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# Modeling the hydro-mechanical reinforcement of vegetation to the stability of a shallow slope based on the dynamic root growth

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**Abstract** **Keywords** Herb

## 1 Introduction

## 2 Methodology

### 2.1 The mechanical reinforcement model

### 2.2 Soil hydrological dynamic model

The SWAP (Soil, Water, Atmosphere, and Plant) model has been applied widely to simulate soil water dynamics in the vegetated slope (Romano et al, 2011). In this study, we employed the Richards' equation to simulate the soils-vegetation-atmosphere interactions under unsaturated conditions; The modified Richards' equation can be written as follows:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}(k\frac{\partial h}{\partial z}) + S(z) \tag{1}$$

where $k$ is the unsaturated soil permeability at a given suction; $h$ is the hydraulic head; $x$, and $z$ are the coordinates in the horizontal and vertical directions, respectively; $\theta$ is the volumetric water content; $t$ is the time; $S$ is the source/sink term.

The soil water characteristic curve (SWCC) and hydraulic conductivity functions (HFC) are written according to the Van-Genuchten Genuchten (1980)

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = [\frac{1}{1 + |\alpha h|^n}]^m \tag{2}$$

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where $\Theta$ is the soil saturation; $\theta$ is the soil water content; $\theta_s$ is the saturated water content; $\theta_r$ is the residual water content; $n$ and $m$ are the shape parameters, if n $> 1$, then $m = 1 - \frac{1}{n}$; $\alpha$ is a scale parameter.

$$K(h) = K_s \Theta^{0.5}[1 - (1 - \Theta^{1/m})^m]^2 \tag{3}$$

where $K(h)$ is the hydraulic conductivity at a given suction head $h$; $K_s$ is the saturated hydraulic conductivity.

## 2.3 Transpiration

$$ET_c = T_{pot} + E_{pot} = (K_{cb} + K_e)ET_0 \tag{4}$$

where:

## 2.4 Dynamic root growth model

### *2.4.1 Root penetration*

Root growth calculation is based on cumulative day-degrees, which is common for root models and has been shown in field trials to be applicable to both monocot and dicot crops Pedersen et al (2009).

$$DD = \begin{cases} 0; \quad T_{min} \geq T_{air} \\ T_{air} - T_{min} \quad T_{min}; \leq T_{air} < T_{max} \\ T_{max} - T_{min} \quad T_{air}; \geq T_{max} \end{cases} \tag{5}$$

The root penetration depth ($R_z$) is calculated as follows:

$$R_z = \begin{cases} R_{z,min}; \quad \sum DD \leq DD_{lag} \\ \sum((DD - DD_{lag})k_{rz} + R_{z,min}); \quad \sum DD > DD_{lag} \\ R_{z,min}; \quad \sum(DD - DD_{lag})k_{rz} + R_{z,min} > R_{z,max} \end{cases} \tag{6}$$

where $DD$ is the Day-degress; $DD_{lag}$ is the lag phase for initiating root growth; $R_{z,max}$ is the maximum rooting depth; $R_{z,min}$ is the rooting depth at sowing or planting; $T_{min}$ is minimum temperature for root growth; $T_{max}$ is the maximum temperature for root growth.

The root density distributions at different growth stages are quite similar, which indicate the feasibility of using a single normalized root density function for the whole growth season of each crop. A third-order polynomial equation was utilized to fit the pooled data of each crop as follows**?**:

$$L_{nrd}(z_r) = R_0 + R_{1z_r} + R^2_{2z_r} + R^3_{3z_r} \tag{7}$$

where $R_i$ ($i$=0,1,2,3) are the polynomial coeffecients.

#### *2.4.2 Root density*

a simple normalized root density function A third-order polynomial equation was utilized to fit the pooled data of each vegetation as follows:

$$L_{nrd}(z_r) = R_0 + R_1 z_r + R_2 z_r^2 + R_3 z_r^3 \tag{8}$$

where $R_i (i = 0, 1, 2, 3)$ are the polynomial coefficients.

### 2.5 Infinite slope model

For many rainfall-induced landslides, the failure surfaces are often shallow and parallel to the slope surface. The infinite slope model serves as an excellent illustration for slope stability analysis (Lu and Godt, 2013; Cho and Lee, 2002). Lu and Godt (2008) present a generalized framework for the stability of infinite slopes under steady unsaturated seepage conditions. This framework allows the water table to be located at any depth below the ground surface and variation of soil suction and moisture content above the water table under steady infiltration conditions. The schematic diagram of infinite slope is shown in Fig. 2.5. The factor of safety $(F_s)$ of a slope is evaluated as fellows:

$$F = \frac{c_s + (\gamma z cos^2\beta - \sigma^s) tan\phi'}{\gamma z sin\beta cos\beta} \tag{9}$$

where $c_s$ is the soil effective cohesion; $\gamma$ is the soil specific weight of soil; $z$ is the depth of soil; $\beta$ is the slope angle; $\sigma^s$ is the suction stress characteristic curve; $\phi$ is the effective soil friction angle.

The suction stress can be expressed in terms of normalized volumetric water content or degree of saturation as (Lu and Godt, 2008):

$$\sigma^s = -\frac{\theta - \theta_r}{\theta_s - \theta_r}(u_a - u_w) \tag{10}$$

where $\theta$ is the volumetric water content, $\theta_r$ is the residual volumetric water content; $\theta_s$ is the saturated volumetric water content; $u_a$ is the pore air pressure; $u_s$ is the pore water pressure.

For vegetated slope, the mechanical reinforcement provided by roots is quantified by means of a root cohesion term $c_r$ (Pollen, 2007; Arnone et al, 2016), so that the total cohesion of root-soil $c_{tot} = c_r + c_s$ and the factor of safety becomes

$$F = \frac{c_r + c_s + (\gamma z cos^2\beta - \sigma^s) tan\phi'}{\gamma z sin\beta cos\beta} \tag{11}$$

which, after rearranging and applying trigonometric identities, becomes:

$$F_s = \underbrace{\frac{tan\phi'}{tan\beta}}_{1} + \underbrace{\frac{2c_s}{\gamma z sin 2\beta}}_{2} + \underbrace{\frac{2c_r}{\gamma z sin 2\beta}}_{3} - \underbrace{\frac{\sigma^s (tan\beta + cot\beta) tan\phi}{\gamma z}}_{4} \tag{12}$$

In the Eq.(12), terms 1 and 2 are, respectively, the slope stability contribution due to the internal frictional $\phi'$ and the soil cohesion $c_s$. Terms 3 and 4 are, respectively, the slope stability contribution due to the root cohesion $c_r$ and the suction stress.

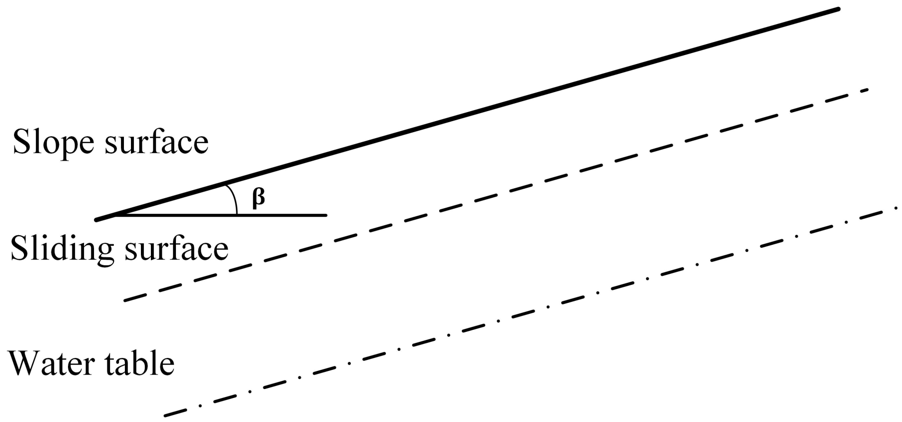


**Fig. 1** Schematic diagram of infinite slope

## 3 Result

### 3.1 Effects of root system on the slope stability

### 3.2 Effects of slope geometry on the slope stability

## 4 Discussion

## 5 Conclusion

## 6 Acknowledgment

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