Project 3 Markov Chains

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Contents

2.1 Markov Chains

A Markov chain is a discrete-time discrete-state proccess.

Given three states with values thus:

State 0: 0.64 0.32 0.04
State 1: 0.40 0.50 0.10
State 2: 0.25 0.50 0.25

We start at state 0. Then we sample the states possibilities based upon the the probabilites provided in the matrix, and this bring us to a new state.

Recording how many times we go to a given state and evaluating how many states we went to total can give us the probability of each state in total.

We then compare them to theoretical results.

```
pi = (a,b,c)

0.64a  +   0.32b  +  0.04c = a

0.40a  +   0.50b  +  0.10c = b

0.25a  +   0.50b  +  0.25c = c
```

or equivilent to:

```
0.32b  +  0.04c = 0.36a

0.40a  +  0.10c = 0.50b

0.25a  +  0.50b = 0.75c
```

multiply the first by 5 and the second by 2

```
1.6b  +  0.20c = 1.8a

0.80a  +  0.20c = b

0.25a  +  0.50b = 0.75c
```

subtracting the two equations first and second we get:

```
1.6b - 0.8a = 1.8a - b
```

simplying we get: 2.6a = 2.6b

or

```
a = b
```

using this in the third equation we get:

```
0.25a + 0.50a = 0.75c

0.75a = 0.75c

a = c
```

so

```
a = b = c = 1
```

so

```
pi = (0.33, 0.33, 0.33)
```

Unless I made a mistake in calculating this

```r
state_0_probability = 0.3333
state_1_probability = 0.3333
state_2_probability = 0.3333

p_00 = 0.64
p_01 = 0.32
p_02 = 0.04
p_10 = 0.40
p_11 = 0.50
p_12 = 0.10
p_20 = 0.25
p_21 = 0.50
p_22 = 0.25

P = matrix(c(p_00, p_01, p_02, p_10, p_11, p_12, p_20, p_21, p_22), nrow=3, byrow=TRU

number_of_transitions = 10000

current_state = 0

states_population = c(0,1,2)

states_count = c(0,0,0)

print(P)
```

```
##         State 0 State 1 State 2
## State 0    0.64    0.32    0.04
## State 1    0.40    0.50    0.10
## State 2    0.25    0.50    0.25
```

```r
for (current_transition in 1:number_of_transitions){

  for (states in states_population){
    if(current_state == states){
```

```r
      states_count[states] = states_count[states] + 1
      current_state = sample(states_population, 1, replace = FALSE, prob = P[states+1
      break
    }
  }

}

cat("\n")
```

```r
cat("Probability of State 0: ", states_count[0]/number_of_transitions * 100.0, "%\n")
```

```
## Probability of State 0:   %
```

```r
cat("Probability of State 1: ", states_count[1]/number_of_transitions * 100.0, "%\n")
```

```
## Probability of State 1:  40.96 %
```

```r
cat("Probability of State 2: ", states_count[2]/number_of_transitions * 100.0, "%\n")
```

```
## Probability of State 2:  8.49 %
```

```r
cat("\n")
```

```r
cat("Theoretical Probability of State 0: ", state_0_probability, "%\n")
```

```
## Theoretical Probability of State 0:  0.3333 %
```

```r
cat("Theoretical Probability of State 1: ", state_1_probability, "%\n")
```

```
## Theoretical Probability of State 1:  0.3333 %
```

```r
cat("Theoretical Probability of State 2: ", state_2_probability, "%\n")
```

```
## Theoretical Probability of State 2:  0.3333 %
```

```r
cat("\n")
```

```r
cat("Distance of Simulated to Theoretical Probability of State 0: ", 100 * (states_co
```

```
## Distance of Simulated to Theoretical Probability of State 0:   %
```

```r
cat("Distance of Simulated to Theoretical Probability of State 1: ", 100 * (states_cou
```

```
## Distance of Simulated to Theoretical Probability of State 1:  7.63 %
```

```r
cat("Distance of Simulated to Theoretical Probability of State 2: ", 100 * (states_cou
```

```
## Distance of Simulated to Theoretical Probability of State 2:  -24.84 %
```