CONTENTS

CHAPTER 1

SETS

1.1 SETS

It is a well known fact that any attempt to define a set has always led mathematicians to unsurmountable difficulties. For example, suppose one defines the term set as "*a well defined collection of objects*". One may then ask what is meant by a collection. If one answers that a collection is an aggregate of objects or things. What is then an aggregate? Perhaps then one may define that an aggregate is a class of things. What is then a class? Now, one may define a class as a collection. In this manner question after question, since our language is finite, we find that after some time we will have to use some words which have already been questioned. The definition thus becomes circular and worthless. Thus, mathematicians realized that there must be some undefined (or primitive) terms. In this chapter, we start with two undefined (or primitive) terms — "element" and "set". We assume that the word "set" is synonymous with the words "collection", "aggregate", "class" and is comprised of elements. The words "element", "object", "member" are synonymous.

If a is an element of a set A, then we write $a \in A$ and say a belongs to A or a is in A or a is a member of A. If a does not belong to A, then we write $a \notin A$. It is assumed here that if A is any set and a is any element, then either $a \in A$ or $a \notin A$ and the two possibilities are mutually exclusive. Thus, one cannot say "consider the set A of some positive integers", because it is not sure whether $3 \in A$ or $3 \notin A$.

Throughout this chapter we shall denote sets by capital alphabets e.g. A, B, C, X, Y, Z etc. and the elements by the small alphabets e.g. a, b, c, x, y, z etc.

The following are some illustrations of sets:

ILLUSTRATION 1 *The collection of vowels in English alphabets. This set contains five elements, namely, a, e, i, o, u.*

ILLUSTRATION 2 *The collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.*

ILLUSTRATION 3 *The collection of all States in the Indian Union is a set.*

ILLUSTRATION 4 *The collection of past presidents of the Indian union is a set.*

ILLUSTRATION 5 *The collection of cricketers in the world who were out for 99 runs in a test match is a set.*

ILLUSTRATION 6 *The collection of good cricket players of India is not a set, since the term "good player is vague and it is not well defined".*

Similarly, collection of good teachers in a school is not a set. However, the collection of all teachers in a school is a set.

In this chapter we will have frequent interaction with some sets, so we reserve some letters for these sets as listed below:

N : for the set of natural numbers.
Z : for the set of integers.
Z^+ : for the set of all positive integers.
Q : for the set of all rational numbers.

Q^+ : for the set of all positive rational numbers.
R : for the set of all real numbers.
R^+ : for the set of all positive real numbers.
C : for the set of all complex numbers.

EXERCISE 1.1

1. What is the difference between a collection and a set? Give reasons to support your answer?
2. Which of the following collections are sets? Justify your answer:
   (i) A collection of all natural numbers less than 50.
   (ii) The collection of good hockey players in India.
   (iii) The collection of all girls in your class.
   (iv) The collection of most talented writers of India. [NCERT]
   (v) The collection of difficult topics in Mathematics.
   (vi) The collectiocollection of novels written by Munshi Prem Chand. [NCERT]
   (viii) The collection of all months of a year beginning with the letter J. [NCERT]
   (vii) A n of all questions in this chapter. [NCERT]
   (ix) A collection of most dangerous animals of the world. [NCERT]
   (x) The collection of prime integers.
3. If $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then insert the appropriate symbol $\in$ or $\notin$ in each of the following blank spaces:
   (i) $4 \ldots\ldots A$ (ii) $-4 \ldots\ldots A$ (iii) $12 \ldots\ldots A$
   (iv) $9 \ldots\ldots A$ (v) $0 \ldots\ldots A$ (vi) $-2 \ldots\ldots A$

ANSWERS

1. Every set is a collection but a collection is not necessarily a set. Only well defined collections are sets. For example, group of good cricket players is a collection but it is not a set.
2. (i), (iii), (vi), (vii), (viii), (x)
3. (i) $\in$ (ii) $\notin$ (iii) $\notin$ (iv) $\in$ (v) $\in$ (vi) $\notin$

HINTS TO NCERT & SELECTED PROBLEMS

2. (iv) The collection of most talented writers of India is not a set as there is no specific criterion to determine whether a writer is talented or not.
   (vi) The collection of all months of a year beginning with the letter J is a set given by {January, June July}.
   (vii) The collection of novels written by Munshi Prem Chand is a set because one can determine whether a novel is written by him or not.
   (viii) The collection of all questions in this chapter is a set because if a question is given one can easily decide whether it is a question of this chapter or not.
   (ix) The collection of most dangerous animals of the world is not a set because there is no criterion to determine whether an animal is most dangerous or not.

1.2 DESCRIPTION OF A SET

A set is often described in the following two forms. One can make use of any one of these two ways according to his (her) convenience.

(i) Roster form or Tabular form (ii) Set-builder form

Let us now discuss these forms.

1.2.1 ROSTER FORM

In this form a set is described by listing elements, separated by commas, within braces { }.

ILLUSTRATION 1 *The set of vowels of English Alphabet may be described as $\{a, e, i, o, u\}$.*

ILLUSTRATION 2 *The set of even natural numbers can be described as $\{2, 4, 6, \ldots\}$. Here the dots stand for 'and so on'.*

ILLUSTRATION 3 *If A is the set of all prime numbers less than 11, then $A = \{2, 3, 5, 7\}$.*

NOTE *The order in which the elements are written in a set makes no difference. Thus, $\{a, e, i, o, u\}$ and $\{e, a, i, o, u\}$ denote the same set. Also, the repetition of an element has no effect. For example, $\{1, 2, 3, 2\}$ is the same set as $\{1, 2, 3\}$.*

1.2.2 SET-BUILDER FORM

In this form, a set is described by a characterizing property $P(x)$ of its elements x. In such a case the set is described by $\{x : P(x) \text{ holds}\}$ or, $\{x \mid P(x) \text{ holds}\}$, which is read as 'the set of all x such that $P(x)$ holds'. The symbol '|' or ':' is read as 'such that'.

In other words, in order to describe a set, a variable x (say) (to denote each element of the set) is written inside the braces and then after putting a colon the common property $P(x)$ possessed by each element of the set is written within the braces.

ILLUSTRATION 4 *The set E of all even natural numbers can be written as*
$E = \{x : x \text{ is a natural number and } x = 2n \text{ for } n \in N\}$
or, $E = \{x : x \in N, x = 2n, n \in N\}$ or, $E = \{x \in N : x = 2n, n \in N\}$

ILLUSTRATION 5 *The set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ can be written as $A = \{x \in N : x \leq 8\}$.*

ILLUSTRATION 6 *The set of all real numbers greater than -1 and less than 1 can be described as $\{x \in R : -1 < x < 1\}$.*

ILLUSTRATION 7 *The set $A = \{0, 1, 4, 9, 16, \ldots\}$ can be written as $A = \{x^2 : \in Z\}$.*

ILLUSTRATIVE EXAMPLES

LEVEL-1

Type I ON DESCRIBING OR REPRESENTING SETS IN TABULAR FORM OR ROSTER FORM

EXAMPLE 1 *Describe the following sets in Roster form:*
(i) *The set of all letters in the word 'MATHEMATICS'*
(ii) *The set of all letters in the word 'ALGEBRA'*
(iii) *The set of all vowels in the word 'EQUATION'*
(iv) *The set of all natural numbers less than 7.*
(v) *The set of squares of integers.*

SOLUTION (i) We observe that distinct letters in the word 'MATHEMATICS' are:
M, A, T, H, E, I, C, S

Since the order in which the elements of a set are written is immaterial and the repetition of elements has no effect. So, required set can be described as follows:
$\{M, A, T, H, E, I, C, S\}$

(ii) We find that the word 'ALGEBRA' has following distinct letters: A, L, G, E, B, R

Hence, required set can be described in Roster form as follows: $\{A, L, G, E, B, R\}$

(iii) Clearly, word 'EQUATION' has following vowels: A, E, I, O, U

So, required set can be described as follows: $\{A, E, I, O, U\}$

(iv) Natural numbers less than 7 are: 1, 2, 3, 4, 5, 6.
Hence, required set can be described as follows: $\{1, 2, 3, 4, 5, 6\}$.

(v) Since square of a negative integer is same as the square of its absolute value. Therefore, squares of integers are 0, 1, 4, 9, 16, 25, ... Hence, required set is $\{0, 1, 4, 9, 16, ...\}$.

Type II ON DESCRIBING OR REPRESENTING SETS IN SET-BUILDER FORM

EXAMPLE 2 *Describe the following sets in set-builder form:*

(i) *The set of all letters in the word 'PROBABILITY'.*
(ii) *The set of reciprocals of natural numbers.* (iii) *The set of all odd natural numbers.*
(iv) *The set of all even natural numbers.*

SOLUTION (i) Given set in set-builder form can be described as follows:

$\{x : x$ is a letter in the word 'PROBABILITY'$\}$

(ii) Given set can be described in set-builder form as follows:

$\{x : x$ is reciprocal of a natural number$\}$ or, $\left\{x : x = \frac{1}{n}, \in N\right\}$ or, $\left\{\frac{1}{n} : \in N\right\}$

(iii) An odd natural number can be written in the form $(2n-1)$. So, given set can be described as follows $\{x : x = 2n-1, n \in N\}$ or, $\{2n-1 : n \in N\}$.

(iv) An even natural number can be written as $2n$, where $n \in N$. Therefore, set of all even natural numbers can be written in the form $\{x : x = 2n, n \in N\}$ or, $\{2n : n \in N\}$

EXAMPLE 3 *Write the set of all integers whose cube is an even integer.*

SOLUTION We know that the cube of an even integer is also an even integer. Hence, the required set is the set of all even integers which can also be written in the set-builder form as $\{2n : n \in Z\}$.

EXAMPLE 4 *Write the set of all real numbers which cannot be written as the quotient of two integers in the set-builder form.*

SOLUTION We know that all rational numbers are expressible as the quotient of two integers. Therefore, the required set is the set of all irrational numbers which can be written as

$\{x : x$ is real and irrational$\}$ or, $\{x : x \in R$ but $x \notin Q\}$.

Type III ON DESCRIBING A SET IN ROSTER FORM WHEN IT IS GIVEN IN SET-BUILDER FORM

EXAMPLE 5 *Describe each of the following sets in Roster form*

(i) $\{x : x$ *is a positive integer and a divisor of* $9\}$ (ii) $\{x : x \in Z$ *and* $|x| \le 2\}$

(iii) $\{x : x$ *is a letter of the word 'PROPORTION'*$\}$ (iv) $\left\{x : x = \frac{n}{n^2+1}\right.$ *and* $1 \le n \le 3$, *where* $\left. n \in N\right\}$

SOLUTION (i) Since x is a positive integer and a divisor of 9. So, x can take values 1, 3, 9.

$\therefore$ $\{x : x$ is a positive integer and a divisor of $9\} = \{1, 3, 9\}$

(ii) We find that x is an integer satisfying $|x| \le 2$.
and, $|x| = 0, 1, 2 \Rightarrow x = 0, \pm 1, \pm 2$
So, x can take values $-2, -1, 0, 1, 2$.
$\therefore$ $\{x : x \in Z$ and $|x| \le 2\} = \{-2, -1, 0, 1, 2\}$

(iii) We find that distinct letters in the word 'PROPORTION' are P, R, O, T, N, I. So, x can be P, R, O, T, I, N.

Hence, $\{x : x$ is a letter in the word 'PROPORTION'$\} = \{P, R, O, T, I, N\}$

(iv) We have,

$$x = \frac{n}{n^2+1} \text{ where } n \in N \text{ and } 1 \le n \le 3.$$

$$\therefore \quad x = \frac{n}{n^2+1}, \text{ where } n = 1, 2, 3.$$

$$\Rightarrow \quad x = \frac{1}{1^2+1}, \frac{2}{2^2+1}, \frac{3}{3^2+1}$$

$$\Rightarrow \quad x = \frac{1}{2}, \frac{2}{5}, \frac{3}{10}$$

Hence, $\left\{x : x = \frac{n}{n^2+1} \text{ and } 1 \le n \le 3, \text{ where } n \in N\right\} = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}\right\}$

EXAMPLE 6 *Write the set of all vowels in English alphabet which precede s.*

SOLUTION The vowels in English alphabet which precede s are a, e, i, o. So, the set $A = \{a, e, i, o\}$ is the set of all vowels in English alphabet which precede s.

EXAMPLE 7 *Write the set* $A = \{x : x \in Z, x^2 < 20\}$ *in the roster form.*

SOLUTION We observe that the integers whose squares are less than 20 are: $0, \pm 1, \pm 2, \pm 3, \pm 4$. Therefore, the set A in roster form is $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$.

EXAMPLE 8 *Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form.* [NCERT]

(i) $\{P, R, I, N, C, A, L\}$	(a) $\{x : x$ *is a positive integer and is a divisor of* $18\}$
(ii) $\{0\}$	(b) $\{x : x$ *is an integer and* $x^2 - 9 = 0\}$
(iii) $\{1, 2, 3, 6, 9, 18\}$	(c) $\{x : x$ *is an integer and* $x + 1 = 1\}$
(iv) $\{-3, 3\}$	(d) $\{x : x$ *is a letter of the word 'PRINCIPAL'*$\}$

SOLUTION (i) Clearly, $\{P, R, I, N, C, A, L\} = \{P, R, I, N, C, I, P, A, L\}$
$= \{x : x$ is a letter of the word 'PRINCIPAL'$\}$

Hence, (i) matches with (d).

(ii) $\{0\} = \{x : x$ is an integer equal to zero$\} = \{x : x$ is an integer and $x + 1 = 1\}$
Hence, (ii) matches with (c).

(iii) $\{1, 2, 3, 6, 9, 18\}$ = Set of all positive divisors of 18
$= \{x : x$ is a positive integer and is a divisor of $18\}$
Hence, (iii) matches with (a).

(iv) Clearly, $\{-3, 3\} = \{x : x$ is an integer and $x^2 - 9 = 0\}$. Hence, (iv) matches with (b).

Type IV ON DESCRIBING A SET IN ROSTER FORM WHEN IT IS GIVEN IN SET-BUILDER FORM

EXAMPLE 9 *Write the set* $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}\right\}$ *in the set-builder form.* [NCERT]

SOLUTION We observe that each element in the given set has the denominator one more than the numerator. Also, the numerator begins from 1 and do not exceed 9. Hence, in the set-builder form the given set can be written as $\left\{x : x = \frac{n}{n+1}, n \in N, n \le 9\right\}$.

EXAMPLE 10 *Write the set* $X = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, ...\right\}$ *in the set -builder form.*

SOLUTION We observe that the elements of set X are the reciprocals of the squares of all natural numbers. So, the set X in set builder form is $X = \left\{\frac{1}{n^2} : n \in N\right\}$.

EXAMPLE 11 *Write the following sets in Roster form:*

(i) $A = \{a_n : n \in N, a_{n+1} = 3a_n \text{ and } a_1 = 1\}$ (ii) $B = \{a_n : n \in N, a_{n+2} = a_{n+1} + a_n, a_1 = a_2 = 1\}$

SOLUTION (i) We have, $a_1 = 1$ and $a_{n+1} = 3a_n$ for all $n \in N$

Putting $n = 1$ in $a_{n+1} = 3a_n$, we get

$$a_2 = 3a_1 = 3 \times 1 = 3 \qquad [\because a_1 = 1]$$

Putting $n = 2$ in $a_{n+1} = 3a_n$, we get

$$a_3 = 3a_2 = 3 \times 3 = 3^2 \qquad [\because a_2 = 3]$$

Putting $n = 3$ in $a_{n+1} = 3a_n$, we get

$$a_4 = 3a_3 = 3 \times 3^2 = 3^3 \qquad [\because a_3 = 3]$$

Similarly, we obtain

$$a_5 = 3a_4 = 3 \times 3^3 = 3^4,\ a_6 = 3a_5 = 3 \times 3^4 = 3^5 \text{ and so on.}$$

Hence, $A = \{a_1, a_2, a_3, a_4, a_5, a_6, \ldots\} = \{1, 3, 3^2, 3^3, 3^4, 3^5, \ldots\ldots\}$

(ii) We have, $a_1 = 1, a_2 = 1$ and $a_{n+2} = a_{n+1} + a_n$.

Putting $n = 1, 2, 3, 4, \ldots\ldots$ in $a_{n+2} = a_{n+1} + a_n$, we get

$$a_3 = a_2 + a_1 = 1 + 1 = 2;\quad a_4 = a_3 + a_2 = 2 + 1 = 3;\ a_5 = a_4 + a_3 = 3 + 2 = 5;$$
$$a_6 = a_5 + a_4 = 5 + 3 = 8 \text{ and so on.}$$

Hence, $B = \{a_1, a_2, a_3, a_4, a_5, a_6, \ldots\ldots\} = \{1, 1, 2, 3, 5, 8, \ldots\ldots\}$

EXERCISE 1.2

LEVEL-1

1. Describe the following sets in Roster form:
   (i) $\{x : x$ is a letter before e in the English alphabet$\}$.
   (ii) $\{x \in N : x^2 < 25\}$.
   (iii) $\{x \in N : x$ is a prime number, $10 < x < 20\}$.
   (iv) $\{x \in N : x = 2n, n \in N\}$.
   (v) $\{x \in R : x > x\}$.
   (vi) $\{x : x$ is a prime number which is a divisor of $60\}$.
   (vii) $\{x : x$ is a two digit number such that the sum of its digits is $8\}$.
   (viii) The set of all letters in the word 'Trigonometry'.
   (ix) The set of all letters in the word 'Better'.
2. Describe the following sets in set-builder form:
   (i) $A = \{1, 2, 3, 4, 5, 6\}$ (ii) $B = \{1, 1/2, 1/3, 1/4, 1/4, \ldots\}$
   (iii) $C = \{0, 3, 6, 9, 12, \ldots\}$ (iv) $D = \{10, 11, 12, 13, 14, 15\}$
   (v) $E = \{0\}$ (vi) $\{1, 4, 9, 16, \ldots, 100\}$
   (vii) $\{2, 4, 6, 8, \ldots\ldots\}$ (viii) $\{5, 25, 125, 625\}$
3. List all the elements of the following sets:
   (i) $A = \{x : x^2 \le 10, x \in Z\}$ (ii) $B = \left\{x : x = \dfrac{1}{2n-1}, 1 \le n \le 5\right\}$
   (iii) $C = \left\{x : x \text{ is an integer}, -\dfrac{1}{2} < x < \dfrac{9}{2}\right\}$
   (iv) $D = \{x : x$ is a vowel in the word "EQUATION"$\}$
   (v) $E = \{x : x$ is a month of a year not having 31 days$\}$
   (vi) $F = \{x : x$ is a letter of the word "MISSISSIPPI"$\}$

4. Match each of the sets on the left in the roster form with the same set on the right described in the set-builder form:

(i) $\{A, P, L, E\}$	(i) $\{x : x + 5 = 5, x \in Z\}$
(ii) $\{5, -5\}$	(ii) $\{x : x$ is a prime natural number and a divisor of $10\}$
(iii) $\{0\}$	(iii) $\{x : x$ is a letter of the word "RAJASTHAN"$\}$
(iv) $\{1, 2, 5, 10\}$	(iv) $\{x : x$ is a natural number and divisor of $10\}$
(v) $\{A, H, J, R, S, T, N\}$	(v) $\{x : x^2 - 25 = 0\}$
(vi) $\{2, 5\}$	(vi) $\{x : x$ is a letter of the word "APPLE"$\}$

5. Write the set of all vowels in the English alphabet which precede q.
6. Write the set of all positive integers whose cube is odd.
7. Write the set $\left\{\dfrac{1}{2}, \dfrac{2}{5}, \dfrac{3}{10}, \dfrac{4}{17}, \dfrac{5}{26}, \dfrac{6}{37}, \dfrac{7}{50}\right\}$ in the set-builder form.

ANSWERS

1. (i) $\{a, b, c, d\}$ (ii) $\{1, 2, 3, 4\}$ (iii) $\{11, 13, 17, 19\}$ (iv) $\{2, 4, 6, 8, \ldots\}$
   (v) ϕ (vi) $\{2, 3, 5\}$ (vii) $\{17, 26, 35, 44, 53, 62, 71, 80\}$
   (viii) $\{T, R, I, G, O, N, M, E, Y\}$ (ix) $\{B, E, T, R\}$
2. (i) $\{x : x \in N, x < 7\}$ (ii) $\{x : x = 1/n, x \in N\}$ (iii) $\{x : x = 3n, n \in Z^+\}$
   (iv) $\{x : x \in N, 9 < x < 16\}$ (v) $\{x : x = 0\}$ (vi) $\{x^2 : x \in N, 1 \le x \le 10\}$
   (vii) $\{x : x = 2n, n \in N\}$ (viii) $\{5^n : n \in N, 1 \le n \le 4\}$
3. (i) $A = \{0, \pm 1, \pm 2, \pm 3\}$ (ii) $B = \left\{1, \dfrac{1}{3}, \dfrac{1}{5}, \dfrac{1}{7}, \dfrac{1}{9}\right\}$
   (iii) $C = \{0, 1, 2, 3, 4\}$ (iv) $D = \{A, E, I, O, U\}$
   (v) $E = \{$Feb., April, June, Sept., November$\}$ (vi) $F = \{M, I, S, P\}$
4. (i)→ (vi); (ii)→ (v); (iii) → (i); (iv) → (iv); (v) → (iii); (vi) → (ii)
5. $\{a, e, i, o\}$
6. $\{2n + 1 : n \in Z, n > 0\}$
7. $\left\{\dfrac{n}{n^2 + 1} : n \in N, n \le 7\right\}$

1.3 TYPES OF SETS

EMPTY SET *A set is said to be empty or null or void set if it has no element and it is denoted by* ϕ.

In Roster method, ϕ *is denoted by { }.*

It follows from this definition that a set A is an empty set if the statement $x \in A$ is not true for any x.

ILLUSTRATION 1 $\{x \in R : x^2 = -2\} = \phi$.

ILLUSTRATION 2 $\{x \in N : 5 < x < 6\} = \phi$.

ILLUSTRATION 3 *The set A given by A = {x: x is an even prime number greater than 2} is an empty set because 2 is the only even prime number.*

A set consisting of at least one element is called a non-empty or non-void set.

NOTE *If A and B are any two empty sets, then* $x \in A$ *iff (if and only if)* $x \in B$ *is satisfied because there is no element x in either A or B to which the condition may be applied. Thus, A = B. Hence, there is only one empty set and we denote it by* ϕ. *Therefore, article 'the' is used before empty set.*

SINGLETON SET *A set consisting of a single element is called a singleton set.*

ILLUSTRATION 4 *The set {5} is a singleton set.*

ILLUSTRATION 5 *The set* $\{x : x \in N$ *and* $x^2 = 9\}$ *is a singleton set equal to* $\{3\}$.

FINITE SET *A set is called a finite set if it is either void set or its elements can be listed (counted, labelled) by natural numbers 1, 2, 3, ... and the process of listing terminates at a certain natural number n (say).*

CARDINAL NUMBER OF A FINITE SET *The number n in the above definition is called the cardinal number or order of a finite set A and is denoted by n(A).*

INFINITE SET *A set whose elements cannot be listed by the natural numbers 1, 2, 3, ..., n, for any natural number n is called an infinite set.*

ILLUSTRATION 6 *Each one of the following sets is a finite set:*

(i) *Set of even natural numbers less than 100.* (ii) *Set of soldiers in Indian army.*
(iii) *Set of even prime natural numbers.* (iv) *Set of all persons on the earth.*

ILLUSTRATION 7 *Each one of the following sets is an infinite set:*

(i) *Set of all points in a plane.* (ii) *Set of all lines in a plane.* (iii) $\{x \in R : 0 < x < 1\}$.

EQUIVALENT SETS *Two finite sets A and B are equivalent if their cardinal numbers are same. i.e.* $n(A) = n(B)$.

EQUAL SETS *Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.*

If sets A and B are equal, we write $A = B$ and $A \neq B$ when A and B are not equal.

If $A = \{1, 2, 5, 6\}$ and $B = \{5, 6, 2, 1\}$. Then $A = B$, because each element of A is an element of B and vice-versa. Note that the elements of a set may be listed in any order.

It follows from the above definition and the definition of equivalent sets that equal sets are equivalent but equivalent sets need not be equal.

For example, $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ are equivalent sets but not equal sets.

ILLUSTRATIVE EXAMPLES

LEVEL-1

Type I **ON IDENTIFYING WHETHER GIVEN SET IS EMPTY OR NOT**

EXAMPLE 1 *Which of the following sets are empty sets?*

(i) $A = \{x : x^2 - 3 = 0$ *and x is rational* $\}$ (ii) $B = \{x : x$ *is an even prime number* $\}$
(iii) $C = \{x : 4 < x < 5, x \in N\}$ (iv) $D = \{x : x^2 = 25,$ *and x is an odd integer* $\}$

SOLUTION (i) We know that there is no rational number whose square is 3. So, $x^2 - 3 = 0$ is not satisfied by any rational number. Hence, A is an empty set.

(ii) We know that 2 is the only even prime number. Therefore, $B = \{2\}$. So, B is not an empty set.

(iii) Since there is no natural number between 4 and 5. So, C is an empty set.

(iv) Since $x = 5, -5$ satisfy $x^2 = 25$ and ± 5 are odd integers. Therefore, $D = \{-5, 5\}$. Thus, D is a non-empty set.

Type II **ON EQUAL SETS**

EXAMPLE 2 *Find the pairs of equal sets, from the following sets, if any, giving reasons:*

$A = \{0\}$, $B = \{x : x > 15$ *and* $x < 5\}$, $C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$

$E = \{x : x$ *is an integral positive root of the equation* $x^2 - 2x - 15 = 0\}$. [NCERT]

SOLUTION We have,

$A = \{0\}$, $B = \{x : x > 15$ and $x < 5\} = \phi$, $C = \{x : x - 5 = 0\} = \{5\}$,

$D = \{x : x^2 = 25\} = \{-5, 5\}$, and, $E = \{5\}$.

Clearly, $C = E$.

EXAMPLE 3 *Which of the following pairs of sets are equal ? Justify your answer.*

(i) $A = \{x : x$ *is a letter in the word "LOYAL"*$\}$, $B = \{x : x$ *is a letter of the word "ALLOY"* $\}$

(ii) $A = \{x : x \in Z$ and $x^2 \leq 8\}$, $B = \{x : x \in R$ *and* $x^2 - 4x + 3 = 0\}$

SOLUTION (i) We have,

$A = \{L, O, Y, A, L\} = \{L, O, Y, A\}$ and, $B = \{A, L, L, O, Y\} = \{L, O, Y, A\}$

Clearly, $A = B$.

(ii) $A = \{x : x \in Z$ and $x^2 \leq 8\} = \{-2, -1, 0, 1, 2\}$ and, $B = \{x : x \in R$ and $x^2 - 4x + 3 = 0\} = \{1, 3\}$.

We observe that $0 \in A$ but $0 \notin B$. So, $A \neq B$.

Type III **ON FINITE AND INFINITE SETS**

EXAMPLE 4 *State which of the following sets are finite and which are infinite:*

(i) $A = \{x : x \in Z$ and $x^2 - 5x + 6 = 0\}$ (ii) $B = \{x : x \in Z$ and x^2 is even $\}$
(iii) $C = \{x : x \in Z$ and $x^2 = 36\}$ (iv) $D = \{x : x \in Z$ and $x > -10\}$

SOLUTION (i) $A = \{x : x \in Z$ and $x^2 - 5x + 6 = 0\} = \{2, 3\}$

So, A is a finite set

(ii) $B = \{x : x \in Z$ and x^2 is even$\} = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$

Clearly, B is an infinite set.

(iii) $C = \{x : x \in Z$ and $x^2 = 36\} = \{6, -6\}$

Clearly, A is a finite set.

(iv) $D = \{x : x \in Z$ and $x > -10\} = \{-9, -8, -7, \ldots\}$

Clearly, D is an infinite set.

EXERCISE 1.3

LEVEL-1

1. Which of the following are examples of empty set ?
   (i) Set of all even natural numbers divisible by 5.
   (ii) Set of all even prime numbers.
   (iii) $\{x : x^2 - 2 = 0$ and x is rational $\}$.
   (iv) $\{x : x$ is a natural number, $x < 8$ and simultaneously $x > 12\}$.
   (v) $\{x : x$ is a point common to any two parallel lines $\}$.
2. Which of the following sets are finite and which are infinite ?
   (i) Set of concentric circles in a plane. (ii) Set of letters of the English Alphabets.
   (iii) $\{x \in N : x > 5\}$ (iv) $\{x \in N : x < 200\}$
   (v) $\{x \in Z : x < 5\}$ (vi) $\{x \in R : 0 < x < 1\}$.
3. Which of the following sets are equal ?
   (i) $A = \{1, 2, 3\}$ (ii) $B = \{x \in R : x^2 - 2x + 1 = 0\}$
   (iii) $C = \{1, 2, 2, 3\}$ (iv) $D = \{x \in R : x^3 - 6x^2 + 11x - 6 = 0\}$.

4. Are the following sets equal ?
$A = \{x : x \text{ is a letter in the word reap}\}$, $B = \{x : x \text{ is a letter in the word paper}\}$,
$C = \{x : x \text{ is a letter in the word rope}\}$.

5. From the sets given below, pair the equivalent sets:
$A = \{1, 2, 3\}$, $B = \{t, p, q, r, s\}$, $C = \{\alpha, \beta, \gamma\}$, $D = \{a, e, i, o, u\}$.

6. Are the following pairs of sets equal ? Give reasons.
(i) $A = \{2, 3\}$, $B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$
(ii) $A = \{x : x \text{ is a letter of the word ''WOLF''}\}$,
$B = \{x : x \text{ is a letter of the word ''FOLLOW''}\}$ [NCERT]

7. From the sets given below, select equal sets and equivalent sets.
$A = \{0, a\}$, $B = \{1, 2, 3, 4\}$ $C = \{4, 8, 12\}$, $D = \{3, 1, 2, 4\}$, $E = \{1, 0\}$, $F = \{8, 4, 12\}$
$G = \{1, 5, 7, 11\}$, $H = \{a, b\}$.

8. Which of the following sets are equal?
$A = \{x : x \in N, x < 3\}$, $B = \{1, 2\}$, $C = \{3, 1\}$,
$D = \{x : x \in N, x \text{ is odd}, x < 5\}$, $E = \{1, 2, 1, 1\}$, $F = \{1, 1, 3\}$.

9. Show that the set of letters needed to spell "CATARACT" and the set of letters needed to spell "TRACT" are equal. [NCERT]

ANSWERS

1. (iii), (iv), (v) 2. (i) Infinite (ii) finite (iii) Infinite (iv) Finite (v) Infinite (vi) Infinite.
3. $A = C = D$ 4. No 5. $A, C ; B, D$ 6. (i) No (ii) Yes
7. Equal sets : $B = D$, $C = F$ Equivalent sets : $A, E, H ; B, D, G ; C, F$
8. $A = B = E$, $C = D = F$

HINTS TO NCERT & SELECTED PROBLEMS

6. (ii) We have,
$A = \{x : x \text{ is a letter of the word ''WOLF''}\} = \{W, O, L, F\}$
$B = \{x : x \text{ is a letter of the word ''FOLLOW''}\} = \{W, O, L, F\}$
Clearly, $A = B$.

9. A = Set of letters of the word "CATARACT" $= \{A, C, R, T\}$
B = Set of letters of the word "TRACT" $= \{A, C, R, T\}$
Clearly, $A = B$.

1.4 SUBSETS

SUBSETS *Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B.*

If A is a subset of B, we write $A \subseteq B$, which is read as "A is a subset of B" or "A is contained in B".

Thus, $A \subseteq B$ iff
$$a \in A \Rightarrow a \in B.$$

The symbol "$\Rightarrow$" stands for "implies".

If A is a subset of B, we say that B contains A or, B is a super set of A and we write $B \supset A$.

If A is not a subset of B, we write $A \not\subseteq B$.

Obviously, every set is a subset of itself and the empty set is subset of every set. A subset A of a set B is called a *proper subset* of B if $A \neq B$ and we write $A \subset B$. In such a case, we also say that B is a super set of A. An improper subset is a subset containing every element of the original set. A proper subset contains some but not all of the elements of the original set. The empty set is a proper subset of a given set.

Thus, if A is a proper subset of B, then there exists an element $x \in B$ such that $x \notin A$.

It follows immediately from this definition and the definition of equal sets that two sets A and B are equal iff $A \subseteq B$ and $B \subseteq A$.

Thus, whenever it is to be proved that two sets A and B are equal, we must prove that $A \subseteq B$ and $B \subseteq A$.

ILLUSTRATION 1 *Clearly $\{1\} \subseteq \{1, 2, 3\}$, but $\{1, 4\} \not\subseteq \{1, 2, 3\}$.*

ILLUSTRATION 2 *Clearly, $N \subset Z \subset Q \subset R \subset C$, where N, Z, Q, R and C have their usual meanings.*

ILLUSTRATION 3 *If A is the set of all divisors of 68 and B is the set of all prime divisors of 68, then B is the subset of A and we write $B \subset A$.*

1.4.1 SOME RESULTS ON SUBSETS

THEOREM 1 *Every set is a subset of itself.*

PROOF Let A be any set. Then, each element of A is clearly in A itself. Hence, $A \subseteq A$.

THEOREM 2 *The empty set is a subset of every set.*

PROOF Let A be any set and ϕ be the empty set. In order to show that $\phi \subseteq A$, we must show that every element of ϕ is an element of A also. But, ϕ contains no element. So, every element of ϕ is in A. Hence, $\phi \subset A$.

THEOREM 3 *The total number of subsets of a finite set containing n elements is 2^n.*

PROOF Let A be a finite set containing n elements. Let $0 \le r \le n$. Consider those subsets of A that have r elements each. We know that the number of ways in which r elements can be chosen out of n elements is ${}^nC_r$. Therefore, the number of subsets of A having r elements each is ${}^nC_r$. Hence, the total number of subsets of A is
$$ {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = (1+1)^n = 2^n \quad \text{[Using binomial theorem]}$$

ILLUSTRATION 1 *Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.* [NCERT EXEMPLAR]

SOLUTION Let A and B be two sets having m and n elements respectively. Then,
Number of subsets of set $A = 2^m$, Number of subsets of set $B = 2^n$.

It is given that,
$$2^m - 2^n = 56$$
$$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^3(2^3 - 1)$$
$$\Rightarrow \quad n = 3 \text{ and } m - n = 3$$
$$\Rightarrow \quad n = 3 \text{ and } m = 6.$$

ILLUSTRATION 2 *If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, prove that $X \subset Y$.*

SOLUTION Let $x_n = 4^n - 3n - 1, n \in N$. Then, $x_1 = 4 - 3 - 1 = 0$.

And, for any $n \geq 2$, we have
$$x_n = 4^n - 3n - 1 = (1+3)^n - 3n - 1$$
$$\Rightarrow \quad x_n = {}^nC_0 + {}^nC_1(3) + {}^nC_2(3^2) + {}^nC_3(3^3) + \ldots + {}^nC_n(3^n) - 3n - 1 \quad \text{[Using Binomial Theorem]}$$
$$\Rightarrow \quad x_n = 1 + 3n + {}^nC_2(3^2) + {}^nC_3(3^3) + \ldots + {}^nC_n(3^n) - 3n - 1 \quad [\because\ {}^nC_0 = 1, {}^nC_1 = n]$$
$$\Rightarrow \quad x_n = 3^2\left\{{}^nC_2 + {}^nC_3(3) + {}^nC_4(3^2) + \ldots + {}^nC_n(3^{n-2})\right\}$$
$$\Rightarrow \quad x_n = 9\left\{{}^nC_2 + {}^nC_3(3) + {}^nC_4(3^2) + \ldots + {}^nC_n(3^{n-2})\right\}.$$

$\Rightarrow \quad x_n$ is some positive integral multiple of 9 for all $n \geq 2$.

Thus, X consists of all those positive integral multiples of 9 which are of the form $9\left\{{}^nC_2 + 3 \times {}^nC_3 + 3^2 \times {}^nC_4 + \ldots + 3^{n-2} \times {}^nC_n\right\}$ together with 0.

Clearly, $Y = \{9(n-1) : n \in N\}$ consists of all integral multiples of 9 together with 0.

Hence, $X \subset Y$.

1.4.2 SUBSETS OF THE SET R OF REAL NUMBERS

Following sets are important subsets of the set R of all real numbers:

(i) The set of all natural numbers $N = \{1, 2, 3, 4, 5, 6, \ldots\}$

(ii) The set of all integers $Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

(iii) The set of all rational numbers $Q = \left\{x : x = \frac{m}{n}, m, n \in Z, n \neq 0\right\}$.

(iv) The set of all irrational numbers. It is denoted by T.

Thus, $\quad T = \{x : x \in R \text{ and } x \notin Q\}$.

Clearly, $N \subset Z \subset Q \subset R, T \subset R$ and $N \not\subset T$.

1.4.3 INTERVALS AS SUBSETS OF R

On real line various types of infinite subsets are designated as intervals as defined below:

CLOSED INTERVAL *Let a and b be two given real numbers such that $a < b$. Then, the set of all real numbers x such that $a \leq x \leq b$ is called a closed interval and is denoted by $[a, b]$.*

Thus, $\quad [a, b] = \{x \in R : a \leq x \leq b\}$.

On the real line, $[a, b]$ may be graphed as shown in Fig. 1.1

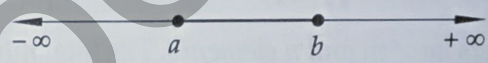

Fig. 1.1

For example, $[-1, 2] = \{x \in R : -1 \leq x \leq 2\}$ is the set of all real numbers lying between -1 and 2 including the end points. Clearly, it is an infinite subset of R.

OPEN INTERVAL *If a and b are two real numbers such that $a < b$, then the set of all real numbers x satisfying $a < x < b$ is called an open interval and is denoted by (a, b) or $]a, b[$.*

Thus, $\quad (a, b) = \{x \in R : a < x < b\}$

On the real line, (a, b) may be graphed as shown in Fig. 1.2.

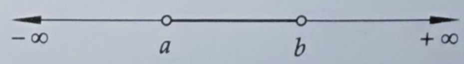

Fig. 1.2

Here, encircling a and b means that a and b are not included in the set.

For example, $(1, 2) = \{x \in R : 1 < x < 2\}$ is the set of all real numbers lying between 1 and 2 excluding the end-points 1 and 2. This is an infinite subset of R.

SEMI-OPEN OR SEMI-CLOSED INTERVAL *If a and b are two real numbers such that $a < b$, then the sets $(a, b] = \{x \in R : a < x \leq b\}$ and $[a, b) = \{x \in R : a \leq x < b\}$ are known as semi-open or semi-closed intervals. $(a, b]$ and $[a, b)$ are also denoted by $]a, b]$ and $[a, b[$ respectively.*

On real line these sets may be graphed as shown in Figs. 1.3 and 1.4 respectively.

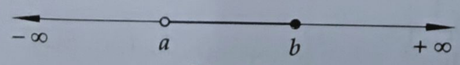

Fig. 1.3

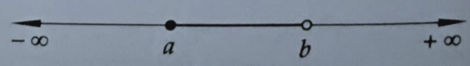

Fig. 1.4

The number $b - a$ is called the length of any of the intervals (a, b), $[a, b]$, $[a, b)$ and $(a, b]$.

These notations provide an alternative way of designating the subsets of the set R of all real numbers. For example, the interval $[0, \infty)$ denotes the set R^+ of all non-negative real numbers, while the interval $(-\infty, 0)$ denotes the set R^- of all negative real numbers. The interval $(-\infty, \infty)$ denotes the set R of all real numbers.

1.5 UNIVERSAL SET

In any discussion in set theory, there always happens to be a set that contains all sets under consideration i.e. it is a super set of each of the given sets. Such a set is called the universal set and is denoted by U.

Thus, a set that contains all sets in a given context is called the universal set.

ILLUSTRATION 1 *When we study two dimensional coordinate geometry, then the set of all points in xy-plane is the universal set.*

ILLUSTRATION 2 *When we are using sets containing natural numbers, then N is the universal set.*

ILLUSTRATION 3 *If $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$ and $C = \{1, 3, 5, 7\}$, then $U = \{1, 2, 3, 4, 5, 6, 7\}$ can be taken as the universal set.*

ILLUSTRATION 4 *When we are using intervals on real line, the set R of real numbers is taken as the universal set.*

1.6 POWER SET

POWER SET *Let A be a set. Then the collection or family of all subsets of A is called the power set of A and is denoted by P (A).*

That is, $P(A) = \{S : S \subset A\}$.

Since the empty set and the set A itself are subsets of A and are therefore elements of $P(A)$. Thus, the power set of a given set is always non-empty.

ILLUSTRATION 1 *Let $A = \{1, 2, 3\}$. Then, the subsets of A are : ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$ and $\{1, 2, 3\}$. Hence, $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.*

ILLUSTRATION 2 *If A is the void set ϕ, then P(A) has just one element ϕ i.e. $P(\phi) = \{\phi\}$.*

ILLUSTRATION 3 *Show that $n\{P[P(P(\phi)]\} = 4$.*

SOLUTION We have ,

$$P(\phi) = \{\phi\}$$

$$\therefore \quad P(P(\phi)) = \{\phi, \{\phi\}\}$$

$$\Rightarrow \quad P[P(P(\phi))] = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}.$$

Hence, $P[P(P(\phi))]$ consists of 4 elements i.e. $n\{P[P(P(\phi))]\} = 4$.

REMARK *We know that a set having n elements has 2^n subsets. Therefore, if A is a finite set having n elements, then P(A) has 2^n elements.*

ILLUSTRATION 4 *If $A = \{a, \{b\}\}$, find P (A).*

SOLUTION Let $B = \{b\}$. Then, $A = \{a, B\}$.

$$\therefore \quad P(A) = \{\phi, \{a\}, \{B\}, \{a, B\}\} = \{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}.$$

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Consider the following sets: ϕ, $A = \{1, 2\}$, $B = \{1, 4, 8\}$, $C = \{1, 2, 4, 6, 8\}$.*

Insert the correct symbol $\subset$ or $\not\subset$ between each of the following pair of sets:

(i) $\phi \ldots B$ (ii) $A \ldots B$ (iii) $A \ldots C$ (iv) $B \ldots C$

SOLUTION (i) Since null set is subset of every set. Therefore, $\phi \subset B$.

(ii) Clearly, $2 \in A$ but $2 \notin B$. So, $A \not\subset B$.

(iii) Since all elements of set A are in C and $A \neq C$. So, $A \subset C$.

(iv) Clearly, all elements of set B are in set C and $B \neq C$. So, $B \subset C$.

EXAMPLE 2 *Let $A = \{a, b, c, d\}$, $B = \{a, b, c\}$ and $C = \{b, d\}$. Find all sets X such that:*

(i) $X \subset B$ and $X \subset C$ (ii) $X \subset A$ and $X \not\subset B$.

SOLUTION (i) We have,

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \ldots\ldots\}$, $P(B) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

and, $P(C) = \{\phi, \{b\}, \{d\}, \{b, d\}\}$

Now, $X \subset B$ and $X \subset C$

$\Rightarrow$ $X \in P(B)$ and $X \in P(C)$

$\Rightarrow$ $X \in \{\phi, \{b\}\}$

$\Rightarrow$ $X = \phi, \{b\}$

(ii) We have,

$X \subset A$ and $X \not\subset B$

$\Rightarrow$ X is a subset of A but X is not a subset of B

$\Rightarrow$ $X \in P(A)$ but $X \notin P(B)$

$\Rightarrow$ $X = \{d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c, d\}$.

EXAMPLE 3 *Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$? If not give an example.*

SOLUTION Consider the following sets: $A = \{a\}$, $B = \{\{a\}, b\}$ and $C = \{\{a\}, b, c\}$.

Clearly, $A \in B$ and $B \subset C$. But, $A \not\subset C$ as $a \in A$ but $a \notin C$. Thus, the given statement is not true.

EXAMPLE 4 *Let $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3\}$ and $C = \{2, 4\}$. Find all sets X satisfying each pair of conditions:*

(i) $X \subset B$ and $X \not\subset C$ (ii) $X \subset B, X \neq B$ and $X \not\subset C$ (iii) $X \subset A, X \subset B$ and $X \subset C$.

SOLUTION (i) We have,

$X \subset B$ and $X \not\subset C$

$\Rightarrow$ X is a subset of B but X is not a subset of C

$\Rightarrow$ $X \in P(B)$ but $X \notin P(C)$

$\Rightarrow$ $X = \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

(ii) We have,

$X \subset B, X \neq B$ and $X \not\subset C$

$\Rightarrow$ X is a subset of B other than B itself and X is not a subset of C

$\Rightarrow$ $X \in P(B), X \notin P(C)$ and $X \neq B$

$\Rightarrow$ $X = \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$

(iii) We have,

$X \subset A, X \subset B$ and $X \subset C$

$\Rightarrow$ $X \in P(A), X \in P(B)$ and $X \in P(C)$

$\Rightarrow$ X is a subset of A, B and C

$\Rightarrow$ $X = \phi, \{2\}$.

EXAMPLE 5 *Let B be a subset of a set A and let $P(A : B) = \{X \in P(A) : X \supset B\}$.*

(i) *Show that: $P(A : \phi) = P(A)$*

(ii) *If $A = \{a, b, c, d\}$ and $B = \{a, b\}$. List all the members of the set $P(A : B)$.*

SOLUTION (i) We have,

$$P(A : B) = \{X \in P(A) : X \supset B\} = \{X \in P(A) : B \subset X\}$$

= Set of all those subsets of A which contain B

$\therefore \quad P(A:\phi) =$ Set of all those subsets of A which contain ϕ

$=$ Set of all subsets of set $A = P(A)$.

(ii) If $A = \{a, b, c, d\}$ and $B = \{a, b\}$. Then,

$P(A:B) =$ Set of all those subsets of set A which contain B

$= \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}$

EXAMPLE 6 *Prove that $A \subset \phi$ implies $A = \phi$.*

SOLUTION We know that two sets A and B are equal iff $A \subset B$ and $B \subset A$. Also, we know that

$\phi \subset A$

and, $\quad A \subset \phi$ [Given]

$\therefore \quad A = \phi$

EXAMPLE 7 *In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.*

(i) If $x \in A$ and $A \in B$, then $x \in B$

(ii) *If $A \subset B$ and $B \in C$, then $A \in C$*

(iii) *If $A \subset B$ and $B \subset C$, then $A \subset C$*

(iv) *If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$*

(v) *If $x \in A$ and $A \not\subset B$, then $x \in B$*

(vi) *If $A \subset B$ and $x \notin B$, then $x \notin A$*

SOLUTION (i) False:

Consider sets $A = \{1\}$ and, $B = \{\{1\}, 2\}$.

Clearly $1 \in A$ and $A \in B$, but $1 \notin B$. So, $x \in A$ and $A \in B$ need not imply that $x \in B$.

(ii) False:

Let $A = \{1\}$, $B = \{1, 2\}$ and $C = \{\{1, 2\}, 3\}$. Then, we observe that $A \subset B$ and $B \in C$ but $A \notin C$.

Thus, $A \subset B$ and $B \in C$ need not imply that $A \in C$.

(iii) True:

Let $x \in A$. Then,

$A \subset B \Rightarrow x \in B \Rightarrow x \in C$ $[\because B \subset C]$

Thus, $x \in A \Rightarrow x \in C$ for all $x \in A$. So, $A \subset C$.

Hence, $A \subset B$ and $B \subset C \Rightarrow A \subset C$.

(iv) False:

Let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{1, 2, 5\}$. Then, $A \not\subset B$ and $B \not\subset C$. But, $A \subset C$.

Thus, $A \not\subset B$ and $B \not\subset C$ need not imply that $A \not\subset C$.

(v) False:

Let $A = \{1, 2\}$ and $B = \{2, 3, 4, 5\}$. Then, we observe that $1 \in A$ and $A \not\subset B$, but $1 \notin B$.

Thus, $x \in A$ and $A \not\subset B$ need not imply that $x \in B$.

(vi) True:

Let $A \subset B$. Then, we observe that

$x \in A \Rightarrow x \in B \Leftrightarrow x \notin B \Rightarrow x \notin A$.

EXAMPLE 8 *Write the following subsets of R as intervals:*

(i) $\{x : x \in R, -4 < x \leq 6\}$

(ii) $\{x : x \in R, -12 < x < -10\}$

(iii) $\{x : x \in R, 0 \leq x < 7\}$

(iv) $\{x : x \in R, 3 \leq x \leq 4\}$.

Also, find the length of each interval.

SOLUTION (i) $\{x : x \in R, -4 < x \leq 6\} = (-4, 6]$. Length $= 6 - (-4) = 10$

(ii) $\{x : x \in R, -12 < x < -10\} = (-12, -10)$. Length $= -10 - (-12) = 2$

(iii) $\{x : x \in R, 0 \leq x < 7\} = [0, 7)$. Length $= 7 - 0 = 7$

(iv) $\{x : x \in R, 3 \leq x \leq 4\} = [3, 4]$. Length $= 4 - 3 = 1$

EXAMPLE 9 *Write the following intervals in the set-builder form:*

(i) $(-7, 0)$ (ii) $[6, 12]$ (iii) $(6, 12]$ (iv) $[-20, 3)$

SOLUTION (i) $(-7, 0) = \{x : x \in R$ and $-7 < x < 0\}$

(ii) $[6, 12] = \{x : x \in R$ and $6 \leq x \leq 12\}$

(iii) $(6, 12] = \{x : x \in R \text{ and } 6 < x \le 12\}$
(iv) $[-20, 3) = \{x : x \in R \text{ and } -20 \le x < 3\}$

EXAMPLE 10 *Let* $A = \{1, 2, \{3, 4\}, 5\}$. *Which of the following statements are incorrect and why?*

(i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ (iii) $\{\{3, 4\}\} \subset A$ (iv) $1 \in A$
(v) $1 \subset A$ (vi) $\{1, 2, 5\} \subset A$ (vii) $\{1, 2, 5\} \in A$ (viii) $\{1, 2, 3\} \subset A$
(ix) $\phi \in A$ (x) $\phi \subset A$ (xi) $\{\phi\} \subset A$

SOLUTION $\{3, 4\}$ is an element of set A. Therefore, $\{3, 4\} \in A$ is correct and $\{3, 4\} \subset A$ is incorrect.

So, (i) is incorrect and (ii) is correct. As $\{3, 4\}$ is an element of set A. Therefore, $\{\{3, 4\}\}$ is a set containing element $\{3, 4\}$ which belongs to A. So, $\{\{3, 4\}\} \subset A$.

Hence, (iii) is correct.

Since 1 is an element of set A. So, $1 \in A$ is correct and $1 \subset A$ is incorrect.

So, (iv) is correct and (v) is incorrect.

Since 1, 2, 5 are elements of set A. Therefore, $\{1, 2, 5\}$ is a subset of set A.

Hence, (vi) is correct and (vii) is incorrect.

As 3 is not an element of set A. So, $\{1, 2, 3\} \subset A$ is incorrect. The null set is subset of every set.

So, $\phi \subset A$ is correct and $\phi \in A$ is incorrect. Hence, (ix) is incorrect and (x) is correct.

As $\phi \subset A$ but $\{\phi\}$ is not a subset of A. So, (xi) is incorrect.

EXERCISE 1.4

LEVEL-1

1. Which of the following statements are true ? Give reason to support your answer.
   (i) For any two sets A and B either $A \subseteq B$ or $B \subseteq A$.
   (ii) Every subset of an infinite set is infinite.
   (iii) Every subset of a finite set is finite.
   (iv) Every set has a proper subset.
   (v) $\{a, b, a, b, a, b, ...\}$ is an infinite set.
   (vi) $\{a, b, c\}$ and $\{1, 2, 3\}$ are equivalent sets.
   (vii) A set can have infinitely many subsets.
2. State whether the following statements are true or false:
   (i) $1 \in \{1, 2, 3\}$ (ii) $a \subset \{b, c, a\}$ (iii) $\{a\} \in \{a, b, c\}$
   (iv) $\{a, b\} = \{a, a, b, b, a\}$ (v) The set $\{x : x + 8 = 8\}$ is the null set.
3. Decide among the following sets, which are subsets of which:
   $A = \{x : x \text{ satisfies } x^2 - 8x + 12 = 0\}$, $B = \{2, 4, 6\}$, $C = \{2, 4, 6, 8, ...\}$, $D = \{6\}$.
4. Write which of the following statements are true ? Justify your answer.
   (i) The set of all integers is contained in the set of all rational numbers.
   (ii) The set of all crows is contained in the set of all birds.
   (iii) The set of all rectangles is contained in the set of all squares.
   (iv) The set of all real numbers is contained in the set of all complex numbers.
   (v) The sets $P = \{a\}$ and $B = \{\{a\}\}$ are equal.
   (vi) The sets $A = \{x : x \text{ is a letter of the word "LITTLE"}\}$
   and, $B = \{x : x \text{ is a letter of the word "TITLE"}\}$ are equal.
5. Which of the following statements are correct? Write a correct form of each of the incorrect statements.
   (i) $a \subset \{a, b, c\}$ (ii) $\{a\} \in \{a, b, c\}$ (iii) $a \in \{\{a\}, b\}$

(iv) $\{a\} \subset \{\{a\}, b\}$ (v) $\{b, c\} \subset \{a, \{b, c\}\}$ (vi) $\{a, b\} \subset \{a, \{b, c\}\}$
(vii) $\phi \in \{a, b\}$ (viii) $\phi \subset \{a, b, c\}$ (ix) $\{x : x + 3 = 3\} = \phi$

6. Let $A = \{a, b, \{c, d\}, e\}$. Which of the following statements are false and why?
(i) $\{c. d\} \subset A$ (ii) $\{c, d\} \in A$ (iii) $\{\{c, d\}\} \subset A$
(iv) $a \in A$ (v) $a \subset A$ (vi) $\{a, b, e\} \subset A$
(vii) $\{a, b, e\} \in A$ (viii) $\{a, b, c\} \subset A$ (ix) $\phi \in A$
(x) $\{\phi\} \subset A$

7. Let $A = \{\{1, 2, 3\}, \{4, 5\}, \{6, 7, 8\}\}$. Determine which of the following is true or false:
(i) $1 \in A$ (ii) $\{1, 2, 3\} \subset A$ (iii) $\{6, 7, 8\} \in A$
(iv) $\{\{4, 5\}\} \subset A$ (v) $\phi \in A$ (vi) $\phi \subset A$

8. Let $A = \{\phi, \{\phi\}, 1, \{1, \phi\}, 2\}$. Which of the following are true?
(i) $\phi \in A$ (ii) $\{\phi\} \in A$ (iii) $\{1\} \in A$
(iv) $\{2, \phi\} \subset A$ (v) $2 \subset A$ (vi) $\{2, \{1\}\} \not\subset A$
(vii) $\{\{2\}, \{1\}\} \not\subset A$ (viii) $\{\phi, \{\phi\}, \{1, \phi\}\} \subset A$ (ix) $\{\{\phi\}\} \subset A$.

9. Write down all possible subsets of each of the following sets:
(i) $\{a\}$ (ii) $\{0, 1\}$ (iii) $\{a, b, c\}$
(iv) $\{1, \{1\}\}$ (v) $\{\phi\}$

10. Write down all possible proper subsets each of the following sets:
(i) $\{1, 2\}$ (ii) $\{1, 2, 3\}$ (iii) $\{1\}$

11. What is the total number of proper subsets of a set consisting of n elements?

12. If A is any set, prove that: $A \subseteq \phi \Leftrightarrow A = \phi$.

13. Prove that: $A \subseteq B$, $B \subseteq C$ and $C \subseteq A \Rightarrow A = C$.

14. How many elements has $P(A)$, if $A = \phi$?

15. What universal set (s) would you propose for each of the following:
(i) The set of right triangles. (ii) The set of isosceles triangles.

LEVEL-2

16. If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n-1) : n \in N\}$, then prove that $X \subseteq Y$.

ANSWERS

1. (i) F, $A = \{1, 2, 3\}$, $B = \{a, b\}$ (ii) F, $A = \{1, 2\}$ is a finite subset of N.
(iii) T (iv) F, ϕ does not have a proper subset
(v) F, Given set $= \{a, b\}$ (vi) T (vii) F

2. (i) T (ii) F (iii) F (iv) T (v) F

3. $D \subset A \subset B \subset C$

4. (i) T (ii) T (iii) F (iv) T (v) F (vi) T

5. (i) $a \in \{a, b, c\}$ (ii) $\{a\} \subset \{a, b, c\}$ (iii) $\{a\} \in \{\{a\}, b\}$
(iv) $\{\{a\}\} \subset \{\{a\}, b\}$ (v) $\{b, c\} \in \{a, \{b, c\}\}$ (vi) $\{a, b\} \not\subset \{a, \{b, c\}\}$
(vii) $\phi \subset \{a, b\}$ (viii) $\phi \subset \{a, b, c\}$ (ix) $\{x : x + 3 = 3\} \neq \phi$

6. (i) F (ii) T (iii) T (iv) T (v) F (vi) T
(vii) F (viii) F (ix) F (x) F

7. (i) F (ii) F (iii) T (iv) T (v) F (vi) T

8. (i) T (ii) T (iii) F (iv) T (v) F (vi) T (vii) T
(viii) T (ix) T

9. (i) $\phi, \{a\}$ (ii) $\phi, \{0\}, \{1\}, \{0, 1\}$ (iii) $\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$
(iv) $\phi, \{1\}, \{\{1\}\}, \{1, \{1\}\}$ (v) $\phi, \{\phi\}$.

10. (i) $\phi, \{1\}, \{2\}$ (ii) $\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}$ (iii) ϕ

11. $2^n - 1$

14. 1

15. (i) The setof all triangles in a plane. (ii) The set of all triangles in a plane.

HINTS TO SELECTED PROBLEMS

16. Let $x_n = 8^n - 7n - 1 = (1+7)^n - 7n - 1 = {}^nC_2\, 7^2 + {}^nC_3\, 7^3 + \ldots + {}^nC_n\, 7^n$

$$= 49\,({}^nC_2 + {}^nC_3\, 7 + \ldots + {}^nC_n\, 7^{n-2}) \text{ for } n \geq 2.$$

For $n = 1$, $x_n = 0$. Thus, X contains all positive integral multiples of 49 of the form $49\,k_n$, where $k_n = {}^nC_2 + {}^nC_3\,(7) + {}^nC_4\,(7^2) + \ldots + {}^nC_n\,(7^{n-2})$.

Also, Y contains all positive integral multiples of 49 including zero. Thus, $X \subseteq Y$.

1.7 VENN DIAGRAMS

Sometimes pictures are very helpful in our thinking. First of all a Swiss mathematician Euler gave an idea to represent a set by the points in a closed curve. Later on British mathematician John-Venn (1834-1883) brought this idea to practice. That is why the diagrams drawn to represent sets are called *Venn-Euler diagrams* or simply Venn-diagrams. In Venn-diagrams the universal set U is represented by points within a rectangle and its subsets are represented by points in closed curves (usually circles) within the rectangle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B as shown in Fig. 1.5 (i). If A and B are not equal but they have some common elements, then to represent A and B we draw two intersecting circles. (See Fig. 1.5 (ii)). Two disjoint sets are represented by two non-intersecting circles. (See Fig. 1.5 (iii))

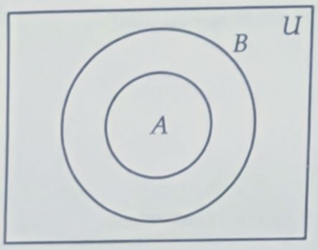

Fig. 1.5 (i)

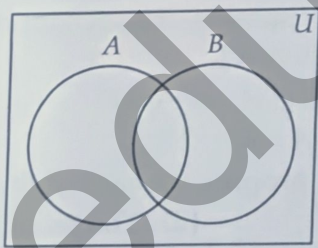

Fig. 1.5 (ii)

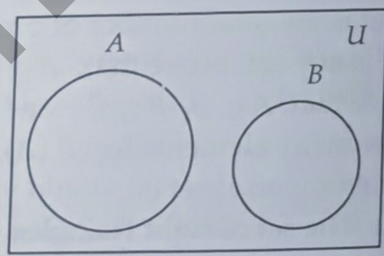

Fig. 1.5 (iii)

1.8 OPERATIONS ON SETS

In this section, we shall introduce some operations on sets to construct new sets from given ones.

UNION OF SETS *Let A and B be two sets. The union of A and B is the set of all those elements which belong either to A or to B or to both A and B.*

We shall use the notation $A \cup B$ (read as "A union B") to denote the union of A and B.

Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Clearly, $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$.

And, $x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$.

In Fig. 1.6 the shaded part represents $A \cup B$. It is evident from the definition that $A \subseteq A \cup B$, $B \subseteq A \cup B$

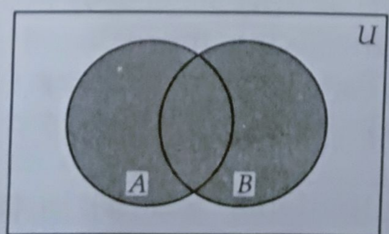

Fig. 1.6

If A and B are two sets such that $A \subset B$, then $A \cup B = B$. Also, $A \cup B = A$, if $B \subset A$.

ILLUSTRATION 1 *If $A = \{1, 2, 3\}$ and $B = \{1, 3, 5, 7\}$, then $A \cup B = \{1, 2, 3, 5, 7\}$.*

ILLUSTRATION 2 *If $A = \{x : x = 2n + 1, n \in Z\}$ and $B = \{x : x = 2n, n \in Z\}$, then*
$A \cup B = \{x : x$ is an odd integer$\} \cup \{x : x$ is an even integer$\} = \{x : x$ is an integer$\} = Z$.

NOTE *If $A_1, A_2, \ldots, A_n$ is a finite family of sets, then their union is denoted by $\bigcup_{i=1}^{n} A_i$ or, $A_1 \cup A_2 \cup A_3 \ldots \cup A_n$.*

ILLUSTRATION 3 *Let $A = \{1, 2, 3\}$, $B = \{3, 5\}$, $C = \{4, 7, 8\}$. Then, $A \cup B \cup C = \{1, 2, 3, 4, 5, 7, 8\}$*

INTERSECTION OF SETS *Let A and B be two sets. The intersection of A and B is the set of all those elements that belong to both A and B.*

The intersection of A and B is denoted by $A \cap B$ (read as "A intersection B").

Thus, $A \cap B = \{x : x \in A$ and $x \in B\}$.

Clearly, $x \in A \cap B \quad \Leftrightarrow \quad x \in A$ and $x \cap B$

In Fig. 1.7 the shaded region represents $A \cap B$. Evidently, $A \cap B \subseteq A$, $A \cap B \subseteq B$.

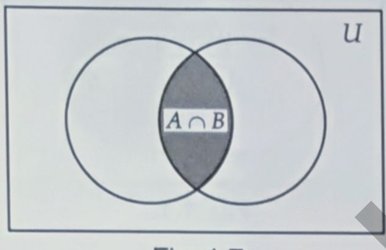

Fig. 1.7

If A and B are two sets, then $A \cap B = A$, if $A \subset B$ and $A \cap B = B$, if $B \subset A$.

NOTE *If $A_1, A_2, \ldots, A_n$ is a finite family of sets, then their intersection is denoted by*
$\bigcap_{i=1}^{n} A_i$ *or,* $A_1 \cap A_2 \cap \ldots \cap A_n$.

ILLUSTRATION 4 If $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 9, 12\}$, then $A \cap B = \{1, 3\}$.

ILLUSTRATION 5 If $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6, 8, 10\}$ *and* $C = \{4, 6, 7, 8, 9, 10, 11\}$, *then* $A \cap B = \{2, 4, 6\}$. *Therefore,* $A \cap B \cap C = \{4, 6\}$.

ILLUSTRATION 6 *If $A = \{x : x = 2n, n \in Z\}$ and $B = \{x : x = 3n, n \in Z\}$, then*

$$
\begin{aligned}
A \cap B &= \{x : x = 2n, n \in Z\} \cap \{x : x = 3n, n \in Z\} \\
&= \{\ldots, -4, -2, 0, 2, 4, 6, \ldots\} \cap \{\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots\} \\
&= \{\ldots, -6, 0, 6, 12, \ldots\} = \{x : x = 6n, n \in Z\}.
\end{aligned}
$$

ILLUSTRATION 7 *If $A = \{x : x = 3n, n \in Z\}$ and $B = \{x : x = 4n, n \in Z\}$, then find $A \cap B$.*

SOLUTION Clearly,

$x \in A \cap B$

$\Leftrightarrow$ $x = 3n$ and $x = 4n, n \in Z$

$\Leftrightarrow$ x is a multiple of 3 and x is a multiple of 4

$\Leftrightarrow$ x is a multiple of 3 and 4 both

$\Leftrightarrow$ x is a multiple of 12.

$\Leftrightarrow$ $x = 12n$, $n \in Z$

Hence, $A \cap B = \{x : x = 12n, n \in Z\}$.

DISJOINT SETS *Two sets A and B are said to be disjoint, if $A \cap B = \phi$*

If $A \cap B \neq \phi$, then A and B are said to be intersecting or overlapping sets.

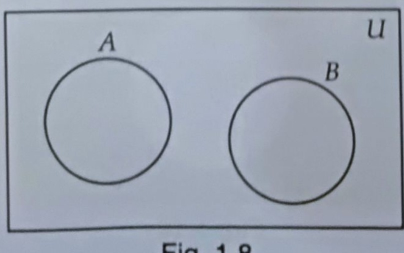

Fig. 1.8

ILLUSTRATION 8 *If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{7, 8, 9, 10, 11\}$ and $C = \{6, 8, 10, 12, 14\}$, then A and B are disjoint sets, while A and C are intersecting sets.*

DIFFERENCE OF SETS *Let A and B be two sets. The difference of A and B, written as $A - B$, is the set of all those elements of A which do not belong to B.*

Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

or, $A - B = \{x \in A : x \notin B\}$.

Clearly, $x \in A - B \Leftrightarrow x \in A$ and $x \notin B$.

In Fig. 1.9, the shaded part represents $A - B$.

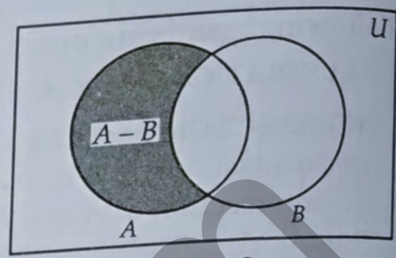

Fig. 1.9

Similarly, the difference $B - A$ is the set of all those elements of B that do not belong to A i.e. $B - A = \{x \in B : x \notin A\}$.

In Fig. 1.10, the shaded part represents $B - A$.

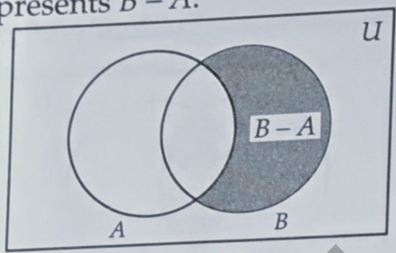

Fig. 1.10

ILLUSTRATION 9 *If $A = \{2, 3, 4, 5, 6, 7\}$ and $B = \{3, 5, 7, 9, 11, 13\}$, then $A - B = \{2, 4, 6\}$ and $B - A = \{9, 11, 13\}$.*

SYMMETRIC DIFFERENCE OF TWO SETS *Let A and B be two sets. The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.*

Thus, $A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}$.

The shaded part in Fig. 1.11 represents $A \Delta B$.

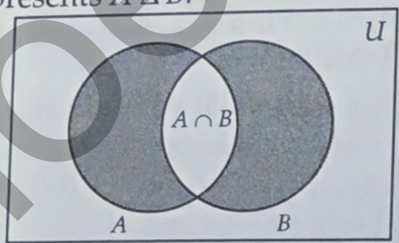

Fig. 1.11

ILLUSTRATION 10 *If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 6, 7, 8, 9\}$, then $A - B = \{2, 4\}$, $B - A = \{9\}$*

$\therefore \quad A \Delta B = \{2, 4, 9\}$.

ILLUSTRATION 11 *If $A = \{x \in R : 0 < x < 3\}$ and $B = \{x \in R : 1 \le x \le 5\}$, then*

$A - B = \{x \in R : 0 < x < 1\}$, $B - A = \{x \in R : 3 \le x \le 3\}$

and, $A \Delta B = \{x \in R : 0 < x < 1\} \cap \{x \in R : 3 \le x \le 5\} = \{x \in R : 0 < x < 1 \text{ or } 3 \le x \le 5\}$.

COMPLEMENT OF A SET *Let U be the universal set and let A be a set such that $A \subset U$. Then, the complement of A with respect to U is denoted by A' or A^c or $U - A$ and is defined the set of all those elements of U which are not in A.*

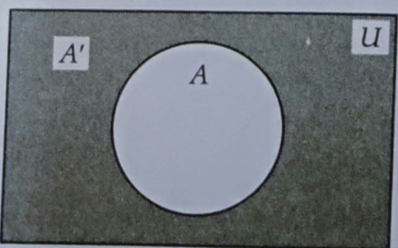

Fig. 1.12

Thus $A' = \{x \in U : x \notin A\}$. Clearly, $x \in A' \Leftrightarrow x \notin A$.

ILLUSTRATION 12 *Let the set of natural numbers N = {1, 2, 3, 4, ...} be the universal set and let* $A = \{2, 4, 6, 8, ...\}$. *Then* $A' = \{1, 3, 5, ...\}$.

ILLUSTRATION 13 *If* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ *and* $A = \{1, 3, 5, 7, 9\}$, *then* $A' = \{2, 4, 6, 8\}$.

Following results are direct consequences of the definition of the complement of a set:

(i) $U' = \{x \in U : x \notin U\} = \phi$

(ii) $\phi' = \{x \in U : x \notin \phi\} = U$

(iii) $(A')' = \{x \in U : x \notin A'\} = \{x \in U : x \in A\} = A$

(iv) $A \cup A' = \{x \in U : x \in A\} \cup \{x \in U : x \notin A\} = U$

(v) $A \cap A' = \{x \in U : x \in A\} \cap \{x \in U : x \notin A\} = \phi$.

EXERCISE 1.5

LEVEL-1

1. If A and B are two sets such that $A \subset B$, then Find:
   (i) $A \cap B$ (ii) $A \cup B$
2. If $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7, 8\}$, $C = \{7, 8, 9, 10, 11\}$ and $D = \{10, 11, 12, 13, 14\}$. Find:
   (i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$
   (iv) $B \cup D$ (v) $A \cup B \cup C$ (vi) $A \cup B \cup D$
   (vii) $B \cup C \cup D$ (viii) $A \cap (B \cup C)$ (ix) $(A \cap B) \cap (B \cap C)$
   (x) $(A \cup D) \cap (B \cup C)$.
3. Let $A = \{x : x \in N\}$, $B = \{x : x = 2n, n \in N\}$, $C = \{x : x = 2n - 1, n \in N\}$ and, $D = \{x : x$ is a prime natural number $\}$. Find:
   (i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$
   (iv) $B \cap C$ (v) $B \cap D$ (vi) $C \cap D$
4. Let $A = \{3, 6, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ and $D = \{5, 10, 15, 20\}$. Find:
   (i) $A - B$ (ii) $A - C$ (iii) $A - D$
   (iv) $B - A$ (v) $C - A$ (vi) $D - A$
   (vii) $B - C$ (viii) $B - D$
5. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find:
   (i) A' (ii) B' (iii) $(A \cap C)'$
   (iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$
6. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that:
   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

ANSWERS

1. (i) A (ii) B
2. (i) $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (ii) $\{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\}$
   (iii) $\{4, 5, 6, 7, 8, 9, 10, 11\}$ (iv) $\{4, 5, 6, 7, 8, 10, 11, 12, 13, 14\}$
   (v) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ (vi) $\{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14\}$
   (vii) $\{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ (viii) $\{4, 5\}$ (ix) ϕ (x) $\{4, 5, 10, 11\}$
3. (i) B (ii) C (iii) D

(iv) ϕ (v) $\{2\}$ (vi) $D - \{2\}$

4. (i) $\{3, 6, 15, 18, 21\}$ (ii) $\{3, 15, 18, 21\}$ (iii) $\{3, 6, 12, 18, 21\}$
(iv) $\{4, 8, 16, 20\}$ (v) $\{2, 4, 8, 10, 14, 16\}$ (vi) $\{5, 10, 20\}$
(vii) $\{20\}$ (viii) $\{4, 8, 12, 16\}$

5. (i) $\{5, 6, 7, 8, 9\}$ (ii) $\{1, 3, 5, 7, 9\}$ (iii) $\{1, 2, 5, 6, 7, 8, 9\}$
(iv) $\{5, 7, 9\}$ (v) A (vi) $\{1, 3, 4, 5, 6, 7, 9\}$

1.9 LAWS OF ALGEBRA OF SETS

In this section, we shall state and prove some fundamental laws of algebra of sets.

THEOREM 1 *(Idempotent Laws) For any set A,*

(i) $A \cup A = A$ (ii) $A \cap A = A$.

PROOF (i) $A \cup A = \{x : x \in A \text{ or } x \in A\} = \{x : x \in A\} = A$

(ii) $A \cap A = \{x : x \in A \text{ and } x \in A\} = \{x : x \in A\} = A$.

THEOREM 2 *(identity Laws) For any set A,*

(i) $A \cup \phi = A$ (ii) $A \cap U = A$.

i.e. ϕ and U are identity elements for union and intersection respectively.

PROOF (i) $A \cup \phi = \{x : x \in A \text{ or } x \in \phi\} = \{x : x \in A\} = A$

(ii) $A \cap U = \{x : x \in A \text{ and } x \in U\} = \{x : x \in A\} = A$

THEOREM 3 *(Commutative Laws) For any two sets A and B*

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

PROOF Recall that two sets X and Y are equal iff $X \subseteq Y$ and $Y \subseteq X$. Also, $X \subseteq Y$ if every element of X belongs to Y.

(i) Let x be an arbitrary element of $A \cup B$. Then,

$$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B \Rightarrow x \in B \text{ or } x \in A \Rightarrow x \in B \cup A$$

$\therefore$ $A \cup B \subseteq B \cup A$.

Similarly, $B \cup A \subseteq A \cup B$.

Hence, $A \cup B = B \cup A$.

THEOREM 4 *(Associative Laws) If A, B and C are any three sets, then*

(i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$ **[NCERT EXEMPLAR]**

i.e. union and intersection are associative.

PROOF (i) Let x be an arbitrary element of $(A \cup B) \cup C$. Then,

$x \in (A \cup B) \cup C$

$\Rightarrow$ $x \in (A \cup B)$ or $x \in C$

$\Rightarrow$ $(x \in A \text{ or } x \in B)$ or $x \in C$

$\Rightarrow$ $x \in A$ or $(x \in B \text{ or } x \in C)$

$\Rightarrow$ $x \in A$ or $x \in (B \cup C)$

$\Rightarrow$ $x \in A \cup (B \cup C)$

$\therefore$ $(A \cup B) \cup C \subseteq A \cup (B \cup C)$.

Similarly, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$.

Hence, $(A \cup B) \cup C = A \cup (B \cup C)$.

(ii) Let x be an arbitrary element of $A \cap (B \cap C)$. Then,

$x \in A \cap (B \cap C)$

$\Rightarrow$ $x \in A$ and $x \in (B \cap C)$

$\Rightarrow \quad x \in A$ and $(x \in B$ and $x \in C)$
$\Rightarrow \quad (x \in A$ and $x \in B)$ and $x \in C$
$\Rightarrow \quad x \in (A \cap B)$ and $x \in C$
$\Rightarrow \quad x \in (A \cap B) \cap C$
$\therefore \quad A \cap (B \cap C) \subseteq (A \cap B) \cap C.$
Similarly, $(A \cap B) \cap C \subseteq A \cap (B \cap C)$.
Hence, $A \cap (B \cap C) = (A \cap B) \cap C$.

THEOREM 5 *(Distributive Laws) If A, B and C are any three sets, then*
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
i.e. union and intersection are distributive over intersection and union respectively.
[NCERT EXEMPLAR

PROOF (i) Let x be an arbitrary element of $A \cup (B \cap C)$. Then,
$x \in A \cup (B \cap C)$
$\Rightarrow \quad x \in A$ or $x \in (B \cap C)$
$\Rightarrow \quad x \in A$ or $(x \in B$ and $x \in C)$
$\Rightarrow \quad (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$ [$\because$ 'or' is distributive over 'and']
$\Rightarrow \quad x \in (A \cup B)$ and $x \in (A \cup C)$
$\Rightarrow \quad x \in ((A \cup B) \cap (A \cup C))$
$\therefore \quad A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$
Similarly, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$.
Hence, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(ii) Let x be an arbitrary element of $A \cap (B \cup C)$. Then,
$x \in A \cap (B \cup C)$
$\Rightarrow \quad x \in A$ and $x \in (B \cup C)$
$\Rightarrow \quad x \in A$ and $(x \in B$ or $x \in C)$
$\Rightarrow \quad (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$
$\Rightarrow \quad x \in (A \cap B)$ or $x \in (A \cap C)$
$\Rightarrow \quad x \in (A \cap B) \cup (A \cap C)$
$\therefore \quad A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$
Similarly, $(A \cup B) \cup (A \cap C) \subseteq A \cap (B \cup C)$.
Hence, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

THEOREM 6 *(De-Morgan's Laws) If A and B are any two sets, then*
(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

PROOF (i) Let x be an arbitrary element of $(A \cup B)'$. Then,
$x \in (A \cup B)'$
$\Rightarrow \quad x \notin (A \cup B)$
$\Rightarrow \quad x \notin A$ and $\notin B$
$\Rightarrow \quad x \in A'$ and $x \in B'$
$\Rightarrow \quad x \in A' \cap B'$.
$\therefore \quad (A \cap B)' \subseteq A' \cap B'$.
Again, let y be an arbitrary element of $A' \cap B'$. Then,
$y \in A' \cap B'$
$\Rightarrow \quad y \in A'$ and $y \in B'$
$\Rightarrow \quad y \notin A$ and $y \notin B$
$\Rightarrow \quad y \notin A \cup B$.
$\Rightarrow \quad y \in (A \cup B)'$
$\therefore \quad A' \cap B' \subseteq (A \cup B)'$

Hence, $(A \cup B)' = A' \cap B'$.

(ii) Let x be an arbitrary element of $(A \cap B)'$. Then,

$$x \in (A \cap B)'$$
$$\Rightarrow \quad x \notin (A \cap B)$$
$$\Rightarrow \quad x \notin A \text{ or } x \notin B$$
$$\Rightarrow \quad x \in A' \text{ or } x \in B'$$
$$\Rightarrow \quad x \in A' \cup B'$$
$$\Rightarrow \quad (A \cap B)' \subseteq A' \cup B'.$$

Again, let y be an arbitrary element of $A' \cup B'$. Then ,

$$y \in A' \cup B'$$
$$\Rightarrow \quad y \in A' \text{ or } y \in B'$$
$$\Rightarrow \quad y \notin A \text{ or } y \notin B$$
$$\Rightarrow \quad y \notin (A \cap B)$$
$$\Rightarrow \quad y \in (A \cap B)'$$
$$\therefore \quad A' \cup B' \subseteq (A \cap B)'.$$

Hence, $(A \cap B)' = A' \cup B'$.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *If $a \in N$ such that $aN = \{ax : x \in N\}$. Describe the set $3N \cap 7N$.*

SOLUTION We have, $aN = \{ax : x \in N\}$

$\therefore \quad 3N = \{3x : x \in N\} = \{3, 6, 9, 12, \ldots\}$ and, $7N = \{7x : x \in N\} = \{7, 14, 21, 28, \ldots\}$

Hence, $3N \cap 7N = \{21, 42, \ldots\} = \{21x : x \in N\} = 21N$.

EXAMPLE 2 *If $A = \{1, 3, 5, 7, 11, 13, 15, 17\}$, $B = \{2, 4, 6, \ldots, 18\}$ and N is the universal set, then find $A' \cup ((A \cup B) \cap B')$.*

SOLUTION Clearly, $(A \cup B) \cap B' = A$ [$\because$ A, B are disjoint sets]

$\therefore \quad A' \cup ((A \cup B) \cap B') = A' \cup A = N$.

EXAMPLE 3 *For any natural number a, we define $aN = \{ax : x \in N\}$. If $b, c, d \in N$ such that $bN \cup cN = dN$, then prove that d is the l.c.m. of b and c.*

SOLUTION We have,

$bN = \{bx : x \in N\}$ = The set of positive integral multiples of b

$cN = \{cx : x \in N\}$ = The set of positive integral multiples of c

$\therefore \quad bN \cap cN$ = The set of positive integral multiples of b and c both.

$\Rightarrow \quad bN \cap cN = \{kx : x \in N\}$, where k is the l.c.m. of b and c.

Hence, d = l.c.m. of b and c.

LEVEL-2

EXAMPLE 4 *Suppose $A_1, A_2, \ldots A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots, B_n$ are n sets each with three elements. Let $\bigcup\limits_{i=1}^{30} A_i = \bigcup\limits_{j=1}^{n} B_j = S$. Assume that each element of S belongs to exactly ten of the A_i's and exactly 9 of B_j's. Find n.*

SOLUTION Since each A_i has 5 elements and each element of S belongs to exactly 10 of A_i's.

$$\therefore \quad S = \bigcup_{i=1}^{30} A_i \Rightarrow n(S) = \frac{1}{10} \sum_{i=1}^{30} n(A_i) = \frac{1}{10}(5 \times 30) = 15 \quad \text{...(i)}$$

Again, each B_j has 3 elements and each element of S belongs to exactly 9 of B_j's

$$\therefore \quad S = \bigcup_{j=1}^{n} B_j \Rightarrow n(S) = \frac{1}{9}\sum_{j=1}^{n} n(B_j) = \frac{1}{9}(3n) = \frac{n}{3} \quad \text{...(ii)}$$

From (i) and (ii), we get : $15 = \frac{n}{3} \Rightarrow n = 45$.

EXAMPLE 5 *For any two sets A and B, prove that $A \cup B = A \cap B \Leftrightarrow A = B$.*

SOLUTION First let $A = B$. Then,

$$A \cup B = A \text{ and } A \cap B = A \Rightarrow A \cup B = A \cap B$$

Thus, $A = B \Rightarrow A \cup B = A \cap B$ ...(i)

Conversely, let $A \cup B = A \cap B$. Then, we have to prove that $A = B$. For this, let

$$\begin{aligned} x \in A &\Rightarrow x \in A \cup B \\ &\Rightarrow x \in A \cap B && [\because A \cup B = A \cap B] \\ &\Rightarrow x \in A \text{ and } x \in B \\ &\Rightarrow x \in B \end{aligned}$$

$\therefore \quad A \subset B$ ...(ii)

Now, let

$$\begin{aligned} y \in B &\Rightarrow y \in A \cup B \\ &\Rightarrow y \in A \cap B && [\because A \cup B = A \cap B] \\ &\Rightarrow y \in A \text{ and } y \in B \\ &\Rightarrow y \in A \end{aligned}$$

$\therefore \quad B \subset A$ ...(iii)

From (ii) and (iii), we get $A = B$.

Thus, $A \cup B = A \cap B \Rightarrow A = B$ ...(iv)

From (i) and (iv), we obtain

$$A \cup B = A \cap B \Leftrightarrow A = B.$$

EXAMPLE 6 *Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.*

SOLUTION We have,

$$\begin{aligned} & A \cup B = A \cup C \\ \Rightarrow\ & (A \cup B) \cap C = (A \cup C) \cap C \\ \Rightarrow\ & (A \cap C) \cup (B \cap C) = C && [\because (A \cup C) \cap C = C] \\ \Rightarrow\ & (A \cap B) \cup (B \cap C) = C && [\because A \cap C = A \cap B] \quad \text{...(i)} \end{aligned}$$

$$\begin{aligned} \text{Again,}\ & A \cup B = A \cup C \\ \Rightarrow\ & (A \cup B) \cap B = (A \cup C) \cap B \\ \Rightarrow\ & B = (A \cap B) \cup (C \cap B) && [\because (A \cup B) \cap B = B] \\ \Rightarrow\ & B = (A \cap B) \cup (B \cap C) && \text{...(ii)} \end{aligned}$$

From (i) and (ii), we get $B = C$.

EXAMPLE 7 *Let A and B be sets, if $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, prove that $A = B$.*

SOLUTION We have,

$$\begin{aligned} & A \cup X = B \cup X \text{ for some set } X \\ \Rightarrow\ & A \cap (A \cup X) = A \cap (B \cup X) \\ \Rightarrow\ & A = (A \cap B) \cup (A \cap X) && [\because A \cap (A \cup X) = A] \\ \Rightarrow\ & A = (A \cap B) \cup \phi && [\therefore A \cap X = \phi \text{(given)}] \\ \Rightarrow\ & A = A \cap B \\ \Rightarrow\ & A \subset B && \text{...(i)} \end{aligned}$$

Again, $A \cup X = B \cup X$

$\Rightarrow \quad B \cap (A \cup X) = B \cap (B \cup X)$ $\quad [\because B \cap (B \cup X) = B]$

$\Rightarrow \quad (B \cap A) \cup (B \cap X) = B$ $\quad [\because B \cap X = \phi \text{(given)}]$

$\Rightarrow \quad (B \cap A) \cup \phi = B$

$\Rightarrow \quad B \cap A = B$

$\Rightarrow \quad A \cap B = B$

$\Rightarrow \quad B \subset A$ $\quad$ ...(ii)

From (i) and (ii), we get $A = B$.

EXAMPLE 8 *For any two sets A and B, prove that:* $P(A) = P(B) \Rightarrow A = B$.

SOLUTION Let x be an arbitrary element of A. Then, there exists a subset, say X, of set A such that $x \in X$.

Now, $\quad X \subset A$

$\Rightarrow \quad X \in P(A)$

$\Rightarrow \quad X \in P(B)$ $\quad [\because P(A) = P(B)]$

$\Rightarrow \quad X \subset B$

$\Rightarrow \quad x \in B$ $\quad [\because x \in X \text{ and } X \subset B \therefore x \in B]$

Thus, $\quad x \in A \Rightarrow x \in B$ for all $x \in B$.

$\therefore \quad A \subset B$ $\quad$ ...(i)

Now, let y be an arbitrary element of B. Then, there exists a subset, say Y, of set B such that $y \in Y$.

Now, $\quad Y \subset B$

$\Rightarrow \quad Y \in P(B)$

$\Rightarrow \quad Y \in P(A)$ $\quad [\because P(A) = P(B)]$

$\Rightarrow \quad Y \subset A$

$\Rightarrow \quad y \in A$

Thus, $\quad y \in B \Rightarrow y \in A$ for all $y \in B$.

$\therefore \quad B \subset A$ $\quad$ ...(ii)

From (i) and (ii), we obtain $A = B$.

EXAMPLE 9 *For any two sets A and B prove that:* $P(A \cap B) = P(A) \cap P(B)$.

SOLUTION In order to prove that $P(A \cap B) = P(A) \cap P(B)$, it is sufficient to prove that

$$P(A \cap B) \subset P(A) \cap P(B) \text{ and } P(A) \cap P(B) \subset P(A \cap B).$$

First let

$\quad X \in P(A \cap B)$

$\Rightarrow \quad X \subset A \cap B$

$\Rightarrow \quad X \subset A$ and $X \subset B$

$\Rightarrow \quad X \in P(A)$ and $X \in P(B)$

$\Rightarrow \quad X \in P(A) \cap P(B)$

$\therefore \quad P(A \cap B) \subset P(A) \cap P(B)$ $\quad$ ...(i)

Now, let

$\quad Y \in P(A) \cap P(B)$. Then,

$\quad Y \in P(A) \cap P(B)$

$\Rightarrow \quad Y \in P(A)$ and $Y \in P(B)$

$\Rightarrow \quad Y \subset A$ and $Y \subset B$

$\Rightarrow \quad Y \subset A \cap B$

$\Rightarrow \quad Y \in P(A \cap B)$

$\therefore \quad P(A) \cap P(B) \subset P(A \cap B)$ $\quad$ ...(ii)

From (i) and (ii), we get: $P(A \cap B) = P(A) \cap P(B)$.

EXAMPLE 10 *For any two sets A and B prove that* $P(A) \cup P(B) \subset P(A \cup B)$. *But,* $P(A \cup B)$ *is not necessarily a subset of* $P(A) \cup P(B)$.

SOLUTION Let $X \in P(A) \cup P(B)$. Then,

$$\begin{aligned} & X \in P(A) \cup P(B) \\ \Rightarrow \quad & X \in P(A) \text{ or } X \in P(B) \\ \Rightarrow \quad & X \subset A \text{ or } X \subset B \\ \Rightarrow \quad & X \subset A \cup B \\ \Rightarrow \quad & X \in P(A \cup B) \\ \therefore \quad & P(A) \cup P(B) \subset P(A \cup B) \end{aligned}$$

Let $A = \{1, 2\}$ and $B = \{3, 4, 5\}$. Then, we find that $X = \{1, 2, 3, 4\} \subset (A \cup B)$. Therefore, $X \in P(A \cup B)$. But, $X \notin P(A)$, $X \notin P(B)$. So, $X \notin P(A) \cup P(B)$. Thus, $P(A \cup B)$ is not necessarily a subset of $P(A) \cup P(B)$.

EXERCISE 1.6

LEVEL-1

1. Find the smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$.
2. Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$. Verify the following identities:
   (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$ (iv) $A - (B \cup C) = (A - B) \cap (A - C)$
   (v) $A - (B \cap C) = (A - B) \cup (A - C)$ (vi) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.
3. If $U = \{2, 3, 5, 7, 9\}$ is the universal set and $A = \{3, 7\}$, $B = \{2, 5, 7, 9\}$, then prove that:
   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

LEVEL-2

4. For any two sets A and B, prove that
   (i) $B \subset A \cup B$ (ii) $A \cap B \subset A$ (iii) $A \subset B \Rightarrow A \cap B = A$
5. For any two sets A and B, show that the following statements are equivalent:
   (i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$.
6. For three sets A, B and C, show that
   (i) $A \cap B = A \cap C$ need not imply $B = C$. (ii) $A \subset B \Rightarrow C - B \subset C - A$
7. For any two sets, prove that:
   (i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$
8. Find sets A, B and C such that $A \cap B$, $A \cap C$ and $B \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.
9. For any two sets A and B, prove that: $A \cap B = \phi \Rightarrow A \subseteq B'$.
10. If A and B are sets, then prove that $A - B$, $A \cap B$ and $B - A$ are pair wise disjoint.
11. Using properties of sets, show that for any two sets A and B, $(A \cup B) \cap (A \cup B') = A$.
12. For any two sets of A and B, prove that:
    (i) $A' \cup B = U \Rightarrow A \subset B$ (ii) $B' \subset A' \Rightarrow A \subset B$
13. Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.
14. Show that for any sets A and B,
    (i) $A = (A \cap B) \cap (A - B)$ (ii) $A \cup (B - A) = A \cup B$
15. Each set X_r contains 5 elements and each set Y_r contains 2 elements and $\bigcup_{r=1}^{20} X_r = S = \bigcup_{r=1}^{n} Y_r$. If each element of S belongs to exactly 10 of the $X_r^{'s}$ and to exactly 4 of $Y_r^{'s}$, then find the value of n.

ANSWERS

1. $A = \{3, 5, 9\}$ 7. $A = \{1, 2\}, B = \{1, 3\}, C = \{2, 3\}$ 12. False 15. 20

HINTS TO SELECTED PROBLEMS

5. (i) $\Rightarrow$ (ii)

We know that, $A - B = \{x \in A : x \notin B\}$

Since $A \subset B$. Therefore, there is no element in A which does not belong to B.

$\therefore \quad A - B = \phi$

Hence, (i) $\Rightarrow$ (ii).

(ii) $\Rightarrow$ (ii)

We have, $A - B = \phi \Rightarrow A \subset B \Rightarrow A \cup B = B$

Hence, (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (iv)

We have, $A \cup B = B \Rightarrow A \subset B \Rightarrow A \cap B = A$

Hence, (iii) $\Rightarrow$ (iv).

(iv) $\Rightarrow$ (i)

We have, $A \cap B = A \Rightarrow A \subset B$

Hence, (iv) $\Rightarrow$ (i).

Consequently, (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iii) $\Leftrightarrow$ (iv).

6. (i) Let $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$ and $C = \{1, 3, 4, 7, 8\}$. Then, $A \cap B = A \cap C$, but $B \neq C$.

(ii) Let $x \in C - B$. Then,

$x \in C - B$

$\Rightarrow x \in C$ and $x \notin B$ $\quad [\because A \subset B]$

$\Rightarrow x \in C$ and $x \notin A$

$\Rightarrow x \in C - A$

$\therefore \quad C - B \subset C - A$

7. (i) We have,

$A \cup (A \cap B) = (A \cup A) \cap (A \cup B)$ $\quad [\because \cup$ is distributive over $\cap]$

$= A \cap (A \cup B) = A$ $\quad [\because A \subset A \cup B]$

(ii) We have,

$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$ $\quad [\because \cap$ is distributive over $\cup]$

$= A \cup (A \cap B) = A$ $\quad [\because A \cap B \subset A]$

8. $A = \{1, 2\}, B = \{1, 3\}, C = \{2, 3\}$

9. $x \in A \Rightarrow x \notin B$ $\quad [\because A \cap B = \phi]$

$\Rightarrow x \in B'$.

So, $A \subset B'$

10. We have,

$A - B = \{x : x \in A$ and $x \notin B\}, B - A = \{x \in B$ and $x \notin A\}$

$\therefore \quad A - B$ and $B - A$ are disjoint sets

Now,

$x \in A - B \Rightarrow x \in A$ and $x \notin B \Rightarrow x \notin A \cap B$

$\therefore \quad (A - B)$ and $A \cap B$ are disjoint sets.

Similarly, $B - A$ and $A \cap B$ are disjoint sets.

11. $(A \cup B) \cap (A \cup B') = ((A \cup B) \cap A) \cup ((A \cup B) \cup B')$

$= A \cup ((A \cup B) \cap B') = A \cup (A \cap B') \cup (B \cap B')$

$= A \cup (A \cap B') = A$

12. (i) Let $x \in A$. Then,

$$x \in A \Rightarrow x \in U \Rightarrow x \in A' \cup B \Rightarrow x \in B \quad [\because\ x \notin A']$$

$\therefore$ $A \subset B$

(ii) Let $x \in A$. Then,

$$x \in A \Rightarrow x \notin A' \Rightarrow x \notin B' \Rightarrow x \in B \quad [\because\ B' \subset A']$$

$\therefore$ $A \subset B$

1.10 MORE RESULTS ON OPERATIONS ON SETS

THEOREM 1 *If A and B are any two sets, then*

(i) $A - B = A \cap B'$ (ii) $B - A = B \cap A'$

(iii) $A - B = A \Leftrightarrow A \cap B = \phi$ (iv) $(A - B) \cup B = A \cup B$

(v) $(A - B) \cap B = \phi$ (vi) $A \subseteq B \Leftrightarrow B' \subseteq A'$

(vii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

PROOF (i) Let x be an arbitrary element of $A - B$. Then,

$x \in (A - B)$

$\Rightarrow$ $x \in A$ and $x \notin B$

$\Rightarrow$ $x \in A$ and $x \in B'$

$\Rightarrow$ $x \in A \cap B'$

$\therefore$ $A - B \subseteq A \cap B'$ ...(i)

Again, let y be an arbitrary element of $A \cap B'$. Then,

$y \in A \cap B'$

$\Rightarrow$ $y \in A$ and $y \in B'$

$\Rightarrow$ $y \in A$ and $y \notin B$

$\Rightarrow$ $y \in A - B$

$\therefore$ $A \cap B' \subseteq (A - B)$ ...(ii)

Hence, from (i) and (ii), we obtain $A - B = A \cap B'$.

(ii) Proceed as in (i).

(iii) In order to prove that $A - B = A \Leftrightarrow A \cap B = \phi$, we shall prove that :

(a) $A - B = A \Rightarrow A \cap B = \phi$ and, (b) $A \cap B = \phi \Rightarrow A - B = A$.

First, let $A - B = A$. Then we have to prove that $A \cap B = \phi$. If possible, let $A \cap B \neq \phi$. Then,

$A \cap B \neq \phi$

$\Rightarrow$ There exists $x \in A \cap B$

$\Rightarrow$ $x \in A$ and $x \in B$

$\Rightarrow$ $x \in A - B$ and $x \in B$ $[\because A - B = A]$

$\Rightarrow$ $(x \in A$ and $x \notin B)$ and $x \in B$ [By def. of $A - B$]

$\Rightarrow$ $x \in A$ and $(x \notin B$ and $x \in B)$

But $x \notin B$ and $x \in B$ both can never be possible simultaneously. Thus, we arrive at a contradiction. So, our supposition is wrong. Therefore, $A \cap B = \phi$.

Hence, $A - B = A \Rightarrow A \cap B = \phi$ ...(i)

Conversely, let $A \cap B = \phi$. Then we have to prove that $A - B = A$. For this we shall show that $A - B \subseteq A$ and $A \subseteq A - B$.

Let x be an arbitrary element of $A - B$. Then,

$x \in A - B \Rightarrow x \in A$ and $x \notin B \Rightarrow x \in A$

$\therefore$ $A - B \subseteq A$.

Again let y be an arbitrary element of A. Then,

$y \in A$

$\Rightarrow$ $y \in A$ and $y \notin B$ $[\because\ A \cap B = \phi]$

$\Rightarrow \quad y \in A - B$ [By def. of $A - B$]

$\therefore \quad A \subseteq A - B.$

So, we have $A - B \subseteq A$ and $A \subseteq A - B$. Therefore, $A - B = A$.

Thus, $\quad A \cap B = \phi \Rightarrow A - B = A$ ...(ii)

Hence, from (i) and (ii), we have

$$A - B = A \Leftrightarrow A \cap B = \phi.$$

(iv) Let x be an arbitrary element of $(A - B) \cup B$. Then,

$x \in (A - B) \cup B$

$\Rightarrow \quad x \in A - B$ or $x \in B$

$\Rightarrow \quad (x \in A$ and $x \notin B)$ or $x \in B$

$\Rightarrow \quad (x \in A$ or $x \in B)$ and $(x \notin B$ or $x \in B)$

$\Rightarrow \quad x \in A \cup B$

$\therefore \quad (A - B) \cup B \subseteq A \cup B$

Let y be an arbitrary element of $A \cup B$. Then,

$y \in A \cup B$

$\Rightarrow \quad y \in A$ or $y \in B$

$\Rightarrow \quad (y \in A$ or $y \in B)$ and $(y \notin B$ or $y \in B)$

$\Rightarrow \quad (y \in A$ and $y \notin B)$ or $y \in B$

$\Rightarrow \quad y \in (A - B) \cup B$

$\therefore \quad A \cup B \subseteq (A - B) \cup B$

Hence, $(A - B) \cup B = A \cup B$

(v) If possible let $(A - B) \cap B \neq \phi$. Then, there exists at least one element x, (say), in $(A - B) \cap B$.

Now, $\quad x \in (A - B) \cap B$

$\Rightarrow \quad x \in (A - B)$ and $x \in B$

$\Rightarrow \quad (x \in A$ and $x \notin B)$ and $x \in B$

$\Rightarrow \quad x \in A$ and $(x \notin B$ and $x \in B)$

But, $x \notin B$ and $x \in B$ both can never be possible simultaneously. Thus, we arrive at a contradiction. So, our supposition is wrong. Hence, $(A - B) \cap B = \phi$.

(vi) First, let $A \subseteq B$. Then we have to prove that $B' \subseteq A'$. Let x be an arbitrary element of B'. Then,

$x \in B'$

$\Rightarrow \quad x \notin B$

$\Rightarrow \quad x \notin A$ [$\because A \subseteq B$]

$\Rightarrow \quad x \in A'$

$\therefore \quad B' \subseteq A'.$

Thus, $\quad A \subseteq B \Rightarrow B' \subseteq A'.$ ...(i)

Conversely, let $B' \subseteq A'$. Then, we have to prove that $A \subseteq B$. Let y be an arbitrary element of A. Then,

$y \in A$

$\Rightarrow \quad y \notin A'$

$\Rightarrow \quad y \notin B'$ [$\because B' \subseteq A'$].

$\Rightarrow \quad y \in B$

$\therefore \quad A \subseteq B.$

Thus, $\quad B' \subseteq A' \Rightarrow A \subseteq B$ ...(ii)

From (i) and (ii), we obtain that $A \subseteq B \Leftrightarrow B' \subseteq A'$.

(vii) Let x be an arbitrary element of $(A - B) \cup (B - A)$. Then ,

$$x \in (A - B) \cup (B - A)$$
$\Rightarrow \quad x \in A - B$ or $x \in B - A$
$\Rightarrow \quad (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \notin A)$
$\Rightarrow \quad (x \in A$ or $x \notin B)$ and $(x \notin B$ or $x \notin A)$
$\Rightarrow \quad x \in (A \cup B)$ and $x \notin (A \cap B)$
$\Rightarrow \quad x \in (A \cup B) - (A \cap B)$
$\therefore \quad (A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$. ...(i)

Again, let y be an arbitrary element of $(A \cup B) - (A \cap B)$. Then,

$$y \in (A \cup B) - (A \cap B)$$
$\Rightarrow \quad y \in A \cup B$ and $y \notin A \cap B$
$\Rightarrow \quad (y \in A$ or $y \in B)$ and $(y \notin A$ or $y \notin B)$
$\Rightarrow \quad (y \in A$ and $y \notin B)$ or $(y \in B$ and $y \notin A)$
$\Rightarrow \quad y \in (A - B)$ or $y \in (B - A)$
$\Rightarrow \quad y \in (A - B) \cup (B - A)$
$\therefore \quad (A \cup B) - (A \cap B) \subseteq (A - B) \cup (B - A)$. ...(ii)

Hence, from (i) and (ii), we have

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B).$$

THEOREM 2 *If A, B and C are any three sets, then prove that:*

(i) $A - (B \cap C) = (A - B) \cup (A - C)$ (ii) $A - (B \cup C) = (A - B) \cap (A - C)$

(iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$ (iv) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

PROOF (i) Let x be any element of $A - (B \cap C)$. Then,

$$x \in A - (B \cap C)$$
$\Rightarrow \quad x \in A$ and $x \notin (B \cap C)$
$\Rightarrow \quad x \in A$ and $(x \notin B$ or $x \notin C)$
$\Rightarrow \quad (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$
$\Rightarrow \quad x \in (A - B)$ or $x \in (A - C)$
$\Rightarrow \quad x \in (A - B) \cup (A - C)$
$\therefore \quad A - (B \cap C) \subseteq (A - B) \cup (A - C)$

Similarly, $(A - B) \cup (A - C) \subseteq A - (B \cap C)$

Hence, $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) Let x be an arbitrary element of $A - (B \cup C)$. Then

$$x \in A - (B \cup C)$$
$\Rightarrow \quad x \in A$ and $x \notin (B \cup C)$
$\Rightarrow \quad x \in A$ and $(x \notin B$ and $x \notin C)$
$\Rightarrow \quad (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$
$\Rightarrow \quad x \in (A - B)$ and $x \in (A - C)$
$\Rightarrow \quad x \in (A - B) \cap (A - C)$
$\therefore \quad A - (B \cup C) \subseteq (A - B) \cap (A - C)$

Similarly, $(A - B) \cap (A - C) \subseteq A - (B \cup C)$

Hence, $A - (B \cup C) = (A - B) \cap (A - C)$

(iii) Let x be any arbitrary element of $A \cap (B - C)$. Then

$$x \in A \cap (B - C)$$
$\Rightarrow \quad x \in A$ and $x \in (B - C)$
$\Rightarrow \quad x \in A$ and $(x \in B$ and $x \notin C)$

$\Rightarrow \quad (x \in A \text{ and } x \in B) \text{ and } (x \in A \text{ and } x \notin C)$

$\Rightarrow \quad x \in (A \cap B) \text{ and } x \notin (A \cap C)$

$\Rightarrow \quad x \in (A \cap B) - (A \cap C)$

$\therefore \quad A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$

Similarly, $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$

Hence, $A \cap (B - C) = (A \cap B) - (A \cap C)$.

(iv) $A \cap (B \Delta C) = A \cap [(B - C) \cup (C - B)]$

$= [A \cap (B - C)] \cup [A \cap (C - B)]$ [By distributive law]

$= [(A \cap B) - (A \cap C)] \cup [(A \cap C) - (A \cap B)]$ [Using (iii)]

$= (A \cap B) \Delta (A \cap C)$

ILLUSTRATIVE EXAMPLES

LEVEL-2

EXAMPLE 1 *Let A and B be two sets. Using properties of sets prove that:*

(i) $A \cap B' = \phi \Rightarrow A \subset B$ (ii) $A' \cup B = U \Rightarrow A \subset B$

SOLUTION (i) We have,

$A = (A \cap U)$

$\Rightarrow \quad A = A \cap (B \cup B')$ [$\because B \cup B' = U$]

$\Rightarrow \quad A = (A \cap B) \cup (A \cap B')$ [$\because \cap$ is distributive over union]

$\Rightarrow \quad A = (A \cap B) \cup \phi$ [$\because A \cap B' = \phi$]

$\Rightarrow \quad A = A \cap B$

$\therefore \quad A \subset B$

(ii) From (i), we have

$A \cap B' = \phi$

$\Leftrightarrow \quad (A \cap B')' = \phi'$

$\Leftrightarrow \quad A' \cup (B')' = U$ [$\because \phi' = U$]

$\Leftrightarrow \quad A' \cup B = U$ [$\because (B')' = B$]

Thus, $A \cap B' = \phi \Leftrightarrow A' \cup B = U$ and, $A \cap B' = \phi \Rightarrow A \subset B$

$\therefore \quad A' \cup B = U \Rightarrow A \subset B$

ALITER We have,

$A' \cup B = U$

$\Rightarrow \quad A \cap (A' \cup B) = A \cap U$ [Taking intersection with A]

$\Rightarrow \quad (A \cap A') \cup (A \cap B) = A$ [$\because A \cap U = A$]

$\Rightarrow \quad \phi \cup (A \cap B) = A$

$\Rightarrow \quad A \cap B = A$

$\Rightarrow \quad A \subset B$

EXAMPLE 2 *Let A and B be two sets. Prove that: $(A - B) \cup B = A$ if and only if $B \subset A$.*

SOLUTION First let, $(A - B) \cup B = A$. Then, we have to prove that $B \subset A$.

Now, $(A - B) \cup B = A$

$\Rightarrow \quad (A \cap B') \cup B = A$ [$\because A - B = A \cap B'$]

$\Rightarrow \quad (A \cup B) \cap (B' \cup B) = A$

$\Rightarrow \quad (A \cup B) \cap U = A$

$\Rightarrow \quad A \cup B = A$

$\Rightarrow \quad B \subset A$.

Conversely, let $B \subset A$. Then, we have to prove that $(A - B) \cup B = A$.

Now, $(A - B) \cup B = (A \cap B') \cup B$

$= (A \cup B) \cap (B' \cup B)$

$$= (A \cup B) \cap U$$
$$= A \cup B$$
$$= A \qquad [\because B \subset A \therefore A \cup B = A]$$

EXAMPLE 3 *Let A, B and C be three sets such that $A \cup B = C$ and $A \cap B = \phi$. Then, prove that $A = C - B$.*

SOLUTION We have, $A \cup B = C$.

$\therefore$
$$C - B = (A \cup B) - B$$
$$= (A \cup B) \cap B' \qquad [\because X - Y = X \cap Y']$$
$$= (A \cap B') \cup (B \cap B')$$
$$= (A \cap B') \cup \phi$$
$$= A \cap B'$$
$$= A - B$$
$$= A \qquad [\because A \cap B = \phi]$$

EXAMPLE 4 *Let A and B be any two sets. Using properties of sets prove that:*

(i) $(A - B) \cup B = A \cup B$ (ii) $(A - B) \cup A = A$

(iii) $(A - B) \cap B = \phi$ (iv) $(A - B) \cap A = A \cap B'$

SOLUTION (i) We have,
$$(A - B) \cup B = (A \cap B') \cup B \qquad [\because A - B = A \cap B']$$
$$= (A \cup B) \cap (B' \cup B) \qquad [\because \cup \text{ is distributive over } \cap]$$
$$= (A \cup B) \cap U \qquad [\because B' \cup B = U]$$
$$= A \cup B$$

(ii) $(A - B) \cup A = A \qquad [\because A - B \subset A]$

(iii) $(A - B) \cap B = (A \cap B') \cap B = A \cap (B' \cap B) = A \cap \phi = \phi$

(iv) $(A - B) \cap A = A - B \qquad [\because A - B \subset A]$
$$= A \cap B'$$

EXAMPLE 5 *For any two sets A and B prove by using properties of sets that:*

(i) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$ (ii) $(A \cap B) \cup (A - B) = A$

(iii) $(A \cup B) - A = B - A$

SOLUTION (i) We have,
$$(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)' \qquad [\because X - Y = X \cap Y']$$
$$= (A \cup B) \cap (A' \cup B') \qquad [\because (A \cap B)' = A' \cup B']$$
$$= X \cap (A' \cup B'), \text{ where } X = A \cup B$$
$$= (X \cap A') \cup (X \cap B')$$
$$= (B \cap A') \cup (A \cap B') \qquad \left[\begin{aligned} &\because X \cap A' = (A \cup B) \cap A' \\ &= (A \cap A') \cup (B \cap A') = \phi \cup (B \cap A') \\ &= B \cap A' \text{ Similarly, } X \cap B' = A \cap B' \end{aligned}\right]$$
$$= (A \cap B') \cup (B \cap A')$$
$$= (A - B) \cup (B - A) \qquad [\because A - B = A \cap B' \text{ and } B - A = B \cap A']$$

(ii)
$$(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$$
$$= X \cup (A \cap B'), \text{ where } X = A \cap B$$
$$= (X \cup A) \cap (X \cup B')$$
$$= A \cap (A \cup B') \qquad \left[\begin{aligned} &\because X \cup A = (A \cap B) \cup A = A\ [\because A \cap B \subset A] \\ &X \cup B' = (A \cap B) \cup B' = (A \cup B') \cap (B \cup B') \\ &= (A \cup B') \cap U = A \cup B' \end{aligned}\right]$$
$$= A \qquad [\because A \subset A \cup B']$$

(iii)
$$(A \cup B) - A = (A \cup B) \cap A' \qquad [\because X - Y = X \cap Y']$$
$$= (A \cap A') \cup (B \cap A')$$
$$= \phi \cup (B \cap A') \qquad [\because A \cap A' = \phi]$$

$= B \cap A'$

$= B - A$ $[\because B - A = B \cap A']$

EXAMPLE 6 *For sets A, B and C using properties of sets, prove that:*

(i) $A - (B \cup C) = (A - B) \cap (A - C)$ **[NCERT EXEMPLAR]**

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

(iii) $(A \cup B) - C = (A - C) \cup (B - C)$

(iv) $(A \cap B) - C = (A - C) \cap (B - C)$ **[NCERT EXEMPLAR]**

SOLUTION (i) We have,

$A - (B \cup C) = A \cap (B \cup C)'$ $[\because X - Y = X \cap Y']$

$= A \cap (B' \cap C')$ $[\because (B \cup C)' = B' \cap C']$

$= (A \cap B') \cap (A \cap C')$

$= (A - B) \cap (A - C)$

(ii) $A - (B \cap C) = A \cap (B \cap C)'$ $[\because X - Y = X \cap Y']$

$= A \cap (B' \cup C')$ $[\because (B \cap C)' = B' \cup C']$

$= (A \cap B') \cup (A \cap C')$ [$\because \cap$ is distributive over $\cup$]

$= (A - B) \cup (A - C)$

(iii) $(A \cup B) - C = (A \cup B) \cap C'$ $[\because X - Y = X \cap Y']$

$= (A \cap C') \cup (B \cap C')$

$= (A - C) \cup (B - C)$

(iv) $(A \cap B) - C = (A \cap B) \cap C'$

$= (A \cap C') \cap (B \cap C')$

$= (A - C) \cap (B - C)$

EXAMPLE 7 *For sets A, B and C using properties of sets, prove that:*

(i) $A - (B - C) = (A - B) \cup (A \cap C)$ (ii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

SOLUTION (i) We have,

$A - (B - C) = A - (B \cap C')$ $[\because B - C = B \cap C']$

$= A \cap (B \cap C')'$ $[\because X - Y = X \cap Y']$

$= A \cap (B' \cup C)$ $[\because (B \cap C')' = B' \cup (C')' = B' \cup C]$

$= (A \cap B') \cup (A \cap C)$

$= (A - B) \cup (A \cap C)$

(ii) $A \cap (B - C) = A \cap (B \cap C')$ $[\because B - C = B \cap C']$

$= (A \cap B) \cap C'$

$= \phi \cup ((A \cap B) \cap C')$

$= ((A \cap B) \cap A') \cup ((A \cap B) \cap C')$ $[\because (A \cap B) \cap A' = \phi]$

$= (A \cap B) \cap (A' \cup C')$

$= (A \cap B) \cap (A \cap C)'$

$= (A \cap B) - (A \cap C)$

EXERCISE 1.7

LEVEL-1

1. For any two sets A and B, prove that : $A' - B' = B - A$
2. For any two sets A and B, prove the following :

   (i) $A \cap (A' \cup B) = A \cap B$ (ii) $A - (A - B) = A \cap B$

   (iii) $A \cap (A \cup B)' = \phi$ (iv) $A - B = A \,\Delta\, (A \cap B)$.
3. If A, B, C are three sets such that $A \subset B$, then prove that $C - B \subset C - A$.

4. For any two sets A and B, prove that
   (i) $(A \cup B) - B = A - B$ (ii) $A - (A \cap B) = A - B$
   (iii) $A - (A - B) = A \cap B$ (iv) $A \cup (B - A) = A \cup B$ [NCERT EXEMPLAR]
   (v) $(A - B) \cup (A \cap B) = A$ [NCERT EXEMPLAR]

HINTS TO SELECTED PROBLEMS

1. We know that $X - Y = X \cap Y'$. So $A' - B' = A' \cap (B')' = A' \cap B = B \cap A' = B - A$
2. (ii) $A - (A - B) = A - (A \cap B') = A \cap (A \cap B')' = A \cap (A' \cup (B')') = A \cap (A' \cup B) = A \cap B$
3. Let $x \in C - B$. Then,
   $x \in C - B \Rightarrow x \in C$ and $x \notin B \Rightarrow x \in C$ and $x \notin A \Rightarrow x \in C - A$ $[\because A \subset B]$
   $\therefore C - B \subset C - A$.

1.11 SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS

If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint non-void sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$ i.e. $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \,\Delta\, B) =$ No. of elements which belong to exactly one of A or B
$= n((A - B) \cup (B - A))$
$= n(A - B) + n(B - A)$ $[\because (A - B)$ and $(B - A)$ are disjoint$]$
$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$
$= n(A) + n(B) - 2\,n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B, C
$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3\,n(A \cap B \cap C)$.

(vii) Number of elements in exactly one of the sets A, B, C
$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(viii) $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *If X and Y are two sets such that $n(X) = 17, n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.*

SOLUTION We know that
$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y) \Rightarrow 38 = 17 + 23 - n(X \cap Y) \Rightarrow n(X \cap Y) = 40 - 38 = 2.$

EXAMPLE 2 *If X and Y are two sets such that $n(X) = 45, n(X \cup Y) = 76$ and $n(X \cap Y) = 12$, find $n(Y)$.*

SOLUTION We know that
$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y) \Rightarrow 76 = 45 + n(Y) - 12 \Rightarrow n(Y) = 43$$
Hence, $n(Y) = 43$.

EXAMPLE 3 *If A and B are two sets such that $n(A) = 17, n(B) = 23$ and $n(A \cup B) = 38$, find*
(i) $n(A \cap B)$ (ii) $n(A - B)$ (iii) $n(B - A)$ (iv) *number of elements is exactly one of A and B.*

SOLUTION (i) We know that
$$n(A \cap B) = n(A) + n(B) - n(A \cup B) \Rightarrow n(A \cap B) = 17 + 23 - 38 = 2$$
(ii) We know that
$$n(A - B) = n(A) - n(A \cap B) \Rightarrow n(A - B) = 17 - 2 = 15$$

(iii) We know that

$$n(B-A)=n(B)-n(A\cap B) \Rightarrow n(B-A)=23-2=21$$

(iv) We know that

Number of elements in exactly one of A and $B = n(A)+n(B)-2n(A\cap B)=17+23-2\times 2=36$

EXAMPLE 4 *Let A and B be two sets such that $n(A)=35, n(A\cap B)=11$ and $n((A\cup B)')=17$. If $n(U)=57$, find*

(i) $n(B)$ (ii) $n(A-B)$ (iii) $n(B-A)$

SOLUTION (i) We have,

$n((A\cup B)')=17$

$\Rightarrow \quad n(U)-n(A\cup B)=17$

$\Rightarrow \quad 57-n(A\cup B)=17$

$\Rightarrow \quad n(A\cup B)=57-17=40$

$\Rightarrow \quad n(A)+n(B)-n(A\cap B)=40 \Rightarrow 35+n(B)-11=40 \Rightarrow n(B)=16$

(ii) We know that

$$n(A-B)=n(A)-n(A\cap B) \Rightarrow n(A-B)=35-11=24$$

(iii) We know that

$$n(B-A)=n(B)-n(A\cap B) \Rightarrow n(B-A)=16-11=5$$

EXAMPLE 5 *Let A and B be two sets and U be the universal set such that $n(A)=25, n(B)=28$ and $n(U)=50$. Find*

(i) the greatest value of $n(A\cup B)$ *(ii) the least value of $n(A\cap B)$.*

SOLUTION (i) We know that

$$A\cup B\subset U \Rightarrow n(A\cup B)\le n(U) \Rightarrow n(A\cup B)\le 50$$

Hence, the greatest value of $n(A\cup B)$ is 50.

(ii) From (i), we have

$$n(A\cup B)\le 50 \Rightarrow n(A)+n(B)-n(A\cap B)\le 50 \Rightarrow 25+28-n(A\cap B)\le 50 \Rightarrow n(A\cap B)\ge 3$$

Hence, the least value of $n(A\cap B)$ is 3.

EXAMPLE 6 *If A and B are two sets and U is the universal set such that $n(U)=700$, $n(A)=200$, $n(B)=300$ and $n(A\cap B)=100$. Find $n(A'\cap B')$.*

SOLUTION We have, $A'\cap B'=(A\cup B)'$

$\therefore \quad n(A'\cap B')=n((A\cup B)')=n(U)-n(A\cup B)$

Thus, $n(A'\cap B')=n(U)-[n(A)+n(B)-n(A\cap B)]=700-(200+300-100)=300.$

EXAMPLE 7 *In a group of 800 people, 550 can speak Hindi and 450 can speak English. How many can speak both Hindi and English?*

SOLUTION Let H denote the set of people speaking Hindi and E denote the set of people speaking English. We are given that: $n(H)=550, n(E)=450$ and $n(H\cup E)=800$ and we have to find $n(H\cap E)$.

Now, $\quad n(H\cup E)=n(H)+n(E)-n(H\cap E)$

$\Rightarrow \quad n(H\cap E)=n(H)+n(E)-n(H\cup E)=550+450-800=200.$

Hence, 200 persons can speak both Hindi and English.

EXAMPLE 8 *In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?*

SOLUTION Let C be the set of students who like to play cricket and F be the set of students who like to play football. Then, $C\cup F$ is the set of students who like to play at least one game and, $C\cap F$ is the set of all students who like to play both games. It is given that $n(C)=24, n(F)=16$, $n(C\cup F)=35$ and we have to find $n(C\cap F)$.

Now, $n(C \cup F) = n(C) + n(F) - n(C \cap F)$

$\Rightarrow \quad n(C \cap F) = n(C) + n(F) - n(C \cup F) = 24 + 16 - 35 = 5.$

EXAMPLE 9 *In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all the people speak at least one of the two languages. How many people speak only English and not Hindi ? How many people speak English?*

SOLUTION Let H denote the set of people speaking Hindi and E the set of people speaking English. Then, it is given that: $n(H \cup E) = 50, n(H) = 35, n(H \cap E) = 25.$

Now, $n(E - H) = n(H \cup E) - n(H) = 50 - 35 = 15$

Thus, the number of people speaking English but not Hindi is 15.

Now, $n(H \cup E) = n(H) + n(E) - n(H \cap E) \Rightarrow 50 = 35 + n(E) - 25 \Rightarrow n(E) = 40$

Hence, the number of people who speak English is 40.

EXAMPLE 10 *There are 40 students in a Chemistry class and 60 students in a Physics class. Find the number of students which are either in Physics class or Chemistry class in the following cases:*

(i) *the two classes meet at the same hour.*

(ii) *the two classes meet at different hours and 20 students are enrolled in both the subjects.*

SOLUTION Let A be the set of students in Chemistry class and B be the set of students in Physics class. It is given that $n(A) = 40$ and $n(B) = 60$. We have to find $n(A \cup B)$ in both the cases.

(i) If two classes meet at the same hour, then there will not be a common student sitting in both the classes. Therefore, $n(A \cap B) = 0.$

$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B) = 40 + 60 - 0 = 100$

(ii) If two classes meet at different timings then there can be some students attending both the classes. It is given that the number of such students is 20 i.e. $n(A \cap B) = 20.$

$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B) = 40 + 60 - 20 = 80.$

EXAMPLE 11 *In a survey of 700 students in a college, 180 were listed as drinking Limca, 275 as drinking Miranda and 95 were listed as both drinking Limca as well as Miranda. Find how many students were drinking neither Limca nor Miranda.*

SOLUTION Let U be the set of all surveyed students, A denote the set of students drinking Limca and B be the set students drinking Miranda. It is given that $n(U) = 700, n(A) = 180, n(B) = 275$ and $n(A \cap B) = 95$. We have to find $n(A' \cap B')$.

Now, $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B) = n(U) - \{n(A) + n(B) - n(A \cap B)\}$

$\Rightarrow \quad n(A' \cap B') = 700 - (180 + 275 - 95) = 700 - 360 = 340.$

LEVEL-2

EXAMPLE 12 *There are 200 individuals with a skin disorder, 120 has been exposed to chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to (i) chemical C_1 or chemical C_2 (ii) chemical C_1 but not chemical C_2 (iii) chemical C_2 but not chemical C_1.*

SOLUTION Let U denote the universal set consisting of individuals suffereing from the skin disorder, A denote the set of individuals exposed to chemical C_1 and B denote the set of individuals exposed to chemical C_2. It is given that: $n(U) = 200, n(A) = 120, n(B) = 50$ and $n(A \cap B) = 30.$

(i) The number of individuals exposed to chemical C_1 or chemical C_2 is given by $n(A \cup B)$.

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 120 + 50 - 30 = 140$

Hence, required number of individuals is 140.

(ii) The number of individuals exposed to chemical C_1 but not chemical C_2 is given by $n(A \cap \overline{B})$.

Now, $n(A \cap \overline{B}) = n(A) - n(A \cap B) = 120 - 30 = 90$

Hence, required number of individuals is 90.

(iii) The number of individuals exposed to chemical C_2 but not chemical C_1 is given by $n(\bar{A} \cap B)$.

Now, $n(\bar{A} \cap B) = n(B) - n(A \cap B) = 50 - 30 = 20$

Hence, required number is 20.

EXAMPLE 13 *Out of 500 car owners investigated, 400 owned Maruti car and 200 owned Hyundai car; 50 owned both cars. Is this data correct?*

SOLUTION Let U be the set of all car owners investigated, M be the set of persons who owned Maruti cars and H be the set of persons who owned Hyundai cars. It is given that $n(U) = 500$, $n(M) = 400$, $n(H) = 200$ and $n(M \cap H) = 50$.

Now, $n(M \cup H) = n(M) + n(H) - n(M \cap H) = 400 + 200 - 50 = 550$

But, $M \cup H \subseteq U$.

$\therefore \quad n(M \cup H) \leq n(U) \Rightarrow n(M \cup H) \leq 500$

This is a contradication. So, the given data is incorrect.

EXAMPLE 14 *If A and B are finite sets such that $n(A) = m_1$ and $n(B) = m_2$, then find the least and greatest values of $n(A \cup B)$.*

SOLUTION We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad n(A \cup B) \leq n(A) + n(B)$ $\quad [\because n(A \cap B) \geq 0]$

$\Rightarrow \quad n(A \cup B) \leq m_1 + m_2$ ...(i)

So, the greatest value of $n(A \cup B)$ is $m_1 + m_2$. It may be noted that $n(A \cup B) = m_1 + m_2$ only when $n(A \cap B) = 0$ i.e. when $A \cap B = \phi$ i.e. when A and B are disjoint sets.

Also, we know that

$A \subseteq A \cup B$ and $B \subseteq A \cup B$

$\Rightarrow \quad n(A) \leq n(A \cup B)$ and $n(B) \leq n(A \cup B)$

$\Rightarrow \quad m_1 \leq n(A \cup B)$ and $m_2 \leq n(A \cup B)$

$\Rightarrow \quad n(A \cup B) \geq m_1$ and $n(A \cup B) \geq m_2$

$\Rightarrow \quad n(A \cup B) \geq \max\{m_1, m_2\}$ ...(ii)

So, the least value of $n(A \cup B)$ is $\max\{m_1, m_2\}$.

It may be noted that $n(A \cup B) = \max\{m_1, m_2\}$ only when either $A \subseteq B$ or $B \subseteq A$.

From (i) and (ii), we obtain

$$\max\{m_1, m_2\} \leq n(A \cup B) \leq m_1 + m_2 \text{ or, } \max\{n(A), n(B)\} \leq n(A \cup B) \leq n(A) + n(B)$$

EXAMPLE 15 *If A and B are two sets such that $n(A) = 35$, $n(B) = 30$ and $n(U) = 50$, then find*

(i) the greatest value of $n(A \cup B)$ *(ii) the least value of $(A \cap B)$*

SOLUTION (i) We know that

$A \cup B \subseteq U \Rightarrow n(A \cup B) \leq n(U) \Rightarrow n(A \cup B) \leq 50$

So, the greatest value of $n(A \cup B)$ is 50.

(ii) From (i), we have

$n(A \cup B) \leq 50$

$\Rightarrow \quad n(A) + n(B) - n(A \cap B) \leq 50 \Rightarrow 35 + 30 - n(A \cap B) \leq 50 \Rightarrow 15 \leq (A \cap B) \Rightarrow n(A \cap B) \geq 15$

So, the least value of $n(A \cap B)$ is 15.

EXAMPLE 16 *If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.*

SOLUTION We have, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

CASE I *When $n(A \cap B)$ is minimum, i.e. $n(A \cap B) = 0$.*

This is possible only when $A \cap B = \phi$. In this case,

$$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9$$

So, maximum number of elements in $A \cup B$ is 9.

CASE II *When $n(A \cap B)$ is maximum.*

This is possible only when $A \subseteq B$. In this case, $n(A \cap B) = 3$.

$$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B) = 3 + 6 - 3 = 6$$

So, minimum number of elements in $A \cup B$ is 6.

ALITER We know that

$$\max\{n(A), n(B)\} \leq n(A \cup B) \leq n(A) + n(B) \quad \max\{3, 6\} \leq n(A \cup B) \leq 3 + 6$$

$$\Rightarrow \quad 6 \leq n(A \cup B) \leq 9$$

Hence, the minimum and maximum number of elements in $A \cup B$ is 6 and 9 respectively.

EXAMPLE 17 *A market research group conducted a survey of 2000 consumers and reported that 1720 consumers liked product P_1 and 1450 consumers liked product P_2. What is the least number that must have liked both the products ?*

SOLUTION Let U be the set of all consumers who were questioned, A be the set of consumers who liked product P_1 and B be the set of consumers who liked the product P_2. It is given that $n(U) = 2000, n(A) = 1720, n(B) = 1450$.

$$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B) = 1720 + 1450 - n(A \cap B) = 3170 - n(A \cap B)$$

Now, $\quad A \cup B \subset U$

$$\Rightarrow \quad n(A \cup B) \leq n(U) \Rightarrow 3170 - n(A \cap B) \leq 2000 \Rightarrow 3170 - 2000 \leq n(A \cap B)$$

$$\Rightarrow \quad n(A \cap B) \geq 1170$$

Thus, the least value of $n(A \cap B)$ is 1170. Hence, the least number of consumer who liked both the products is 1170.

EXAMPLE 18 *A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, find the value of x.*

SOLUTION Let A denote the set of Americans who like cheese and let B denote those who like apples. Let the population of America be 100. Then, $n(A) = 63, n(B) = 76$.

Now, $\quad n(A \cap B) = n(A) + n(B) - n(A \cup B) \Rightarrow n(A \cap B) = 63 + 76 - n(A \cup B) = 139 - n(A \cup B)$

But, $\quad n(A \cup B) \leq 100$.

$$\Rightarrow \quad -n(A \cup B) \geq -100$$

$$\Rightarrow \quad 139 - n(A \cup B) \geq 139 - 100 \Rightarrow 139 - n(A \cup B) \geq 39 \Rightarrow n(A \cap B) \geq 39 \qquad \text{...(i)}$$

Now, $\quad A \cap B \subseteq A$ and $A \cap B \subseteq B$

$$\Rightarrow \quad n(A \cap B) \leq n(A) \text{ and } n(A \cap B) \leq n(B)$$

$$\Rightarrow \quad n(A \cap B) \leq 63 \text{ and } n(A \cap B) \leq 76 \Rightarrow n(A \cap B) \leq 63 \qquad \text{...(ii)}$$

From (i) and (ii), we obtain

$$39 \leq n(A \cap B) \leq 63 \Rightarrow 39 \leq x \leq 63.$$

EXAMPLE 19 *In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. Find the number of students who have taken both Mathematics and Economics and the number of students who have taken Economics but not Mathematics, if it is given that each student has taken either Mathematics or Economics or both.*

SOLUTION Let A denote the set of students who have taken Mathematics and B be the set of students who have taken Economics. It is given that $n(A \cup B) = 35, n(A) = 17$ and $n(A - B) = 10$.

Now, $\quad n(A) = n(A - B) + n(A \cap B) \Rightarrow 17 = 10 + n(A \cap B) \Rightarrow n(A \cap B) = 7$

Thus, 7 students have taken both Mathematics and Economics.

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) \Rightarrow 35 = 17 + n(B) - 7 \Rightarrow n(B) = 25$

Also, $n(B) = n(B - A) + n(A \cap B) \Rightarrow 25 = n(B - A) + 7 \Rightarrow n(B - A) = 18.$

Thus, 18 students have taken Economics but not Mathematics.

EXAMPLE 20 *In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three news papers, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C.*

SOLUTION Let P, Q and R denote the sets of families buying newspaper A, B and C respectively. Let U be the universal set. Then,

$n(P) = 40\%$ of $10{,}000 = 4000$, $n(Q) = 20\%$ of $10{,}000 = 2000$, $n(R) = 10\%$ of $10{,}000 = 1000$,

$n(P \cap Q) = 5\%$ of $10{,}000 = 500$, $n(Q \cap R) = 3\%$ of $10{,}000 = 300$, $n(R \cap P) = 4\%$ of $10{,}000 = 400$

$n(P \cap Q \cap R) = 2\%$ of $10{,}000 = 200$ and $n(U) = 10{,}000$.

(i) Required number $= n(P \cap Q' \cap R') = n(P \cap (Q \cup R)')$

$= n(P) - n[P \cap (Q \cup R)]$ $\quad [\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(P) - n[(P \cap Q) \cup (P \cap R)]$

$= n(P) - [n(P \cap Q) + n(P \cap R) - n\{(P \cap Q) \cap (P \cap R)\}]$

$= n(P) - [n(P \cap Q) + n(P \cap R) - n(P \cap Q \cap R)]$

$= 4000 - (500 + 400 - 200) = 3300$

(ii) Required number $= n(P' \cap Q \cap R') = n(Q \cap P' \cap R') = n(Q \cap (P \cup R)')$

$= n(Q) - n(Q \cap (P \cup R))$ $\quad [\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(Q) - n[(Q \cap P) \cup (Q \cap R)]$

$= n(Q) - [n(Q \cap P) + n(Q \cap R) - n\{(Q \cap P) \cap (Q \cap R)\}]$

$= n(Q) - [n(P \cap Q) + n(Q \cap R) - n(P \cap Q \cap R)]$

$= 2000 - (500 + 300 - 200) = 1400$

(iii) Required number $= n(P' \cap Q' \cap R') = n[(P \cup Q \cup R)']$

$= n(U) - n(P \cup Q \cup R)$

$= n(U) - [n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(R \cap P) + n(P \cap Q \cap R)]$

$= 10000 - [4000 + 2000 + 1000 - 500 - 300 - 400 + 200] = 4000.$

ALITER It is given that $n(P \cap Q \cap R) = 200$ and $n(P \cap Q) = 500$. So, the number of families buying newspaper A and B only is $500 - 200 = 300$. Similarly, the number of families buying news papers B and C only is $300 - 200 = 100$ and news papers C and A only is $400 - 200 = 200$.

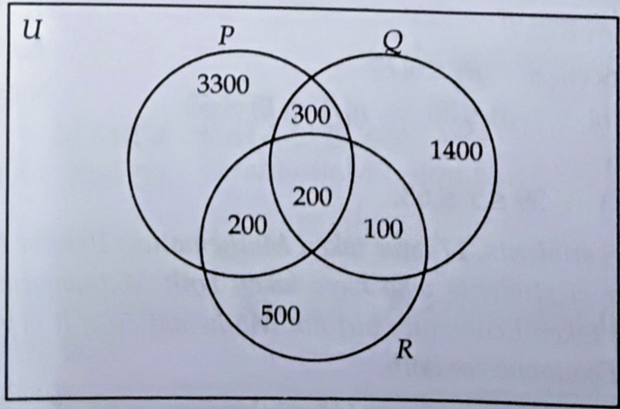

Fig. 1.13

(i) It is given that 4000 families buy news paper A. So, the number of families buying news paper A only $= 4000 - (300 + 200 + 200) = 3300$.

(ii) Similarly,

The number of families buying news paper B only $= 2000 - (300 + 200 + 100) = 1400$

The number of families buying news paper C only $= 1000 - (200 + 200 + 100) = 500$

(iii) The number of families buying none of the news papers

$$= n(U) - n(P \cup Q \cup R) = 10000 - (3300 + 300 + 200 + 200 + 100 + 1400 + 500) = 4000$$

EXAMPLE 21 *A college awarded 38 medals in Football, 15 in Basketball and 20 to Cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?*

SOLUTION Let F denote the set of men who received medals in Football, B the set of men who received medals in Basketball and C the set of men who received medals in Cricket. It is given that $n(F) = 38$, $n(B) = 15$, $n(C) = 20$, $n(F \cup B \cup C) = 58$ and $n(F \cap B \cap C) = 3$.

Now,

$$n(F \cup B \cup C) = n(F) + n(B) + n(C) - n(F \cap B) - n(B \cap C) - n(F \cap C) + n(F \cap B \cap C)$$

$$\Rightarrow \quad 58 = 38 + 15 + 20 - n(F \cap B) - n(B \cap C) - n(F \cap C) + 3$$

$$\Rightarrow \quad n(F \cap B) + n(B \cap C) + n(F \cap C) = 76 - 58 = 18$$

Number of men who received medals in exactly two of the three sports

$$= n(F \cap B) + n(B \cap C) + n(F \cap C) - 3\,n(F \cap B \cap C) = 18 - 3 \times 3 = 9$$

Thus, 9 men received medals in exactly two of the three sports.

EXAMPLE 22 *In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken*

(i) *only Chemistry.*
(ii) *only Mathematics.*
(iii) *only Physics.*
(iv) *Physics and Chemistry but not Mathematics.*
(v) *Mathematics and Physics but not Chemistry.*
(vi) *only one of the subjects.*
(vii) *at least one of the three subjects.*
(viii) *none of the subjects.*

SOLUTION Let M denote the set of students who had taken Mathematics, P the set of students who had taken Physics and C the set of students who had taken Chemistry. It is given that

$$n(U) = 25, n(M) = 15, n(P) = 12, n(C) = 11, n(M \cap C) = 5, n(M \cap P) = 9, n(P \cap C) = 4$$

and, $\quad n(M \cap P \cap C) = 3$

(i) Number of students who had opted Chemistry only

$$= n(M' \cap P' \cap C) = n((M \cup P)' \cap C)$$

$$= n(C) - n((M \cup P) \cap C) \qquad [\because n(A \cap B') = n(A) - n(A \cap B)]$$

$$= n(C) - n((M \cap C) \cup (P \cap C))$$

$$= n(C) - \{n(M \cap C) + n(P \cap C) - n(M \cap P \cap C)\} = 11 - (5 + 4 - 3) = 5$$

(ii) The number of students who had opted Mathematics only.

$$= n(M \cap P' \cap C')$$

$$= n(M \cap (P \cap C)')$$

$$= n(M) - n(M \cap (P \cup C))$$

$$= n(M) - n((M \cap P) \cup (M \cap C))$$

$$= n(M) - \{n(M \cap P) + n(M \cap C) - n(M \cap P \cap C)\} = 15 - (9 + 5 - 3) = 4$$

(iii) The number of students who had opted Physics only

$$= n(P \cap M' \cap C')$$

$$= n(P \cap (M \cup C)')$$

$= n(P) - n(P \cap (M \cup C))$

$= n(P) - n((P \cap M) \cup (P \cap C))$

$= n(P) - \{n(P \cap M) + n(P \cap C) - n(P \cap M \cap C)\} = 12 - (9 + 4 - 3) = 2.$

(iv) Required number of students $= n(P \cap C \cap M')$

$= n(P \cap C) - n(P \cap C \cap M)$

$= 4 - 3 = 1$ $\quad [\because n(A \cap B') = n(A) - n(A \cap B)]$

(v) Required number of students $= n(M \cap P \cap C') = n(M \cap P) - n(M \cap P \cap C) = 9 - 3 = 6$

(vi) Required number of students

$= n(M) + n(P) + n(C) - 2\{n(M \cap P) + n(P \cap C) + n(M \cap C) + 3n(M \cap P \cap C)\}$

$= 15 + 12 + 11 - 2(9 + 4 + 5) + 3 \times 3 = 38 - 36 + 9 = 11$

(vii) Required number of students $= n(M \cup P \cup C)$

$= n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$

$= 15 + 12 + 11 - 9 - 4 - 5 + 3 = 23$

(viii) Required number of students

$= n(M' \cap P' \cap C') = n(M \cup P \cup C)' = n(U) - n(M \cup P \cup C) = 25 - 23 = 2.$

ALITER 1 Consider the Venn diagram shown in Fig. 1.14. Let a, b, c, d, e, f, g denote the number of students in the respective regions.

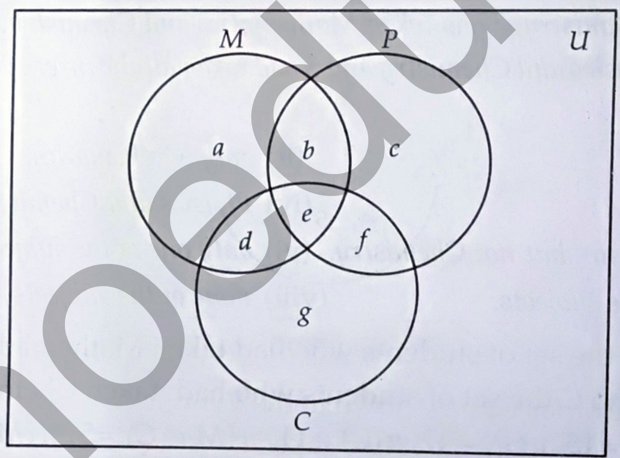

Fig. 1.14

From the Venn-daigram, we have

$n(M) = a + b + d + e, n(P) = b + c + e + f, n(C) = d + e + f + g,$

$n(M \cap P) = b + e\, n(P \cap C) = e + f\, n(M \cap C) = d + e$ and, $n(M \cap P \cap C) = e$

It is given that

$n(M \cap P \cap C) = 3 \Rightarrow e = 3$

$n(M \cap P) = 9 \Rightarrow b + e = 9 \Rightarrow b + 3 = 9 \Rightarrow b = 6$

$n(P \cap C) = 4 \Rightarrow e + f = 4 \Rightarrow 3 + f = 4 \Rightarrow f = 1$

$n(M \cap C) = 5 \Rightarrow d + e = 5 \Rightarrow d + 3 = 5 \Rightarrow d = 2$

$n(M) = 15 \Rightarrow a + b + d + e = 15 \Rightarrow a + 6 + 2 + 3 = 15 \Rightarrow a = 4$

$n(P) = 12 \Rightarrow b + c + e + f = 12 \Rightarrow 6 + c + 3 + 1 = 12 \Rightarrow c = 2$

$n(C) = 11 \Rightarrow d + e + f + g = 11 \Rightarrow 2 + 3 + 1 + g = 11 \Rightarrow g = 5$

Thus, we have $a = 4, b = 6, c = 2, d = 2, e = 3, f = 1$ and $g = 5$.

Now,

(i) Required number of students $= g = 5$

(ii) Required number of students $= a = 4$

(iii) Required number of students $= c = 2$

(iv) Required number of students $= f = 1$

(v) Required number of students $= b = 6$

(vi) Required number of students $= a + c + g = 4 + 2 + 5 = 11$

(vii) Required number of students $= a + b + c + d + e + f + g = 23$

(viii) Required number of students $= 25 - (a + b + c + d + e + f + g) = 25 - 23 = 2.$

ALITER 2 It is given that 3 students had taken all three subjects and 9 had taken Mathematics and Physics. So, number of students who had taken Mathematics and Physics but not Chemistry is 6 as shown in the Venn diagram. 5 students had taken Mathematics and Chemistry and 3 students had taken all the three subjects. So, 2 students had taken Mathematics and Chemistry but not Physics. It is given that 15 students had taken Mathematics. So, number of students who had taken only Mathematics $= 15 - (6 + 3 + 2) = 4$.

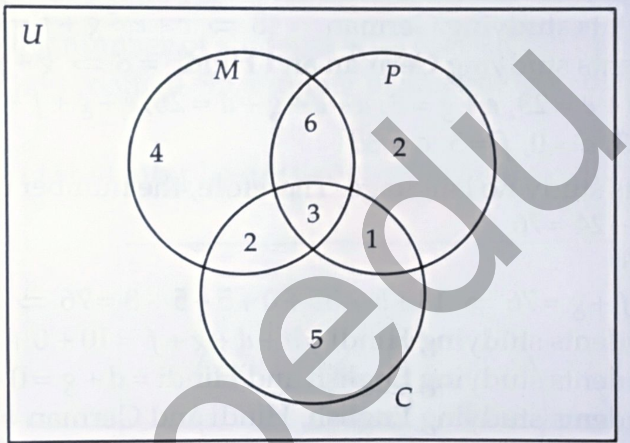

Fig. 1.15

Similarly, we compute the other values shown in the Venn diagram. It is evident from the Venn diagram that the number of students that had taken

(i) Chemistry only is 5.

(ii) only Mathematics is 4.

(iii) only Physics is 2.

(iv) Physics and Chemistry but not Mathematics is 1.

(v) Mathematics and Physics but not Chemistry is 6.

(vi) only one of the subjects is $4 + 2 + 5 = 11$

(vii) at least one of three subjects is $4 + 6 + 2 + 2 + 3 + 1 + 5 = 23$

(viii) none of the subjects $= 25 - 23 = 2$.

EXAMPLE 23 *In a survey of 100 students, the number of students studying the various languages were found to be : English only 18, English but not Hindi 23, English and German 8, English 26, German 48, German and Hindi 8, no language 24. Find the number of students who were studying (i) Hindi (ii) English and Hindi (iii) English, Hindi and German.*

SOLUTION Let E, H and G be the sets of students studying English, Hindi and German respectively. Let U be the set of students surveyed i.e. the universal set.

In the above Venn diagram, let a, b, c, d, e, f and g denote the number of students in the respective regions.

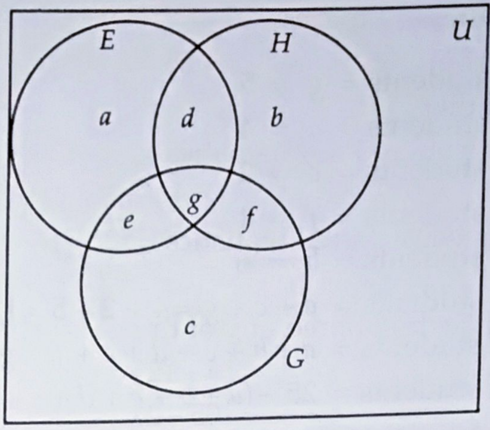

Fig. 1.16

Clearly, $n(U) = 100$. It is given that:

Number of students studying English only = 18 $\Rightarrow a = 18$

Number of students studying English but not Hindi = 23 $\Rightarrow a + e = 23$

Number of students studying English and German = 8 $\Rightarrow e + g = 8$

Number of students studying English = 26 $\Rightarrow a + d + e + g = 26$

Number of students studying German = 48 $\Rightarrow c + e + g + f = 48$

Number of students studying German and Hindi = 8 $\Rightarrow g + f = 8$

Thus, we obtain $a = 18$, $a + e = 23$, $e + g = 8$, $a + e + g + d = 26$, $e + g + f + c = 48$ and $g + f = 8$

$\Rightarrow \quad a = 18, e = 5, g = 3, d = 0, f = 5, c = 35$

It is given that 24 students study no language. Therefore, the number of students who study at least one language is $100 - 24 = 76$

i.e. $\quad n(E \cup H \cup G) = 76$

$\Rightarrow \quad a + b + c + d + e + f + g = 76 \Rightarrow 18 + b + 35 + 0 + 5 + 5 + 3 = 76 \Rightarrow b = 10$

(i) The number of students studying Hindi $= b + d + g + f = 10 + 0 + 3 + 5 = 18$

(ii) The number of students studying English and Hindi $= d + g = 0 + 3 = 3$

(iii) The number of students studying English, Hindi and German $= g = 3$.

EXAMPLE 24 *In an university, out of 100 students 15 offered Mathematics only; 12 offered Statistics only; 8 offered Physics only; 40 offered Physics and Mathematics ; 20 offered Physics and Statistics; 10 offered Mathematics and Statistics; 65 offered Physics. Find the number of students who*

(i) offered Mathematics *(ii) offered statistics* *(iii) did not offer any of the above three subjects.*

SOLUTION Let M, S and P be the sets of students who offered Mathematics, Statistics and Physics respectively. Let x be the number of students who offered all the three subjects. It is given that 10 students offered Mathematics and Statistics. Therefore, number of students who offered Mathematics and Statistics but not Physics is $10 - x$. Similarly, number of students in different regions are marked in Fig. 1.17.

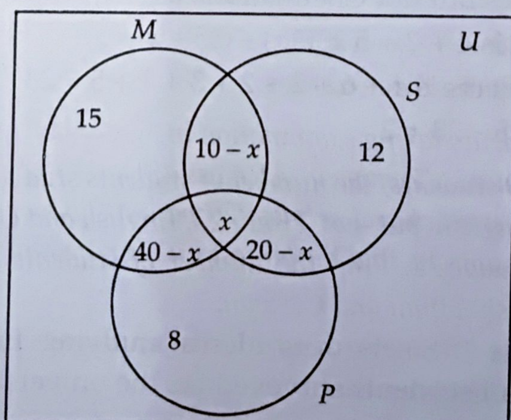

Fig. 1.17

It is given that 65 students offered Physics.

$\therefore \quad (40-x)+x+(20-x)+8=65 \Rightarrow 68-x=65 \Rightarrow x=3$

(i) From Fig. 1.16, we find that

The number of students who offered Mathematics $=15+(10-x)+x+40-x$

$=65-x=65-3=62$

(ii) The number of students who offered Statistics $=12+(10-x)+x+(20-x)$

$=42-x=42-3=39$

(iii) The number of students who offered any of three subjects

$=15+12+8+(10-x)+(40-x)+(20-x)+x$

$=105-2x=105-2\times 3=99$

$\therefore$ Number of students who did not offer any of the three subjects = 100 − 99 = 1.

EXAMPLE 25 *Out of 280 students in class XII of a school, 135 play Hockey, 110 play football, 80 play volleyball, 35 of these play hockey and football, 30 play volleyball and hockey, 20 play football and volleyball. Also, each students plays at least one of the three games. How many students play all the three games?*

SOLUTION Let H, F and V be the sets of students who play hockey, football and volleyball respectively. Let x be the number of students who play all the three games. It is given that 35 students play hockey and football. So, number of student who play hockey and football only is $(35-x)$.

Similarly, the number of students playing various games are written in the regions representing them in Fig. 1.17.

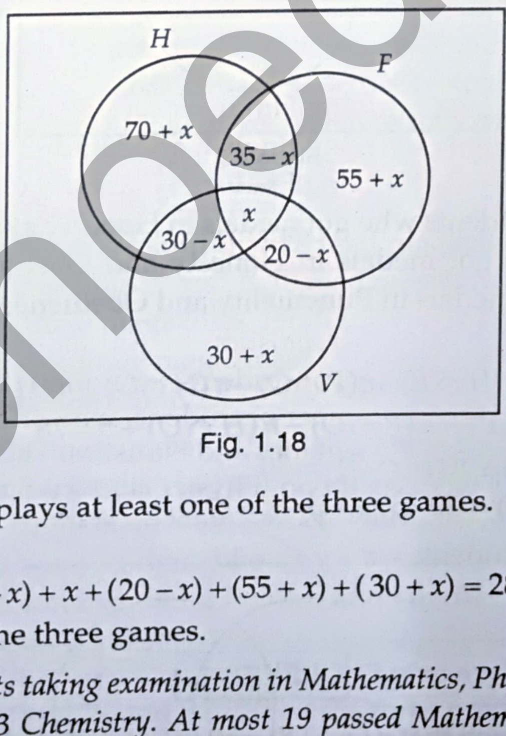

Fig. 1.18

It is given that each student plays at least one of the three games.

$\therefore \quad n(H\cup F\cup V)=280$

$\Rightarrow \quad (70+x)+(35-x)+(30-x)+x+(20-x)+(55+x)+(30+x)=280 \Rightarrow 240+x=280 \Rightarrow x=40$

Hence, 40 students play all the three games.

EXAMPLE 26 *From 50 students taking examination in Mathematics, Physics and Chemistry, 37 passed Mathematic, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics; at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. If each student has passed in at least one of the subjects, find the largest number of students who could have passed in all the three subjects.*

SOLUTION Let M, P and C be the sets of students passing in Mathematics, Physics and Chemistry respectively. It is given that

$n(M\cup P\cup C)=50,\ n(M)=37,\ n(P)=24,\ n(C)=43,\ n(M\cap P)\leq 19,\ n(M\cap C)\leq 29$ and $n(P\cap C)\leq 20$

We know that

$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$$

$\Rightarrow \quad 50 = 37 + 24 + 43 - \{n(M \cap P) + n(M \cap C) + n(P \cap C)\} + n(M \cap P \cap C)$

$\Rightarrow \quad 50 = 104 - \{n(M \cap P) + n(M \cap C) + n(P \cap C)\} + n(M \cap P \cap C)$

$\Rightarrow \quad 54 + n(M \cap P \cap C) = n(M \cap P) + n(M \cap C) + n(P \cap C)$

$\Rightarrow \quad 54 + n(M \cap P \cap C) \leq 19 + 29 + 20$

$\Rightarrow \quad n(M \cap P \cap C) \leq 14 \qquad [\because n(M \cap P) \leq 19, n(M \cap C) \leq 29, n(P \cap C) \leq 20]$

Hence, the largest number of students that could have passed in all the three subjects is 14.

EXAMPLE 27 *A school awarded 58 medals for Honesty, 20 for Punctuality and 25 for Obedience. If these medals were bagged by a total of 78 students and only 5 students got meadals for all the three values, find the number of students who received medals for exactly two of the three values.*

SOLUTION Let H, P and O be the sets of students who bagged medals in Honesty, Punctuality and Obedience respectively. It is given that $n(H) = 58$, $n(P) = 20$, $n(O) = 25$, $n(H \cup P \cup O) = 78$ and $n(H \cap P \cap O) = 5$.

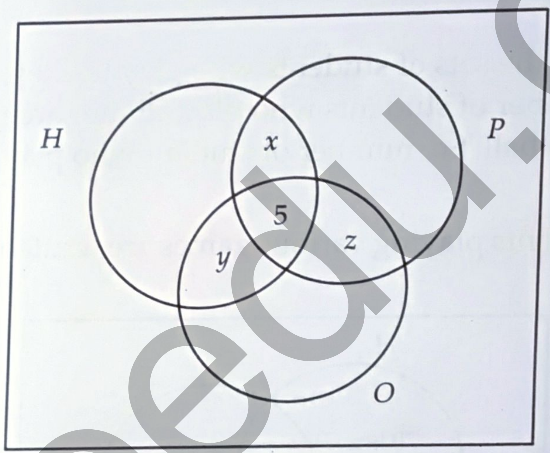

Fig. 1.19

Let x denote the number of students who got medals in Honesty and Punctuality only, y denote the number of students who got medals in Honesty and Obedience only and z denote the number of students who got medals in Punctuality and Obedience only.

Now, $\quad n(H \cup P \cup O) = 78$

$\Rightarrow \quad n(H) + n(P) + n(O) - n(H \cap P) - n(P \cap O) - n(H \cap O) + n(H \cap P \cap O) = 78$

$\Rightarrow \quad 58 + 20 + 25 - n(H \cap P) - n(P \cap O) - n(H \cap O) + 5 = 78$

$\Rightarrow \quad n(H \cap P) + n(P \cap O) + n(H \cap O) = 30$

$\Rightarrow \quad (x+5) + (y+5) + (z+5) = 30 \Rightarrow x + y + z = 15$

Hence, required number of students $= x + y + z = 15$

EXERCISE 1.8

LEVEL-1

1. If A and B are two sets such that $n(A \cup B) = 50$, $n(A) = 28$ and $n(B) = 32$, find $n(A \cap B)$.
2. If P and Q are two sets such that P has 40 elements, $P \cup Q$ has 60 elements and $P \cap Q$ has 10 elements, how many elements does Q have?
3. In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach physics and mathematics. How many teach physics?
4. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many like both coffee and tea?

5. Let A and B be two sets such that : $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$. Find
   (i) $n(B)$ (ii) $n(A - B)$ (iii) $n(B - A)$

6. A survey shows that 76% of the Indians like oranges, whereas 62% like bananas. What percentage of the Indians like both oranges and bananas?

7. In a group of 950 persons, 750 can speak Hindi and 460 can speak English. Find:
   (i) how many can speak both Hindi and English
   (ii) how many can speak Hindi only (iii) how many can speak English only.

8. In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find:
   (i) how may drink tea and coffee both (ii) how many drink coffee but not tea.

9. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:
   (i) the numbers of people who read at least one of the newspapers.
   (ii) the number of people who read exactly one newspaper.

10. Of the members of three athletic teams in a certain school, 21 are in the basketball team, 26 in hockey team and 29 in the football team. 14 play hockey and basket ball, 15 play hockey and football, 12 play football and basketball and 8 play all the three games. How many members are there in all?

11. In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali ? How many can speak both Hindi and Bengali?

12. A survey of 500 television viewers produced the following information; 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How many watch all the three games? How many watch exactly one of the three games?

LEVEL-2

13. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find:
    (i) How many read none of three magazines?
    (ii) How many read magazine C only?

14. In a survey of 100 students, the number of students studying the various languages were found to be : English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find:
    (i) How many students were studying Hindi?
    (ii) How many students were studying English and Hindi?

15. In a survey it was found that 21 persons liked product P_1, 26 liked product P_2 and 29 liked product P_3. If 14 persons liked products P_1 and P_2 ; 12 persons liked product P_3 and P_1 ; 14 persons liked products P_2 and P_3 and 8 liked all the three products. Find how many liked product P_3 only.

ANSWERS

1. 10 **2.** 30 **3.** 12 **4.** 19 **5.** (i) 26 (ii) 16 (iii) 22
6. 38% **7.** (i) 260 (ii) 490 (iii) 200 **8.** (i) 16 (ii) 20
9. (i) 52 (ii) 30 **10.** 43 **11.** (i) 600 (ii) 250 (iii) 150
12. 20, 325 **13.** (i) 20 (ii) 30 **14.** (i) 18, (ii) 3 **15.** 11

HINTS TO NCERT AND SELECTED PROBLEMS

5. (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B) \Rightarrow n(B) = 26$
   (ii) $n(A - B) = n(A) - n(A \cap B) \Rightarrow n(A - B) = 16$
   (iii) $n(B - A) = n(B) - n(A \cap B) \Rightarrow 22$

7. (iii) Let A and B denote the sets of persons who can speak Hindi and English respectively. Then, $n(A \cup B) = 950$, $n(A) = 750$ and $n(B) = 460$.
   (i) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 750 + 460 - 950 = 260$
   (ii) Required number $= n(A - B) = n(A) - n(A \cap B)$
   (iii) Required number $= n(B - A) = n(B) - n(A \cap B)$.

8. (ii) Let A and B be sets of persons who drink tea and coffee respectively. Then
   $n(A \cup B) = 50, n(A - B) = 14, n(A) = 30.$
   (i) $n(A - B) = 14 \Rightarrow n(A) - n(A \cap B) = 14 \Rightarrow n(A \cap B) = n(A) - 14 = 30 - 14 = 16$
   (ii) Required number $= n(B - A) = n(B) - n(A \cap B)$.
   Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) \Rightarrow 50 = 30 + n(B) - 16 \Rightarrow n(B) = 36.$
   $\therefore\ n(B - A) = n(B) - n(A \cap B) \Rightarrow n(B - A) = 36 - 16 = 20$

10. Let A, B and C be the sets of members of basketball, hockey and football teams respectively. Then, $n(A) = 21$, $n(B) = 26$, $n(C) = 29$, $n(A \cap B) = 14$, $n(B \cap C) = 15$, $n(A \cap C) = 12$ and $n(A \cap B \cap C) = 8$.
   Required number $= n(A \cup B \cup C)$
   $= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

11. Let A and B be the sets of persons who can speak Hindi and Bengali respectively. Then, $n(A \cup B) = 1000$, $n(A) = 750$ and $n(B) = 400$.
   No. of persons who can speak Hindi only $= n(A - B) = n(A) - n(A \cap B)$
   No. of persons who can speak Bengali only $= n(B - A) = n(B) - n(B \cap A)$
   No. of persons who can speak both Hindi and Bengali $= n(A \cap B) = n(A) + n(B) - n(A \cup B)$.

12. N = Total number of television viewers $= 500$, $n(F) = 285$, $n(H) = 195$, $n(F \cap B) = 45$, $n(F \cap H) = 70$, $n(H \cap B) = 50$, $n(F' \cap H' \cap B') = 50$.
   Now, $n(F' \cap H' \cap B') = 50$
   $\Rightarrow\ n[(F \cup H \cup B)'] = 50$
   $\Rightarrow\ N - n(F \cup H \cup B) = 50$
   $\Rightarrow\ 500 - [n(F) + n(H) + n(B) - n(F \cap H) - n(F \cap B) - n(H \cap B) + n(F \cap H \cap B)] = 50$
   $\Rightarrow\ n(F \cap H \cap B) = 500 - 285 - 195 - 115 + 70 + 50 + 45 - 50 = 20.$
   $\therefore$ Required number $= n(F \cap H \cap B) = 20$
   Required number $= n(F \cap H' \cap B') + n(F' \cap H' \cap B) + n(F' \cap H \cap B')]$
   $= n(F) + n(H) + n(B) - 2[n(F \cap H) + n(H \cap B) + n(B \cap F)\} + 3n(F \cap H \cap B)$

14. We have, $a = 18$, $a + b = 23$, $d + e = 8$, $a + b + d + e = 26$, $d + e + f + g = 48$,
   and, $a + b + c + d + e + f + g = 100 - 24 = 76$
   $\therefore\ a = 18, b = 0, c = 10, d = 5, e = 3, f = 5$ and, $g = 35$
   (i) $n(H) = b + c + e + f = 18$
   (ii) $n(H \cap E) = b + e = 3$

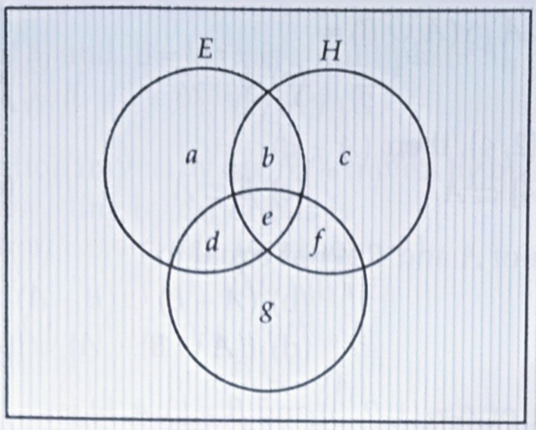

Fig. 1.20

VERY SHORT ANSWER QUESTIONS (VSAQs)

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

1. If a set contains n elements, then write the number of elements in its power set.
2. Write the number of elements in the power set of null set.
3. Let $A = \{x : x \in N, x \text{ is a multiple of } 3\}$ and $B = \{x : x \in N \text{ and } x \text{ is a multiple of } 5\}$. Write $A \cap B$.
4. Let A and B be two sets having 3 and 6 elements respectively. Write the minimum number of elements that $A \cup B$ can have.
5. If $A = \{x \in C : x^2 = 1\}$ and $B = \{x \in C : x^4 = 1\}$, then write $A - B$ and $B - A$.
6. If A and B are two sets such that $A \subset B$, then write $B' - A'$ in terms of A and B.
7. Let A and B be two sets having 4 and 7 elements respectively. Then write the maximum number of elements that $A \cup B$ can have.
8. If $A = \{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\}$ and $B = \{(x, y) : y = -x, x \in R\}$, then write $A \cap B$.
9. If $A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = e^{-x}, x \in R\}$, then write $A \cap B$.
10. If A and B are two sets such that $n(A) = 20, n(B) = 25$ and $n(A \cup B) = 40$, then write $n(A \cap B)$.
11. If A and B are two sets such that $n(A) = 115, n(B) = 326, n(A - B) = 47$, then write $n(A \cup B)$.

ANSWERS

1. 2^n 2. 1 3. $\{x : x \in N, x \text{ is a multiple of } 15\}$ 4. 6
5. $A - B = \phi, B - A = \{i, -i\}$ 6. ϕ 7. 11 8. ϕ 9. $\{(0, 1)\}$
10. 5 11. 373

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. For any set A, $(A')'$ is equal to
   (a) A' (b) A (c) ϕ (d) none of these
2. Let A and B be two sets in the same universal set. Then, $A - B =$
   (a) $A \cap B$ (b) $A' \cap B$ (c) $A \cap B'$ (d) none of these
3. The number of subsets of a set containing n elements is
   (a) n (b) $2^n - 1$ (c) n^2 (d) 2^n

4. For any two sets A and B, $A \cap (A \cup B) =$
(a) A (b) B (c) ϕ (d) none of these

5. If $A = \{1, 3, 5, B\}$ and $B = \{2, 4\}$, then
(a) $4 \in A$ (b) $\{4\} \subset A$ (c) $B \subset A$ (d) none of these

6. The symmetric difference of A and B is not equal to
(a) $(A - B) \cap (B - A)$ (b) $(A - B) \cup (B - A)$
(c) $(A \cup B) - (A \cap B)$ (d) $\{(A \cup B) - A\} \cup \{(A \cup B) - B\}$

7. The symmetric difference of $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ is
(a) $\{1, 2\}$ (b) $\{1, 2, 4, 5\}$ (c) $\{4, 3\}$ (d) $\{2, 5, 1, 4, 3\}$

8. For any two sets A and B, $(A - B) \cup (B - A) =$
(a) $(A - B) \cup A$ (b) $(B - A) \cup B$
(c) $(A \cup B) - (A \cap B)$ (d) $(A \cup B) \cap (A \cap B)$

9. Which of the following statement is false :
(a) $A - B = A \cap B'$ (b) $A - B = A - (A \cap B)$
(c) $A - B = A - B'$ (d) $A - B = (A \cup B) - B$

10. For any three sets A, B and C
(a) $A \cap (B - C) = (A \cap B) - (A \cap C)$ (b) $A \cap (B - C) = (A \cap B) - C$
(c) $A \cup (B - C) = (A \cup B) \cap (A \cup C')$ (d) $A \cup (B - C) = (A \cup B) - (A \cup C)$.

11. Let $A = \{x : x \in R, x > 4\}$ and $B = \{x \in R : x < 5\}$. Then, $A \cap B =$
(a) $(4, 5]$ (b) $(4, 5)$ (c) $[4, 5)$ (d) $[4, 5]$

12. Let U be the universal set containing 700 elements. If A, B are sub-sets of U such that $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then, $n(A' \cap B') =$
(a) 400 (b) 600 (c) 300 (d) none of these.

13. Let A and B be two sets such that $n(A) = 16$, $n(B) = 14$, $n(A \cup B) = 25$. Then, $n(A \cap B)$ is equal to
(a) 30 (b) 50 (c) 5 (d) none of these

14. If $A = \{1, 2, 3, 4, 5\}$, then the number of proper subsets of A is
(a) 120 (b) 30 (c) 31 (d) 32

15. In set-builder method the null set is represented by
(a) $\{\ \}$ (b) Φ (c) $\{x : x \neq x\}$ (d) $\{x : x = x\}$

16. If A and B are two disjoint sets, then $n(A \cup B)$ is equal to
(a) $n(A) + n(B)$ (b) $n(A) + n(B) - n(A \cap B)$
(c) $n(A) + n(B) + n(A \cap B)$ (d) $n(A)\, n(B)$

17. For two sets $A \cup B = A$ iff
(a) $B \subseteq A$ (b) $A \subseteq B$ (c) $A \neq B$ (d) $A = B$

18. If A and B are two sets such that $n(A) = 70$, $n(B) = 60$, $n(A \cup B) = 110$, then $n(A \cap B)$ is equal to
(a) 240 (b) 50 (c) 40 (d) 20

19. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to
(a) A (b) B (c) Φ (d) $A \cap B^c$

20. If $A = \{x : x \text{ is a multiple of } 3\}$ and, $B = \{x : x \text{ is a multiple of } 5\}$, then $A - B$ is
(a) $A \cap B$ (b) $A \cap \overline{B}$ (c) $\overline{A} \cap \overline{B}$ (d) $\overline{A \cap B}$

21. In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus is
(a) 80% (b) 40% (c) 60% (d) 70%

22. If $A \cap B = B$, then
(a) $A \subseteq B$ (b) $B \subseteq A$ (c) $A = \Phi$ (d) $B = \Phi$

23. An investigator interviewed 100 students to determine the performance of three drinks: milk, coffee and tea. The investigator reported that 10 students take all three drinks milk, coffee and tea; 20 students take milk and coffee; 25 students take milk and tea; 20 students take coffee and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of three drinks is
(a) 10 (b) 20 (c) 25 (d) 30

24. Two finite sets have m and n elements. The number of elements in the power set of first set is 48 more than the total number of elements in power set of the second set. Then, the values of m and n are:
(a) 7, 6 (b) 6, 3 (c) 6, 4 (d) 7, 4

25. In a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone?
(a) 35 (b) 48 (c) 60 (d) 22

26. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots, B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the $A_i^{'s}$ and exactly 9 of the $B_j^{'s}$, then n is equal to
(a) 15 (b) 3 (c) 45 (d) 35

27. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second. The values of m and n are respectively
(a) 4, 7 (b) 7, 4 (c) 4, 4 (d) 7, 7

28. For any two sets A and B, $A \cap (A \cup B)'$ is equal to
(a) A (b) B (c) ϕ (d) $A \cap B$

29. The set $(A \cup B')' \cup (B \cap C)$ is equal to
(a) $A' \cup B \cup C$ (b) $A' \cup B$ (c) $A' \cup C'$ (d) $A' \cap B$

30. Let F_1 be the set of all parallelograms, F_2 the set of all rectangles, F_3 the set of all rhombuses, F_4 the set of all squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to
(a) $F_2 \cap F_3$ (b) $F_3 \cap F_4$ (c) $F_2 \cup F_3$ (d) $F_2 \cup F_3 \cup F_4 \cup F_1$

ANSWERS

1. (b) 2. (c) 3. (d) 4. (a) 5. (d) 6. (a) 7. (b) 8. (c) 9. (c)
10. (a), (b), (c) 11. (b) 12. (c) 13. (c) 14. (c) 15. (c) 16. (a) 17. (a)
18. (d) 19. (d) 20. (b) 21. (c) 22. (b) 23. (b) 24. (c) 25. (c) 26. (c)
27. (b) 28. (c) 29. (b) 30. (d)

SUMMARY

1. A set is a well defined collection of objects.
2. A set is described either in set builder form or tabular form.
3. A set consisting of no element is called the null set and is denoted by ϕ
4. A set consisting of a single element is called a singleton set.
5. A set consisting of a definite number of elements is called a finite set, otherwise the set is called an infinite set.
6. The number of elements in a finite set A is called its cardinal number or order and is denoted by $n(A)$.
7. Two sets A and B are equal if they have exactly the same elements.
8. A set A is said to be a subset of a set B, if every element of A is also an element of B.
9. If a, b are real numbers such that $a < b$, then the set
   (i) $\{x : x \in R$ and $a \le x \le b\}$ is called the closed interval $[a, b]$
   (ii) $\{x : x \in R$ and $a < x < b\}$ is called the open interval (a, b)
   (iii) $\{x : x \in R$ and $a \le x < b\}$ is called the semi-open or semi-closed interval $[a, b)$.
   (iv) $\{x : x \in R$ and $a < x \le b\}$ is called the semi-open or semi-closed interval $(a, b]$.
10. The total number of subsets of a finite set consisting of n elements is 2^n.
11. The collection of all subsets of a set A is called the power set of A and is denoted by $P(A)$.
12. The union of two sets A and B is the set of all those elements which are either in A or in B or in both and is denoted by $A \cup B$. Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$
13. The intersection of two sets A and B is the set of all those elements which are common to both A and B and is denoted by $A \cap B$. Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$.
14. The difference $A - B$ of two sets A and B is the set of all those elements of A which do not belong to B i.e. $A - B = \{x : x \in A \text{ and } x \notin B\}$. Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.
15. The symmetric difference of two sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.
16. The complement of a subset A of universal set U is the set of all those elements of U which are not the elements of A. The complement of A is denoted by A' or A^c.
17. For any three sets A, B and C, we have
   (i) $A \cup A = A$ and $A \cap A = A$ (Idempotent laws)
   (ii) $A \cup \phi = A$ and $A \cap U = A$ (Identity laws)
   (iii) $A \cup B = B \cup A$ and $A \cap B = B \cap A$ (Commutative laws)
   (iv) $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative laws)
   (v) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive laws)
   (vi) $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$ (De' Morgan's laws)
18. If A, B and C are finite sets and U be the finite universal set, then
   (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   (ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint non-void sets
   (iii) $n(A - B) = n(A) - n(A \cap B)$ i.e., $n(A - B) + n(A \cap B) = n(A)$
   (iv) $n(A \Delta B) = n(A - B) + n(B - A) = n(A) + n(B) - 2n(A \cap B)$
   (v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
   (vi) Number of elements in exactly two of sets A, B and C
   $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
   (vii) Number of elements in exactly one of sets A, B and C
   $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$.