Homework 2 of Dynamic Programming and Optimal Control

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1 Exercise 4.16

Suppose that a person wants to sell a house and an offer comes at the beginning of each day. We assume that successive offers are independent and an offer is w_j with probability p_j, $j = 1, \ldots, n$, where w_j are given nonnegative scalars. Any offer not immediately accepted is not lost but may be accepted at any later date. Also, a maintenance cost c is incurred for each day that the house remains unsold. The objective is to maximize the price at which the house is sold minus the maintenance costs. Consider the problem when there is a deadline to sell the house within N days and characterize the optimal policy.

Solution:

- State x_k: the offer under consideration. $x_k = T$ means the house has been sold.
- Action u_k: u^1 (sell) and u^2 (do not sell)
- State transition:

$$\begin{aligned} x_{k+1} &= f_k(x_k, u_k, w_k) \\ &= \begin{cases} T & \text{if } x_k = T, \text{ or } x_k \neq T \text{ and } u_k = u^1 \\ w_k & \text{otherwise} \end{cases} \end{aligned}$$

  where $x_0 = 0$, w_k are i.i.d.

- Per-stage reward:

$$g_N(x_N) = \begin{cases} x_N & \text{if } x_N \neq T \\ 0 & \text{otherwise} \end{cases}$$

$$g_k(x_k, u_k, w_k) = \begin{cases} \max\{x_1, \ldots, x_k\} - kc & \text{if } x_k \neq T \text{ and } u_k = u^1 \\ 0 & \text{otherwise} \end{cases}$$

- Start with

$$J_N(x_N) = g_N(x_N) = \begin{cases} x_N & \text{if } x_N \neq T \\ 0 & \text{otherwise} \end{cases}$$

and go backwards using

$$J_k(x_k) = \max_{u_k} E_{w_k}\{g_k(x_k, u_k, w_k) + J_{k+1}(f_k(x_k, u_k, w_k))\}$$
$$= \begin{cases} \max[\max\{x_1, \ldots, x_k\} - kc, E\{J_{k+1}(w)\}] & \text{if } x_k \neq T \\ 0 & \text{otherwise} \end{cases}$$

- Optimal policy

$$\text{accept the offer } x_k \text{ if } x_k > \alpha_k$$
$$\text{reject the offer } x_k \text{ if } x_k < \alpha_k$$

where

$$\alpha_k = \frac{EJ_{k+1}(w)}{\max\{x_1, \ldots, x_k\} - kc}$$