Global Stability Analysis of Filamentary Molecular Clouds: Logatropic Filament

Global linear stability analysis

In linear perturbation analysis, small perturbations are added to the equilibrium quantities. Then all equations are linearized.

The perturbed quantities can be decomposed to theri Fourier moeds.

If the amplitude of the modes are considered to be some functions of radial distance, this type of perturbation analysis is called global.

This method can be used for finding dispersion relation. Dispersion relation tell us for which modes the fluid is unstable, what is the fastest growing mode, critical mode and so on. It also gives us essential information about the fragmentation and structure formation in the medium under study.

Logatropic equation of state

$P = \rho + \kappa \log \rho$

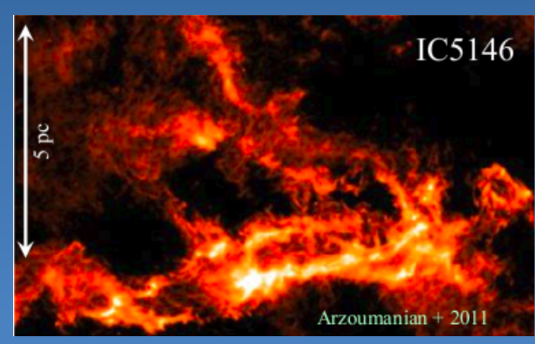
Isothermal density profile for a cylinder in hydrostatic equilibrium is proportional to $\sim r^{-4}$ [1].

This is not consistent with observations of molecular clouds (MCs). As a solution, it was suggested to use shallower density profiles for filamentary MCs.

The concurrence effect of magnetohydrodynamics (MHD) and turbulence probably results in softer equation of state, so one interesting case is the logatropic form of equation of state which is softer than its isothermal counterpart.

Filamentary molecular clouds

Filamentary star-forming structures are ubiquitous in interstellar medium (e.g [2], [3]). Their key role in star-formation process was highlighted by Schneider, S. and Elmegreen, B. G. [4] for the first time. Recently it has been revealed by Herschel space observatory that they commonly have a characteristic width of about 0.1 pc and show non-thermal line broadening which could be evidence for existence of turbulent motions.



Perturbed Fluid Equations

$$\frac{\partial \rho_1}{\partial t} + \nabla \rho_0 \cdot \mathbf{u}_1 + \rho_0 \nabla \cdot \mathbf{u}_1 = 0,$$

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} + \nabla p_1 + \rho_0 \nabla \psi_1 + \rho_1 \nabla \psi_0 - \mathcal{L}_1 = 0,$$

$$\frac{\partial \mathbf{B}_1}{\partial t} + \nabla \times (\mathbf{B}_0 \times \mathbf{u}_1) = 0,$$

$$\nabla^2 \psi_1 = \rho_1,$$

$$\mathcal{L}_1 = (\mathbf{B}_0 \cdot \nabla)\mathbf{B}_1 + (\mathbf{B}_1 \cdot \nabla)\mathbf{B}_0 - \nabla(\mathbf{B}_0 \cdot \mathbf{B}_1)$$

$$p_1 = P'(\rho_0)\rho_1.$$

+

Fourier decomposition

$$\begin{pmatrix} \rho_1(\mathbf{x},t) \\ \mathbf{u}_1(\mathbf{x},t) \\ \mathbf{B}_1(\mathbf{x},t) \\ \psi_1(\mathbf{x},t) \end{pmatrix} = \Re \left[\begin{pmatrix} f(r) \\ \mathbf{v}(r) \\ \mathbf{b}(r) \\ \phi(r) \end{pmatrix} \exp(ikz - i\omega t) \right]$$

=

Disguised eigen value problem

$$AU = \omega U$$

Boundary conditions

$$f = 1, \quad \frac{d\phi}{dr} = 0, \quad w = 0, \quad at \quad r = 0,$$
$$f = 0, \quad w = 0, \quad at \quad r = \infty.$$

Newton-Raphson-Kantorovich Loop

This algorithm, uses the second-order finite-difference method to discretize the system of ODEs as well as BCs over a mesh grid. The resultant system of linear algebraic equations are then solved by inverting the matrix of coefficients. This procedure is iterated while an error threshold test is performed after each iteration until the solutions are converged. The reader can find more details of the algorithm in [5]. See also [6] for other application.

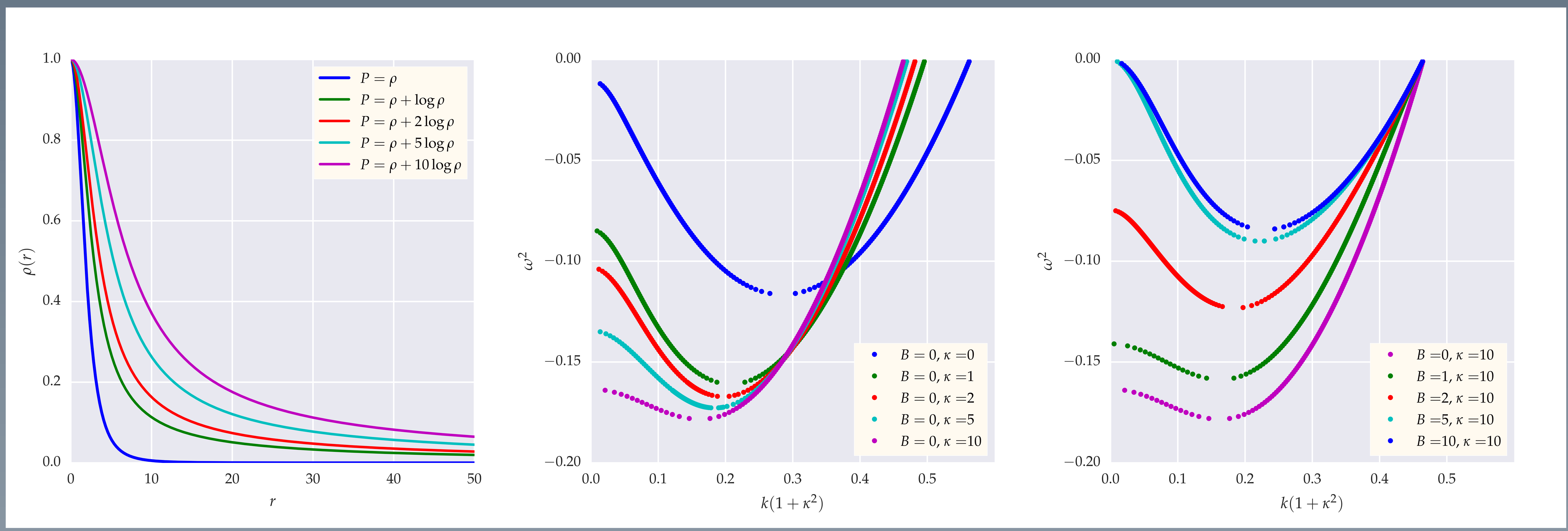


Figure 1. Left: Comparison of the isothermal and logatropic density profile. The density and radial distance are scaled by typical values for a filament.
Middle: Dispersion relation for a filament in different turbulence regime κ when the magnetic field is not present.
Right: Dispersion relation for a filament in different magnetic field strength when the turbulence parameter has a constant value (κ = 10).
The horizontal axis is the wave number k multiplied by effective sound speed at the filament center and the vertical axis is ω^2 that are normalized in the units of $(4\pi G\rho)^{1/2}/c_s$ and $4\pi G\rho$ respectively. c_s is the typical sound speed.

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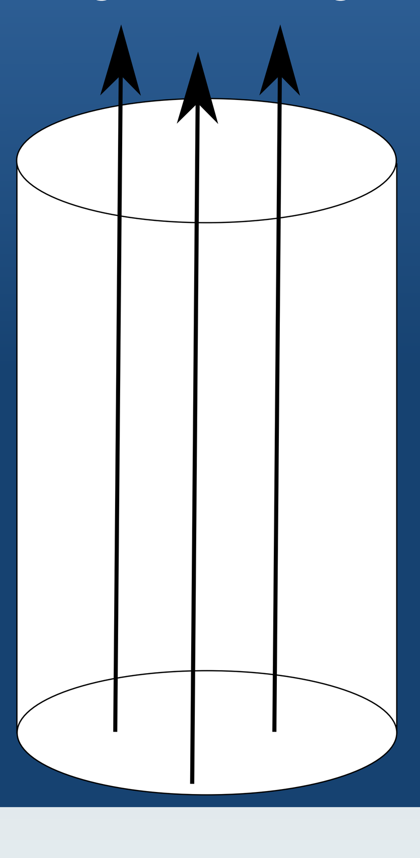
Magnetic Field configuration

$$B_0 = B_0 \hat{z}$$



Results

The density profile of the isothermal filament is compared with logatropic filament in Fig.1 (left). The density of logatropic filaments is reduced more smoothly than the isothermal one.

In the middle, the effect of turbulence parameter on the fragmentation of the filament is showed. It is obvious that the filament is destabilized by increasing the turbulence parameter for a fixed magnetic field strength.

The effect of magnetic field strength in a fixed turbulence regime is depicted in the right panel. Here, the fastest growing mode is decreased by increasing of magnetic field but the critical fragmentation wavelength remained constant. Therefore the filament is more stable in stronger magnetic field regime.

In subsequent study, we will investigate this problem in non-ideal MHD framework (see [6]).

References

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