

Newtonian Telescope Mirrors Made from Deformed Reflective Membranes

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Contents

Background and Introduction	3
Geometry.....	4
Material Model.....	5
Developing the Lumped Material Model.....	6
Boundary Conditions	7
The Mesh	8
Convergence Study	9
Dealing with Nonlinearity.....	9
Model Verification.....	10
Model Validation	12
Uncertainty Quantification.....	13
Results.....	13
Discussion	15
Pressure to Achieve a 1200mm Focal Length	15
Suitability as a Newtonian Telescope Primary Mirror	15
Areas for Improvement.....	17
Sources	18
Appendix A: Abaqus Python Script.....	19
Appendix B: Center Deflection to Achieve a 1200mm Focal Length.....	43

Appendix C: Applying Snell's Law to Judge Mirror Quality.....	44
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Background and Introduction

Primary mirrors in Newtonian telescopes are usually made from blanks of borosilicate glass that have been painstakingly ground to have a concave profile. The specific profile determines how well the primary mirror focuses parallel rays of light from a distant source to a single point, which determines image quality. For example, spherical profiles are relatively cheap to make but come with spherical aberration that produces blurry images. Parabolic mirrors are ideal because they have no aberration, but they are more difficult to make and thus more expensive.

This project explores an experimental method for forming parabolic mirrors: pulling a vacuum on one side of a reflective membrane. The appeal is that this could be a cheaper, faster way to make primary mirrors without aberrations. This concept is illustrated in Figures 1 and 2. This project aims to determine the vacuum pressure needed to deform a 200mm diameter aluminized PET membrane to have a focal length of 1200mm, and to investigate the mirror quality by how closely it matches a perfect parabola. To achieve a 1200mm focal length, the membrane mirror's center must deflect by 2.0833mm, which is determined in [Appendix B: Center Deflection to Achieve a 1200mm Focal Length](#). Python scripting within Abaqus is used to implement a Newton-Raphson root finding strategy to run several simulations autonomously and determine

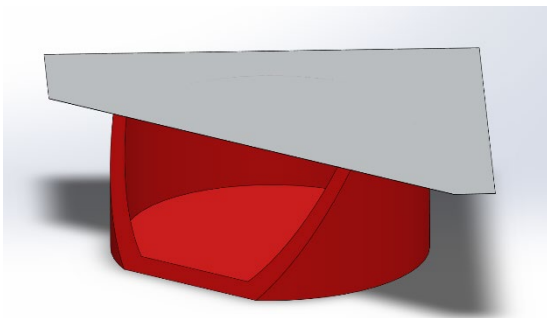


Figure 1. The experimental telescope mirror involves a kind of open-top pressure vessel (red) and a reflective membrane (gray). When the ambient pressure and vessel pressure are identical, the membrane does not deform.

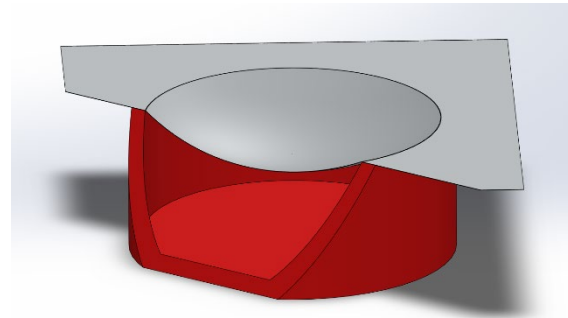


Figure 2. Decreasing pressure in the vessel (red) produces a pressure load that causes the reflective membrane (gray) to deform inward, forming a concave surface. Depending on how close the resulting surface is to a parabola, the mirror may or may not be suitable for use in a Newtonian telescope.

the differential pressure needed to achieve that target center deflection (and thus the target focal length). The Python script also exports data to files for analysis. It is available in [Appendix A: Abaqus Python Script](#).

Geometry

The membrane mirror involves an open-top pressure vessel to apply a vacuum force and a plastic film with aluminum deposited on the surface that is deformed by the vacuum. The point of concern is the membrane itself and how it deforms when the vacuum pressure is applied, so this is the modeled geometry. Since telescope mirrors are radially symmetric, an axisymmetric profile capturing the membrane profile's radius is used. This is shown in Figure 3. The pressure vessel is not considered.

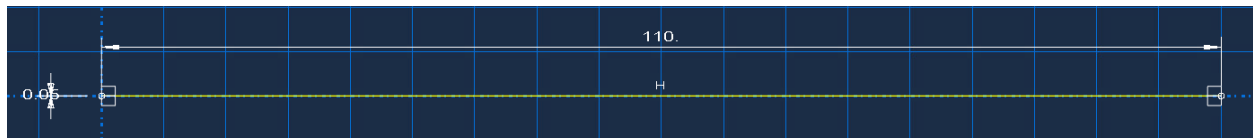


Figure 3. The mirror membrane is modeled as the axisymmetric profile shown. Although the goal is to model the deflection of a 200mm diameter mirror, there needs to be some room for boundary conditions to clamp the membrane to the pressure vessel to form a vacuum-tight seal, and that is the purpose of the extra 10mm radius in this sketch.

The geometry is created by defining a sketch containing a rectangle with a thickness of 0.0508 and a length of 110. These dimensions are in millimeters and correspond to the membrane's thickness (2mil) and radius from the axis of symmetry, respectively. After the sketch is defined, a new axisymmetric part is created with a shell base feature. The membrane sketch is the

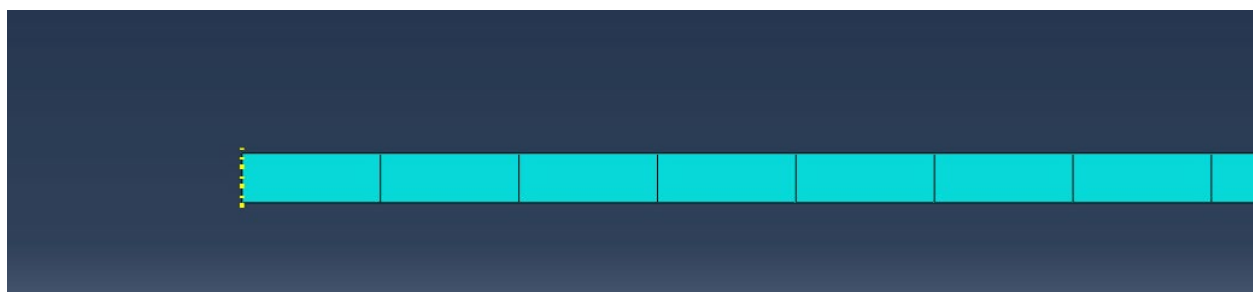


Figure 4. The sketch in Figure 3 is used to generate an axisymmetric shell, which can then be meshed with axisymmetric elements. The CAX4 elements used here are solved with 2nd-order Gauss quadrature and use linear shape functions.

revolved profile for the shell representing the membrane, the inner edge of which is shown with a preliminary CAX4 mesh in Figure 4.

The membrane geometry does not capture a bonded layer of aluminum with the PET film because it would be extremely thin in comparison and thus challenging to mesh with the thicker PET film. However, its contribution to the membrane's stiffness is not negligible at approximately 17% of that contributed by the thicker PET membrane. This is calculated in Equation (1). Instead, I will factor this extra stiffness due to deposited aluminum into the membrane's material model. $t_{Al} = 0.5\mu m$ is an estimate of the aluminum film's thickness from surface vapor deposition [1]. E_{Al} and E_{PET} are estimates from [2, 3].

$$\frac{k_{Al}}{k_{PET}} = \frac{\frac{AE_{Al}}{L}}{\frac{AE_{PET}}{L}} = \frac{t_{Al}E_{Al}}{t_{PET}E_{PET}} = \frac{(0.5 \times 10^{-6}m)(70 \times 10^9Pa)}{(0.0508 \times 10^{-3}m)(4 \times 10^9Pa)} = 0.1722 \quad (1)$$

Material Model

The component being modeled in this project is the aluminized PET membrane that will deform to approximate a paraboloid. Thus, the material model needs to capture the behavior of PET plastic and the vapor-deposited aluminum on its surface. The argument for a homogenous, lumped material model over multiple zones in the membrane mesh was made in [Geometry](#), and this is now developed further.

The material model itself is linear elastic. Many polymers behave linearly before yielding, just like metals, so this will be the material model of choice before experiments can validate that the aluminized PET has a linear elastic region [4]. An important disclaimer is that the Poisson's ratios and Young's moduli used will require calibration since at the time of writing I have been able to find neither a reputable database of polymer material properties nor studies on the

properties of physical vapor deposited aluminum. Instead, the relevant properties for PET and 1100 aluminum are used and provided in Table 1 [5]. Note that 1100 aluminum is an alloy, but elemental aluminum is what is on the membrane surface.

Table 1. Average material properties for PET and 1100 Aluminum are reported from [5]. The “Combined Model” row contains the result of lumping these properties from two materials into a single material that can be applied to the entire membrane mesh.

Material	E (GPa)	ν	t (mm)
PET	3.450	0.33	0.0508
1100 Aluminum	69	0.33	0.5×10^{-3}
Combined Model	4.089	0.33	N/A

Developing the Lumped Material Model

Given a strip of aluminized PET taking the assumed dimensions from [Geometry](#) (Figure 5), the stress contributed by each component of the membrane can be resolved using their individual Young’s moduli E_{PET}, E_{Al} for a certain elastic strain ϵ . The axial stress in each material is $\sigma_{Al} = E_{Al}\epsilon$ and $\sigma_{PET} = E_{PET}\epsilon$, and the total

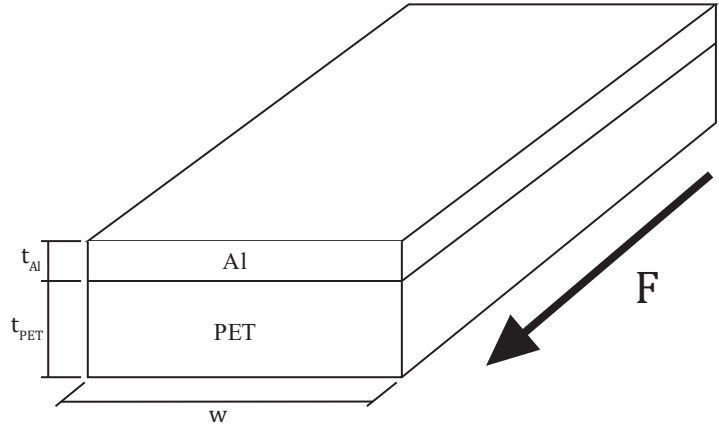


Figure 5. The aluminized PET membrane has a PET layer and a significantly thinner PVD aluminum layer. Considering the total stress for a given strain along the direction of an applied force F yields a Young’s modulus for the bulk material model.

applied force is the sum of those stresses times the respective cross sectional areas: $F = \sigma_{Al}t_{Al}w + \sigma_{PET}t_{PET}w$. The total stress is this force divided by the total area, $\sigma_t =$

$F/(t_{Al} + t_{PET})w$, and this can be divided by the axial strain to find the combined Young's modulus for the membrane. After expanding and canceling, the result is:

$$E_t = \frac{E_{Al}t_{Al} + E_{PET}t_{PET}}{t_{Al} + t_{PET}} \quad (2)$$

Equation 2 is evaluated for the first two rows of table 1 (component materials) to get the combined material Young's modulus in the third row. Since the Poisson's ratio is common to both component materials, it is assumed to be the same in the combined material model.

Boundary Conditions

The goal is to model a 200mm diameter mirror, but the axisymmetric profile for the membrane has been given an extra 10mm at the outer edge. This edge is meant to be the area where the membrane is clamped to a pressure vessel, forming a vacuum-tight seal. The boundary condition to model this clamping is simply Abaqus' Encastre to prevent rotation or displacement at the mirror's outer 10mm of radius. To accomplish this, a face partition was created on the axisymmetric mesh's outer 10mm and the Encastre boundary condition was applied to the top and bottom edge of this partition, simulating clamping between two surfaces. The boundary conditions are illustrated in Figure 6. Figure 6 also shows how the differential pressure load is applied. Pulling a vacuum on one side of the mirror causes the atmosphere to push on the other side, hence the downward-facing pressure load.



Figure 6. Boundary conditions and loads on the axisymmetric profile are shown. Encastre boundary conditions simulate the membrane's clamped outer radius (right). At the profile's far left is a Displacement/Rotation boundary condition that captures that this portion of the mirror is bonded to the material meeting at the center. The downward facing arrows are a uniform pressure load due to a vacuum being pulled on the opposite side of the mirror.

The Mesh

The membrane mirror being modeled in this project uses an axisymmetric profile to save simulation time. Since this profile is naturally rectangular when undeformed, it makes sense to use quadrilateral elements that easily capture the shape without distortion. Abaqus provides options for the element geometric order and an option called “reduced integration.” I achieved convergence with the geometric order set to linear and turned off the reduced integration option because this corresponds to fewer Gauss points used. As expected, experimentation with reduced integration showed that it negatively affected convergence. These options led to my selection of CAX4 elements to comprise the mesh.

The membrane geometry is quite simple, so meshing was similarly trivial. I simply seed the edges with consistent spacing between nodes on all dimensions (the membrane radius and its thickness) so that meshing based on those seeds produces a clean grid of elements with aspect ratios close to 1, as shown in Figure 7. I

judged the mesh quality to be good by the elements’ squareness. The final mesh (with which I achieved convergence) is shown in Figure 8.

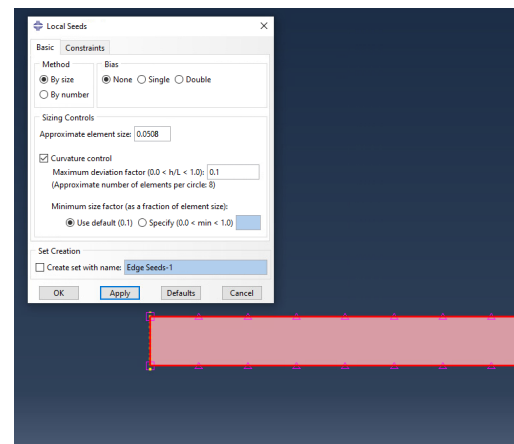


Figure 7. Meshing involves seeding the edges of the membrane in such a way that square or approximately square elements will result. Here the membrane profile’s inner edge is shown having been seeded with an element size equal to the membrane’s thickness.

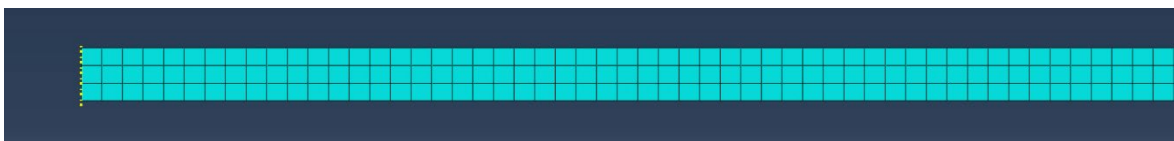


Figure 8. Node spacing of 0.02mm with CAX4 mesh elements is the combination that resulted in convergence. Shown is the left (inner radius) edge of the membrane’s axisymmetric profile.

Convergence Study

I used both H and P refinement to find a mesh that converges quickly for a given vacuum pressure. A plot of the center deformation for a pressure of 100Pa and different meshes is shown in Figure 9. The CAX4 elements surprisingly converged faster than the CAX8 elements, so CAX4 elements with node spacing of 0.02mm were the obvious choice.

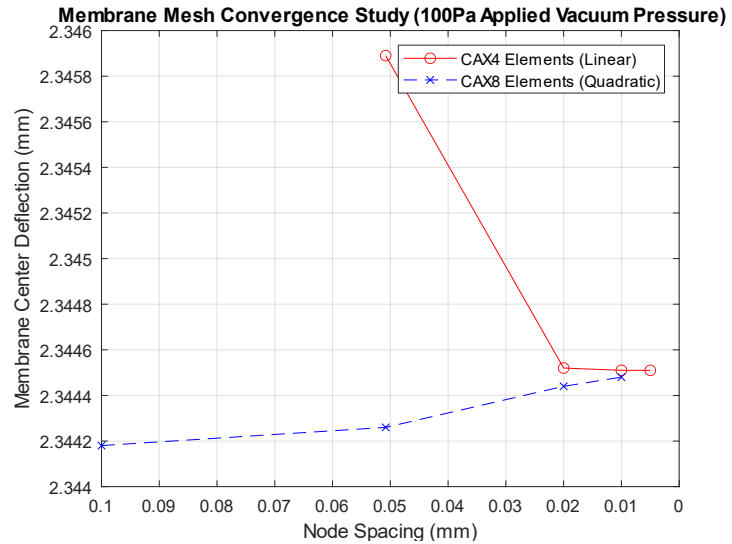


Figure 9. H and P refinement of the membrane mesh revealed that a mesh of axisymmetric linear quadrilateral elements (CAX4) converges with node spacing of 0.02mm. Nonlinear solving with Abaqus' Nlgeom option was required due to the membrane's relatively large displacements.

Dealing with Nonlinearity

Abaqus elements are built on the assumption of small strains, which is completely necessary to prevent stiffness matrices from being functions of displacements themselves. Such displacement-dependent stiffness matrices would require more advanced methods to solve.

The nature of the membrane being studied here is that it experiences large displacements/strains to form a concave surface. Consequently, Abaqus' linear solver produced completely unreasonable deflection results. The Nlgeom option in Abaqus splits the solution to a step into several sequential frames at which a small portion of the loads resulting in large displacements are applied. This solves the problem with large displacements and produces much more reasonable results.

The Nlgeom solver often makes multiple attempts to solve a frame at smaller and smaller load increments if it continues to show large displacements, and defaults to aborting the simulation after more than five of these unsuccessful attempts. This model consistently required more than 5

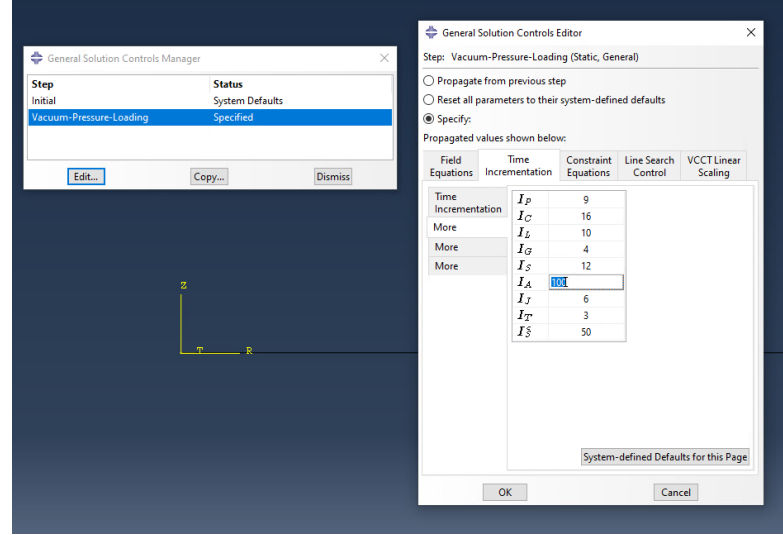


Figure 10. The number of allowable unsuccessful attempts to solve a frame using Nlgeom needed to be increased by editing the I_A parameter in the General Solutions Controls Manager. This parameter defaults to 5.

attempts to solve the first frame, which required changing the number of allowable attempts in the General Solutions Controls Editor, as shown in Figure 10.

Model Verification

It happens that Alfonso Hermida at the Goddard Space Flight Center developed a model for the deflection of a membrane under pressure that can be applied to verifying the FE model results.

Hermida presents the following:

$$\pi E h w_0^3 + 2 a \alpha^3 p w_0 - \pi a^4 \alpha^3 q = 0, \quad (3)$$

$$\alpha^3 = \frac{6615(v^2 - 1)}{2(2791v^2 - 4250v - 7505)}, \quad (4)$$

where E is the Young's modulus, h is the membrane thickness, a is the membrane's radius up to its clamped edge, p is a preload force, q is the applied differential pressure, and ν is Poisson's ratio [6]. w_0 is the deflection at the membrane's center, which can be compared to FE results to build confidence in the simulation [6]. The FE model assumes $p = 0$ (no preload/pretension force), so Equation 3 can be rearranged into

$$w_0 = \sqrt[3]{\frac{a^4 \alpha^3 q}{Eh}}. \quad (5)$$

The deflection predicted by Hermida's model is compared to that produced by the FE model in Figure 11. Both models show a linear trend at the same slope and predict deflections within approximately 0.2mm of each other. The common linear trend is reassuring, but I cannot explain the offset in central displacements at this time. This would probably require a deeper dive into Hermida's model to understand any assumptions or simplifications possibly being made that could affect its predictions.

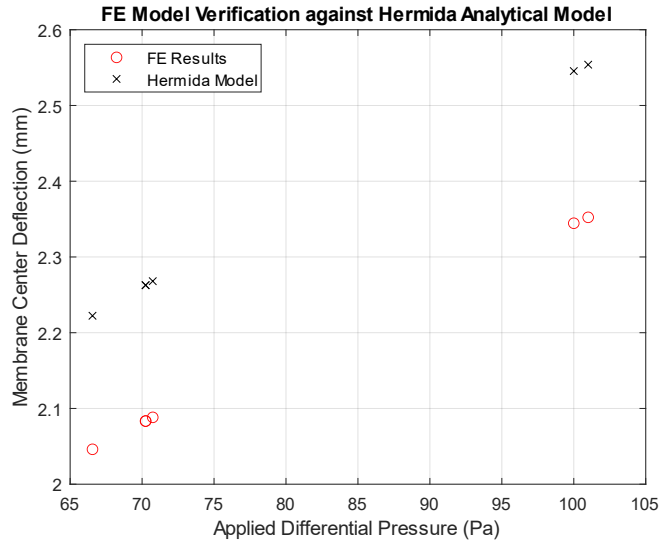


Figure 11. The FE model results are plotted with Hermida's analytical solution for the deflection of a membrane for the same differential pressures. The common linear trend in the data (at this scale at least) and the models producing results within approximately 0.2mm of each other is reassuring, although I do not presently have an explanation for the discrepancy.

Model Validation

Recall that the membrane is a sheet of PET with a very thin layer of aluminum deposited on it. Although I described a method of finding a Young's modulus for the entire aluminized membrane, this result comes from textbook PET properties and an assumed aluminum layer thickness of $0.5\mu\text{m}$. Without much confidence in these material properties, validation of the material aspect of the model would be useful. This would involve testing a sample of the membrane material in a tensile testing machine.

The experiment would be as follows: Cut out a small rectangular sample of membrane material and fix it to the grips of a tensile testing machine. Jog the machine such that the sample is taut, but with as little pretension as possible. The material is thin and compliant enough that strain gauges should not be used to measure strain in the tensile tests to follow—they would contribute non-negligible stiffness to the samples. Instead, the testing machine's built-in displacement measurement capabilities should be used. Perform standard tensile strength tests, stretching the material and recording displacement and load cell readings before the samples break. True strain can be calculated from the samples' original lengths and their displacements throughout the test using $\epsilon = \ln(L_i/L_0)$, where L_0 is the sample's undeformed length and $L_i = d + L_0$ is its stretched length throughout the test, d being the measured displacement. Axial engineering stress can be calculated from the known sample thickness t , width w , and measured force F using $s = F/(tw)$, but then must be converted to true stress using $\sigma = s(1 + \epsilon)$. Plotting the resulting engineering stress-strain curve is expected to show a linear elastic region, the slope of which is the calibrated Young's modulus that should be loaded into the FE simulation (after several trials and statistical analysis).

Uncertainty Quantification

It was already discussed in [Model Validation](#) that the membrane material properties are uncertain at this time. Another source of uncertainty is the pretension applied to the membrane. This pretension is the force being applied to the mirror's edge parallel to the membrane's undeformed plane.

Experimentation with variations of the membrane model in Abaqus revealed that the pretension does significantly affect the membrane's deflection for the same differential pressure, but this is not something that I had time to explore or simulate in detail. As such, the results presented here assume no pretension (i.e., the undeformed membrane is barely taut over the pressure vessel).

With more time, I would simulate the effect of pretension with a new step in the FE model. Part of the complexity of considering pretension is deciding exactly what magnitude of force to use. The extra simulation step I propose would simulate the manufacturing process of clamping the membrane over the pressure vessel, stretching it slightly in the act of clamping its edge over a lip, for example. This step would occur before the vacuum pressure loading step.

Results

Newton-Raphson root finding
implemented via Python script
determined that the membrane mirror
could be deformed to have a 1200mm
focal length when a differential
pressure of 70.240096Pa is applied.

The Python code used to accomplish

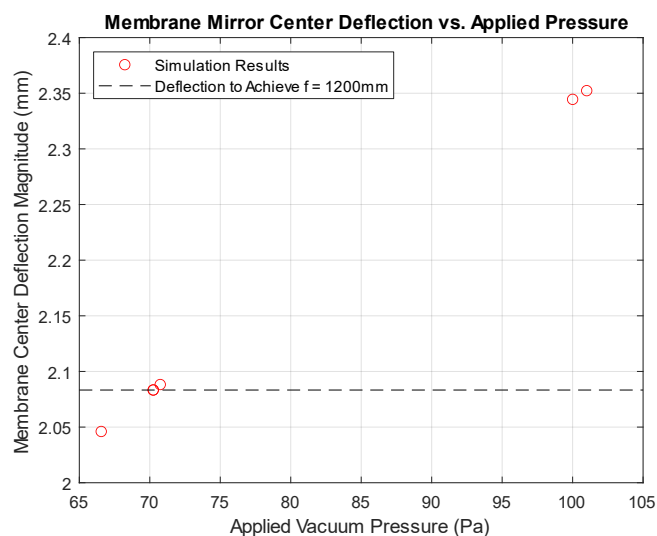


Figure 12. Newton-Raphson root finding implemented via Python script in Abaqus determined that 70.240096Pa of differential pressure yields a 1200mm focal length mirror for an initial guess of 100Pa.

this is available in [Appendix A: Abaqus Python Script](#). Progression of the automated root finding code is illustrated in Figure 12. The initial pressure guess was 100Pa, and five Newton-Raphson iterations were completed before converging to the target focal length.

Figure 13 shows a partially revolved rendering of the deformed axisymmetric profile at the target focal length, with displacements magnified by ten times. In [Discussion](#), the shape of this profile is compared to a perfect parabola and a spherical surface to judge this mirror's quality.

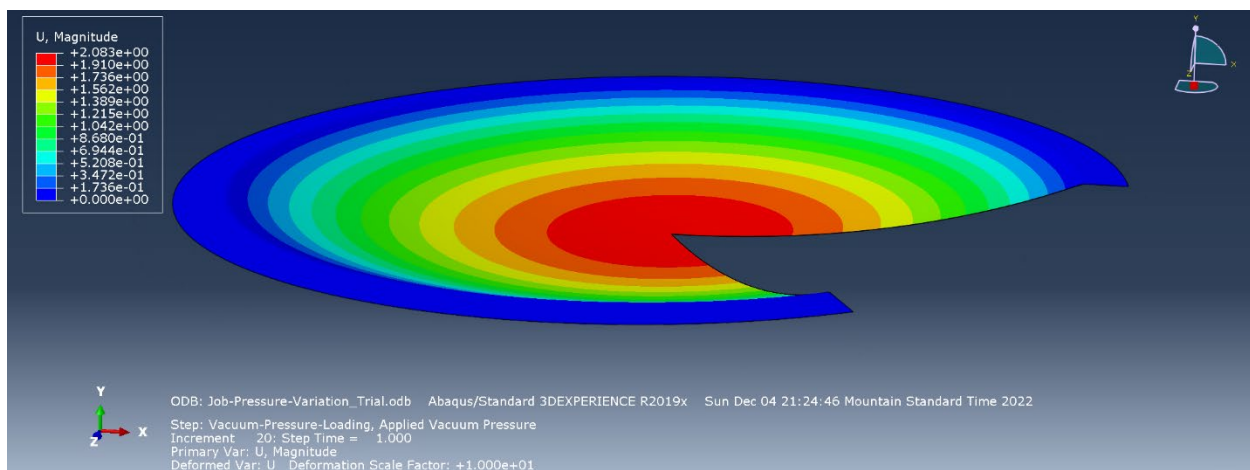


Figure 13. The axisymmetric profile is partially swept about the axis of symmetry to show the mirror's 3D shape after a vacuum pressure is used to deform it. The central deflection of 2.083mm gives this mirror a focal length of 1200mm, which was the design goal. The contour plot shows the magnitude of displacements, which are exaggerated by a factor of ten here.

Visually inspecting the surface of the membrane mirror compared to a spherical and parabolic mirror designed to have the same focal length shows some very clear and discouraging deviations. The parabolic, spherical, and membrane mirror profiles are plotted in Figure 14.

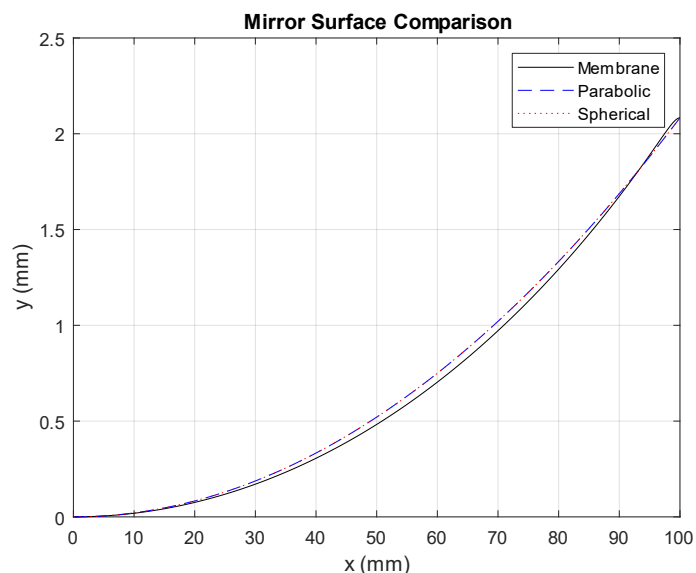


Figure 14. The membrane mirror surface very clearly deviates from both the parabolic and spherical mirrors, especially at the outer edge and midway between the center and edge.

Discussion

There were two questions that the simulation discussed in this report aims to answer. Firstly, what differential pressure is needed to deform the 0.0508mm thick, 200mm diameter membrane such that it has a 1200mm focal length. Secondly, what is the quality of the 1200mm focal length mirror (in terms of optical aberrations present) compared to a perfect parabolic mirror.

Pressure to Achieve a 1200mm Focal Length

It was determined via automated Newton-Raphson root finding in Abaqus that the mirror is deformed to have a 1200mm focal length when 70.240096Pa. This pressure acting over the mirror's area is equivalent to the weight of a 0.22kg mass. 70Pa is probably a good starting point for attempting to build this mirror, since the exact pressure applied to this mirror in the real world would need to change with the fluctuating atmospheric pressure.

Suitability as a Newtonian Telescope Primary Mirror

In [Background and Introduction](#), spherical and parabolic mirrors were mentioned as two standard shapes used as Newtonian telescope mirrors. Idealized parabolic mirrors are perfect because they focus parallel rays of light to a single point—they have no optical aberration. Spherical mirrors are sometimes suitable for amateur astronomers because they are cheaper, but they exhibit spherical aberration that corresponds to blurriness in images produced by them.

Optical aberration refers to the degree of inconsistency of where light reflected from the mirror intersects the mirror's optical axis. In a parabolic, 1200mm focal length mirror, all rays of light will intersect the optical axis 1200mm from the mirror surface. There will be a certain distribution around 1200mm for the spherical mirror, which I will call the focal distribution of the spherical mirror. The goal here is to compare the membrane mirror focal distribution to that

of the parabolic and spherical mirrors. If this focal distribution is less concentrated than the parabolic mirror, but more concentrated than the spherical mirror, then we can deem the membrane mirror to be of an acceptable shape. But if the membrane mirror has a wider focal distribution than the spherical mirror, then it means it has more optical aberrations than even a spherical mirror and is thus unacceptable. Figures 15 and 16 below show these distributions.

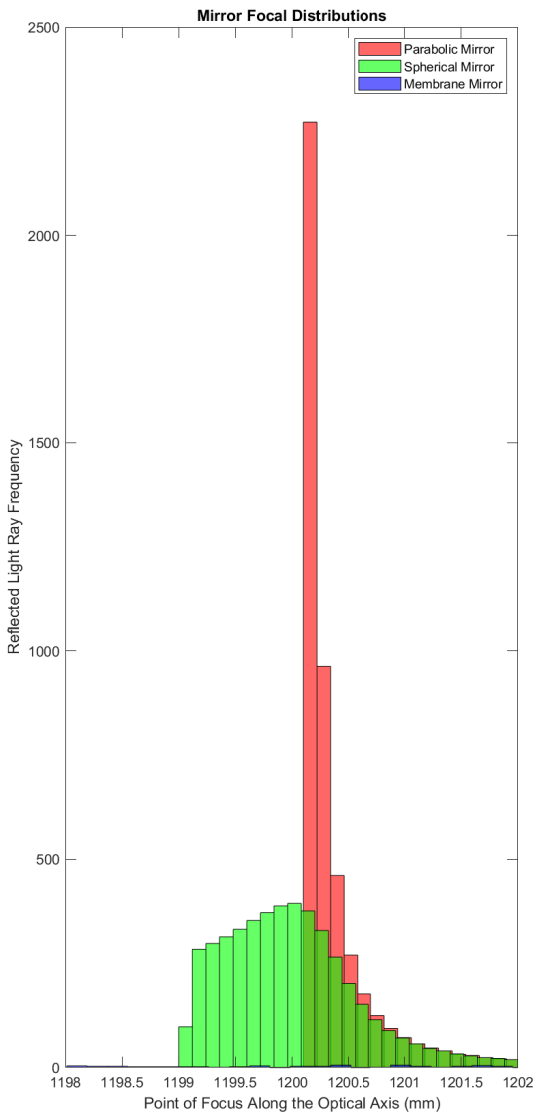


Figure 15. A parabolic mirror has a very tight focal distribution, and the spherical mirror has a wider distribution due to its spherical aberration. The membrane mirror focal distribution is so wide as to be barely distinguishable on this histogram.

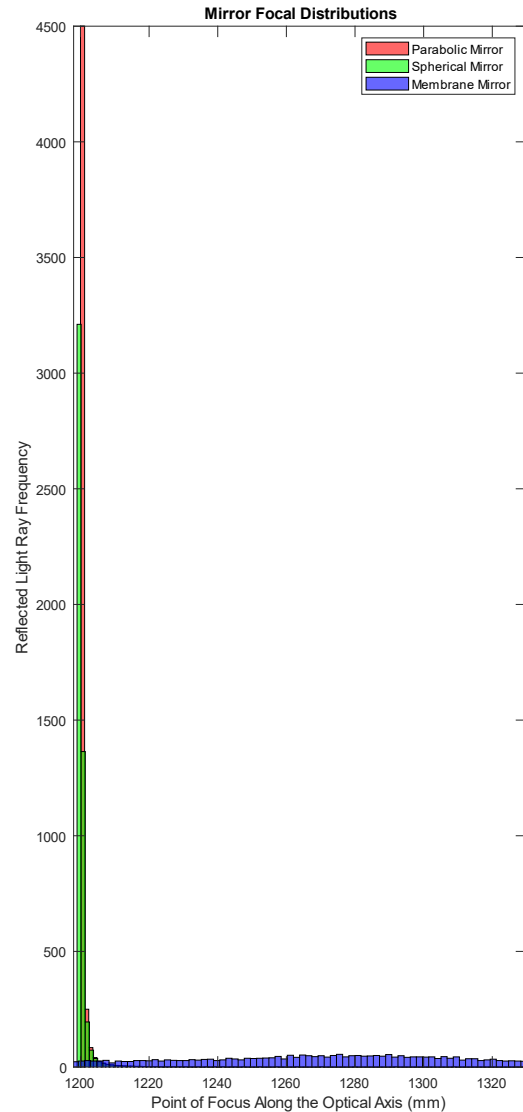


Figure 16. The range of focal distances is increased in this plot to show the focal distribution of the membrane mirror. It is much, much wider than that of the spherical mirror, indicating the presence of much, much more optical aberration.

The conclusion that can be drawn from Figures 15 and 16 is that the deformed membrane is an unacceptable shape for use in a Newtonian telescope. It has much more optical aberration than even a spherical mirror and would not be suitable for that reason, even for an amateur astronomer. The MATLAB code and derivation used to perform this analysis is available in [Appendix C: Applying Snell's Law to Judge Mirror Quality](#).

There are several other factors besides the surface shape that contribute to a good telescope mirror. This analysis does not consider the effects of the mirror surface's surface finish, for example. The need for a control system to maintain mirror pressure and react to changes in atmospheric pressure is yet another challenge. What's more, the applied pressure may need to be ever-increasing to account for creep in the membrane material. With all of this, the membrane mirror's shape is extremely problematic and would come with lots of optical aberration. So overall, this is not a feasible way to make a good telescope mirror.

Areas for Improvement

A weakness of this analysis is the fact that membrane pretension is not considered. A proposal for studying pretension's effect on the mirror's shape is detailed in [Uncertainty Quantification](#), but this is beyond the project scope at this time. Further work could also consider the implications of creep in holding the mirror's shape for long periods of time or exploring its susceptibility to problematic vibrations due to its lower mass than a traditional mirror.

Sources

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Appendix A: Abaqus Python Script

The following Python script was executed in Abaqus to find the differential pressure needed to achieve a 1200mm focal length with the membrane mirror. It accomplishes this task with a Newton-Raphson root finding scheme. The root finding problem is posed as

$$w_0(P) - w_{desired} = 0,$$

where $w_0(P)$ represents the central deflection output from the FE simulation for a given pressure P and $w_{desired}$ is the central deflection corresponding to a 1200mm focal length in the 200mm diameter mirror being studied. $w_{desired} = 2.0833\text{mm}$ is found in [Appendix B: Center](#)

Deflection to Achieve a 1200mm Focal Length.

```
from abaqus import *
from abaqusConstants import *
import __main__
import math
import section
import regionToolset
import displayGroupMdbToolset as dgm
import part
import material
import assembly
import step
import interaction
import load
import mesh
import optimization
import job
import sketch
import visualization
import xyPlot
import displayGroupOdbToolset as dgo
import connectorBehavior
import numpy as np
import os
import time

# This function handles copying and modifying a model with a
# specific vacuum pressure. It then submits a job for that
# model, extracts the relevant data, and deletes the model and
# job to avoid cluttering Abaqus' user interface. Although the
# model and job are deleted, the output database file is not, so
# data is not lost.
def simulate_deflection(pressure_Pa):

    # Convert the vacuum pressure in Pa to vacuum pressure in simulation units,
    # which is N/mm^2:
    square_m_to_square_mm = pow(1000, 2)
    vacuum_pressure_sim = pressure_Pa/square_m_to_square_mm;

    # Copy the base model. Rather than build a model from scratch in code,
    # This allows easily resimulating the pressure variations if loads, boundary
```

```

# conditions, etc. need to change about the model. This model used as a base
# is base_model.
variation_model_name = 'Model-Pressure-Variation-Trial'
base_model_name = 'Model-1'
mdb.Model(name=variation_model_name, objectToCopy=mdb.models[base_model_name])
a = mdb.models[variation_model_name].rootAssembly

# Change the vacuum pressure to the current variation to be tested:
mdb.models[variation_model_name].loads['Vacuum-Pressure-Load'].setValues(
    magnitude=vacuum_pressure_sim)

# Create a new job corresponding to this model capturing the current variation
# in vacuum pressure:
variation_job_name = 'Job-Pressure-Variation_Trial'
current_job = mdb.Job(name=variation_job_name, model=variation_model_name, description='',
    type=ANALYSIS, atTime=None, waitMinutes=0, waitHours=0, queue=None,
    memory=90, memoryUnits=PERCENTAGE, getMemoryFromAnalysis=True,
    explicitPrecision=SINGLE, nodalOutputPrecision=SINGLE, echoPrint=OFF,
    modelPrint=OFF, contactPrint=OFF, historyPrint=OFF, userSubroutine='',
    scratch='', resultsFormat=ODB, multiprocessingMode=DEFAULT, numCpus=1,
    numGPUs=0)

# Submit the job and then wait for it to finish:
current_job.submit(consistencyChecking=OFF)
current_job.waitForCompletion()

# Given a pressure in Pa for labeling and categorizing purposes, this function
# writes to an rpt file directly from abaqus containing a table of all nodes'
# x coordinates (membrane upper surface) and their U2 deflections.
def write_deflection(pressure_Pa):

    # Define the names of various elements of the simulation result/output
    # directories that are used to resolve the appropriate data:
    variation_job_name = 'Job-Pressure-Variation_Trial'
    odb_directory = 'X:/win_desktop/FEA Lab/Final Project/%s.odb' % variation_job_name
    instance_name = 'MIRROR-MEMBRANE-PART-1' # Will always be this.
    path_name = 'Membrane-Surface-Path'
    name = '%.6f' % pressure_Pa # The output data is named after the differential pressure.
    name = name.replace('.', '_')
    filename = '%s.rpt' % name
    data_name = 'Membrane-U2-Deflection-%s' % name

    # Open the output database for the simulation just ran. The job and model
    # names are always the same. The viewport changes seem to be necessary to
    # avoid VisError exceptions.
    session.viewports['Viewport: 1'].view.setValues(nearPlane=0.255994,
        farPlane=0.36626, width=0.00127647, height=0.000604286,
        viewOffsetX=-0.0448981, viewOffsetY=5.75847e-05)
    a = mdb.models['Model-1'].rootAssembly
    session.viewports['Viewport: 1'].setValues(displayedObject=a)
    o3 = session.openOdb(name=odb_directory)
    session.viewports['Viewport: 1'].setValues(displayedObject=o3)
    session.viewports['Viewport: 1'].makeCurrent()
    a = mdb.models['Model-1'].rootAssembly
    session.viewports['Viewport: 1'].setValues(displayedObject=a)
    session.mdbData.summary()
    session.viewports['Viewport: 1'].setValues(
        displayedObject=session.odbs[odb_directory])
    session.viewports['Viewport: 1'].view.setValues(nearPlane=256.059,
        farPlane=366.195, width=0.654404, height=0.309798, viewOffsetX=45.1057,
        viewOffsetY=0.0496219)

    # Set up a path on the mesh that is the entire upper edge of the membrane.
    # This will allow resolving the deflection of the mirrored surface as a
    # function of radius.
    session.Path(name=path_name, type=EDGE_LIST, expression=((
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[illegible]

[illegible]

[illegible]

[illegible]

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# Create XY data in table format where the first column is the x coordinate
# of the sample points and the second column is the vertical deflection.
session.viewports['Viewport: 1'].odbDisplay.setPrimaryVariable(
    variableLabel='U', outputPosition=NODAL, refinement=(COMPONENT, 'U2'))
pth = session.paths[path_name]
session.XYDataFromPath(name=data_name, path=pth,
    includeIntersections=False, projectOntoMesh=False,
    pathStyle=PATH_POINTS, numIntervals=10, projectionTolerance=0,
    shape=DEFORMED, labelType=X_COORDINATE, removeDuplicateXYPairs=True,
    includeAllElements=False)

# Write the XY data just created to a file in the pressure_variation_output
# folder. The output_file is named according to pressures tested.
output_file = 'pressure_variation_output/' + filename
x0 = session.xyDataObjects[data_name]
session.xyReportOptions.setValues(layout=SINGLE_TABLE)
session.writeXYReport(fileName=output_file, xyData=(x0, ))

# Given a pressure in Pa used to find the correct output file produced by
# write_output, this function opens the RPT file corresponding to the pressure
# variation test at pressure_Pa, finds the maximum deflection at the center
# of the membrane, and then appends this maximum deflection along with the
# applied pressure to a separate CSV file, max_deflection.csv.
def append_max_deflection(pressure_Pa):

    # Assemble the XY data filename (produced in write_output) to get maximum
    # deflection data from given the pressure passed as an argument. Then read
    # that file into a matrix.
    name = '%.6f' % pressure_Pa
    name = name.replace('.', '_')
    raw_filename = 'pressure_variation_output/%s.rpt' % name
    file_in = np.loadtxt(raw_filename, skiprows=4) # The first 4 rows are headers.

    # Find the maximum value of the 2nd column. This is the maximum deflection of
    # this membrane (occurring at the center of the mirror):
    vertical_deflection = file_in[:, 1]
    max_deflection = np.max(np.abs(vertical_deflection))

    # Append the maximum deflection to a separate file where the first column is
    # the applied pressure in Pa and the second column is the max deflection:
    out_filename = 'pressure_variation_output/max_deflection.csv'
    new_row = np.array([[pressure_Pa, max_deflection]])

    # Append new_row containing the pressure and max deflection of the latest
    # simulation to the file at out_filename. It is possible that the file does not
    # exist yet, so this first conditional will create it if it does not.
    if not os.path.exists(out_filename):
        open(out_filename, 'w')
        appended_file_contents = new_row
    else:
        current_file = np.loadtxt(out_filename, delimiter=',')

    # Now check that the file being read in was read in with rows still being

```

```

        # rows. If the CSV file only has one row, that row will be read in as a
        # column vector (Checked here by looking for anything other than a 2D
        # array) unless the following is done:
        if current_file.ndim != 2:
            current_file = np.array([current_file])
        appended_file_contents = np.concatenate((current_file, new_row), axis=0)
        np.savetxt(out_filename, appended_file_contents, delimiter=',')
        return max_deflection

# Delete all files in the pressure_variation_output directory in preparation
# for a new test.
def delete_all_output_files():
    output_dir = 'pressure_variation_output'
    all_output_files = [file for file in os.listdir('pressure_variation_output')]
    for file in all_output_files:
        os.remove(os.path.join(output_dir, file))

# This function involves all of the previous function definitions to simulate
# and log results for a membrane mirror subjected to the differential pressure
# passed as a parameter. It also times this entire process and provides some
# console output that appears in Abaqus to help monitor a series of
# simulations.
def full_mirror_pressure_test(pressure_Pa):
    start_time = time.time()

    print('\nStarting a pressure variation test with P = %.6fPa' % pressure_Pa)
    simulate_deflection(pressure_Pa);

    file_name = '%.6f' % pressure_Pa
    file_name = file_name.replace('.', '_')
    print('Writing mirror surface deflection to %s.rpt.' % file_name)
    write_deflection(pressure_Pa);

    print('Resolving maximum deflection at the mirrors center and appending to
max_deflection.csv.')
    max_deflection = append_max_deflection(pressure_Pa);

    end_time = time.time();
    test_duration = end_time-start_time;
    print('Finished test.      t = %ds      P = %.6fPa      w0 = %.6fmm' % (test_duration,
        pressure_Pa, max_deflection))

    return max_deflection

# *****
# * CONTROL STRUCTURE *
# *****
#
# Using the function definitions above, a Newton-Raphson root finding scheme
# is implemented below to find the differential pressure needed to achieve
# a mirror with a 1200mm focal length, corresponding to a central deflection
# of 2.0833mm.
#
# The root finding problem being solved is
#
#  $w_0(P) - \text{target\_}w_0 = 0 = f(P)$ ,
#
# where target_w0 is the target center deflection and w0(P) is the simulated center
# deflection for a certain differential pressure P. The function f(P) is found in
# the Newton-Raphson implementation below to refer to the difference in current
# center deflection from the target value.

target_w0 = 2.0833 # (mm) Yields a focal length of 1200mm.
guess_P = 100; # (Pa) Initial pressure guess to achieve the target deflection.
dP = 1; # (Pa) The is the initial change in pressure to estimate a derivative.
err_tolerance = 1e-5; # allowable relative error.

# Don't attempt to overwrite existing output files, just delete everything:

```

```

delete_all_output_files()

# Evaluate the initial step before proceeding with Newton-Raphson:
nr_start_time = time.time()
last_P = guess_P;
P = guess_P+dP;
last_w0 = full_mirror_pressure_test(guess_P)
last_f = last_w0-target_w0

err = 1
iters = 0
while err > err_tolerance:
    w0 = full_mirror_pressure_test(P)
    f = w0-target_w0

    # Calculate the next pressure to test.  $P = \text{last\_P} - f(\text{last\_P})/f'(\text{last\_P})$ .
    f_deriv = (f-last_f)/(P-last_P)
    last_P = P
    last_w0 = w0
    last_f = last_w0-target_w0
    P = last_P - f/f_deriv

    # Evaluate the current absolute error:
    err = np.abs(f)
    iters += 1
    print('Absolute Relative Error: %.6f' % err)
    print('Num Newton-Raphson Iterations: %d' % iters)

nr_end_time = time.time()
runtime = nr_end_time-nr_start_time
print('\nMirror center deflection has converged to the target at P = %.6fPa' %last_P)
print('    after %d Newton-Raphson iterations and %ds of runtime.' %(iters, runtime))

```

Appendix B: Center Deflection to Achieve a 1200mm Focal Length

The following MATLAB script is used to find the center deflection of a paraboloid with a 1200mm focal length.

```
% Stefen Gill
% ME EN 6510
%
% This script finds the target center deflection to achieve a focal length
% of 1200mm.

f = 1200; % (mm)
edge_radius = 100; % (mm)

% The equation 4*f*y = x^2 describes a parabola with focal length f. This
% is evaluated to solve for the mirror's center deflection, which will be
% equivalent to the y value at the mirror's edge evaluated with this
% formula (at 100mm radius).
y = @(x) (1/(4*f))*x.^2
w0 = y(100)
```

Appendix C: Applying Snell's Law to Judge Mirror Quality

Snell's law describes how rays of light reflect off of a surface, and says that, when there is no change in refractive index present (as in the case of a telescope mirror), the incident and reflected angles of a light ray are equal.

The goal for a telescope mirror is to reflect parallel rays of light from a distant source to a single point. To judge the quality of the membrane mirror, I simulate the reflection of such rays of light using MATLAB and compare the distribution of the focus point for a parabolic mirror, a spherical mirror, and the membrane mirror. Please see [Suitability as a Newtonian Telescope Primary Mirror](#) for more another explanation of this analysis.

The MATLAB script provided in this appendix applies Snell's law to find the distance along the optical axis where reflected rays of light intersect (with the optical axis). What follows is a derivation of this scheme.

The angle ϕ is the slope of

the mirror with respect to the horizontal, and determines the incident angle θ . It is found easily using an inverse tangent and a change in the y coordinate with respect to the x of the mirror surface:

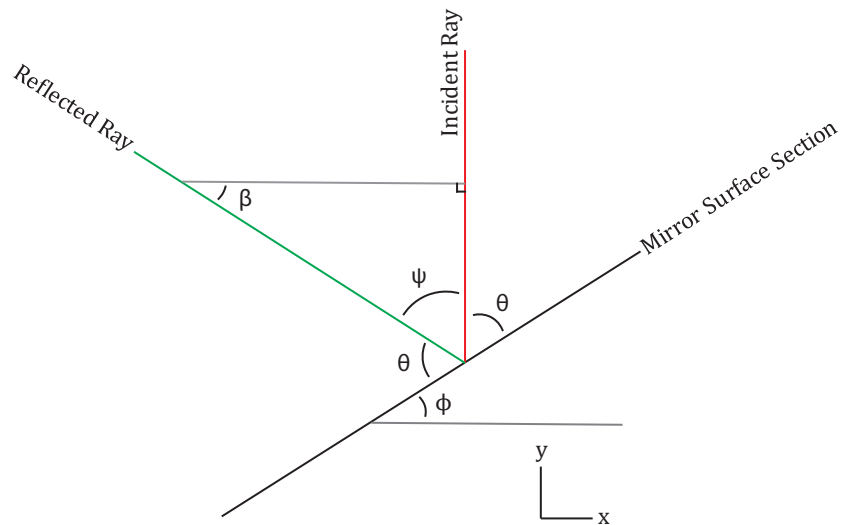


Figure 17. This is the diagram that pairs with the derivation of the focal distance from a mirror's surface. ϕ is the angle of the mirror surface's section from the horizontal, θ is both the incident and reflected angle of a ray of light (due to Snell's law), ψ is the angle between those rays of light, and β is the angle from the horizontal to the reflected light ray. Where the light ray intersects the optical axis (y axis) is the focal distance, which is also the y intercept of the equation for the reflected ray.

$$\phi = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right).$$

And the incident angle θ is simply ϕ subtracted from $\pi/2$, since the incident light ray is parallel to the optical axis (at $\pi/2$ rad from a horizontal surface).

$$\theta = \frac{\pi}{2} - \phi$$

If we zoom closely enough into the mirror surface, it appears flat. At this point, we can find ψ , the angle between the incident and reflected light rays, from:

$$\psi = \pi - 2\theta$$

since there are π radians in a straight line. The right triangle in Figure 17 now allows finding β :

$$\beta = \frac{\pi}{2} - \psi.$$

The equation of the reflected ray of light is simply that of a line with slope $-\tan(\beta)$:

$$y(x) = -x \tan(\beta) + y_f,$$

where y_f is the distance from the mirror's surface to the reflected ray's intersection with the optical axis and x and $y(x)$ are the mirror surface's x and y coordinates, which are known. This equation is easily rearranged to solve for y_f , which is the desired quantity.

For a parabolic mirror, all rays should intersect at the same y_f because it has no optical aberration. Plotting the distribution of y_f for a parabolic mirror actually showed a non-point distribution that I attribute to the estimate of the mirror slope used to calculate ϕ . The code used to compare the spherical, parabolic, and membrane mirrors (and generate Figures 15 and 16) is provided on the following pages.

```

% Stefen Gill
% ME EN 6510
%
% This script analyses the surface of the membrane mirror, a parabolic
% mirror, and a spherical mirror, and then makes plots to show how they
% compare in terms of aberrations present. The parabolic mirror is meant as
% a reference to an absolutely perfect mirror. The spherical mirror is
% acceptable for some amateur astronomers, but anything more extreme than
% the spherical aberration exhibited by this mirror is unacceptable. For
% this reason, the membrane mirror will be deemed of an acceptable shape if
% its distribution of reflected light destinations on the optical axis lies
% between those of the parabolic and spherical mirrors.
close all

% Start by loading the membrane mirror data:
w0 = 2.0833; % Center deflection of the membrane/parabolic mirror to achieve 1200mm focal length.
f6_mirror_profile_data = readmatrix('pressure_variation_output\70_240096.rpt', ...
    'FileType', 'text');
f6_mirror_profile_data = sortrows(f6_mirror_profile_data, 1);
x = f6_mirror_profile_data(:, 1);
y_membrane = f6_mirror_profile_data(:, 2) + w0;

% Trim the data down to not include the clamped part of the mirror (up to
% the 100mm radius mark):
mirror_radius = 100; % (mm)
edge_index = find(x == mirror_radius);
x = x(1:edge_index);
y_membrane = y_membrane(1:edge_index);

% Create the parabolic mirror. The x coordinates for all three mirrors
% considered will be the same.
f = 1200; % (mm) Focal length.
y_parabola_func = @(x) (1/(4*f))*x.^2;
y_parabola = y_parabola_func(x);

% Create the spherical mirror. The focal length of a spherical mirror is
% half its radius. So the sphere's radius must be 2*f to make a mirror that
% can be compared to the other two.
R = 2*f; % (mm) Spherical mirror radius.
y_sphere_func = @(x) -sqrt(R^2 - x.^2) + 2*f;
y_sphere = y_sphere_func(x);

% Plot the three mirror surfaces together as a sanity check:
plot(x, y_membrane, 'Color', 'k', 'DisplayName', 'Membrane')
hold on
plot(x, y_parabola, 'Color', 'b', 'DisplayName', 'Parabolic', ...
    'LineStyle', '--')
plot(x, y_sphere, 'Color', 'r', 'DisplayName', 'Spherical', ...
    'LineStyle', ':')
xlabel('x (mm)')
ylabel('y (mm)')
title('Mirror Surface Comparison')
grid
legend
saveas(gcf, 'mirror_shapes.svg')

% Calculate the distribution of light reflected onto the optical axis for
% each mirror, this can be used characterize the aberrations of each
% mirror.
y_membrane_reflected = ray_trace_mirror_surface(x, y_membrane);
y_parabola_reflected = ray_trace_mirror_surface(x, y_parabola);
y_sphere_reflected = ray_trace_mirror_surface(x, y_sphere);

% The strange membrane mirror shape causes several reflected rays to end
% up on the order of 10^16 mm from the surface, and this causes problems
% when trying to plot a histogram. Since I will limit showing the histogram
% to within a few mm of the target focal length anyway, I am scrubbing
% those extremely high points from the data.
membrane_focal_points_are_within_2000 = y_membrane_reflected <= 2000;

```

```

y_membrane_reflected = y_membrane_reflected(membrane_focal_points_are_within_2000);

% Plot histograms showing the distribution of light that is reflected back
% to the optical axis. The tighter these distributions, the better. The
% parabolic mirror should be a perfect point at 1200mm.
figure
histogram(y_parabola_reflected, 10000, 'FaceColor', 'r', 'DisplayName', ...
    'Parabolic Mirror')
hold on
histogram(y_sphere_reflected, 10000, 'FaceColor', 'g', 'DisplayName', ...
    'Spherical Mirror')
histogram(y_membrane_reflected, 10000, 'FaceColor', 'b', 'DisplayName', ...
    'Membrane Mirror')
title('Mirror Focal Distributions')
xlabel('Point of Focus Along the Optical Axis (mm)')
ylabel('Reflected Light Ray Frequency')
xlim([1198, 1202])
legend
set(gcf, 'position', [10, 10, 550, 1200])
saveas(gcf, 'focal_dist_1.svg')

% Plot a second histogram that highlights how poor the membrane mirror
% shape is compared to the other two.
figure
histogram(y_parabola_reflected, 1000, 'FaceColor', 'r', 'DisplayName', ...
    'Parabolic Mirror')
hold on
histogram(y_sphere_reflected, 1000, 'FaceColor', 'g', 'DisplayName', ...
    'Spherical Mirror')
histogram(y_membrane_reflected, 1000, 'FaceColor', 'b', 'DisplayName', ...
    'Membrane Mirror')
title('Mirror Focal Distributions')
xlabel('Point of Focus Along the Optical Axis (mm)')
ylabel('Reflected Light Ray Frequency')
xlim([1198, 1330])
legend
set(gcf, 'position', [10, 10, 550, 1200])
saveas(gcf, 'focal_dist_2.svg')

% This function returns a distribution of y coordinates where reflected
% parallel rays of light end up on the optical axis, essentially applying
% Snell's law to many parallel rays of light approaching a mirror's
% surface. For a parabolic mirror, these y_reflected will be on the same
% place on the mirror, because parabolic mirrors have no optical
% aberration. For the spherical mirror, they will be spread out a bit due to
% spherical aberration. surface_x and surface_y are the x and y coordinates
% of the mirror's surface, passed as vectors.
function y_reflected = ray_trace_mirror_surface(surface_x, surface_y)
% The set of coordinates where parallel rays of light reflect off the
% mirror surface are simply the x and y coordinates of the mirror surface,
% excluding the point in the very center because the light will not be
% reflected at all (incidence angle of pi/2rad):
x_incidence = surface_x(2:end);
y_incidence = surface_y(2:end);

% The mirror surface's angle at each of these incidence points, phi, is
% necessary. It is calculated from a finite difference estimation of the
% surface's derivative with respect to x.
delta_x = diff(surface_x);
delta_y = diff(surface_y);
phi = atan(delta_y./delta_x);

% Using the mirror surface coordinates, solve for the y intercept of the
% equation describing the reflected ray of light from the mirror. Beta is
% the angle from the positive x axis to a reflected ray of light.
beta = pi/2 - 2*phi;
y_reflected = y_incidence + x_incidence.*tan(beta);
end

```