#1

$$3\left(\begin{bmatrix}-2, & 0\\ 1, & 3\end{bmatrix} - 2X\right) = 2\begin{bmatrix}-2 & 0\\ 1 & 3\end{bmatrix}$$

$$\begin{bmatrix}-6 & 0\\ 3 & 9\end{bmatrix} - 6X = \begin{bmatrix}-4 & 0\\ 2 & 6\end{bmatrix}$$

$$6X = \begin{bmatrix}2 & 0\\ -1 & -3\end{bmatrix}$$

$$X = \begin{bmatrix}1/3 & 0\\ -1/6 & -1/2\end{bmatrix}$$

#2

a) $$\begin{bmatrix}-2 & 0 & 3\\ 4 & 1 & -1\end{bmatrix}\begin{bmatrix}2 & -1\\ 0 & 6\\ 2 & -3\end{bmatrix}$$

$$\begin{bmatrix}-4+6, & 2-9\\ 8-2, & -4+6+3\end{bmatrix} = \begin{bmatrix}2 & -7\\ 6 & 5\end{bmatrix}$$

b) $$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}-2 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}-2+2, & 0+2\\ -6+4, & 0+4\end{bmatrix} = \begin{bmatrix}0 & 2\\ -2 & 4\end{bmatrix}$$

c) $$\begin{bmatrix}2 & 0 & 2\\ 0 & -1 & -3\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} = \begin{bmatrix}2+0+2\\ 0-2-3\\ 0+0+1\end{bmatrix} = \begin{bmatrix}4\\ -5\\ 1\end{bmatrix}$$

#3

a) $$[1, 2, 3]^T = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

b) $$\begin{bmatrix}x & y\\ z & w\end{bmatrix}^T = \begin{bmatrix}x & z\\ y & w\end{bmatrix}$$

c) $$\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\\ 7 & 8\end{bmatrix}^T = \begin{bmatrix}1 & 3 & 5 & 7\\ 2 & 4 & 6 & 8\end{bmatrix}$$

#4 1)

$$V = \begin{bmatrix} 1 & -5 & 2 \\ 2 & 0 & -2 \\ 3 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$$

2)

$$V = \begin{bmatrix} 2 & 1 & -2 & 1 \\ -4 & 4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -1 \\ 5 \end{bmatrix}$$

#5

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31}, & A_{11}B_{12} + A_{12}B_{22} + A_{13}B_{32}, & A_{11}B_{13} + A_{12}B_{23} + A_{13}B_{33} \\ A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}, & A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32}, & A_{21}B_{13} + A_{22}B_{23} + A_{23}B_{33} \\ A_{31}B_{11} + A_{32}B_{21} + A_{33}B_{31}, & A_{31}B_{12} + A_{32}B_{22} + A_{33}B_{32}, & A_{31}B_{13} + A_{32}B_{23} + A_{33}B_{33} \end{bmatrix}$$

$$= \begin{bmatrix} A_1 \cdot B_{*1}, & A_1 \cdot B_{*2}, & A_1 \cdot B_{*3} \\ A_2 \cdot B_{*1}, & A_2 \cdot B_{*2}, & A_2 \cdot B_{*3} \\ A_3 \cdot B_{*1}, & A_3 \cdot B_{*2}, & A_3 \cdot B_{*3} \end{bmatrix} = \begin{bmatrix} A_1 \cdot {}_*B \\ A_2 \cdot {}_*B \\ A_3 \cdot {}_*B \end{bmatrix}$$

#6

$$Au = \begin{bmatrix} A_{11}x + A_{12}y + A_{13}z \\ A_{21}x + A_{22}y + A_{23}z \\ A_{31}x + A_{32}y + A_{33}z \end{bmatrix} = xA_{*,1} + yA_{*,2} + zA_{*,3}$$

#7

$$u \times v = (u_y v_z - u_z v_y,\ u_z v_x - u_x v_z,\ u_x v_y - u_y v_x)$$

$$[v_x, v_y, v_z] \begin{bmatrix} 0 & u_z & -u_y \\ -u_z & 0 & u_x \\ u_y & -u_x & 0 \end{bmatrix} = [v_x \cdot 0 - u_z v_y + u_y v_z,\ v_x u_z - v_z u_x,\ -v_x u_y + v_y u_x]$$

$$= [u_y v_z - u_z v_y,\ u_z v_x - u_x v_z,\ u_x v_y - u_y v_x]$$

#8

$$AB = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & -1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & 1 \end{bmatrix} = I$$

$$B = A^{-1}$$

#9

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1, & 2 \\ 0, & 5 \end{bmatrix} \neq I \qquad B \neq A^{-1}$$

#10

$$\det \begin{bmatrix} 21 & -4 \\ 10 & 7 \end{bmatrix} = 21 \cdot 7 + 40 = 187$$

$$\det \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{bmatrix} = 2 \cdot \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix} - 0 \cdot \begin{bmatrix} 0 & 0 \\ 0 & 7 \end{bmatrix} + 0 \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$$

$$= 2 \cdot (21 - 0) = 42$$

#11

$$\begin{bmatrix} 21 & -4 \\ 10 & 7 \end{bmatrix}^{-1} = \frac{1}{187}\begin{bmatrix} 7 & -10 \\ 4 & 21 \end{bmatrix} = \begin{bmatrix} \frac{7}{187}, & -\frac{10}{187} \\ \frac{4}{187}, & \frac{21}{187} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{bmatrix}^{-1} = \frac{1}{42}\begin{bmatrix} \det\begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix}, & -\det\begin{bmatrix} 0 & 0 \\ 0 & 7 \end{bmatrix}, & \det\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \\ -\det\begin{bmatrix} 0 & 0 \\ 0 & 7 \end{bmatrix}, & \det\begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix}, & -\det\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \\ \det\begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}, & -\det\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, & \det\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \end{bmatrix}$$

$$= \frac{1}{42}\begin{bmatrix} 21 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1/2, & 0, & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/7 \end{bmatrix}$$

#12

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 0 \end{bmatrix} = 1 \cdot \cancel{\det\begin{bmatrix} 4 & 5 \\ 0 & 0 \end{bmatrix}} - 2 \cdot \cancel{\det\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}} + 3 \cdot \cancel{\det\begin{bmatrix} 0 & 4 \\ 0 & 0 \end{bmatrix}}$$

$$= 0 \quad \therefore \text{not invertible}$$

#13

$$(A^{-1})^T = \frac{1}{\det A}\begin{bmatrix} A_{11}\det[\ ] & \cdots & A_{1j}\det[\ ] \\ \vdots & & \\ | & & \end{bmatrix}^T = \frac{1}{\det A^T}\begin{bmatrix} A_{11}\det[\ ] & & A_{i1}\det[\ ] \\ \vdots & & \\ A_{1j}\det[\ ] & \cdots & A_{ij}\det \end{bmatrix}$$

$\det A = \det A^T$

$$= (A^T)^{-1}$$

#14

$$\det(AA^{-1}) = \det I = 1 = \det A \cdot \det A^{-1}$$

$$\det A = \frac{1}{\det A^{-1}}$$

#15

$$\det\begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} = u_x v_y - u_y v_x = u \times v = \|u\|\|v\|\sin\theta = \|u\| \times h$$

if u rotates clockwise, $\theta > 0$, $\sin\theta > 0 \rightarrow \det\begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} > 0$

else " counterclockwise, $\theta < 0$, $\sin\theta < 0 \rightarrow \det\begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} < 0$

#16

1) $\det\begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} = 3$

2) $\det\begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} = -1$

#17

$$A(BC) = A\begin{pmatrix} B_{11}C_{11} + B_{12}C_{21}, & B_{11}C_{12} + B_{12}C_{22} \\ B_{21}C_{11} + B_{22}C_{21}, & B_{21}C_{12} + B_{22}C_{22} \end{pmatrix}$$

$$= \begin{pmatrix} A_{11}B_{11}C_{11} + A_{11}B_{12}C_{21} + A_{12}B_{21}C_{11} + A_{12}B_{22}C_{21}, & A_{11}B_{11}C_{12} + A_{11}B_{12}C_{22} + A_{12}B_{21}C_{12} + A_{12}B_{22}C_{22} \\ A_{21}B_{11}C_{11} + A_{31}B_{12}C_{21} + A_{22}B_{21}C_{11} + A_{22}B_{22}C_{21}, & A_{21}B_{11}C_{12} + A_{21}B_{12}C_{22} + A_{22}B_{21}C_{12} + A_{22}B_{22}C_{22} \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21}, & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21}, & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix} C$$

$$= \begin{pmatrix} A_{11}B_{11}C_{11} + A_{11}B_{12}C_{21} + A_{12}B_{21}C_{11} + A_{12}B_{22}C_{21}, & A_{11}B_{11}C_{12} + A_{11}B_{12}C_{22} + A_{12}B_{21}C_{12} + A_{12}B_{22}C_{22} \\ A_{21}B_{11}C_{11} + A_{31}B_{12}C_{21} + A_{22}B_{21}C_{11} + A_{22}B_{22}C_{21}, & A_{21}B_{11}C_{12} + A_{21}B_{12}C_{22} + A_{22}B_{21}C_{12} + A_{22}B_{22}C_{22} \end{pmatrix}$$