#4 $2((1,2,3)-x)-(-2,0,4)=(-2,-4,-6)$

$(2,4,6)-(-2,0,4)-2x=(-2,-4,-6)$

$(4,4,2)-2x=(-2,-4,-6)$

$(6,8,8)=2x$

$x=(3,4,4)$

#5 $\|u\|=\sqrt{(-1)^2+3^2+2^2}=\sqrt{14}$

$\frac{u}{\|u\|}=\left(-\frac{\sqrt{14}}{14},\frac{3\sqrt{14}}{14},\frac{\sqrt{14}}{7}\right)$

$\|v\|=\sqrt{3^2+(-4)^2+1^2}=\sqrt{26}$

$\frac{v}{\|v\|}=\left(\frac{3\sqrt{26}}{26},\frac{-2\sqrt{26}}{13},\frac{\sqrt{26}}{26}\right)$

#6 $ku=(ku_x,ku_y,ku_z)$

$\|ku\|=\sqrt{k^2u_x^2+k^2u_y^2+k^2u_z}=k\sqrt{u_x^2+u_y^2+u_z^2}$

$=k\|u\|=|k|\|u\|$

#7 1) $u\cdot v=\|u\|\cdot\|v\|\cdot\cos\theta=1\cdot2+1\cdot3+1\cdot4=9$, $\|u\|=\sqrt{3}$, $\|v\|=\sqrt{29}$

$\cos\theta=\frac{9}{\sqrt{87}}$ → acute

2) $u\cdot v=\|u\|\cdot\|v\|\cdot\cos\theta=1\cdot(-2)+1\cdot2+0\cdot0=0$

$\|u\|\cdot\|v\|\neq0$ 이므로 $\cos\theta=0$ → orthogonal

3) $u\cdot v=\|u\|\cdot\|v\|\cdot\cos\theta=-1\cdot3+-1\cdot1+-1\cdot0=-4$

$-1<\cos\theta<0$ → obtuse

#8

$$u \cdot v = \|u\| \cdot \|v\| \cdot \cos\theta$$

$$\|u\| = \sqrt{(-1)^2 + 3^2 + 2^2} = \sqrt{14} \qquad \|v\| = \sqrt{3^2 + (-4)^2 + 1^2} = \sqrt{26}$$

$$\|u\| \cdot \|v\| \cdot \cos\theta = 2\sqrt{91} \cdot \cos\theta = (-1) \cdot 3 + 3 \cdot (-4) + 2 \cdot 1 = -14$$

$$\cos\theta = -\frac{14}{\sqrt{91}} = -\frac{2\sqrt{7}}{\sqrt{13}}$$

$$\theta = \cos^{-1}\left(-\frac{2\sqrt{7}}{\sqrt{13}}\right) \quad (0 < \theta < \pi)$$

#9

1) $u \cdot v = u_x \cdot v_x + u_y \cdot v_y + u_z \cdot v_z$

$= v_x \cdot u_x + v_y \cdot u_y + v_z + u_z$

$= v \cdot u$

2) $u \cdot (v + w) = (u_x, u_y, u_z) \cdot (v_x + w_x, v_y + w_y, v_z + w_z)$

$= u_x \cdot (v_x + w_x) + u_y \cdot (v_y + w_y) + u_z \cdot (v_z + w_z)$

$= u_x \cdot v_x + u_y \cdot v_y + u_z \cdot v_z + u_x \cdot w_x + u_y \cdot w_y + u_z \cdot w_z$

$= u \cdot v + u \cdot w$

3) $k(u \cdot v) = k(u_x v_x + u_y v_y + u_z v_z)$

$= k u_x v_x + k u_y v_y + k u_z v_z$

$k u_x v_x + k u_y v_y + k u_z v_z$

$= (k u_x) \cdot v_x + (k u_y) \cdot v_y + (k u_z) \cdot v_z$

$= (ku) \cdot v$

$k u_x v_x + k u_y v_y + k u_z v_z$

$= u_x \cdot (k v_x) + u_y \cdot (k v_y) + u_z \cdot (k v_z)$

$= u \cdot (kv)$

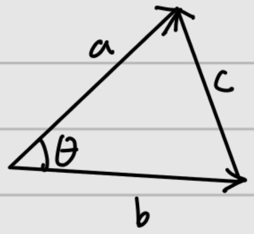
4) $V \cdot V = V_x \cdot V_x + V_y \cdot V_y + V_z \cdot V_z$

$$= V_x^2 + V_y^2 + V_z^2$$

$$= \sqrt{V_x^2 + V_y^2 + V_z^2}^{\,2} = ||V||^2$$

5) $0 \cdot V = 0 \cdot V_x + 0 \cdot V_y + 0 \cdot V_z = 0$

#10




$||c||^2 = ||a||^2 + ||b||^2 - 2||a||\,||b|| \cdot \cos\theta$ $\quad c = a - b$

$$= (a_x - b_x,\ a_y - b_y,\ a_z - b_z)$$

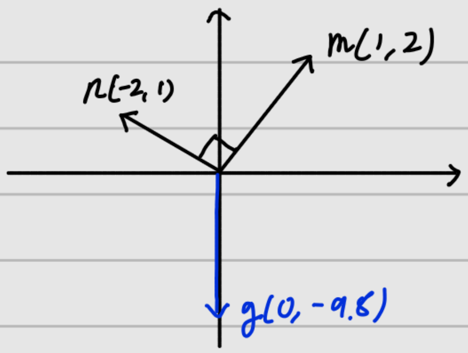
$$(a_x - b_x)^2 + (a_y - b_y)^2 + (a_z - b_z)^2 = a_x^2 + a_y^2 + a_z^2 + b_x^2 + b_y^2 + b_z^2$$

$$- 2\sqrt{a_x^2 + a_y^2 + a_z^2} \cdot \sqrt{b_x^2 + b_y^2 + b_z^2} \cdot \cos\theta$$

$$\cancel{-2}(a_x b_x + a_y b_y + a_z b_z) = \cancel{-2}\sqrt{a_x^2 + a_y^2 + a_z^2}\sqrt{b_x^2 + b_y^2 + b_z^2} \cdot \cos\theta$$

$$a \cdot b = ||a|| \cdot ||b|| \cdot \cos\theta$$

#11




$a, b \equiv$ constant scalar

$an + bm = g$

$a(-2,1) + b(1,2) = (0, -9.8)$

$-2a + b = 0 \rightarrow b = 2a$

$a + 2b = -9.8$

$5a = -9.8 \quad a = -1.96, \quad b = -3.92$

$$g = -1.96 \cdot n - 3.92m$$

#12 $w = u \times v = (1 - (-16),\ 12 + 2,\ 8 - 3) = (17,\ 14,\ 5)$

$w \cdot u = -34 + 14 + 20 = 0$

$w \cdot v = 51 - 56 + 5 = 0$

#13 $a = \overrightarrow{AB} = (0,1,3)$ $\quad a\times b = (0-3,\ 15-0,\ 0-5) = (-3, 15, -5)$

$b = \overrightarrow{AC} = (5,1,0)$

$(a\times b)\cdot a = 0+15-15=0$ $\quad \therefore (-3, 15, -5)$

#14 $u\times v = (u_y v_z - u_z v_y,\ u_z v_x - u_x v_z,\ u_x v_y - u_y v_x)$

$$\|u\times v\| = \{(u_y v_z - u_z v_y)^2 + (u_z v_x - u_x v_z)^2 + (u_x v_y - u_y v_x)^2\}^{\frac{1}{2}}$$

$$= (u_y^2 v_z^2 + u_z^2 v_y^2 + u_z^2 v_x^2 + u_x^2 v_z^2 + u_x^2 v_y^2 + u_y^2 v_x^2 - 2u_y u_z v_y v_z - 2u_x u_z v_x v_z - 2u_x u_y v_x v_y)^{1/2}$$

$$\|u\|\|v\|\sin\theta = \|u\|\|v\|\sqrt{1-\cos^2\theta}$$

$$= \|u\|\|v\|\sqrt{1-\frac{(u\cdot v)^2}{\|u\|^2\|v\|^2}}$$

$$= \cancel{\|u\|\|v\|}\sqrt{\frac{\|u\|^2\|v\|^2-(u\cdot v)^2}{\cancel{\|u\|^2\|v\|^2}}}$$

$$= \{(u_x^2+u_y^2+u_z^2)(v_x^2+v_y^2+v_z^2) - (u_x v_x + u_y v_y + u_z v_z)^2\}^{1/2}$$

$$= (\cancel{u_x^2 v_x^2} + u_x^2 v_y^2 + u_x^2 v_z^2 + u_y^2 v_x^2 + \cancel{u_y^2 v_y^2} + u_y^2 v_z^2 + u_z^2 v_x^2 + u_z^2 v_y^2 + \cancel{u_z^2 v_z^2} - \cancel{u_x^2 v_x^2} - \cancel{u_y^2 v_y^2} - \cancel{u_z^2 v_z^2} - 2u_x u_y v_x v_y - 2u_x u_z v_x v_z - 2u_y u_z v_y v_z)^{1/2}$$

$$= \|u\times v\|$$

#15 $\text{Area} = h\cdot\|v\| = \|u\|\sin\theta\,\|v\| = \|u\|\|v\|\sin\theta$

$$= \|u\times v\|$$

$u_x \qquad u_y \qquad u_z$

#16

$u\times(v\times w) = u\times(v_y w_z - v_z w_y,\ v_z w_x - v_x w_z,\ v_x w_y - v_y w_x)$

$= (u_y v_x w_y - u_y v_y w_x - u_z v_z w_x + u_z v_x w_z,$

$u_z v_y w_z - u_z v_z w_y - u_x v_x w_y + u_x v_y w_x,$

$u_x v_z w_x - u_x v_x w_z - u_y v_y w_z + u_y v_z w_y)$

$(u\times v)\times w = (u_y v_z - u_z v_y,\ u_z v_x - u_x v_z,\ u_x v_z - u_y v_x)\times w$ diffreat

$w_x \qquad w_y \qquad w_z$

$= (u_z v_x w_z - u_x v_z w_z - u_x v_z w_y + u_y v_x w_y,$

$u_x v_z w_x - u_y v_z w_z - u_y v_z w_z + u_z v_y w_z,$

$u_y v_z w_y - u_z v_y w_y - u_z v_x w_x + u_x v_z w_z)$

#17

$u\times ku = (u_x, u_y, u_z)\times(ku_x, ku_y, ku_z)$

$= (\cancel{ku_y u_z - ku_z u_y}, \cancel{ku_x u_z - ku_z u_x}, \cancel{ku_y u_x - ku_x u_y})$

$= (0, 0, 0)$

#18

$V_1 = (1,0,0),\ V_2 = (1,5,0),\ V_3 = (2,1,-4)$

$u_1 = (1,0,0)$

$V_2 - \mathrm{proj}(u_1) = (0,5,0)\quad u_2 = (0,1,0)$

$V_3 - \mathrm{Proj}(u_1) - \mathrm{Proj}(u_2) = (0,0,4)\quad u_3 = (0,0,1)$