Problem 1. (10 points). Please select T/F (True or False) for the following statements.

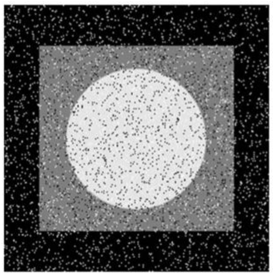
- [True] if available hardware, or software routines, have only the capability to perform the DFT, we can use it to compute the inverse DFT using $f[x] = \frac{1}{M}\,\overline{DFT\{F[\mu]\}}$.
- [True] In either domain, time or frequency, the function is not periodic, then the argument in the other domain runs continuously. If in either domain, time or frequency, the function has a discrete argument, then the transformed function in the other domain is periodic.
- [False] Fourier spectrum carry much of the information about where discernable objects are located in an image.
- [False] To construct a Gaussian image pyramid for a given image, we first down-sample it using bilinear interpolation method, and then apply a Gaussian filter to smooth potential artifacts introduced by the down-sampling.
- [True] Bag-of-SIFT descriptors is an effective model capturing rough layout/configuration information of scene or objects in an image due to the nice properties (scale and orientation invariance) of SIFT descriptors.
- [True] To improve accuracy performance in test and avoid overfitting, we should not always seek the classifier whose training error is zero (if possible).
- [True] To handle wraparound error in filtering in frequency domain, we need to pad a given input image before filtering. Either centered padding or left-top-based padding works.
- [False] Image restoration is usually posed as an objective process which utilizes a criterion of goodness that will yield an optimal estimate of the desired result.
- [True] An edge can be caused by depth discontinuity or surface color discontinuity. Corner locations are co-variant w.r.t. translation, rotation and scaling.
- [False] For the Canny edge detector, we will obtain more and stronger edges if we use larger Gaussian kernel size.

Problem 2. (10 points).

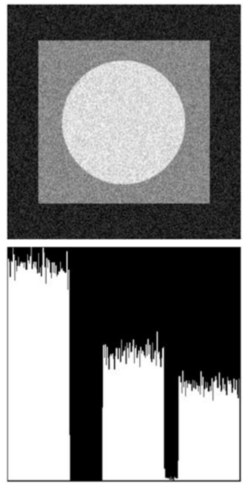
- Select the noise type from (a)~ (f) which describe the noise best in each figures:

(a) $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$, (b) $p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \ge a \\ 0 & \text{for } z < a \end{cases}$, (c) $p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$

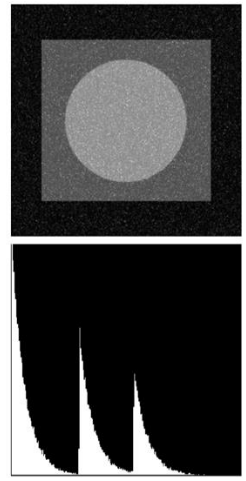
(d) $p(z) = \begin{cases} ae^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$, (e) $p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$, (f) $p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$



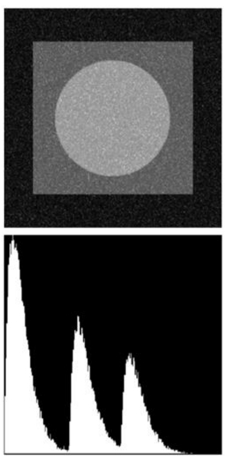
Noise type [f]



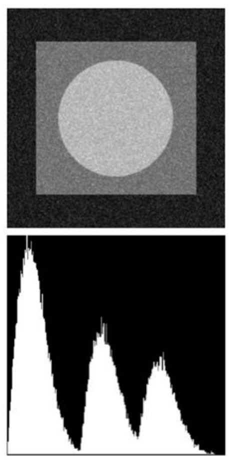
Noise type [e]



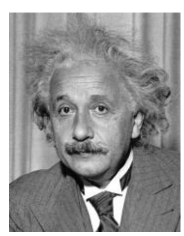
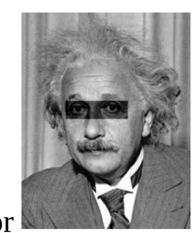
Noise type [d]



Noise type [c]

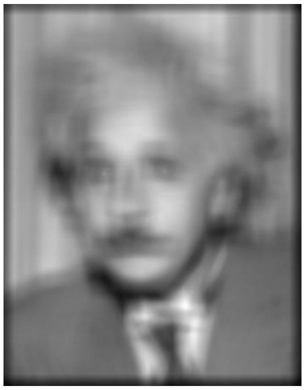


Noise type [b]

- To find $g[x,y]$  in a given image $f[x,y]$  or , select the best method from (a)~(d) which generate the result images $h[x,y]$. Let $\bar{g}$ be the mean value of $g[x,y]$.

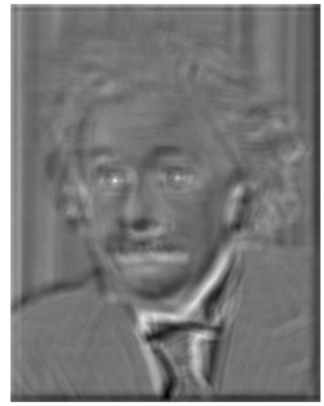
(a) $h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$, (b) $h[m,n] = \sum_{k,l} \{g[k,l] - \bar{g}\} f[m+k, n+l]$

(c) $h[m,n] = \sum_{k,l}(g[k,l] - f[m+k,n+l])^2$, (d) $h[m,n] = \dfrac{\sum_{k,l}(g[k,l] - \bar{g})(f[m-k,n-l] - \bar{f}_{m,n})}{\left(\sum_{k,l}(g[k,l] - \bar{g})^2 \sum_{k,l}(f[m-k,n-l] - \bar{f}_{m,n})^2\right)^{0.5}}$



The result of [a]

The result of [d]



The result of [b]

The result of [c]

The result of [d]

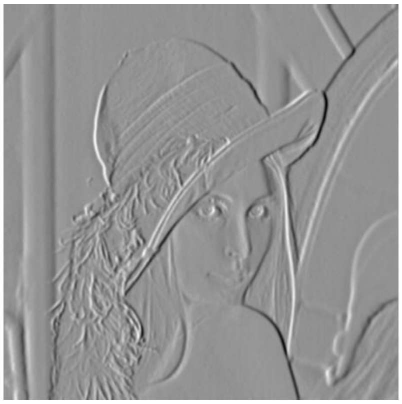
Problem 3. (10 Points)

- Find one instance for each of the four types of edges in the given image: (a) illumination discontinuity, (b) depth discontinuity, (c) surface normal discontinuity, and (d) surface color discontinuity. Draw a small circle for each instance with the type (a)~(d) indicated in the circle.

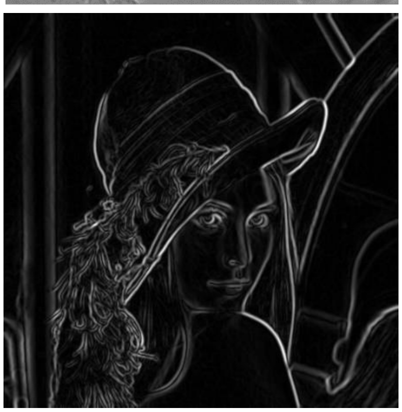
Aa

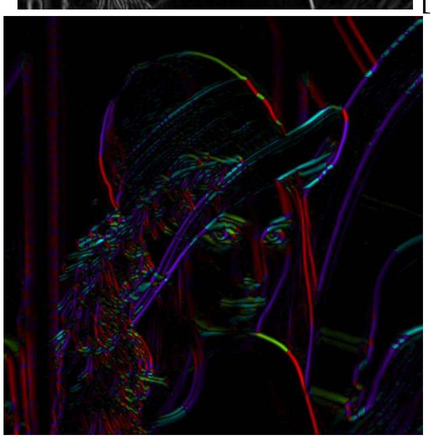
- Write the operation name (e.g., "x-derivative of Gaussian") for each of the steps of the Canny edge detector

a)  [input image]

b)  [X-Derivative of Gaussian]

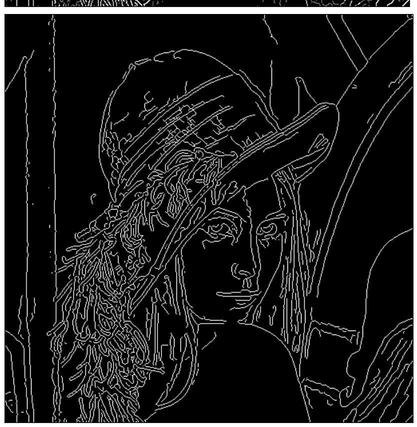
c)  [Y_Derivative of Gaussian]

d)  [Gradient Magnitude]

e)  [Get Orientation]

f)  [Non_Max suppresion]

g)  [Hysteresis Thresholding]

h)  [Final Canny Edges]

Q. 4

$$g(x,y) = \iint_{-\infty}^{\infty} f(\alpha, B)\, h(x-\alpha, y-B)\, d\alpha\, dB$$

$$f(x,y) = \delta(x-a)$$

$$f(\alpha, B) = \delta(\alpha - a)$$

$$g(x,y) = \iint_{-\infty}^{\infty} \delta(\alpha-a) e^{-[(x-\alpha)^2 + (y-B)^2]} d\alpha\, dB$$

$$= \iint_{-\infty}^{\infty} \delta(\alpha-a) e^{-[(x-\alpha)^2]} e^{-[(y-B)^2]} d\alpha\, dB$$

$$= \int_{-\infty}^{\infty} \delta(\alpha-a) e^{-[(x-\alpha)^2]} d\alpha \int_{-\infty}^{\infty} e^{-[(y-B)^2]} dB$$

$$= e^{-[(x-a)^2]} \int_{-\infty}^{\infty} e^{-[(y-B)^2]} dB$$

$$\int_{-\infty}^{\infty} e^{-[(y-B)^2]} = \int_{-\infty}^{\infty} e^{-[(B-y)^2]} dB$$

$$e^{-[(B-y)^2]} = \sqrt{2\pi \times \frac{1}{2}} \left[\frac{1}{\sqrt{2\pi 1/2}} e^{-1/2\left[\left(\frac{B-y^2}{1/2}\right)\right]} \right]$$

Integral of this term inside brackets is one

So $g(x,y) = \sqrt{\pi}\, e^{-[(x-a)^2]}$

which is a blurred version of the original image.

Pr 5. $F(u,v) = \iint_{-\infty}^{\infty} f(x,y)\, e^{-2\pi j(ux+vy)}\, dx\, dy$

$= \iint_{-\infty}^{\infty} A \sin(u_0 x + v_0 y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$

$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

$F(u,v) = \frac{-jA}{2} \iint_{-\infty}^{\infty} \left[e^{j(u_0 x + v_0 y)} - e^{-j(u_0 x + v_0 y)} \right]$

$e^{-2\pi j(ux+vy)}\, dx\, dy$

$= \frac{jA}{2} \left[\iint_{-\infty}^{\infty} e^{-2\pi j(u_0 x/2\pi + v_0 y/2\pi)} \right.$

$e^{-2\pi j(ux+vy)}\, dx\, dy$

These are the fourier transforms of the functions

$1 \times e^{2\pi j(u_0 x/2\pi + v_0 y/2\pi)}$

and $1 \times e^{-2\pi j(u_0 x/2\pi + v_0 y/2\pi)}$

F.T. of 1 gives an impulse at the origin and the exponentials shift the origin of the impulse

$$F(u,v) = \frac{-jA}{2}\left[\delta\left(u - \frac{u_0}{2\pi}, v - \frac{v_0}{2\pi}\right) - \delta\left(u + \frac{u_0}{2\pi}, v + \frac{v_0}{2\pi}\right)\right]$$

Pr. 6 The complex conjugate simply changes j to $-j$ in the inverse transform, so image on right is

$$F^{-1}\left[F^{*}(u,v)\right] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)\, e^{-2\pi j(ux/M + vy/N)}$$

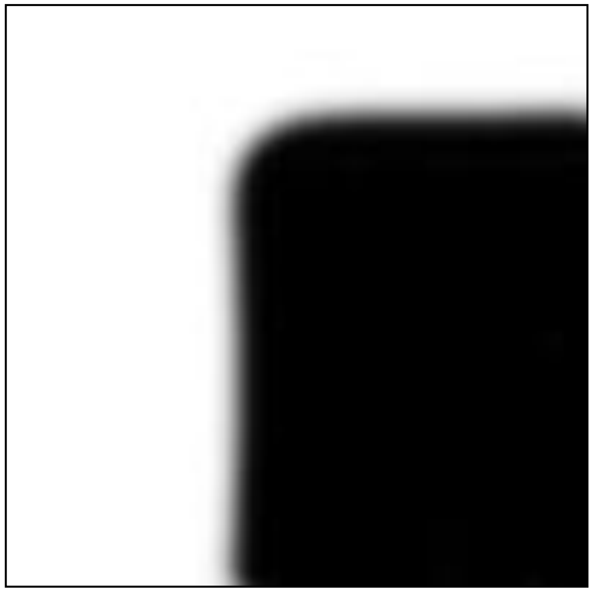
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)\, e^{2\pi j(u(-x)/M + v(-y)/N)}$$

$$= f(-x,-y)$$

which simply mirrors $f(x,y)$ about the origin thus producing the image on the right.

Problem 7. (15 points).

- Show the basic idea of detecting corners using the given toy image.



Basic Idea:

1) We try and recognize the point by looking through a small window
2) Shifting a window in any direction should give a large change in intensity which is used for corner detection.

- Show why the second moment matrix is important in detecting corners using detailed derivation. ***Define your nations and explain steps in the derivation.***

Q 7.

$$E(u,v) = \sum_{x,y \in W} [I(x+u, y+v) - I(x,y)]^2$$

We have to find how this function behaves for small shifts

Using Taylor approx.

$$I(x+u, y+v) \approx I(x,y) + I_x u + I_y v$$

$$E(u,v) \approx \sum [(\cancel{I(x,y)} + I_x u + I_y v - \cancel{I(x,y)})^2$$

$$= \sum [I_x u + I_y v]^2$$

$$= \sum I_x^2 u^2 + 2 I_x I_y uv + I_y^2 v^2$$

$$E(u,v) \approx [u \ \ v] \, M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

Based on eigen values of M matrix, corners can be interpreted as

λ_1 & λ_2 small - Flat region

$\lambda_2 >> \lambda_1$ → Edge

λ_1 & λ_2 large, $\lambda_1 \sim \lambda_2$, E increases all directions

$\lambda_1 >> \lambda_2$ → Edge

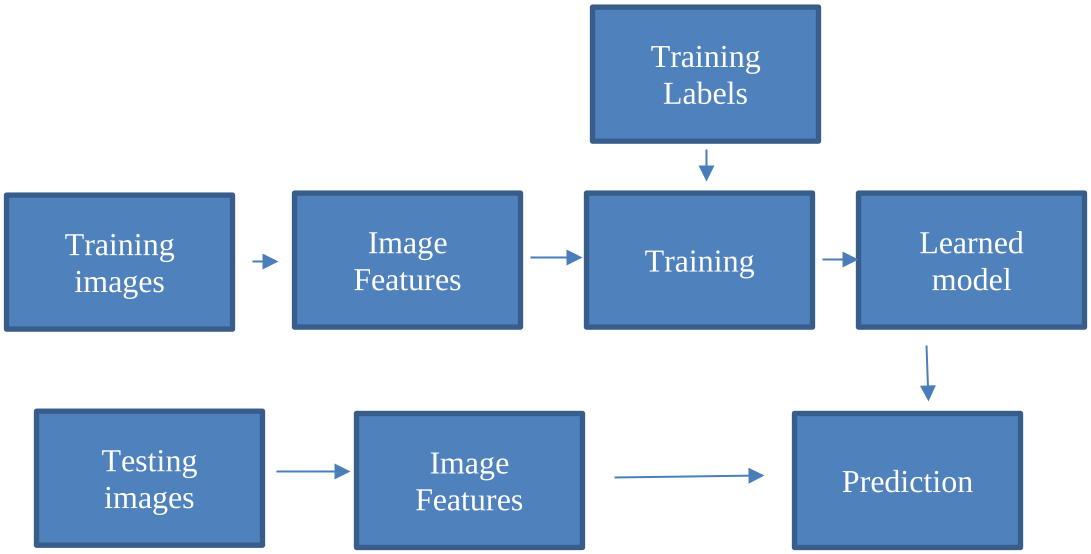
Problem 8. (10 points). Write down the steps of Lowe's SIFT algorithm. **Your steps should include the SIFT point detection and the SIFT descriptor generation.** Explain why SIFT keypoints are scale invariant to certain degree, and why SIFT descriptor is robust w.r.t. scale changes, illumination changes, orientation changes and viewpoint changes up to certain degree.

1) First step is to detect points of interest or keypoints. The image is convolved with Gaussian filters at different scales and then the difference of successive Gaussian blurred images is taken. Keypoints are taken as maxima/minima of the Difference of Gaussians that occur at multiple scales.
2) Then key points are localize to perform a detailed fit to the nearby data for accurate location, scale and ratio of principal curvatures.
3) Then a descriptor vector for each key point if found such that the descriptor is highly distinctive and partially invariant to the variations such as illumination etc.

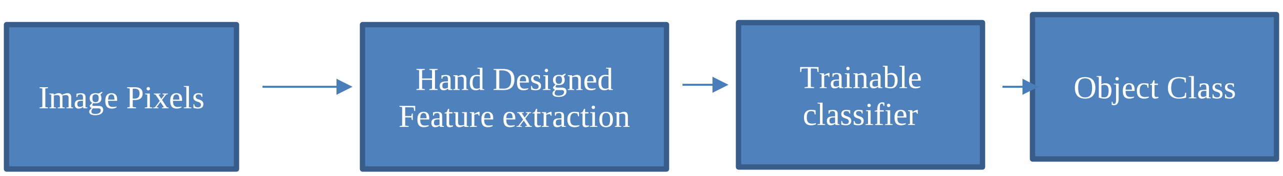
Because the key point locations are found at particular scales and orientations are assigned to them, this ensures invariance to orientation changes. For orientation invariance, each key point is assigned one or more orientations based on local image gradient directions.

Problem 9. (15 points). Draw the workflows and explain the workflows for the following three tasks.

a) Instance-based (object or image patch) matching or recognition.



b) Image / object categorization with hand-crafted features. The workflow should include both training and test stages.



c) RANSAC algorithm for line fitting.
   1) Choose a small subset of points uniformly at random
   2) Fit a model to that subset
   3) Find all remaining points that are close to the model and reject the outliers
   4) Repeat this many times and choose the best model.