Homework 3 of Dynamic Programming and Optimal Control

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Consider the scalar system

$$
\begin{aligned}
x_{k+1} &= x_k + u_k + w_k \\
z_k &= x_k + v_k,
\end{aligned}
$$

3 x_0, and the disturbances w_k and v_k are all independent. Let the cost be

$$
E\left\{x_N^2 + \sum_{k=0}^{N-1}(x_k^2 + u_k^2)\right\},
$$

and let the given probability distributions be

$$
\begin{aligned}
p(x_0 = 2) = \frac{1}{2}, \quad p(w_k = 1) = \frac{1}{2}, \quad p(v_k = \frac{1}{4}) = \frac{1}{2} \\
p(x_0 = -2) = \frac{1}{2}, \quad p(w_k = -1) = \frac{1}{2}, \quad p(v_k = -\frac{1}{4}) = \frac{1}{2}
\end{aligned}
$$

(a) Determine the optimal policy. *Hint*: For this problem, $E\{x_k|I_k\}$ can be determined from $E\{x_{k-1}|I_{k-1}\}, u_{k-1}$ and z_k.

Solution:

Since

$$
A_k = 1, \quad B_k = 1, \quad C_k = 1, \quad Q_k = 1, \quad R_k = 1
$$

we have

$$
\begin{aligned}
L_k &= -(R_k + B_k' K_{K+1} B_k)^{-1} B_k' K_{k+1} A_k \\
&= -\frac{1 + K_{k+1}}{K_{k+1}} \\
P_k &= A_k' K_{k+1} B_k (R_k + B_k' K_{k+1} B_k)^{-1} B_k' K_{k+1} A_k \\
&= \frac{K_{k+1}^2}{1 + K_{k+1}} \\
K_k &= K_{k+1} - P_k + 1 \\
&= \frac{1 + 2K_{k+1}}{1 + K_{k+1}}
\end{aligned}
$$

According to optimal control law:

$$\mu_k^*(I_k) = L_k E\{x_k|I_k\}$$

For this problem, $E\{x_k|I_k\}$ can be determined from $E\{x_{k-1}|I_{k-1}\}, u_{k-1}$ and z_k. First,

$$\begin{aligned} &z_k = x_k + v_k = x_{k-1} + u_{k-1} + w_{k-1} + v_k \\ &\Downarrow \\ &z_k - x_{k-1} - u_{k-1} = w_{k-1} + v_k \end{aligned}$$

The value of $w_{k-1} + v_k$ can be only one of $\{\frac{3}{4}, \frac{5}{4}, -\frac{3}{4}, -\frac{5}{4}\}$. If $z_k - x_{k-1} - u_{k-1} = \frac{3}{4}$ or $\frac{5}{4}$, then $w_k = 1$ and $x_k = x_{k-1} + u_{k-1} + 1$ can be known. It is the same as $z_k - x_{k-1} - u_{k-1} = -\frac{3}{4}$ or $-\frac{5}{4}$. Thus

$$E\{x_0|I_0\} = \begin{cases} 2, & \text{if } z_0 = 2 \pm \frac{1}{4} \\ -2, & \text{if } z_0 = -2 \pm \frac{1}{4} \end{cases}$$

$$E\{x_k|I_k\} = \begin{cases} E\{x_{k-1}|I_{k-1}\} + u_{k-1} + 1, & \text{if } z_k - E\{x_{k-1}|I_{k-1}\} - u_{k-1} = 1 \pm \frac{1}{4} \\ E\{x_{k-1}|I_{k-1}\} + u_{k-1} - 1, & \text{if } z_k - E\{x_{k-1}|I_{k-1}\} - u_{k-1} = -1 \pm \frac{1}{4} \end{cases}$$

Now, we can obtain L_k and $E\{x_k|I_k\}$ and calculate the optimal policy $u_k^* = L_k E\{x_k|I_k\}$.