Отчет по работе
по курсу «Компьютерная алгебра»

Смолов Виктор, Зенцев Фёдор, 4057/2

17 декабря 2010 г.

1 Задание

Пусть задана система A биномиальных уравнений:

$$\begin{cases} y_1 - m_1 = 0 \\ y_2 - m_2 = 0 \\ \cdots \\ y_k - m_k = 0 \end{cases}$$

$$m_1, m_2, \cdots, m_k \in M \subset \mathbb{K}[x_1, x_2, x_3]$$

Требовалось исключить из такой системы x_1, x_2, x_3, другими словами найти $\alpha = (A) \cap \mathbb{K}[y_1, y_2, \cdots, y_k]$. Затем найти базис Грёбнера идеала α, который является *торическим*. Последний пункт задания - визуализация сечения пространства порядков.

2 Использованный инструментарий

Использовалась многопользовательская система компьютерной алгебры `SAGE` и язык программирования `Python`, в частности. Для визуализации был использован пакет `Gfan`.

3 Решения задач и полученные изображения

1. Система:

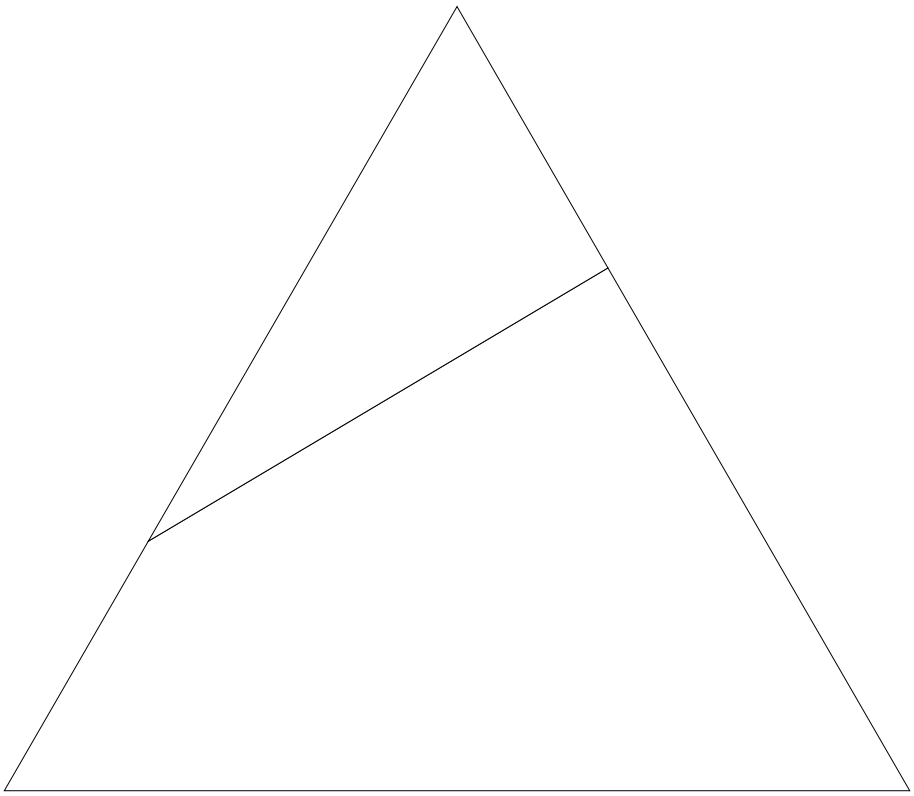
$$\begin{cases} y_1 - x_1^2 x_2 = 0 \\ y_2 - x_1^7 x_2^5 = 0 \\ y_3 - x_1^4 x_2^3 = 0 \end{cases}$$

Исходный код программы:

Listing 1: 1.py

```
R.<x1,x2,y1,y2,y3> = PolynomialRing(QQ, 5, order = 'lex')
I = ideal(y1 - x1^2 * x2,
          y2 - x1^7 * x2^5,
          y3 - x1^4 * x2^3)

B = I.groebner_basis()
print "Groebner basis:"
for b in B:
    print b

print "\nToric ideal basis:"
for b in B:
    if set(b.variables()).intersection([x1, x2]) == set():
        print b
```

Изображение проекций сечения конусами основного конуса, выводы программы `gfan_render`:



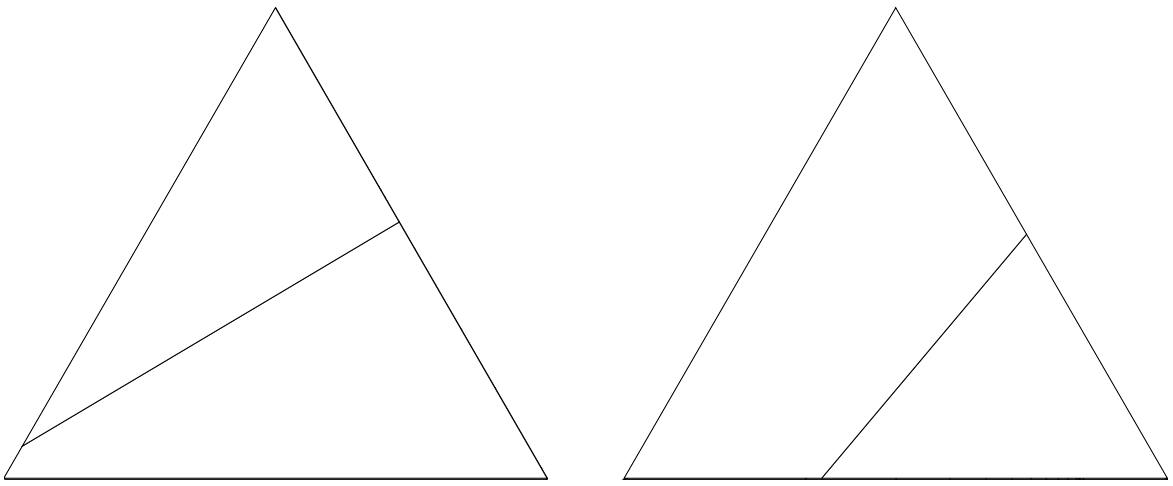
2. Система:

$$\begin{cases} y_1 - x_1 x_3^2 = 0 \\ y_2 - x_2^5 = 0 \\ y_3 - x_1^2 x_2^3 = 0 \\ y_4 - x_3^3 x_4 = 0 \\ y_5 - x_2 x_4^2 = 0 \end{cases}$$

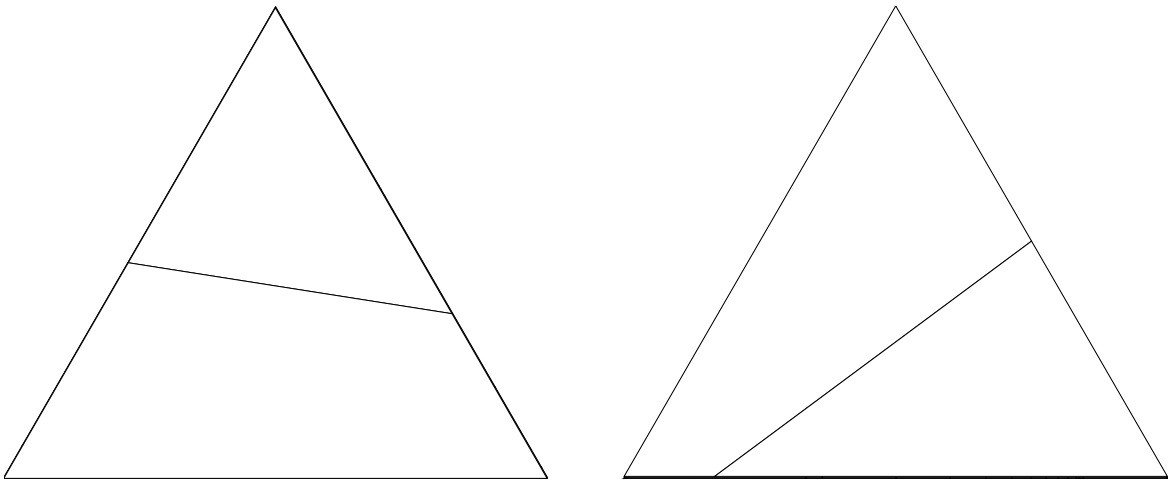
Исходный код программы:

Listing 2: 2.py

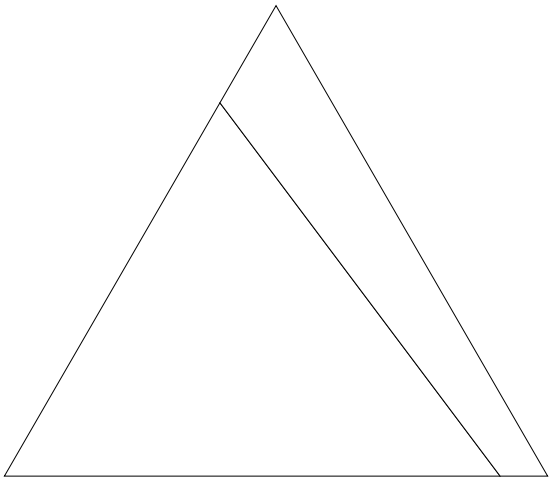
```
R2.<x1, x2, x3, x4, y1, y2, y3, y4, y5> = PolynomialRing(QQ, 9, order
    = 'lex')
I2 = ideal(y1 - x1 * x3^2,
           y2 - x2^5,
           y3 - x1^2 * x2^3,
           y4 - x3^3 * x4,
           y5 - x2 * x4^2)
B2 = I2.groebner_basis()
print "Groebner basis:"
for b in B2:
    print b

print "\nToric ideal basis:"
for b in B2:
    if set(b.variables()).intersection([x1, x2, x3, x4]) == set():
        print b
```



`-shiftVariables 0, 1`



`-shiftVariables 2, 3`



`-shiftVariables 4`

3. Система

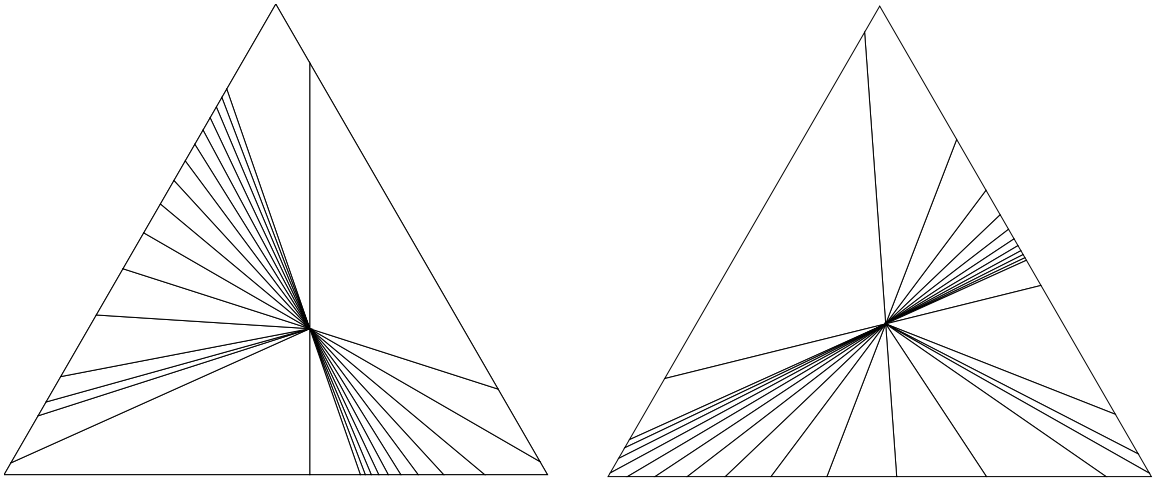
$$\begin{cases} y_1 - x_1^3 x_3^5 = 0 \\ y_2 - x_1^2 x_3 x_2^4 = 0 \\ y_3 - x_2^2 x_3^3 = 0 \\ y_4 - x_2^3 x_1 = 0 \\ y_5 - x_3 x_2 x_1 x_5 = 0 \\ y_6 - x_4 x_5^2 = 0 \\ y_7 - x_5 x_2^2 x_1^3 = 0 \end{cases}$$

Исходный код программы:

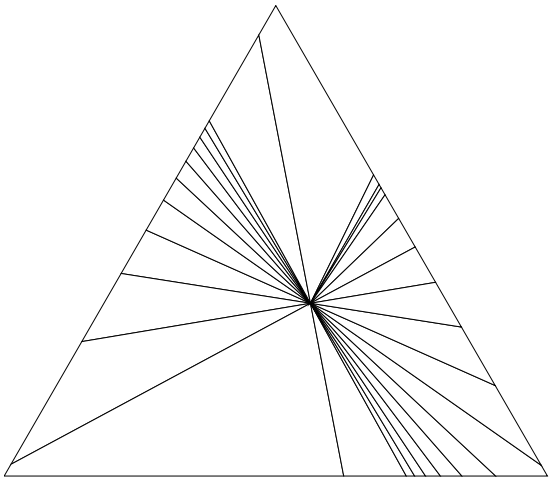
Listing 3: 2.py

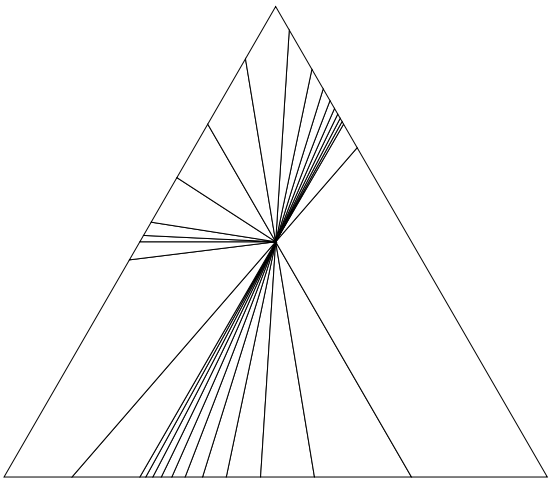
```
R2.<x1, x2, x3, x4, y1, y2, y3, y4, y5> = PolynomialRing(QQ, 9, order
    = 'lex')
I2 = ideal(y1 - x1 * x3^2,
           y2 - x2^5,
           y3 - x1^2 * x2^3,
           y4 - x3^3 * x4,
           y5 - x2 * x4^2)
B2 = I2.groebner_basis()
print "Groebner basis:"
for b in B2:
    print b

print "\nToric ideal basis:"
for b in B2:
    if set(b.variables()).intersection([x1, x2, x3, x4]) == set():
        print b
```

Изображение проекций сечения конусами основного конуса, вывод программы `gfan_render`:

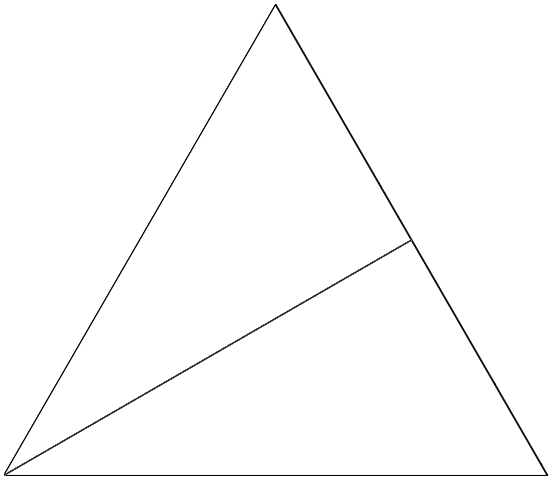


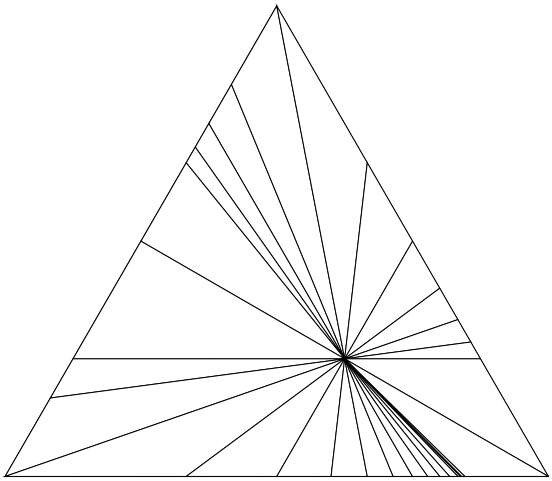
`-shiftVariables 0, 1`



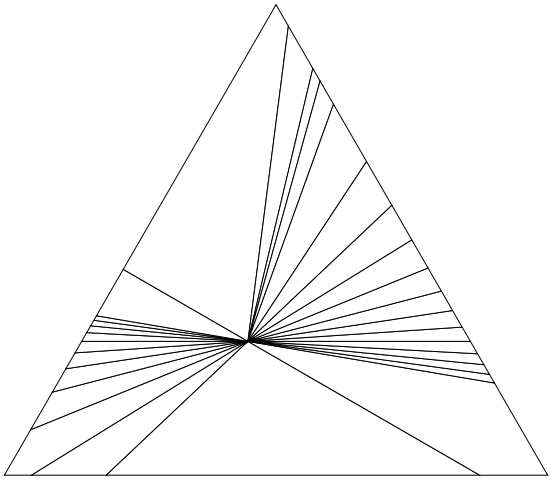


-shiftVariables 2, 3





-shiftVariables 4, 5



-shiftVariables 6

4 Результаты

1.

Listing 4: res1.txt

```
Groebner basis:
x1^2*x2 - y1
x1^2*y3 - y1^3
x1*x2*y1*y3 - y2
x1*y2 - y1^2*y3
x1*y3^2 - y1*y2
x2*y1^2 - y3
x2*y1*y2^2 - y3^4
x2*y2^4 - y3^7
y1*y3^3 - y2^2

Toric ideal basis:
y1*y3^3 - y2^2
```

2.

Listing 5: res2.txt

```
Groebner basis:
x1^2*x2^3 - y3
x1^2*x2^2*y5 - x4^2*y3
x1^2*x2*y5^2 - x4^4*y3
x1^2*y2 - x2^2*y3
x1^2*y5^3 - x4^6*y3
x1*x2^3*y1 - x3^2*y3
x1*x2^2*y1*y5 - x3^2*x4^2*y3
x1*x2*y1*y5^2 - x3^2*x4^4*y3
x1*x3^2 - y1
x1*y1*y2 - x2^2*x3^2*y3
x1*y1*y5^3 - x3^2*x4^6*y3
x1*y4 - x3*x4*y1
x2^5 - y2
x2^4*y5 - x4^2*y2
x2^3*y1^2 - x3^4*y3
x2^3*y5^2 - x4^4*y2
x2^2*x3^4*y3 - y1^2*y2
x2^2*x3*y3*y4 - x4*y1^2*y2
x2^2*y1^2*y5 - x3*x4*y3*y4
x2^2*y5^3 - x4^6*y2
x2*x3^12*y3^3 - y1^6*y2^2
x2*x3^9*y3^3*y4 - x4*y1^6*y2^2
x2*x3^4*y3*y5 - x4^2*y1^2*y2
x2*x3*y3*y4*y5 - x4^3*y1^2*y2
x2*x4^2 - y5
x2*x4*y4 - x3^3*y5
x2*y1^4*y2 - x3^8*y3^2
x2*y1^2*y5^2 - x3*x4^3*y3*y4
x2*y4^2 - x3^6*y5
x2*y5^4 - x4^8*y2
x3^20*y3^5 - y1^10*y2^3
x3^17*y3^5*y4 - x4*y1^10*y2^3
x3^12*y3^3*y5 - x4^2*y1^6*y2^2
x3^9*y3^3*y4*y5 - x4^3*y1^6*y2^2
x3^4*y3*y5^2 - x4^4*y1^2*y2
x3^3*x4 - y4
```

```
x3^3*y5^5 - x4^9*y2*y4
x3^2*x4^6*y1^2*y2 - y3*y4^2*y5^2
x3^2*y1^2*y5^3 - x4^4*y3*y4^2
x3^2*y3^2*y4^2 - y1^4*y2*y5
x3*x4^5*y3*y4 - y1^2*y5^3
x3*x4*y1^4*y2*y5 - y3^2*y4^3
x3*y1^10*y2^2*y5^5 - x4^3*y3^5*y4^7
x3*y1^4*y5^6 - x4^9*y3^2*y4^3
x3*y3^4*y4^5 - x4*y1^8*y2^2*y5^2
x3*y3*y4*y5^2 - x4^5*y1^2*y2
x4^10*y2 - y5^5
x4^6*y1^6*y2^2 - y3^3*y4^4*y5
x4^4*y3^3*y4^4 - y1^6*y2*y5^4
x4^2*y1^12*y2^3*y5^3 - y3^6*y4^8
x4^2*y3^9*y4^12 - y1^18*y2^4*y5^7
y1^30*y2^7*y5^10 - y3^15*y4^20

Toric ideal basis:
y1^30*y2^7*y5^10 - y3^15*y4^20
```

Listing 6: res3.txt

3.

```
1
2  Groebner basis:
3  x1^3*x2^2*x5 - y7
4  x1^3*x2^2*y6 - x4*x5*y7
5  x1^3*x3^5 - y1
6  x1^3*x3^2*y3 - x2^2*y1
7  x1^2*x2*y5 - x3*y7
8  x1^2*x3^5*y4 - x2^3*y1
9  x1^2*x3^4*y5 - x2*x5*y1
10 x1^2*x5*y4 - x2*y7
11 x1^2*y3*y5 - x2*x3^4*y7
12 x1^2*y4*y6 - x2*x4*x5*y7
13 x1*x2^3 - y4
14 x1*x2*x3^5*y7 - x5*y1*y4
15 x1*x2*x3*x5 - y5
16 x1*x2*x3*y4 - y2
17 x1*x2*x3*y6 - x4*x5*y5
18 x1*x2*x5^3*y1*y4 - x3^3*y5^2*y7
19 x1*x2*x5*y1*y4*y6 - x3^3*x4*y5^2*y7
20 x1*x2*y3 - x3^3*y4
21 x1*x3^6*y7 - x2*y1*y5
22 x1*x3^4*y2 - x2^4*y1
23 x1*x3^3*y2^2 - x2^2*y1*y4^2
24 x1*x3^2*x5^2*y4 - x2*y5^2
25 x1*x3^2*y2^3 - y1*y4^4
26 x1*x3^2*y3*y7 - x2*x5*y1*y4
27 x1*x3^2*y4^3 - x2*y2^2
28 x1*x3^2*y4*y6 - x2*x4*y5^2
29 x1*x3*y1*y4^8 - y2^6
30 x1*x3*y2*y3 - x2^6*y1
31 x1*x5*y3 - x2*x3^2*y5
32 x1*x5*y4^2 - x2^4*y7
33 x1*y1^3*y4^20 - y2^15
34 x1*y2^2*y3 - x2^4*y1*y4^2
```

```
x1*y2*y5 - x3^2*y4*y7
x1*y2*y6 - x2^2*x3*x4*x5*y7
x1*y3^2 - x2*x3^6*y4
x1*y3*y4 - x2*x3^2*y2
x1*y3*y6 - x2*x3^2*x4*x5*y5
x1*y4^2*y6 - x2^4*x4*x5*y7
x1*y4*y5 - x2^2*x3*y7
x1*y5^2 - x3^2*x5*y7
x2^7*y1 - x3^4*y2*y4
x2^7*y7 - x5*y4^3
x2^5*x3*y7 - y4^2*y5
x2^5*y1*y3 - x3^7*y2*y4
x2^5*y1*y4 - x3^3*y2^2
x2^3*x3^2*y7 - y2*y5
x2^3*y1*y3^2 - x3^10*y2*y4
x2^3*y1*y3*y4 - x3^6*y2^2
x2^3*y1*y4^3 - x3^2*y2^3
x2^2*x3^3 - y3
x2^2*x5*y1 - x3^5*y7
x2^2*y1*y6 - x3^5*x4*x5*y7
x2^2*y2 - x3*y4^2
x2^2*y5 - x3*x5*y4
x2*x3^16*y2*y4 - y1*y3^4
x2*x3^12*y2^2 - y1*y3^3*y4
x2*x3^8*y2^3 - y1*y3^2*y4^3
x2*x3^7*y7^2 - y1*y4*y5^2

...

x4*y2^2*y5^21 - x5^12*y1*y3^6*y4*y6*y7^7
x4*y2*y3*y5*y7 - y1*y4^3*y6
x4*y2*y5^22 - x5^13*y1*y3^6*y6*y7^7
x4*y3^10*y7^11 - y1^3*y2^6*y4^3*y5^9*y6
x4*y3^9*y4^2*y7^10 - y1^2*y2^9*y5^8*y6
x4*y3^8*y4^7*y7^9 - y1*y2^12*y5^7*y6
x4*y3^7*y4^12*y7^8 - y2^15*y5^6*y6
x4*y3*y5^2*y7 - x5*y1*y4^2*y6
x4*y4^2*y5^2 - y2^2*y6
x4*y4*y5^23 - x5^14*y1*y3^6*y6*y7^7
x4*y5^25 - x5^17*y1^2*y3^5*y4*y6*y7^6
x5^19*y1^2*y3^5*y4*y7^6 - y5^25
x5^15*y1*y3^6*y7^7 - y2*y5^22
x5^14*y1*y3^6*y4*y7^7 - y2^2*y5^21
x5^10*y3^7*y7^8 - y2^3*y5^18
x5^9*y3^7*y4*y7^8 - y2^4*y5^17
x5^8*y3^7*y4^2*y7^8 - y2^5*y5^16
x5^7*y3^7*y4^3*y7^8 - y2^6*y5^15
x5^6*y3^7*y4^4*y7^8 - y2^7*y5^14
x5^5*y3^8*y7^9 - y1*y2^5*y5^14
x5^5*y3^7*y4^5*y7^8 - y2^8*y5^13
x5^4*y3^8*y4*y7^9 - y1*y2^6*y5^13
x5^4*y3^7*y4^6*y7^8 - y2^9*y5^12
x5^3*y1*y4^2 - y3*y5^2*y7
x5^3*y3^8*y4^2*y7^9 - y1*y2^7*y5^12
x5^3*y3^7*y4^7*y7^8 - y2^10*y5^11
x5^2*y1*y4^3 - y2*y3*y5*y7
```

```
x5^2*y3^8*y4^3*y7^9 - y1*y2^8*y5^11
x5^2*y3^7*y4^8*y7^8 - y2^11*y5^10
x5*y1*y4^4 - y2^2*y3*y7
x5*y2 - y4*y5
x5*y3^9*y7^10 - y1^2*y2^6*y4*y5^11
x5*y3^8*y4^4*y7^9 - y1*y2^9*y5^10
x5*y3^7*y4^9*y7^8 - y2^12*y5^9
y1^8*y4^50 - y2^37*y3
y1^7*y4^45*y7 - y2^34*y5
y1^6*y3*y4^40*y7^2 - y2^31*y5^2
y1^5*y3^2*y4^35*y7^3 - y2^28*y5^3
y1^4*y3^3*y4^30*y7^4 - y2^25*y5^4
y1^3*y3^4*y4^25*y7^5 - y2^22*y5^5
y1^2*y2^7*y5^10 - y3^9*y7^10
y1^2*y3^5*y4^20*y7^6 - y2^19*y5^6
y1*y2^10*y5^9 - y3^8*y4^5*y7^9
y1*y3^6*y4^15*y7^7 - y2^16*y5^7
y1*y4^5*y5 - y2^3*y3*y7
y2^13*y5^8 - y3^7*y4^10*y7^8

Toric ideal basis:
y1^8*y4^50 - y2^37*y3
y1^7*y4^45*y7 - y2^34*y5
y1^6*y3*y4^40*y7^2 - y2^31*y5^2
y1^5*y3^2*y4^35*y7^3 - y2^28*y5^3
y1^4*y3^3*y4^30*y7^4 - y2^25*y5^4
y1^3*y3^4*y4^25*y7^5 - y2^22*y5^5
y1^2*y2^7*y5^10 - y3^9*y7^10
y1^2*y3^5*y4^20*y7^6 - y2^19*y5^6
y1*y2^10*y5^9 - y3^8*y4^5*y7^9
y1*y3^6*y4^15*y7^7 - y2^16*y5^7
y1*y4^5*y5 - y2^3*y3*y7
y2^13*y5^8 - y3^7*y4^10*y7^8
```