n(v) = \cos\theta\, v + (1-\cos\theta)(n\cdot v)n + \sin\theta(n\times v)$ 덧셈 보존, 스칼라 보존이 되므로 선형 변환 조건 충족

$R = \cos\theta I + (1-\cos\theta)nn^T + \sin\theta$

#13 $R_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $R_y^T = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_y^{-1} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $R_y^{-1}R_y = R_y^TR_y = I$ → orthonormal

#14 if M is orthogonal → $M^TM = I$ → $M^T = M^{-1}$

#15 $[x, y, z, 1]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ b_x & b_y & b_z & 1 \end{bmatrix} = [x+b_x, y+b_y, z+b_z, 1]$

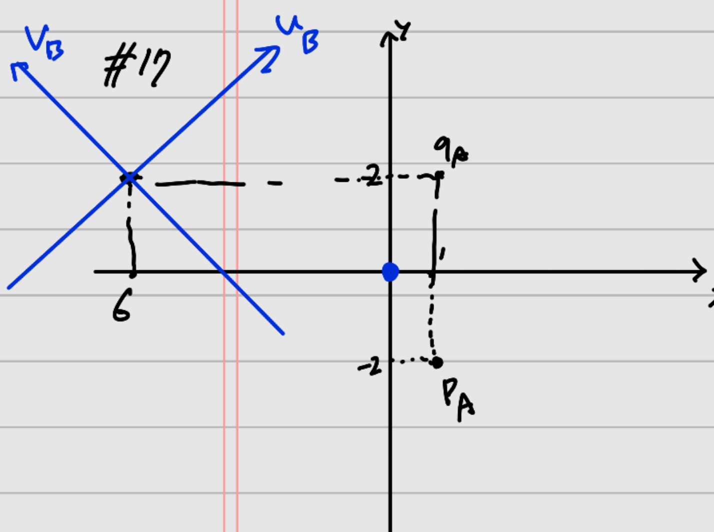
$[x, y, z, 0]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ b_x & b_y & b_z & 1 \end{bmatrix} = [x, y, z, 0]$ w값이 0 일때는 element 반영이 안된다.

#16 $S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $S^{-1} = \begin{bmatrix} \frac{1}{S_x} & 0 & 0 & 0 \\ 0 & \frac{1}{S_y} & 0 & 0 \\ 0 & 0 & \frac{1}{S_z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $SS^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$ $S^{-1}S = I$

$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$ $T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -T_x & -T_y & -T_z & 1 \end{bmatrix}$ $TT^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$

$T^{-1}T = I$

#17



$A \times R \times T = B$

$\begin{cases} R: \text{CW } 45° \\ T: (6, -2, 0) \end{cases}$

R: $\begin{bmatrix} \cos(45) & \sin(45) & 0 & 0 \\ -\sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

T: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix}$

passive transformation 이므로, 반대

$$RT = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix}$$

$$p_B = p_A RT = [-1, 2, 0, 1] \times \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix} = \left[\frac{12-3\sqrt{2}}{2}, \frac{-4+\sqrt{2}}{2}, 0, 1\right]$$

$$q_B = q_A RT = [1, 2, 0, 1] \times \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix} = \left[\frac{12-\sqrt{2}}{2}, \frac{-4+3\sqrt{2}}{2}, 0, 1\right]$$

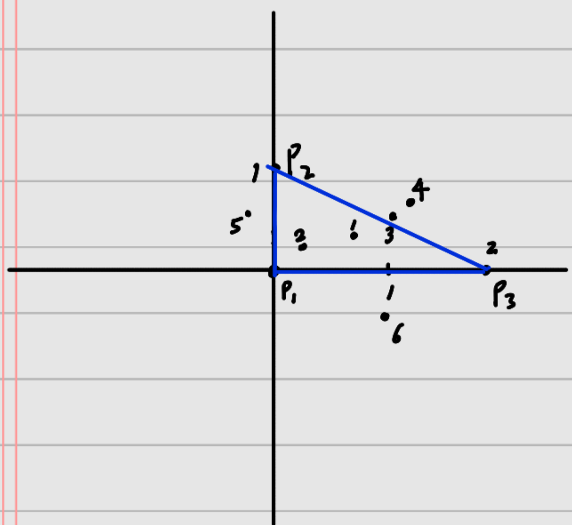
#18

$$p = p_1 + a_2(p_2 - p_1) + \cdots a_n(p_n - p_1)$$

$$= (1 - a_2 - a_3 \cdots - a_n)p_1 + a_2 p_2 + a_3 p_3 + \cdots a_n p_n$$

$$= a_1 p_1 + a_2 p_2 + \cdots a_n p_n \quad \because a_1 + a_2 + \cdots a_n = 1$$

#19



1) $\frac{1}{3}P_1 + \frac{1}{3}P_2 + \frac{1}{3}P_3$

$= (\frac{2}{3}, \frac{1}{3}, 0)$

2) $0.7P_1 + 0.2P_2 + 0.1P_3$

$= (0.2, 0.2, 0)$

3) $0.0P_1 + 0.5P_2 + 0.5P_3$

$= (1, \frac{1}{2}, 0)$

4) $-0.2P_1 + 0.6P_2 + 0.6P$

$= (\frac{6}{5}, \frac{3}{5}, 0)$

5) $0.6P_1 + 0.5P_2 - 0.1P_3$
$= (-\frac{1}{5}, \frac{1}{2}, 0)$

6) $0.8P_1 - 0.3P_2 + 0.5P_3$
$= (1, -\frac{3}{10}, 0)$

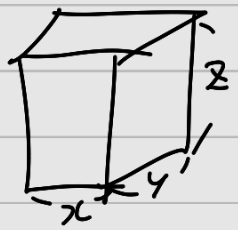
$\frac{1}{3}P_1 + \frac{1}{3}P_2 + \frac{1}{3}P_3$ 는 $\triangle P_1P_2P_3$의 무게중심.
$(1, 0, 0)$에서 P_2의 weight는 0. 하나라도 weight가 음수인 경우
점은 삼각형 바깥에 위치한다.

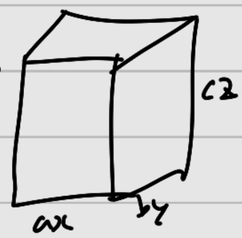
#20 행렬 곱셈에서 상수는 위치이동이 가능하므로,
$\alpha(a_1P_1) = a_1\alpha(P_1)$ and 작은 행렬로 분리 가능 $\alpha(P_1 + P_2) = \alpha(P_1) + \alpha(P_2)$
$\rightarrow \alpha(a_1P_1 + \cdots a_nP_n) = a_1\alpha(P_1) + a_2\alpha(P_2) + \cdots a_n\alpha(P_n)$

#21 x 방향, y방향 0.5씩 이동, scale x방향 0.5, y방향 -0.5

$$\text{so, } \alpha = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.5 & 0.5 & 0 & 1 \end{bmatrix}$$

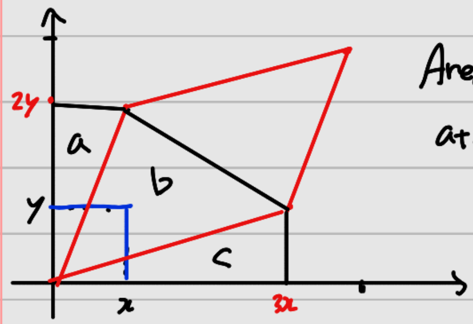
#22
$$S = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \det(S) = a \cdot b \cdot c$$



$V = xyz \rightarrow$


$V' = abc(xyz)$

#23
$$C = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \det(C) = 3(2) - 1(1) + 0(0)$$
$$= 5$$



Area $= 2b$ $\quad a = xy$, $c = \frac{3}{2}xy$
$a + b + c = 3xy + 2xy = 5xy$
$b = (5 - \frac{3}{2} - 1)xy = \frac{5}{2}xy$
Area $= \underline{\underline{5xy}}$

#24

$$R_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \det(R_y) = \cos\theta \det \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$- \sin\theta \det \begin{bmatrix} 0 & 1 & 0 \\ \sin\theta & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det(R_y) = \cos\theta\left(1 \cdot (\cos\theta)\right) - \sin\theta\left(-1 \cdot (\sin\theta)\right)$$

$$= \cos^2\theta + \sin^2\theta = 1$$

#25

$$R_1 = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ 0 & 1 & 0 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 R_2 = R = \begin{bmatrix} c_1c_2 & s_1 & -c_1s_2 & 0 \\ -s_1c_2 & c_1 & s_1s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

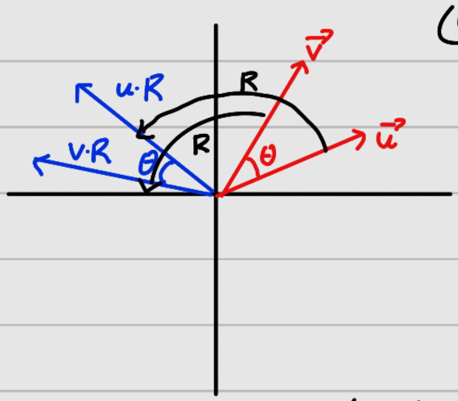
$$R^T = \begin{bmatrix} c_1c_2 & -s_1c_2 & s_2 & 0 \\ s_1 & c_1 & 0 & 0 \\ -c_1s_2 & s_1s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R^T R = \begin{bmatrix} c_1^2c_2^2 + s_1^2c_2^2 + s_1^2 & c_1s_1c_2 - c_1s_1c_2 & 0 & 0 \\ c_1s_1c_2 - c_1s_1c_2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$RR^T = I.$$

#26

a) $(uR)\cdot(vR) = u\cdot v$



$(uR)\cdot(vR) = |uR|\,|vR|\cos\theta$

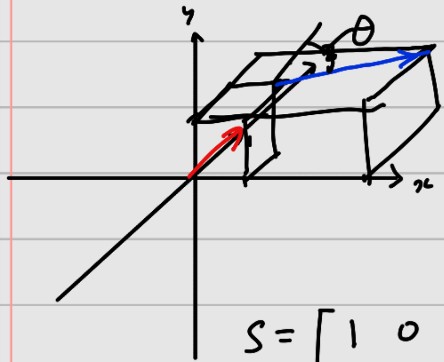
$= |u||v|\cos\theta = u\cdot v$

당연한 것..

(b) $\|uR\| = \|u\|$ 회전행렬을 곱해도 크기는 변하지 않음.

(c) $\theta(uR, vR) = \theta(u, v)$ 같은 R은 곱하면 θ 변하지 않음.

#27



S : z방향 $\sqrt{10}$배 증가

R : $-\theta$ 만큼 y축 기준 회전

T : (1, 1, 1) 이동

$\cos(-\theta) = \frac{\sqrt{10}}{10}$

$\sin(-\theta) = -\frac{3\sqrt{10}}{10}$

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \frac{\sqrt{10}}{10} & 0 & \frac{3\sqrt{10}}{10} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3\sqrt{10}}{10} & 0 & \frac{\sqrt{10}}{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$SR = \begin{bmatrix} \frac{\sqrt{10}}{10} & 0 & \frac{3\sqrt{10}}{10} & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad SRT = \begin{bmatrix} \frac{\sqrt{10}}{10} & 0 & \frac{3\sqrt{10}}{10} & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

#28

원점 이동 ⟶ 스케일링 → 다시 이동.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x & -y & -z & 1 \end{bmatrix} \quad S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ S_x x & S_y y & S_z z & 1 \end{bmatrix}$$

$$TS = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ -x & -y & -z & 1 \end{bmatrix} \quad TST' = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ (S_x-1)x & (S_y-1)y & (S_z-1)z & 1 \end{bmatrix}$$