# LOSSLESS 2D DISCRETE WALSH-HADAMARD TRANSFORM

*Kunitoshi KOMATSU and Kaoru SEZAKI*

Institute of Industrial Science, University of Tokyo
7-22-1 Roppongi Minato-ku Tokyo, 106-8558, Japan
E-mail: komatsu@mcl.iis.u-tokyo.ac.jp

**ABSTRACT**

The 64-point separable lossless two dimensional (2D) WHT is composed of the 8-point lossless one dimensional WHT. The latter is obtained by first decomposing the 8-point WHT into the 2-point WHTs and second replacing every 2-point WHT by a ladder network. Since the coefficients in the ladder network then become real, the advantage of multiplier-free vanishes. This paper therefore proposes a 64-point non-separable lossless 2D WHT without multiplication as follows. First the 64-point separable 2D WHT is decomposed into the 4-point 2D WHTs. Second every 4-point 2D WHT is replaced by a 2D ladder network which is multiplier-free. It is also shown that the transform coefficients of the proposed transform are closer to those of the 64-point lossy 2D WHT than those of the 64-point separable lossless 2D WHT.

## 1. INTRODUCTION

A unified lossless/lossy image coding system is useful for various applications, for example, medical images, satellite images and images in the art world, since we can reconstruct lossy and lossless images from a part and the whole of an encoded data, respectively. In JPEG, lossless and lossy image coding systems are not unified, that is, the discrete cosine transform (DCT) coding system is used for lossy reconstruction and the DPCM coding system is used for lossless reconstruction.

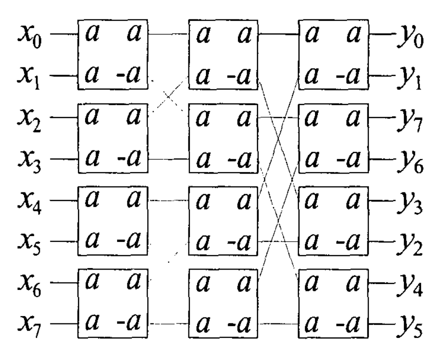
The unified lossless/lossy image coding system can be realized by using the lossless block transforms [1]-[2], the lossless lapped orthogonal transform [3] or the lossless wavelet transforms [4]-[8]. In these lossless transforms, integer input signals are transformed into integer transform coefficients and losslessly reconstructed. The mean first order entropy of the integer transform coefficients is smaller than that of input for image compression. The progressive transmission is possible by transmitting the integer transform coefficients decomposed into bit planes from significant bit plane to insignificant one or by transmitting the coefficients from low-frequency component to high-frequency one.

Thanks to International Communications Foundation for funding.

In the unified lossless/lossy image coding system, the number of computations is desired to be small. The 8-point lossless one dimensional (1D) Walsh-Hadamard transform (WHT) has multiplications, although the lossy (non-lossless) WHT is multiplier-free [1]. The reason is that the coefficients in ladder network [9] become real, when every 2-point WHT into which the normalized 8-point WHT is decomposed is replaced by a ladder network.

On the other hand, the normalized 4-point WHT is able to be replaced by a ladder network which is multiplier-free [10]. The reason is that the elements of the normalized 4-point WHT are +0.5 or -0.5. This paper therefore proposes a 64-point nonseparable lossless two dimensional (2D) WHT without multiplication as follows. First the 64-point separable 2D WHT is decomposed into the 4-point 2D WHTs. Second every 4-point 2D WHT is replaced by a 2D ladder network which is multiplier-free.

The compatibility of the lossless transform with the corresponding lossy transform is desired to be high. That is, the difference between the transform coefficients of a lossy transform and its lossless version is desired to be small. We therefore investigate the compatibility of the proposed lossless 2D WHT with the WHT.




$$a = \frac{1}{\sqrt{2}}$$

**Fig. 1.** 8-point WHT.

0-7803-7041-4/01/$10.00 ©2001 IEEE

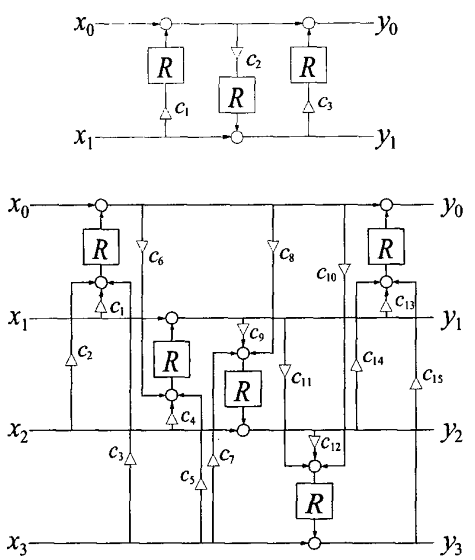



**Fig. 2.** 2-point and 4-point ladder networks.

## 2. LOSSLESS 1D WHT

The 8-point lossless 1D WHT which we name 1D LWHT is obtained by first decomposing the 8-point WHT into the 2-point WHTs as shown in fig. 1 and second replacing every 2-point WHT by a ladder network, where the absolute value of the determinant of the 2-point WHT must be 1 [9]. The 2-point and 4-point ladder networks are shown in fig. 2, where quantizer, R, represents rounding to the nearest integer. The coefficients in the ladder network corresponding to the normalized 2-point WHT, $c_1$, $c_2$ and $c_3$, are -0.41421, 0.70711 and -0.41421, respectively. Each number of multiplications, adds and quantizers in the 1D LWHT is then 36. Note that the advantage of multiplier-free vanishes.

## 3. LOSSLESS 2D WHT

Now consider a 1D transform $\boldsymbol{S}$ that consists of four 2-point WHTs and an 8-point transform $\boldsymbol{A}$ as shown in fig. 3. The transform coefficients $y_{i,j}$ of an 8x8 block of image pixels $x_{i,j}$ are then given by

$$y_{i,j} = \sum_{k=1}^{4} \{a_{i,2k-1}(x_{2k,j} + x_{2k-1,j}) + a_{i,2k}(x_{2k,j} - x_{2k-1,j})\} \tag{1}$$

where the $\boldsymbol{S}$ is applied in the horizontal direction and $a_{i,j}$ are elements of the $\boldsymbol{A}$. The transform coefficients $z_{i,j}$ of an 8x8 block of $y_{i,j}$ are given by

$$z_{l,m} = \sum_{n=1}^{4} \{a_{m,2n-1}(y_{l,2n} + y_{l,2n-1}) + a_{m,2n}(y_{l,2n} - y_{l,2n-1})\} \tag{2}$$

where the $\boldsymbol{S}$ is applied in the vertical direction. Substituting

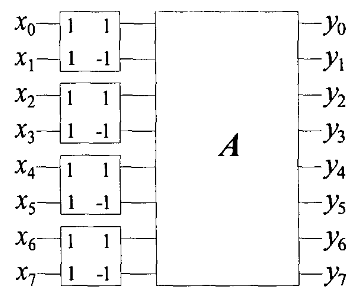



**Fig. 3.** Transform $\boldsymbol{S}$.

(1) into (2) gives

$$\begin{aligned} z_{l,m} = \sum_{k=1}^{4}\sum_{n=1}^{4} \{ & \\ & a_{m,2n-1}a_{l,2k-1}(x_{2k-1,2n-1} + x_{2k,2n-1} + x_{2k-1,2n} + x_{2k,2n}) \\ & + a_{m,2n-1}a_{l,2k}(-x_{2k-1,2n-1} + x_{2k,2n-1} - x_{2k-1,2n} + x_{2k,2n}) \\ & + a_{m,2n}a_{l,2k-1}(-x_{2k-1,2n-1} - x_{2k,2n-1} + x_{2k-1,2n} + x_{2k,2n}) \\ & + a_{m,2n}a_{l,2k}(x_{2k-1,2n-1} - x_{2k,2n-1} - x_{2k-1,2n} + x_{2k,2n})\} \end{aligned} \tag{3}$$

On the other hand, the transform coefficients $y'_{i,j}$ of an 8x8 block of $x'_{i,j}$, where the $\boldsymbol{A}$ is applied in the horizontal direction, are given by

$$y'_{i,j} = \sum_{k=1}^{8} a_{i,k} x'_{k,j} \tag{4}$$

The transform coefficients $z'_{i,j}$ of an 8x8 block of $y'_{i,j}$, where the $\boldsymbol{A}$ is applied in the vertical direction, are given by

$$z'_{l,m} = \sum_{n=1}^{8} a_{m,n} y'_{l,n} \tag{5}$$

Substituting (4) into (5) gives

$$\begin{aligned} z'_{l,m} &= \sum_{k=1}^{8}\sum_{n=1}^{8} a_{m,n}a_{l,k}x'_{k,n} \\ &= \sum_{k=1}^{4}\sum_{n=1}^{4} \{a_{m,2n-1}a_{l,2k-1}x'_{2k-1,2n-1} + a_{m,2n-1}a_{l,2k}x'_{2k,2n-1} \\ &\quad + a_{m,2n}a_{l,2k-1}x'_{2k-1,2n} + a_{m,2n}a_{l,2k}x'_{2k,2n}\} \end{aligned} \tag{6}$$

It is derived from comparing (3) and (6) that applying the $\boldsymbol{S}$ in the vertical direction after the horizontal direction is equivalent to first applying the 4-point 2D WHT and second applying the $\boldsymbol{A}$ to its output in the vertical direction after the horizontal direction.

It is seen from the above result that applying the 8-point 1D WHT in the vertical direction after the horizontal direction is equivalent to applying the sixteen 4-point 2D WHTs three times. The 64-point 2D WHT is shown in fig. 4.

The 4-point lossless 2D WHT is obtained by using $\{c_1=1, c_2=-1, c_3=1, c_4=-1, c_5=1, c_6=-0.5, c_7=0, c_8=0.5, c_9=0, c_{10}=-0.5, c_{11}=0, c_{12}=0, c_{13}=-1, c_{14}=-1, c_{15}=1\}$ in the 4-point ladder network. The number of multiplications, adds, shifts and quantizers of the 4-point lossless 2D WHT is 0, 11, 2

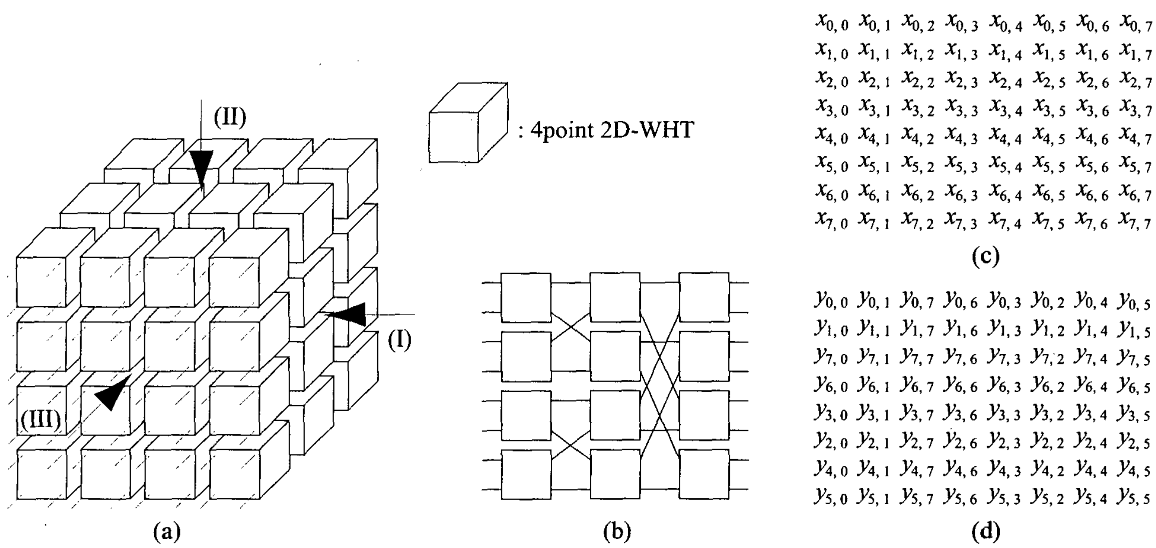



**Fig. 4.** 2D WHT: (a) two dimensional representation, (b) 2D WHT seen from directions (I) or (II), (c) input and (d) output seen from direction (III).

and 3, respectively. The 64-point lossless 2D WHT which we name 2D LWHT is obtained by replacing every 4-point 2D WHT by the 4-point lossless 2D WHT. The number of computations per an 8x8 block of image pixels is shown in table 1. Note that the number of multiplications of the 2D LWHT is 0.

### 4. COMPATIBILITY WITH WHT

In this section the compatibility of the 2D LWHT with the WHT is investigated. Entropies of the transform coefficients of the 2D LWHT, 1D LWHT, and WHT are shown in table 2, where the transform coefficients of the WHT are quantized to integer values. PSNRs [dB] are also shown there, where the transform coefficients are transformed by the inverse WHT. The six input images are 512x512 and 8bit/pixel grayscale. It is seen from the table that the compatibility of the 2D LWHT with the WHT is higher than that of the 1D LWHT.

The distribution of the difference between the transform coefficients of the 2D LWHT or the 1D LWHT and those of the WHT which are quantized to integer values are shown in table 3. The transform coefficients of the 2D LWHT are closer to those of the WHT than those of the 1D LWHT.

Relations between PSNR and entropy of the transform coefficients are shown in figure 5, where the transform coefficients are quantized by uniform quantizers and then transformed by the inverse WHT. The input is an image "Lenna". The performance of the 2D LWHT is superior to that of the 1D LWHT. The reason for high compatibility of the 2D

**Table 1.** Number of computations per an 8x8 block of image pixels.

| | 1D LWHT | 2D LWHT |
|---|---|---|
| round. | 576 | 144 |
| multi. | 576 | 0 |
| add | 576 | 528 |
| shift | 0 | 96 |

**Table 2.** Entropy of transform coefficients (bit/pel, upper row) and PSNR (dB, lower row) of the images resconstructed by the inverse WHT.

| | 1D LWHT | 2D LWHT | WHT |
|---|---|---|---|
| Aerial | 5.88<br>48.99 | 5.87<br>52.61 | 5.87<br>58.33 |
| Airplane | 4.85<br>48.98 | 4.82<br>52.59 | 4.81<br>58.32 |
| Baboon | 6.46<br>48.99 | 6.46<br>52.60 | 6.46<br>58.39 |
| Couple | 4.95<br>48.99 | 4.93<br>52.63 | 4.92<br>58.30 |
| Lenna | 5.05<br>48.99 | 5.03<br>52.64 | 5.03<br>58.41 |
| Peppers | 5.18<br>49.07 | 5.17<br>52.68 | 5.17<br>58.28 |
| Average | 5.40<br>49.00 | 5.38<br>52.63 | 5.38<br>58.34 |

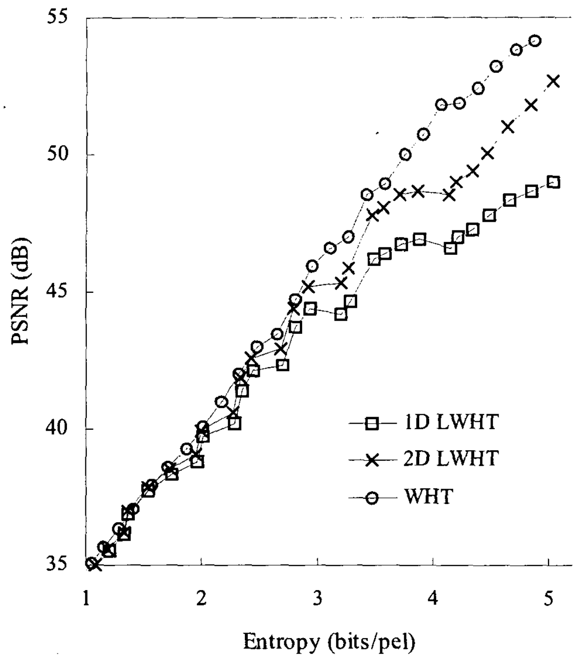



**Fig. 5.** Lossy compression performance.

**Table 3.** Distribution of the difference between the transform coefficients of the lossless WHT and those of the WHT.

| | Baboon | | Lenna | |
|---|---|---|---|---|
| | 1D LWHT | 2D LWHT | 1D LWHT | 2D LWHT |
| -5 | 0 | 0 | 0 | 0 |
| -4 | 9 | 0 | 2 | 0 |
| -3 | 502 | 0 | 563 | 0 |
| -2 | 11562 | 403 | 11779 | 380 |
| -1 | 67646 | 87376 | 68156 | 86761 |
| 0 | 114822 | 162976 | 114921 | 163685 |
| 1 | 58539 | 8660 | 58033 | 8683 |
| 2 | 8707 | 2617 | 8418 | 2547 |
| 3 | 354 | 112 | 267 | 88 |
| 4 | 3 | 0 | 5 | 0 |
| 5 | 0 | 0 | 0 | 0 |

LWHT with the WHT is that the number of quantizers is small.

## 5. CONCLUSION

A 64-point lossless 2D WHT without multiplication was proposed. This is based on the 4-point lossless 2D WHT that is multiplier-free. It was shown that the transform coefficients of the proposed transform were closer to those of the WHT than those of the lossless 1D WHT, and it therefore had high lossy compression efficiency. The lossless 2D WHT seems to be useful for a unified lossless/lossy image coding system to which low computation cost is necessary.

## 6. REFERENCES


[1] K. Komatsu and K. Sezaki, "Reversible Discrete Cosine Transform", *in Proc. IEEE ICASSP98*, pp.1769-1772, May 1998.

[2] K.Komatsu and K.Sezaki, "Design of Lossless Block Transforms and Filter Banks for Image Coding", *IEICE Transactions*, Vol.E82-A No.8 p.1656-1664, Aug. 1999.

[3] K.Komatsu and K.Sezaki, "Design of Lossless LOT and Its Performance Evaluation", *in Proc. IEEE ICASSP2000*, pp.2119-2122, June 2000.

[4] K.Komatsu and K.Sezaki, "Reversible Subband Coding of Images", *in Proc. SPIE VCIP*, vol.2501, pp.676-684, May 1995.

[5] K. Komatsu and K. Sezaki, "Lossless filter banks based on two point transform and interpolative prediction", *in Proc. IEEE ICASSP99*, pp.1469-1472, Mar. 1999.

[6] A. Said and W. Perlman, "Reversible image compression via multiresolution representation and predictive coding", *in Proc. SPIE VCIP*, vol. 2094, pp. 664-674, Nov. 1993.

[7] A. Zandi, J. D. Allen, E L. Schwartz and M. Boliek, "CREW: Compression with reversible embedded wavelets", *in Proc. Data Compression Conf.*, pp.212-221, Mar. 1995.

[8] A. R. Calderbank, I. Daubechies, W. Sweldens and B. Yeo, "Lossless image compression using integer to integer wavelet transforms", *in Proc. IEEE ICIP*, vol. 1, pp.596-599, Oct. 1997.

[9] F. Bruekers and A. Enden, "New Networks for Perfect Inversion and Perfect Reconstruction", *IEEE JSAC*, vol. 10, no.1. pp.130-137, Jan. 1992.

[10] S. Fukuma, K. Ohyama, M. Iwahashi and N. Kambayashi, "Lossless 8-point fast discrete cosine transform using lossless hadamard transform", *IEI CE Technical Report*, DSP99-103, Oct. 1999. (in Japanese)